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# TEMPERATURE CORRECTIONS FOR ELECTROMAGNETIC INDUCTION MEASUREMENTS IN FIELD SOILS


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**ABSTRACT:**

Apparent electrical conductivities ($EC_a$) of field soils vary considerably with variations in temperature. Reliable comparisons between soil electrical conductivity measurements at different dates are only possible after conversion of the measured electrical conductivities to those at a reference temperature of 25 degrees Celsius. The objective of this study is to investigate in which manner conductivities measured with the electromagnetic induction method can be corrected for soil temperature. Theoretical temperature profiles were generated for homogeneous soil profiles with a uniform electrical conductivity of 20 milliseimens per meter (mS/m) at 25 $^{o}$C. Electromagnetic (EM) responses were calculated and correction factors determined. During the warmest and coldest periods of the year, the correction factors were not accurately predicted and resulted in corrected EM responses that exhibited considerable error, whereas, during the more moderate times of the year correction factors were more accurately predicted and the corrected EM responses closely matched the original 20 mS/m electrical conductivity. Methods of determining an average soil profile temperature resulted in correction factors that, when applied to the measured electrical conductivities, the resulting EM responses remained within 10% of the original 20 mS/m electrical conductivity.

## INTRODUCTION:

Worldwide, electromagnetic induction techniques (EM) are being used to measure soil apparent electrical conductivities ($EC_a$) for agricultural, ecological and environmental investigations. Cameron et al. (1981), Wollenhaupt et al. (1986), Rhodes et al. (1990), Slavich and Petterson (1990), Hendrickx et al. (1992) and Sheets et al. (1994) used EM measurements for the assessment of soil salinity. Kachanoski et al. (1988) and Sheets and Hendrickx (1995) measured total soil water content in the profile with EM techniques. Barlow and Ryan (1984), Grady and Haeni (1984), Ketelle and Pin (1984) and Saunders and Cox (1987) investigated subsurface groundwater contamination and its migration while Jordan et al. (1991) and Sherwin and Witten (1991) mapped buried waste sites with it. All these studies demonstrate the usefulness of the EM method for quick, non-invasive assessment of the apparent electrical conductivity of soils and groundwaters. In spite of so many reported EM applications, very few studies have been published that use the EM method to monitor specific sites over time to see whether the apparent electrical conductivity is increasing or decreasing. The most probable reason is that EM measurements need to be corrected for variations in soil temperature. Slavich and Petterson (1990) have determined that $EC_a$ measurements are very sensitive to variations in soil temperature. With all other properties (soil type, water content and thermal properties) being held constant and varying temperature only, the $EC_a$ readings can be as much as 50 - 75% in error. Therefore, comparisons of $EC_a$ measurements made over time are only possible if they are corrected for the variations in soil temperature to a specified reference standard of 25 degrees Celsius ($EC_{25}$).

The difficulty in making the conversion from $EC_a$ to $EC_{25}$ arises in determining the proper temperature to use. Slavich and Petterson (1990) found that the temperature at 50 centimeters depth during winter months and 70 centimeters depth during summer months resulted in coefficients that accurately converted $EC_a$ values to $EC_{25}$. Sheets and Hendrickx (1994) empirically determined that the temperature corresponding to 50 centimeters depth resulted in adequate correction factors.

The purpose of this paper is to determine the best means to arrive at a temperature that yields reliable correction factors for the conversion of $EC_a$ values to the standard reference temperature of 25 degrees Celsius ($EC_{25}$).

## METHOD:

Our approach involves five steps: 1.) The generation of soil temperature profiles; 2.) Conversion of the soil temperatures with depth to correction factors with depth; 3.) The generation of apparent electrical conductivity ($EC_a$) profiles for homogeneous soils that have a uniform electrical conductivity of 20 milliseimens per meter (mS/m) at 25 degrees Celsius; 4.) Modeling the $EC_a$ profiles to determine the electromagnetic device response at the surface; 5.) Correcting the EM reading back to the reference temperature of 25 degrees Celsius and analyzing the correction factors.

**Step 1: Generation of Soil Temperature Profiles:**

Numerous methods exist for the determination of soil temperature profiles. Most are based on some form of the solution to the one dimensional Fourier heat flow equation. Wierenga and de Wit (1970) describe a computer model that solves the heat flow equation in small time steps for thin layers and is based on soil physical properties. The analytic model of Matthias and Warrick (1987) is the sum of a deterministic component (mean annual wave plus diurnal amplitude) and a stochastic component (daily fluctuation about the mean annual wave). Parton (1984) describes an explicit discrete approximation to the heat flow equation that requires maximum and minimum air temperatures and plant biomass as the primary input data. Campbell (1977) and van Wijk (1963) predict temperature variations with depth using an analytic solution of the heat flow equation derived by assuming sinusoidal temperature oscillations that are damped with depth. Campbell's method is based on annual averages where van Wijk uses both annual and daily averages. Although each approach has their own advantages, we opted for the simple harmonic method of van Wijk (1963). If correction factors for this method are possible, other more realistic methods for heterogeneous soil profiles may be considered. The soil temperature with depth and time in a homogeneous soil profile is given as:

$$T(z,t) = \overline{T} + A_{oy} \exp(\frac{-z}{D_y}) \sin(\omega_y t - \frac{z}{D_y}) + A_{od} \exp(\frac{-z}{D_d}) \sin(\omega_d t - \frac{z}{D_d}) \qquad (1)$$

where: $T(z,t)$ is temperature at depth and time ($^oC$), $T$ is average annual temperature ($^oC$), $A_{oy}$ is yearly amplitude of surface temperature ($^oC$), $A_{od}$ is daily amplitude of surface temperature ($^oC$), $D_y$ is yearly damping depth (cm), $D_d$ is daily damping depth (cm), $\omega_y$ is yearly radial frequency ($seconds^{-1}$), $\omega_d$ is daily radial frequency ($seconds^{-1}$), $z$ is depth of interest (cm) and $t$ is time (seconds).

Seventy-two different temperature profiles were created using this model. They resulted from three different homogeneous soil types (sand, clay and peat), each with three varying degrees of water saturation (0%, 50% and 100% saturated) at four different days of the year (1, 90, 180 and 270) for two sets of average annual temperatures of 20.5 and 10 $^oC$ with respective yearly amplitudes of 32 and 25 $^oC$ and daily amplitudes of 12.5 and 15 $^oC$. These values have been chosen because they are representative of field soils throughout the arid southwest. The soil thermal properties are included in this model as damping depths. They are related to the frequency of the variations, the thermal conductivity ($\lambda$) and the volumetric heat capacity ($C$) of each soil type as:

$$D = \left[\frac{2\lambda}{C\omega}\right]^{\frac{1}{2}} \qquad (2)$$

For the three specific soil types used in this study (sand, clay and peat), the thermal conductivities are .7, .6 and .14 ($10^{-3}$ $cal \cdot cm^{-1} \cdot sec^{-1} \cdot {}^oC^{-1}$) and the volumetric heat capacities are .3, .3 and .35 (cal·cm$^{-3}$·$^{o}$C$^{-1}$) respectively. These seventy-two temperature profiles cover a wide variety of field conditions and are representative of many sites where EM surveys have been conducted. Figure 1 is an illustration of how the temperature varies with depth for the three soil types, at 50% saturation, for day 1, with an average temperature equal to 20.5 degrees Celsius and yearly and daily amplitudes equal to 32 and 12.5 degrees Celsius respectively.

Four techniques of averaging soil temperature profiles are also being considered. The first method utilized the average depth of investigation of two EM devices for each mode of operation. These are the EM-38 and the EM-31. For the EM-38 the depth of investigation is 1.5 meters in the vertical mode and 0.75 meters in the horizontal mode. For the EM-31 the depth of investigation is 6.0 meters in the vertical mode and 4.0 meters in the horizontal mode. The modeled temperatures were averaged to each of the EM devices' respective depth of investigation. This average temperature was then used to determine a correction factor value. The second method combined the tool sensitivity of each mode at each depth, with the temperature at that depth. These corrected temperature values were then averaged to the specific EM devices' depth of investigation and that average was used to determine a correction factor value. Richards et al. (1952) and Blanc (1958) recommend that, whenever possible, temperatures should be measured at 10, 20, 50, 100, 150 and 300 centimeters. The third method averages the temperatures from those depths and that average was used to determine a correction factor value. The fourth and final method was based on the subsurface temperatures that are regularly measured at weather stations. We recommend that the average of the temperatures from 10, 20, 50, 100, 200, 400, 600 and 1000 centimeters depth be used. Again, that average was used to determine a correction factor value.

**Step 2: Temperature Corrections:**

In general, soils are electrical insulators with low electrical conductivity. Therefore, in the absence of any highly conductive minerals such as magnetite, hematite or pyrite, the electrical conductivity of soil profiles is mainly a function of the electrical conductivity of the fluid contained within the pore spaces. The U.S.D.A. Agricultural Handbook #60 (1954) gives a table of correction factors to convert the $EC_a$ values of solutions, at a specified temperature, back to a standard reference temperature of 25 degrees Celsius ($EC_{25}$). An exponential curve was fit to the data resulting in the following equation:

$$f_T = .4470 + 1.4034 \exp\left(\frac{-T}{26.815}\right) \qquad r^2 = .999 \qquad (3)$$

where: $f_T$ is correction factor and $T$ is temperature ($^{o}$C).

Applying this equation to the successive depths of the seventy-two temperature profiles results in seventy-two correction factor profiles. Figure 2 is an example of this result when applied to the temperature profiles of Figure 1. Since all temperatures in Figure 1 are less than 25 $^{o}$C, all correction factors exceed unity. The maximum correction factor in Figure 2 is 1.4.

**Step 3: Generation of Apparent Electrical Conductivity ($EC_a$) Profiles:**

A homogeneous soil profile that has a uniform temperature and constant degree of saturation will exhibit a uniform electrical conductivity with depth (for the purposes of this study we chose $EC_{25}$ = 20 mS/m). Applying the correction factor profiles to this value (as $EC_a = EC_{25} / f_T$) results in apparent electrical conductivity profiles ($EC_a$) that show how the electrical conductivity varies with depth due solely to variations in temperature (Figure 3 and 4).

**Step 4: Calculation of the Electromagnetic Induction Measurement Response:**

The electromagnetic induction method utilizes an alternating current that is passed through a transmitter coil which produces a local magnetic field termed the primary field. This primary magnetic field induces small current flows in the earth. The small current flows induce a secondary magnetic field which varies in amplitude and phase and is detected by the receiver coil.

The secondary magnetic field is a function of the spacing between the transmitter coil and the receiver coil, the operating frequency of the alternating current and the conductivity of the earth. The ratio of the secondary magnetic field to the primary magnetic field is proportional to the conductivity of the earth and is measured in milliseimens per meter (mS/m). This is the measurement that the EM device produces and is the value for which we will be modeling.

The measurements obtained by the EM device, in response to a multi-layered earth, can be represented by (McNeill 1980):

$$EM_V = \int_0^{Z_V} \phi_V(Z) EC_a(Z) dZ \tag{4}$$

$$EM_H = \int_0^{Z_H} \phi_H(Z) EC_a(Z) dZ \tag{5}$$

where: $\phi_V(Z)$ is vertical response at depth Z, $\phi_H(Z)$ is horizontal response at depth Z, $Z$ is ratio of depth to intercoil spacing in meters, $Z_V$ is depth of measurement for vertical dipole mode, $Z_H$ is depth of measurement for horizontal dipole mode and $EC_a(Z)$ is apparent electrical conductivity at depth Z.

The vertical and horizontal response functions are:

$$\phi_V(Z) = \frac{4Z}{(4Z^2+1)^{1.5}} \tag{6}$$

$$\phi_H(Z) = 2 - \frac{4Z}{(4Z^2+1)^{0.5}} \tag{7}$$

Figure 5 shows these response functions with depth for the EM-38 with a separation of 1 meter between the transmitter and receiver coils.

McNeill (1980) also indicates that for a multi-layered earth, the cumulative response function can be given as:

$$R_V(Z) = \int_Z^{\infty} \phi_V(Z)dZ = \frac{1}{(4Z^2+1)^{0.5}} \quad (8)$$

$$R_H(Z) = \int_Z^{\infty} \phi_H(Z)dZ = (4Z^2+1)^{0.5} - 2Z \quad (9)$$

Figure 6 illustrates the cumulative response functions with depth.

Integration of equations 4 and 5, by steps, result in the EM response for a N-layered earth:

$$EM_V = \sum_{i=1}^{N} EC_a(Z_i)[R_V(Z_i) - R_V(Z_{i-1})] \quad (10)$$

$$EM_H = \sum_{i=1}^{N} EC_a(Z_i)[R_H(Z_i) - R_H(Z_{i-1})] \quad (11)$$

Equations 10 and 11 were used to determine the surface response ($EM_V$ and $EM_H$) of the electromagnetic induction tool based on the apparent electrical conductivity profiles

($EC_a$) given for the three soil profiles (V and H denote vertical and horizontal dipole modes respectively).

**Step 5: Calculation of the Correction Factor:**

The calculated vertical and horizontal electromagnetic responses ($EM_V$ and $EM_H$) were used to determine the correction factors based on the $EC_{25}$ standard value of 20mS/m as:

$$f_{T(V)} = \frac{EC_{25}}{EM_V} \tag{12}$$

$$f_{T(H)} = \frac{EC_{25}}{EM_H} \tag{13}$$

where: $f_{T(V)}$ and $f_{T(H)}$ are the correction factors corresponding to the vertical and horizontal modes respectively.

## RESULTS AND DISCUSSION:

**Soil Temperature Variations:**

Variations in soil temperature result from two main factors. The first is the thermal properties, volumetric heat capacity and thermal conductivity, of each individual soil type. These thermal properties have been included in this study as damping depths. Varying degrees of saturation create differences in the damping depths by altering the volumetric heat capacity and thermal conductivity. The second, and most important factor, is the climatic changes. Daily and yearly, cyclical changes in atmospheric temperatures create cyclical soil temperatures. Combining the above two factors result in cyclical soil temperature variations which are damped with depth. Figure 1 shows the differences in temperatures with depth for three homogeneous soils, each with 50 % saturation, during the same day of the year (day 1). This illustration shows how the soil thermal properties of each soil type create temperature differences at depth. Figure 7 shows how varying degrees of saturation change the soil thermal properties which results in varying temperatures. This illustration is for a sandy soil with 0%, 50% and 100% saturation. Clay and peat soils show the same trends as the sand soil, slight differences in temperature with depth from 50 to 100 % saturations and larger differences from 0 to 50 % saturations. This indicates that as degree of saturation increase, variations in soil thermal properties decrease. Figure 8 shows the differences in temperature created by the seasonal variations for a sandy soil which is 50% saturated for day 1 and day 180 - six months apart. Again the same trends are seen for the clay and peat soils.

**Correction Factors Based on Single Temperature Values:**

With the wide diversity of temperature variations created, the question becomes - what temperature should we use for a basis to calculate the appropriate correction factor? It has been suggested that a temperature corresponding to a specific depth can be used to determine this correction factor. Figure 2 is an illustration of the correction factors calculated for each depth of the temperature profiles of Figure 1. The correction factors for temperatures corresponding to depths covering the range of 10 to 150 centimeters were analyzed in an attempt to determine an optimum depth/temperature combination which would result in appropriate values. Considering the three different soil types, for the three degrees of saturation, covering all four days of the year, the results varied depending upon the mode of operation (vertical or horizontal) which was used for each of the electromagnetic devices (EM-38 or EM-31). For the EM-38 in the vertical mode, the temperatures that correspond to the 90 - 110 centimeter range resulted in correction factors that kept predicted electromagnetic responses within 15% error. In the horizontal mode, the temperatures that correspond to the 40 - 60 centimeter range resulted in correction factors that kept predicted electromagnetic responses within 15% error. For the EM-31, no temperature values to depths of 150 centimeters consistently predicted electromagnetic responses that remained within the 15% threshold. Therefore, to pick a single depth and determine its corresponding temperature, then calculate a correction factor that can be used to correct $EC_a$ values back to a standard of $EC_{25}$ is very inconsistent and can result in considerable error.

**Correction Factors Based on Average Temperature Values:**

Figures 1, 7 and 8 show that the largest variations in temperature occur in the top 200 cm of soil and that stability in temperature is achieved at approximately 900 cm below the ground surface. Average temperatures calculated from near surface depths will also show much greater variations than average temperatures that include the more stable values. The EM devices considered in this study vary immensely in the depth to which they measure. Therefore, one would expect that the resulting electromagnetic response would be related to those average depths of investigation and the temperatures associated with them. This does not seem to be the case. The EM-38, in the vertical mode, measures to an average depth of 150 cm. In the horizontal mode, it measures to an average depth of 75 cm. These depths include the extreme values of the temperature curves only and exhibit the highest degree of variations in average temperatures. Corrected EM responses based on temperatures averaged to these depths resulted in considerable error. A 10 % margin of error was maintained when deeper averages were used. For the EM-38 in the vertical mode, the correction factors calculated from the average temperatures of method four (to 1000 cm) resulted in predicted electromagnetic responses that remained within 10% error. For the horizontal mode, the correction factors calculated from the average temperatures of method three (to 300 cm) gave results within 10%. The EM-31 responded closer to what was expected when considering its average depth of investigation. The average depth of investigation for the vertical mode is 600 cm and the horizontal mode is 400 cm. Average temperatures to these depths include a wider range of temperature values and approach the more stable values at 900 cm depth. Averaging the temperatures to these respective depths of investigation (method one) consistently predicted electromagnetic responses that remained within 10% error.

All of the above correction factors were compared with the actual value as the ratio of temperature based correction factor to actual correction factor. A typical example of this is shown in Table 1 for a sand soil, for both vertical and horizontal modes, for various days and saturations.

**CONCLUSIONS:**

For the simple case of a homogeneous soil profile without variation of $EC_{25}$ with depth, the results indicate that a temperature corresponding to one depth, for all times, for calculation of a correction factor is not feasible, as it has been suggested. To arrive at correction factors that are within 10% error, some type of average temperature needs to be determined. In reality, we realize that most soil profiles are very heterogeneous. For the non changing heterogeneity cases, such as desert soils, the methods proposed in this paper are most likely quite adequate. But for the heterogeneous cases that change over time, such as agricultural fields where leaching can change the $EC_a$, combinations of different methods (EM in combination with Time Domain Reflectometry) needs to be employed for calibration purposes.

For EM monitoring projects, it is best to take the measurements on the same day each year. This will minimize the variations in soil temperature.

If temperature corrections are neglected, the measured $EC_a$ values at extreme times (summer and winter) can be as much as 50 - 75% in error. Finding an appropriate temperature to determine a correction factor for these extreme days is quite difficult. It is recommended that the extremes be avoided if at all possible. The best measurement times are during the moderate times of the year (spring and fall).

The finding that no consistent trend was detected for the homogeneous soils considered in this study indicates that no trend exists for the more complicated heterogeneous soil profiles.

**Figure 1:** Temperature variations with depth for three soil types at 50% saturation for day 1.

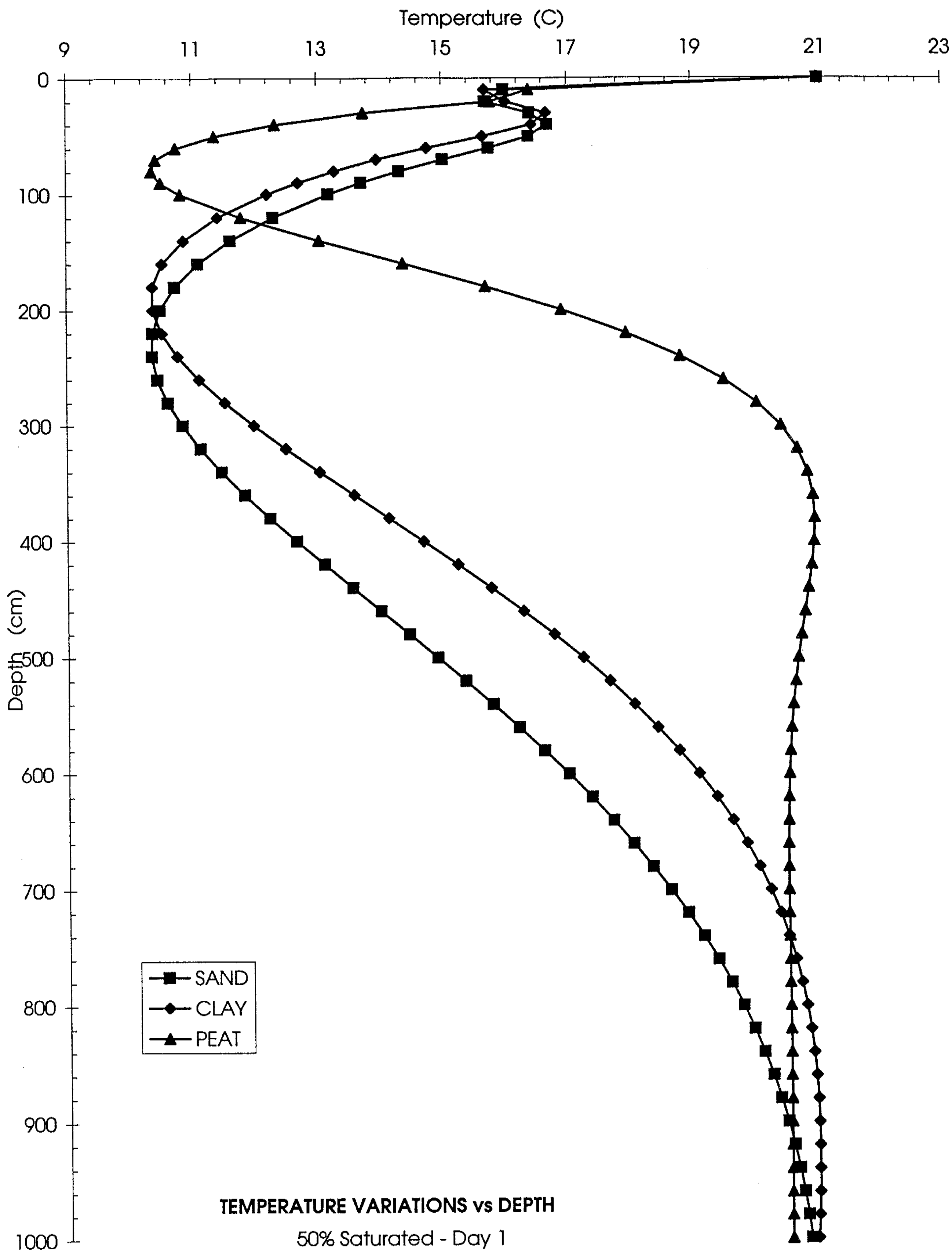

**Figure 2:** Correction factors with depth for three soil types at 50% saturation for day 1

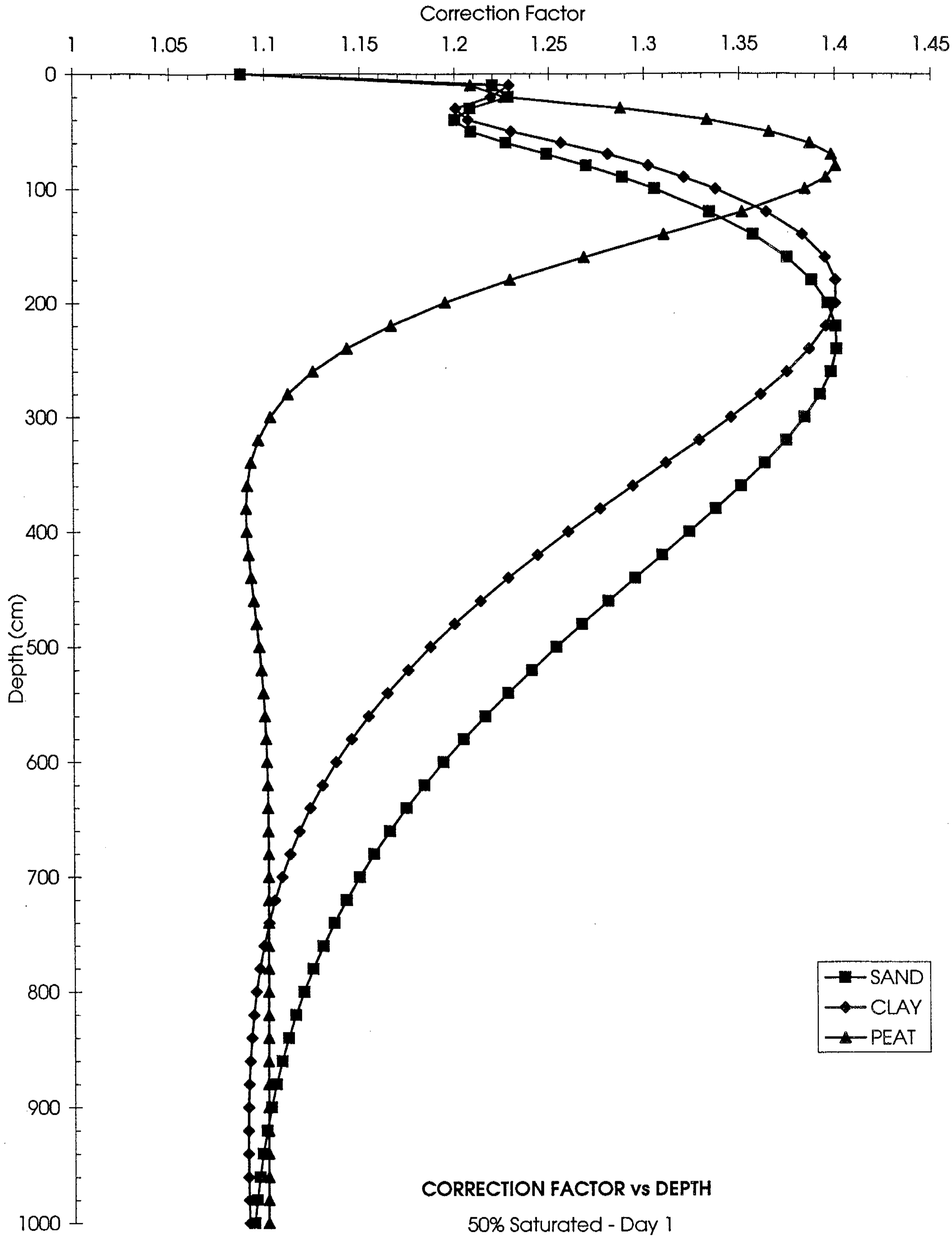

**Figure 3:** Electrical conductivity with depth for a homogeneous soil with a uniform temperature and constant degree of saturation

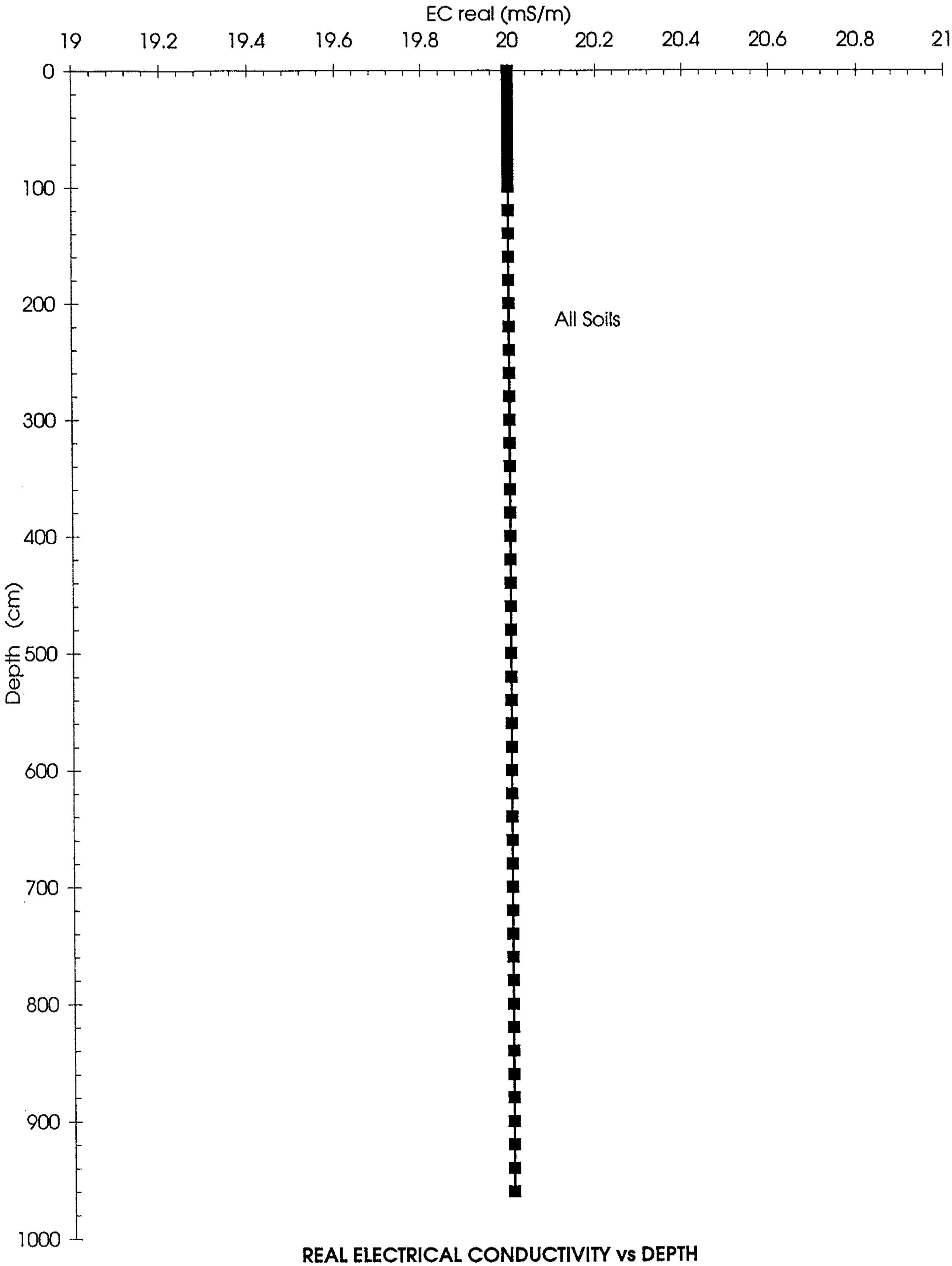

**Figure 4:** Apparent electrical conductivity with depth for three soil types at 50% saturation for day 1

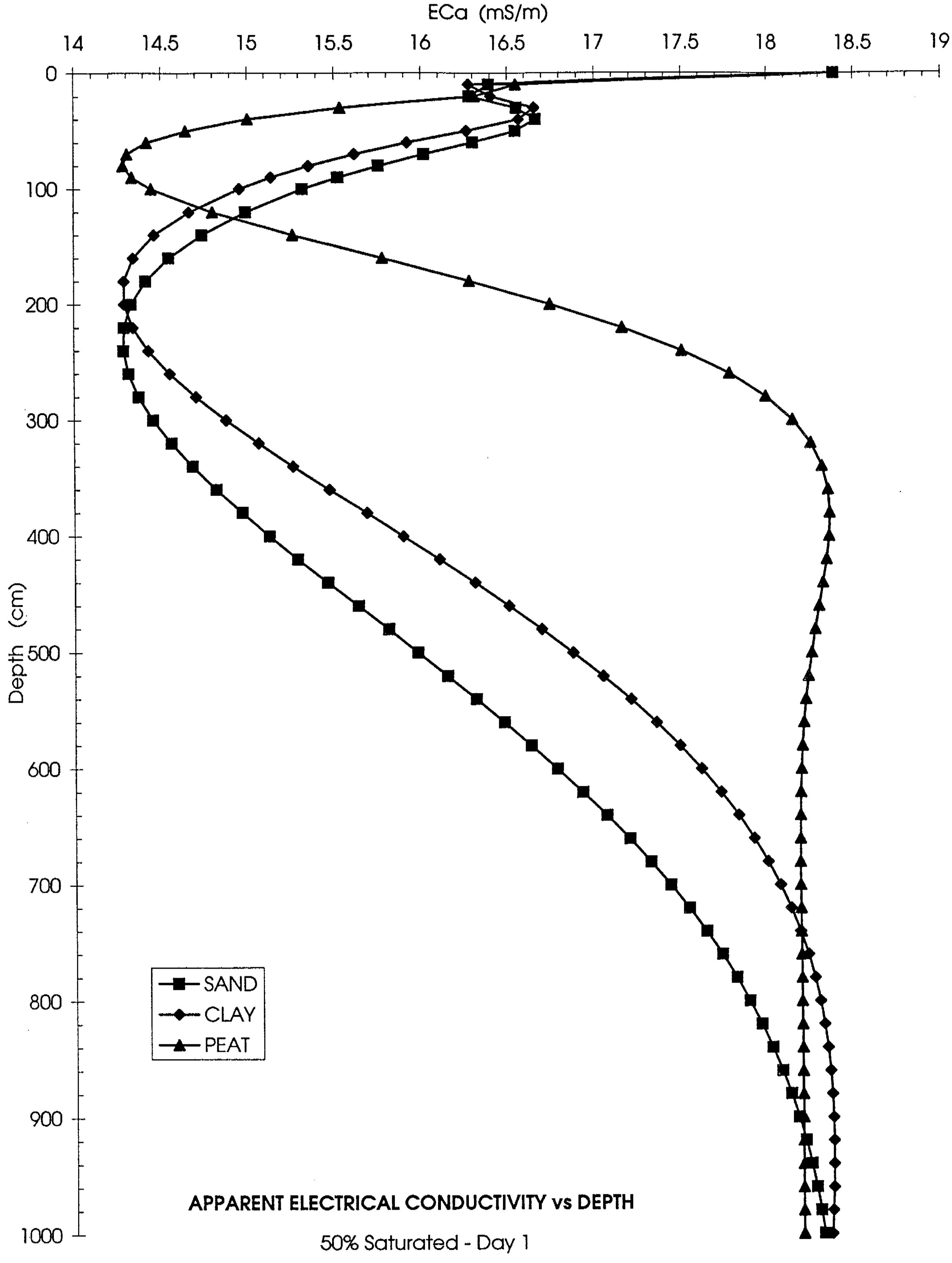

**Figure 5:** Relative response with depth for the verical and horizontal dipole modes

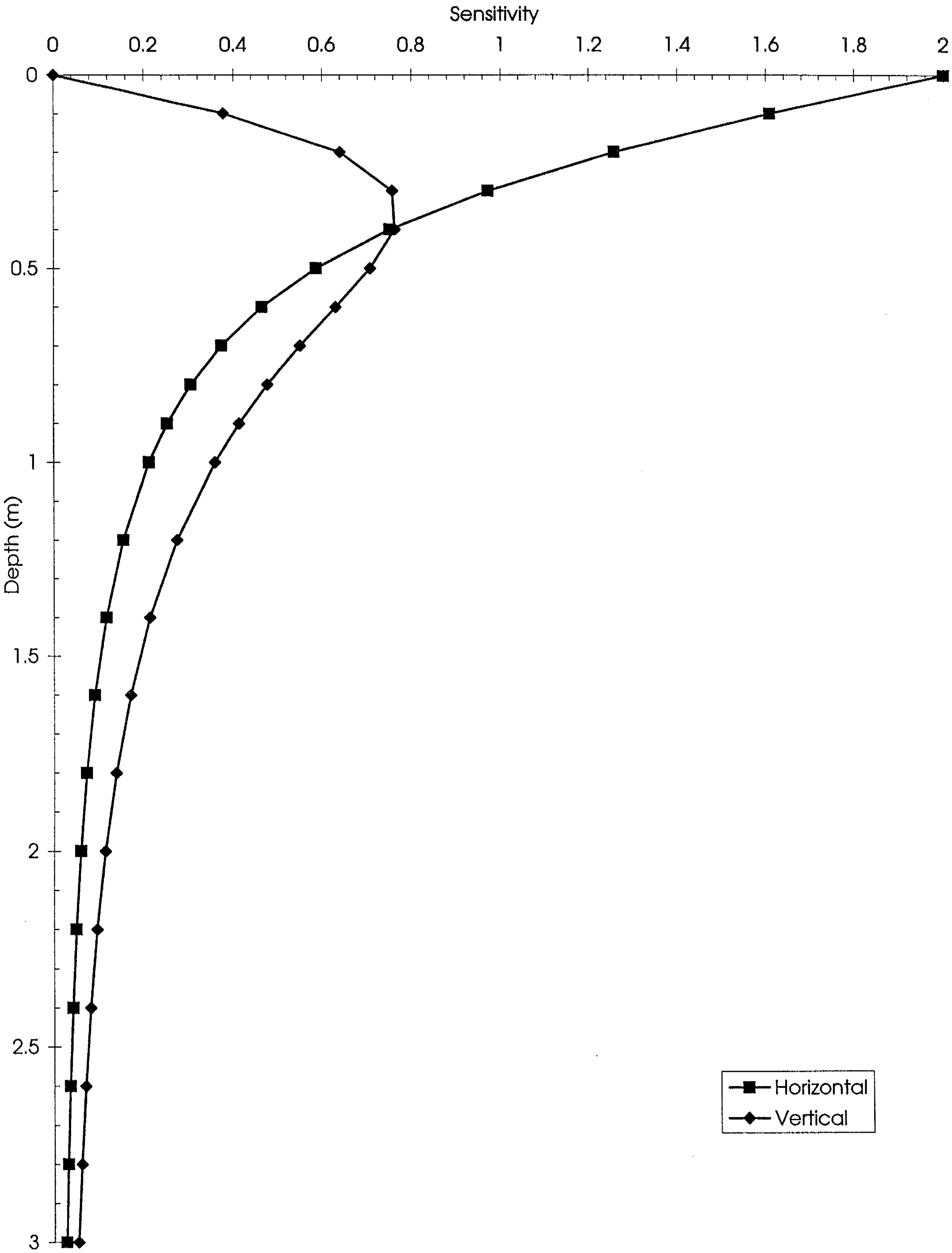

**Figure 6:** Cumulative response with depth for the vertical and horizontal dipole modes

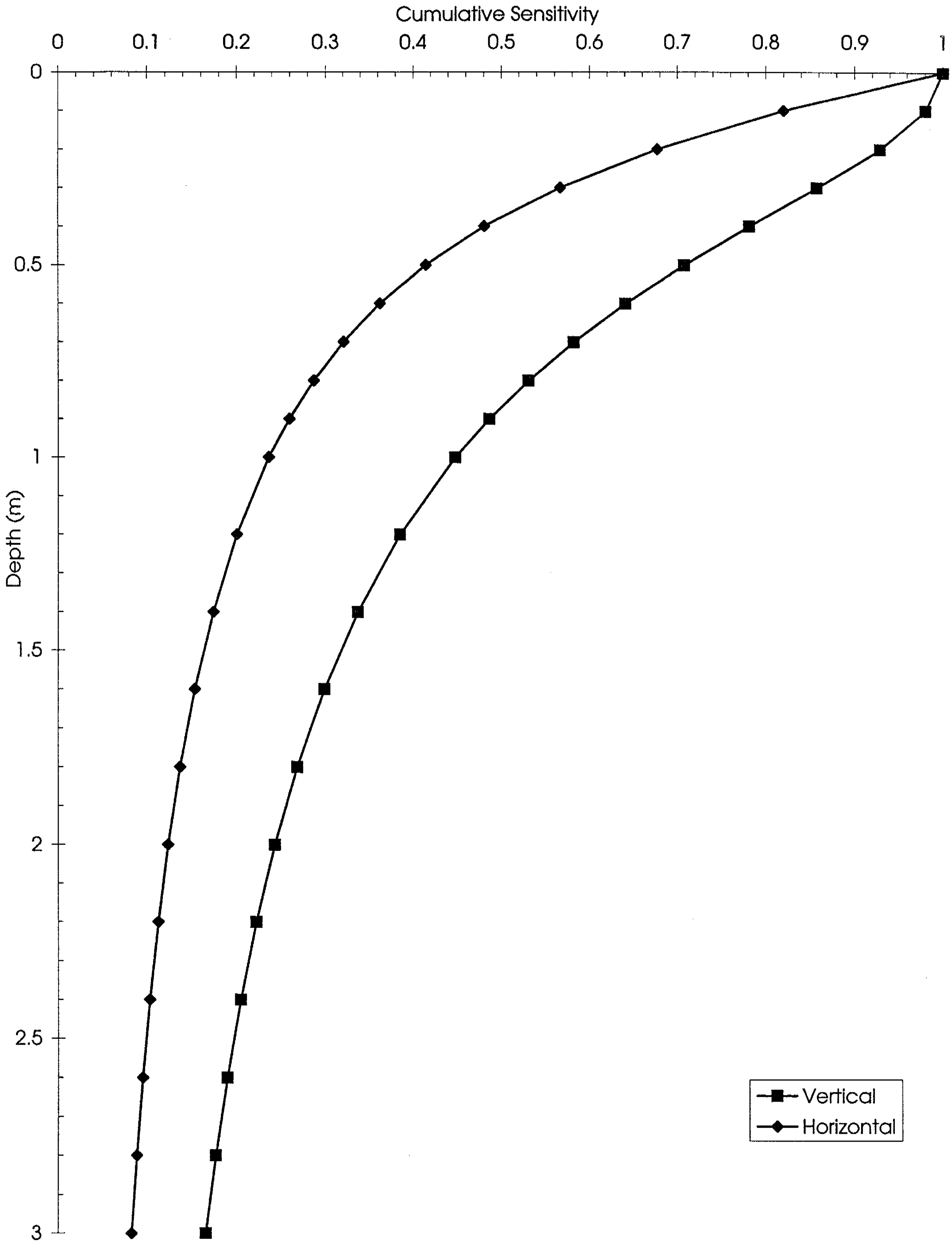

**Figure 7:** Temperature variation with depth for a sand soil on day 1 for three degrees of saturation

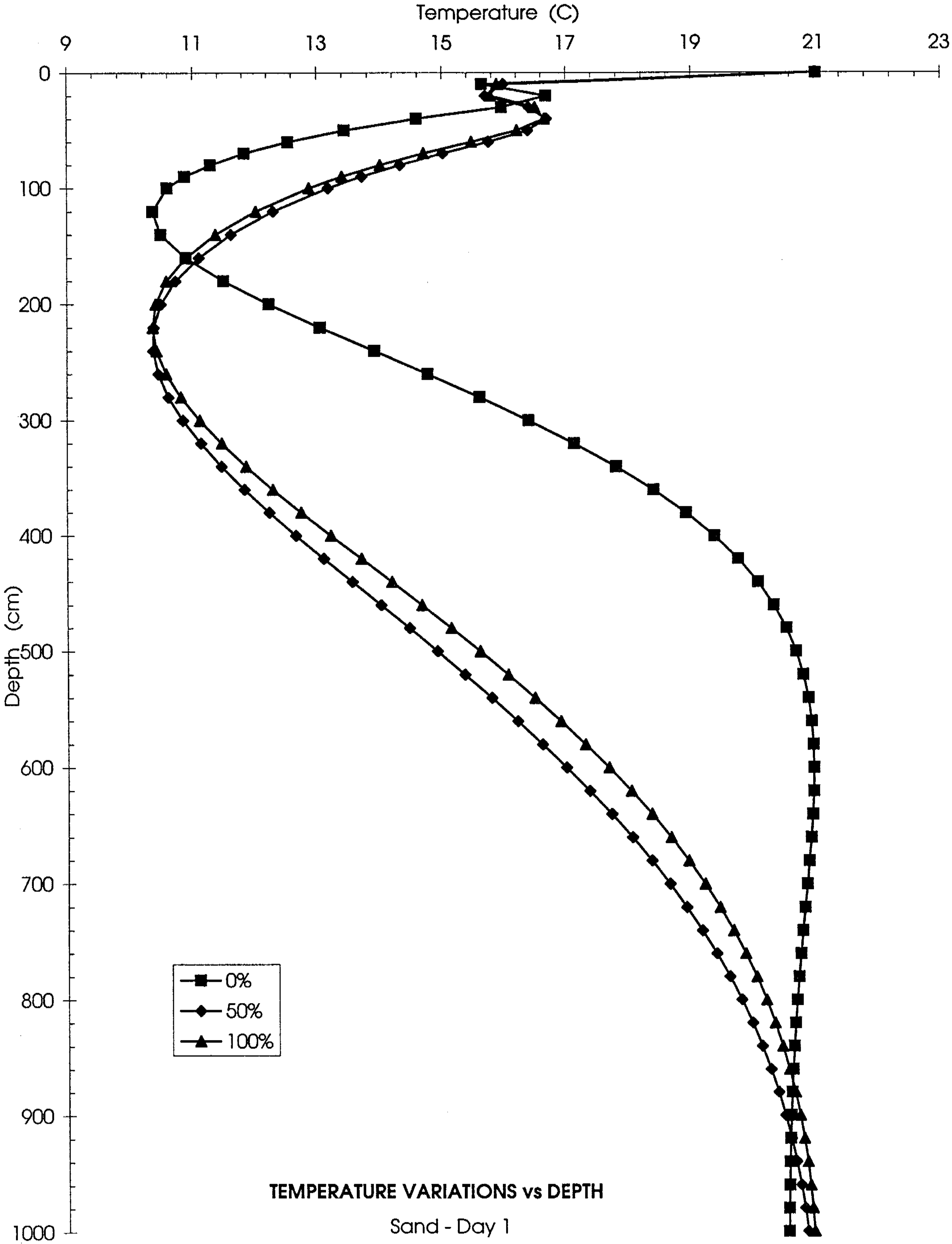

**Figure 8:** Temperature variations with depth for a sand soil at 50% saturation for the warmest and coldest days

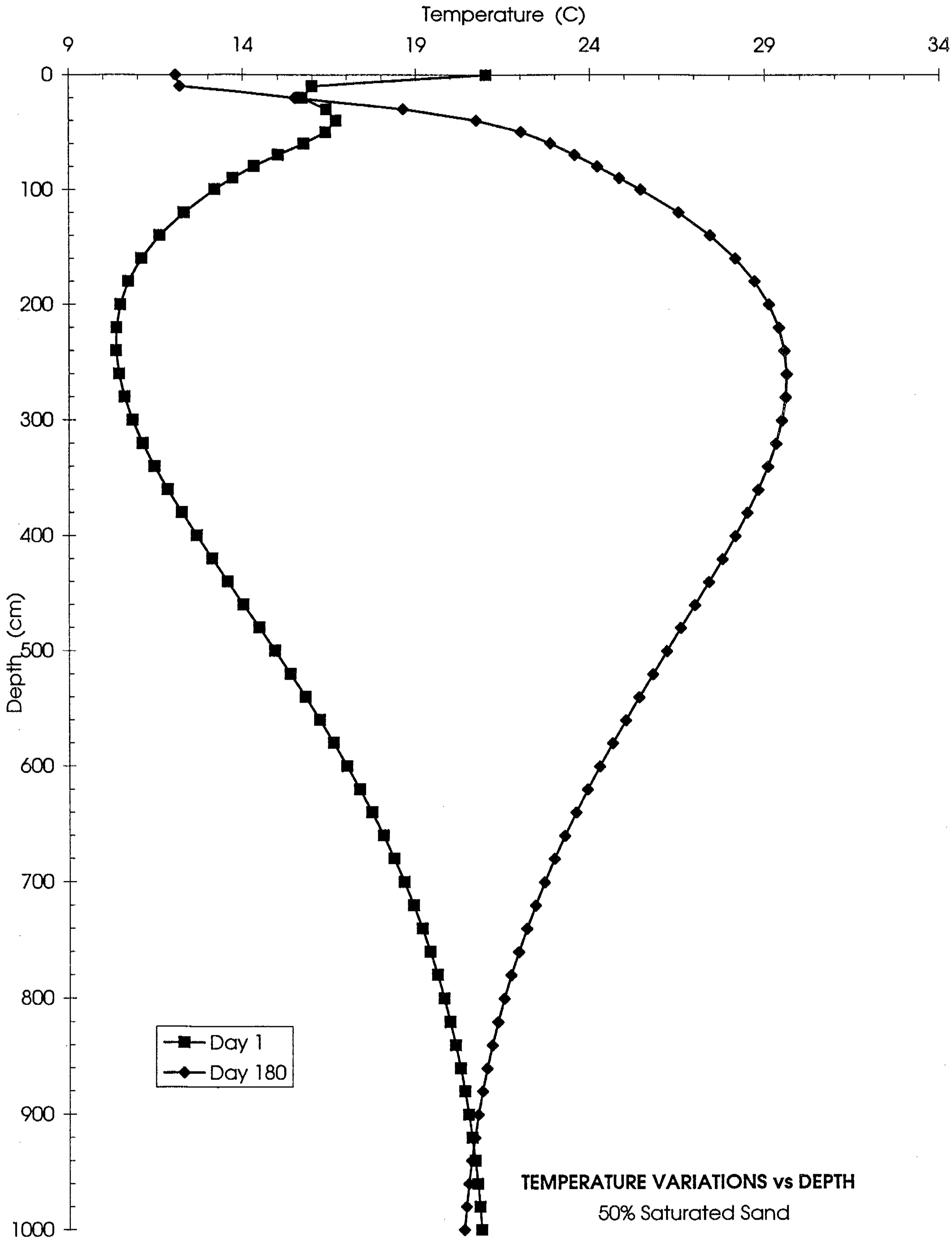

Table 1: Ratio of - (correction factor)/(real correction factor)

EM-31 Vertical
Soil Type = Sand

| Day | Degree of Saturation | Method 1 | Method 2 | Method 3 | Method 4 | Temperature at 10 cm | 30 cm | 50 cm | 70 cm | 90 cm | 110 cm | 130 cm | 150 cm |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.00 | 1.00 | 1.05 | 1.07 | 1.01 | 1.02 | 1.01 | 1.08 | 1.12 | 1.15 | 1.16 | 1.16 | 1.15 |
| | 0.50 | 1.04 | 1.06 | 1.02 | 0.99 | 0.97 | 0.96 | 0.96 | 0.99 | 1.03 | 1.05 | 1.07 | 1.09 |
| | 1.00 | 1.03 | 1.06 | 1.03 | 0.99 | 0.98 | 0.96 | 0.97 | 1.01 | 1.04 | 1.06 | 1.08 | 1.10 |
| 100 | 0.00 | 0.98 | 0.96 | 0.79 | 0.86 | 0.68 | 0.68 | 0.72 | 0.75 | 0.79 | 0.83 | 0.88 | 0.92 |
| | 0.50 | 0.97 | 0.92 | 0.80 | 0.89 | 0.75 | 0.73 | 0.74 | 0.76 | 0.78 | 0.80 | 0.82 | 0.84 |
| | 1.00 | 0.97 | 0.93 | 0.80 | 0.89 | 0.74 | 0.73 | 0.73 | 0.75 | 0.78 | 0.80 | 0.82 | 0.85 |
| 190 | 0.00 | 1.01 | 0.97 | 1.00 | 1.04 | 1.23 | 1.03 | 0.98 | 0.94 | 0.92 | 0.90 | 0.90 | 0.90 |
| | 0.50 | 0.98 | 0.97 | 1.07 | 1.07 | 1.35 | 1.16 | 1.07 | 1.04 | 1.01 | 0.99 | 0.97 | 0.95 |
| | 1.00 | 0.98 | 0.97 | 1.07 | 1.13 | 1.34 | 1.14 | 1.07 | 1.03 | 1.00 | 0.98 | 0.96 | 0.95 |
| 270 | 0.00 | 1.04 | 1.04 | 1.42 | 1.25 | 2.32 | 1.81 | 1.62 | 1.46 | 1.33 | 1.23 | 1.15 | 1.08 |
| | 0.50 | 1.06 | 1.12 | 1.55 | 1.27 | 2.36 | 1.88 | 1.69 | 1.59 | 1.50 | 1.42 | 1.34 | 1.27 |
| | 1.00 | 1.05 | 1.11 | 1.54 | 1.27 | 2.37 | 1.87 | 1.69 | 1.59 | 1.49 | 1.40 | 1.32 | 1.26 |

EM-38 Vertical
Soil Type = Sand

| Day | Degree of Saturation | Method 1 | Method 2 | Method 3 | Method 4 | Temperature at 10 cm | 30 cm | 50 cm | 70 cm | 90 cm | 110 cm | 130 cm | 150 cm |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.00 | 1.03 | 1.03 | 1.01 | 0.95 | 0.96 | 0.96 | 1.02 | 1.06 | 1.08 | 1.10 | 1.10 | 1.09 |
| | 0.50 | 1.00 | 0.99 | 1.02 | 0.98 | 0.97 | 0.96 | 0.96 | 0.99 | 1.02 | 1.05 | 1.07 | 1.08 |
| | 1.00 | 1.00 | 0.99 | 1.02 | 0.98 | 0.97 | 0.95 | 0.96 | 0.99 | 1.03 | 1.05 | 1.07 | 1.09 |
| 100 | 0.00 | 0.92 | 0.90 | 0.95 | 1.04 | 0.82 | 0.82 | 0.87 | 0.91 | 0.96 | 1.01 | 1.06 | 1.11 |
| | 0.50 | 0.92 | 0.91 | 0.96 | 1.07 | 0.90 | 0.88 | 0.89 | 0.91 | 0.93 | 0.96 | 0.99 | 1.01 |
| | 1.00 | 0.92 | 0.91 | 0.96 | 1.07 | 0.89 | 0.88 | 0.88 | 0.91 | 0.94 | 0.96 | 0.99 | 1.02 |
| 190 | 0.00 | 1.00 | 1.00 | 1.03 | 1.06 | 1.26 | 1.06 | 1.00 | 0.97 | 0.94 | 0.93 | 0.92 | 0.92 |
| | 0.50 | 1.03 | 1.04 | 1.04 | 1.04 | 1.31 | 1.12 | 1.04 | 1.01 | 0.98 | 0.96 | 0.94 | 0.92 |
| | 1.00 | 1.03 | 1.04 | 1.04 | 1.04 | 1.31 | 1.11 | 1.04 | 1.00 | 0.98 | 0.95 | 0.93 | 0.92 |
| 270 | 0.00 | 1.18 | 1.22 | 1.13 | 1.00 | 1.85 | 1.45 | 1.30 | 1.16 | 1.06 | 0.98 | 0.91 | 0.86 |
| | 0.50 | 1.23 | 1.25 | 1.17 | 0.96 | 1.78 | 1.42 | 1.27 | 1.20 | 1.13 | 1.07 | 1.01 | 0.96 |
| | 1.00 | 1.22 | 1.25 | 1.16 | 0.96 | 1.79 | 1.42 | 1.28 | 1.20 | 1.13 | 1.06 | 1.00 | 0.95 |

EM-31 Horizontal
Soil Type = Sand

| Day | Degree of Saturation | Method 1 | Method 2 | Method 3 | Method 4 | Temperature at 10 cm | 30 cm | 50 cm | 70 cm | 90 cm | 110 cm | 130 cm | 150 cm |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.00 | 1.02 | 1.03 | 1.03 | 0.97 | 0.99 | 0.98 | 1.04 | 1.08 | 1.11 | 1.12 | 1.12 | 1.12 |
| | 0.50 | 1.05 | 1.02 | 1.02 | 0.98 | 0.97 | 0.96 | 0.96 | 0.99 | 1.02 | 1.05 | 1.07 | 1.08 |
| | 1.00 | 1.05 | 1.02 | 1.02 | 0.98 | 0.97 | 0.96 | 0.96 | 1.00 | 1.03 | 1.05 | 1.07 | 1.09 |
| 100 | 0.00 | 1.06 | 0.93 | 0.90 | 0.98 | 0.78 | 0.78 | 0.82 | 0.86 | 0.91 | 0.96 | 1.00 | 1.05 |
| | 0.50 | 1.01 | 0.92 | 0.91 | 1.01 | 0.85 | 0.83 | 0.84 | 0.86 | 0.88 | 0.91 | 0.93 | 0.96 |
| | 1.00 | 1.01 | 0.92 | 0.91 | 1.01 | 0.84 | 0.83 | 0.83 | 0.86 | 0.88 | 0.91 | 0.93 | 0.96 |
| 190 | 0.00 | 0.98 | 1.00 | 1.01 | 1.04 | 1.23 | 1.04 | 0.99 | 0.95 | 0.92 | 0.91 | 0.90 | 0.90 |
| | 0.50 | 0.95 | 1.01 | 1.04 | 1.04 | 1.31 | 1.12 | 1.04 | 1.01 | 0.98 | 0.96 | 0.94 | 0.92 |
| | 1.00 | 0.95 | 1.01 | 1.04 | 1.10 | 1.31 | 1.11 | 1.04 | 1.00 | 0.98 | 0.95 | 0.93 | 0.92 |
| 270 | 0.00 | 0.94 | 1.15 | 1.21 | 1.07 | 1.99 | 1.55 | 1.39 | 1.25 | 1.14 | 1.05 | 0.98 | 0.93 |
| | 0.50 | 0.99 | 1.22 | 1.28 | 1.05 | 1.95 | 1.56 | 1.40 | 1.32 | 1.24 | 1.17 | 1.11 | 1.05 |
| | 1.00 | 0.99 | 1.21 | 1.27 | 1.05 | 1.96 | 1.55 | 1.40 | 1.32 | 1.24 | 1.16 | 1.10 | 1.04 |

EM-38 Horizontal
Soil Type = Sand

| Day | Degree of Saturation | Method 1 | Method 2 | Method 3 | Method 4 | Temperature at 10 cm | 30 cm | 50 cm | 70 cm | 90 cm | 110 cm | 130 cm | 150 cm |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.00 | 0.99 | 0.96 | 1.02 | 0.97 | 0.98 | 0.97 | 1.03 | 1.07 | 1.10 | 1.11 | 1.11 | 1.10 |
| | 0.50 | 0.97 | 0.95 | 1.03 | 1.00 | 0.98 | 0.97 | 0.97 | 1.00 | 1.04 | 1.06 | 1.08 | 1.10 |
| | 1.00 | 0.97 | 0.95 | 1.03 | 0.99 | 0.98 | 0.97 | 0.98 | 1.01 | 1.04 | 1.07 | 1.09 | 1.10 |
| 100 | 0.00 | 0.92 | 0.90 | 1.05 | 1.14 | 0.90 | 0.90 | 0.95 | 1.00 | 1.05 | 1.11 | 1.16 | 1.22 |
| | 0.50 | 0.94 | 0.93 | 1.02 | 1.13 | 0.95 | 0.93 | 0.94 | 0.96 | 0.99 | 1.02 | 1.04 | 1.07 |
| | 1.00 | 0.94 | 0.93 | 1.02 | 1.13 | 0.95 | 0.93 | 0.94 | 0.96 | 0.99 | 1.02 | 1.05 | 1.08 |
| 190 | 0.00 | 1.03 | 1.09 | 0.98 | 1.01 | 1.19 | 1.01 | 0.95 | 0.92 | 0.89 | 0.88 | 0.87 | 0.88 |
| | 0.50 | 1.04 | 1.10 | 0.96 | 0.96 | 1.21 | 1.04 | 0.97 | 0.93 | 0.91 | 0.89 | 0.87 | 0.86 |
| | 1.00 | 1.04 | 1.10 | 0.96 | 0.97 | 1.21 | 1.04 | 0.96 | 0.93 | 0.91 | 0.89 | 0.87 | 0.86 |
| 270 | 0.00 | 1.23 | 1.36 | 0.94 | 0.83 | 1.54 | 1.21 | 1.08 | 0.97 | 0.88 | 0.81 | 0.76 | 0.72 |
| | 0.50 | 1.20 | 1.31 | 0.97 | 0.80 | 1.49 | 1.19 | 1.06 | 1.00 | 0.95 | 0.89 | 0.84 | 0.80 |
| | 1.00 | 1.20 | 1.31 | 0.97 | 0.80 | 1.50 | 1.18 | 1.07 | 1.00 | 0.94 | 0.89 | 0.84 | 0.79 |

Table 2: Ratio of - (correction factor)/(real correction factor)

EM-31 Vertical
Soil Type = Clay

| Day | Degree of Saturation | Method 1 | Method 2 | Method 3 | Method 4 | Temperature at 10 cm | 30 cm | 50 cm | 70 cm | 90 cm | 110 cm | 130 cm | 150 cm |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.00 | 1.00 | 1.04 | 1.07 | 1.01 | 1.03 | 1.03 | 1.09 | 1.14 | 1.16 | 1.17 | 1.17 | 1.15 |
| | 0.50 | 1.03 | 1.06 | 1.04 | 0.99 | 0.99 | 0.97 | 0.99 | 1.03 | 1.06 | 1.09 | 1.11 | 1.12 |
| | 1.00 | 1.03 | 1.06 | 1.04 | 0.99 | 0.99 | 0.97 | 0.99 | 1.04 | 1.07 | 1.09 | 1.11 | 1.12 |
| 100 | 0.00 | 0.98 | 0.96 | 0.79 | 0.85 | 0.67 | 0.68 | 0.72 | 0.76 | 0.80 | 0.85 | 0.89 | 0.93 |
| | 0.50 | 0.98 | 0.94 | 0.79 | 0.88 | 0.73 | 0.71 | 0.73 | 0.75 | 0.78 | 0.80 | 0.83 | 0.86 |
| | 1.00 | 0.98 | 0.94 | 0.79 | 0.88 | 0.73 | 0.71 | 0.72 | 0.75 | 0.78 | 0.80 | 0.83 | 0.86 |
| 190 | 0.00 | 1.01 | 0.97 | 1.00 | 1.03 | 1.20 | 1.02 | 0.97 | 0.93 | 0.91 | 0.90 | 0.89 | 0.90 |
| | 0.50 | 0.99 | 0.96 | 1.05 | 1.06 | 1.32 | 1.11 | 1.05 | 1.01 | 0.98 | 0.96 | 0.94 | 0.93 |
| | 1.00 | 0.99 | 0.96 | 1.05 | 1.12 | 1.32 | 1.11 | 1.04 | 1.01 | 0.98 | 0.96 | 0.94 | 0.93 |
| 270 | 0.00 | 1.04 | 1.04 | 1.40 | 1.25 | 2.30 | 1.80 | 1.60 | 1.43 | 1.30 | 1.20 | 1.12 | 1.06 |
| | 0.50 | 1.04 | 1.09 | 1.51 | 1.26 | 2.38 | 1.85 | 1.69 | 1.57 | 1.46 | 1.37 | 1.29 | 1.22 |
| | 1.00 | 1.04 | 1.09 | 1.51 | 1.26 | 2.38 | 1.85 | 1.69 | 1.57 | 1.46 | 1.36 | 1.28 | 1.21 |

EM-38 Vertical
Soil Type = Clay

| Day | Degree of Saturation | Method 1 | Method 2 | Method 3 | Method 4 | Temperature at 10 cm | 30 cm | 50 cm | 70 cm | 90 cm | 110 cm | 130 cm | 150 cm |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.00 | 1.03 | 1.03 | 1.00 | 0.95 | 0.96 | 0.96 | 1.03 | 1.07 | 1.09 | 1.10 | 1.09 | 1.08 |
| | 0.50 | 1.01 | 1.00 | 1.02 | 0.97 | 0.97 | 0.95 | 0.97 | 1.01 | 1.04 | 1.07 | 1.08 | 1.10 |
| | 1.00 | 1.01 | 1.00 | 1.02 | 0.97 | 0.97 | 0.95 | 0.97 | 1.01 | 1.04 | 1.07 | 1.08 | 1.10 |
| 100 | 0.00 | 0.92 | 0.90 | 0.95 | 1.03 | 0.81 | 0.82 | 0.86 | 0.91 | 0.97 | 1.02 | 1.07 | 1.12 |
| | 0.50 | 0.92 | 0.91 | 0.96 | 1.07 | 0.88 | 0.86 | 0.88 | 0.91 | 0.94 | 0.97 | 1.00 | 1.04 |
| | 1.00 | 0.92 | 0.91 | 0.96 | 1.06 | 0.88 | 0.86 | 0.88 | 0.91 | 0.94 | 0.97 | 1.00 | 1.04 |
| 190 | 0.00 | 1.00 | 1.00 | 1.03 | 1.06 | 1.24 | 1.05 | 1.00 | 0.96 | 0.94 | 0.92 | 0.92 | 0.93 |
| | 0.50 | 1.02 | 1.03 | 1.03 | 1.05 | 1.30 | 1.10 | 1.03 | 1.00 | 0.97 | 0.94 | 0.93 | 0.92 |
| | 1.00 | 1.02 | 1.03 | 1.03 | 1.05 | 1.30 | 1.09 | 1.03 | 1.00 | 0.97 | 0.94 | 0.93 | 0.92 |
| 270 | 0.00 | 1.18 | 1.21 | 1.13 | 1.01 | 1.85 | 1.45 | 1.29 | 1.15 | 1.05 | 0.97 | 0.90 | 0.85 |
| | 0.50 | 1.22 | 1.25 | 1.16 | 0.97 | 1.82 | 1.42 | 1.29 | 1.20 | 1.12 | 1.05 | 0.98 | 0.93 |
| | 1.00 | 1.22 | 1.25 | 1.15 | 0.97 | 1.82 | 1.42 | 1.29 | 1.20 | 1.12 | 1.04 | 0.98 | 0.93 |

EM-31 Horizontal
Soil Type = Clay

| Day | Degree of Saturation | Method 1 | Method 2 | Method 3 | Method 4 | Temperature at 10 cm | 30 cm | 50 cm | 70 cm | 90 cm | 110 cm | 130 cm | 150 cm |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.00 | 1.01 | 1.03 | 1.03 | 0.98 | 0.99 | 0.98 | 1.04 | 1.09 | 1.11 | 1.13 | 1.13 | 1.12 |
| | 0.50 | 1.05 | 1.02 | 1.02 | 0.98 | 0.98 | 0.95 | 0.98 | 1.02 | 1.05 | 1.07 | 1.09 | 1.10 |
| | 1.00 | 1.05 | 1.02 | 1.02 | 0.98 | 0.98 | 0.95 | 0.98 | 1.02 | 1.05 | 1.08 | 1.09 | 1.11 |
| 100 | 0.00 | 1.06 | 0.93 | 0.90 | 0.98 | 0.77 | 0.77 | 0.82 | 0.87 | 0.92 | 0.97 | 1.02 | 1.06 |
| | 0.50 | 1.03 | 0.92 | 0.91 | 1.00 | 0.83 | 0.81 | 0.83 | 0.86 | 0.88 | 0.91 | 0.95 | 0.98 |
| | 1.00 | 1.03 | 0.92 | 0.91 | 1.00 | 0.83 | 0.81 | 0.83 | 0.86 | 0.89 | 0.92 | 0.95 | 0.98 |
| 190 | 0.00 | 0.99 | 1.00 | 1.01 | 1.04 | 1.22 | 1.03 | 0.98 | 0.94 | 0.92 | 0.91 | 0.90 | 0.91 |
| | 0.50 | 0.95 | 1.00 | 1.03 | 1.04 | 1.30 | 1.09 | 1.03 | 0.99 | 0.96 | 0.94 | 0.92 | 0.91 |
| | 1.00 | 0.95 | 1.00 | 1.03 | 1.10 | 1.29 | 1.09 | 1.03 | 0.99 | 0.96 | 0.94 | 0.92 | 0.91 |
| 270 | 0.00 | 0.94 | 1.15 | 1.21 | 1.07 | 1.98 | 1.55 | 1.38 | 1.23 | 1.12 | 1.03 | 0.96 | 0.91 |
| | 0.50 | 0.97 | 1.20 | 1.26 | 1.06 | 1.98 | 1.55 | 1.41 | 1.31 | 1.22 | 1.14 | 1.07 | 1.01 |
| | 1.00 | 0.97 | 1.19 | 1.26 | 1.06 | 1.98 | 1.55 | 1.41 | 1.31 | 1.22 | 1.14 | 1.07 | 1.01 |

EM-38 Horizontal
Soil Type = Clay

| Day | Degree of Saturation | Method 1 | Method 2 | Method 3 | Method 4 | Temperature at 10 cm | 30 cm | 50 cm | 70 cm | 90 cm | 110 cm | 130 cm | 150 cm |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.00 | 0.99 | 0.96 | 1.01 | 0.96 | 0.98 | 0.97 | 1.03 | 1.07 | 1.10 | 1.11 | 1.11 | 1.10 |
| | 0.50 | 0.97 | 0.95 | 1.03 | 0.99 | 0.98 | 0.96 | 0.99 | 1.03 | 1.06 | 1.08 | 1.10 | 1.11 |
| | 1.00 | 0.97 | 0.95 | 1.03 | 0.99 | 0.98 | 0.96 | 0.99 | 1.03 | 1.06 | 1.08 | 1.10 | 1.11 |
| 100 | 0.00 | 0.92 | 0.90 | 1.05 | 1.14 | 0.89 | 0.90 | 0.95 | 1.01 | 1.06 | 1.12 | 1.18 | 1.24 |
| | 0.50 | 0.93 | 0.93 | 1.03 | 1.14 | 0.94 | 0.92 | 0.94 | 0.97 | 1.00 | 1.04 | 1.07 | 1.11 |
| | 1.00 | 0.93 | 0.92 | 1.03 | 1.14 | 0.94 | 0.92 | 0.94 | 0.97 | 1.00 | 1.04 | 1.08 | 1.11 |
| 190 | 0.00 | 1.03 | 1.09 | 0.98 | 1.01 | 1.18 | 1.01 | 0.95 | 0.91 | 0.89 | 0.88 | 0.88 | 0.88 |
| | 0.50 | 1.04 | 1.10 | 0.97 | 0.98 | 1.22 | 1.02 | 0.96 | 0.93 | 0.90 | 0.88 | 0.87 | 0.86 |
| | 1.00 | 1.04 | 1.10 | 0.97 | 0.98 | 1.22 | 1.02 | 0.96 | 0.93 | 0.90 | 0.88 | 0.87 | 0.86 |
| 270 | 0.00 | 1.23 | 1.37 | 0.94 | 0.84 | 1.54 | 1.21 | 1.07 | 0.96 | 0.87 | 0.80 | 0.75 | 0.71 |
| | 0.50 | 1.21 | 1.33 | 0.96 | 0.81 | 1.51 | 1.18 | 1.07 | 1.00 | 0.93 | 0.87 | 0.82 | 0.78 |
| | 1.00 | 1.21 | 1.33 | 0.96 | 0.81 | 1.52 | 1.18 | 1.08 | 1.00 | 0.93 | 0.87 | 0.82 | 0.77 |

## Table 3: Ratio of - (correction factor)/(real correction factor)

EM-31 Vertical
Soil Type = Peat

| | Degree of | Method | | | | | | | Temperature at | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Day | Saturation | 1 | 2 | 3 | 4 | 10 cm | 30 cm | 50 cm | 70 cm | 90 cm | 110 cm | 130 cm | 150 cm |
| 1 | 0.00 | 1.01 | 1.02 | 1.09 | 1.05 | 1.06 | 1.20 | 1.24 | 1.21 | 1.16 | 1.10 | 1.05 | 1.02 |
| | 0.50 | 1.00 | 1.03 | 1.08 | 1.03 | 1.04 | 1.11 | 1.17 | 1.20 | 1.20 | 1.18 | 1.14 | 1.11 |
| | 1.00 | 1.00 | 1.03 | 1.08 | 1.03 | 1.04 | 1.09 | 1.16 | 1.19 | 1.20 | 1.18 | 1.15 | 1.12 |
| 100 | 0.00 | 0.97 | 0.99 | 0.82 | 0.86 | 0.63 | 0.71 | 0.80 | 0.89 | 0.97 | 1.02 | 1.04 | 1.05 |
| | 0.50 | 0.97 | 0.98 | 0.80 | 0.85 | 0.64 | 0.68 | 0.74 | 0.80 | 0.86 | 0.92 | 0.97 | 1.01 |
| | 1.00 | 0.97 | 0.98 | 0.80 | 0.85 | 0.65 | 0.68 | 0.73 | 0.79 | 0.85 | 0.90 | 0.96 | 1.00 |
| 190 | 0.00 | 1.00 | 0.99 | 0.95 | 0.97 | 1.15 | 0.98 | 0.92 | 0.89 | 0.87 | 0.86 | 0.85 | 0.86 |
| | 0.50 | 1.01 | 0.98 | 0.97 | 1.00 | 1.10 | 0.96 | 0.90 | 0.88 | 0.87 | 0.88 | 0.89 | 0.91 |
| | 1.00 | 1.01 | 0.98 | 0.97 | 1.06 | 1.12 | 0.97 | 0.91 | 0.89 | 0.88 | 0.88 | 0.89 | 0.91 |
| 270 | 0.00 | 1.04 | 1.01 | 1.23 | 1.17 | 1.98 | 1.52 | 1.24 | 1.09 | 1.01 | 0.97 | 0.95 | 0.95 |
| | 0.50 | 1.04 | 1.02 | 1.32 | 1.22 | 2.15 | 1.71 | 1.45 | 1.27 | 1.15 | 1.07 | 1.01 | 0.97 |
| | 1.00 | 1.04 | 1.02 | 1.33 | 1.22 | 2.17 | 1.73 | 1.48 | 1.30 | 1.17 | 1.08 | 1.02 | 0.98 |

EM-38 Vertical
Soil Type = Peat

| | Degree of | Method | | | | | | | Temperature at | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Day | Saturation | 1 | 2 | 3 | 4 | 10 cm | 30 cm | 50 cm | 70 cm | 90 cm | 110 cm | 130 cm | 150 cm |
| 1 | 0.00 | 1.03 | 1.06 | 0.99 | 0.96 | 0.97 | 1.09 | 1.13 | 1.11 | 1.06 | 1.01 | 0.96 | 0.93 |
| | 0.50 | 1.04 | 1.05 | 0.99 | 0.95 | 0.95 | 1.02 | 1.08 | 1.10 | 1.10 | 1.08 | 1.05 | 1.02 |
| | 1.00 | 1.04 | 1.05 | 0.99 | 0.95 | 0.95 | 1.01 | 1.07 | 1.10 | 1.10 | 1.09 | 1.06 | 1.03 |
| 100 | 0.00 | 0.95 | 0.93 | 0.92 | 0.97 | 0.70 | 0.80 | 0.90 | 1.00 | 1.08 | 1.14 | 1.17 | 1.18 |
| | 0.50 | 0.93 | 0.90 | 0.94 | 1.00 | 0.76 | 0.80 | 0.87 | 0.94 | 1.01 | 1.08 | 1.14 | 1.19 |
| | 1.00 | 0.93 | 0.90 | 0.94 | 1.00 | 0.76 | 0.80 | 0.86 | 0.93 | 1.00 | 1.07 | 1.13 | 1.18 |
| 190 | 0.00 | 0.99 | 0.96 | 1.02 | 1.04 | 1.23 | 1.05 | 0.99 | 0.95 | 0.93 | 0.92 | 0.92 | 0.92 |
| | 0.50 | 0.99 | 0.97 | 1.02 | 1.06 | 1.16 | 1.02 | 0.96 | 0.93 | 0.92 | 0.93 | 0.95 | 0.97 |
| | 1.00 | 0.99 | 0.98 | 1.03 | 1.06 | 1.18 | 1.02 | 0.96 | 0.93 | 0.93 | 0.93 | 0.94 | 0.96 |
| 270 | 0.00 | 1.09 | 1.10 | 1.12 | 1.06 | 1.80 | 1.37 | 1.12 | 0.99 | 0.91 | 0.87 | 0.86 | 0.86 |
| | 0.50 | 1.13 | 1.16 | 1.12 | 1.03 | 1.83 | 1.45 | 1.23 | 1.08 | 0.98 | 0.91 | 0.86 | 0.83 |
| | 1.00 | 1.14 | 1.17 | 1.12 | 1.03 | 1.83 | 1.46 | 1.24 | 1.09 | 0.99 | 0.91 | 0.86 | 0.83 |

EM-31 Horizontal
Soil Type = Peat

| | Degree of | Method | | | | | | | Temperature at | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Day | Saturation | 1 | 2 | 3 | 4 | 10 cm | 30 cm | 50 cm | 70 cm | 90 cm | 110 cm | 130 cm | 150 cm |
| 1 | 0.00 | 0.97 | 1.02 | 1.03 | 1.00 | 1.03 | 1.02 | 1.09 | 1.13 | 1.16 | 1.17 | 1.17 | 1.16 |
| | 0.50 | 0.98 | 1.03 | 1.03 | 0.98 | 0.99 | 1.05 | 1.12 | 1.14 | 1.14 | 1.12 | 1.09 | 1.06 |
| | 1.00 | 0.99 | 1.03 | 1.03 | 0.98 | 0.99 | 1.04 | 1.11 | 1.14 | 1.14 | 1.13 | 1.10 | 1.07 |
| 100 | 0.00 | 1.04 | 0.94 | 0.91 | 0.95 | 0.69 | 0.78 | 0.88 | 0.98 | 1.06 | 1.12 | 1.15 | 1.16 |
| | 0.50 | 1.06 | 0.94 | 0.90 | 0.96 | 0.73 | 0.77 | 0.83 | 0.90 | 0.97 | 1.04 | 1.09 | 1.14 |
| | 1.00 | 1.06 | 0.93 | 0.90 | 0.96 | 0.73 | 0.77 | 0.83 | 0.89 | 0.96 | 1.02 | 1.08 | 1.13 |
| 190 | 0.00 | 1.03 | 1.00 | 0.99 | 1.01 | 1.05 | 0.93 | 0.89 | 0.89 | 0.92 | 0.95 | 0.99 | 1.02 |
| | 0.50 | 1.01 | 1.00 | 1.00 | 1.03 | 1.13 | 0.99 | 0.93 | 0.91 | 0.90 | 0.91 | 0.92 | 0.94 |
| | 1.00 | 1.01 | 1.00 | 1.00 | 1.09 | 1.15 | 1.00 | 0.94 | 0.91 | 0.90 | 0.90 | 0.92 | 0.93 |
| 270 | 0.00 | 0.98 | 1.10 | 1.13 | 1.08 | 1.82 | 1.39 | 1.14 | 1.00 | 0.93 | 0.89 | 0.87 | 0.87 |
| | 0.50 | 0.95 | 1.12 | 1.17 | 1.08 | 1.91 | 1.52 | 1.29 | 1.13 | 1.02 | 0.95 | 0.90 | 0.86 |
| | 1.00 | 0.95 | 1.12 | 1.17 | 1.08 | 1.92 | 1.53 | 1.30 | 1.15 | 1.03 | 0.96 | 0.90 | 0.87 |

EM-38 Horizontal
Soil Type = Peat

| | Degree of | Method | | | | | | | Temperature at | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Day | Saturation | 1 | 2 | 3 | 4 | 10 cm | 30 cm | 50 cm | 70 cm | 90 cm | 110 cm | 130 cm | 150 cm |
| 1 | 0.00 | 1.03 | 0.99 | 0.98 | 0.94 | 0.97 | 0.97 | 1.03 | 1.07 | 1.10 | 1.11 | 1.11 | 1.10 |
| | 0.50 | 1.01 | 0.97 | 0.99 | 0.95 | 0.96 | 1.02 | 1.08 | 1.11 | 1.10 | 1.08 | 1.05 | 1.02 |
| | 1.00 | 1.01 | 0.97 | 1.00 | 0.95 | 0.96 | 1.01 | 1.07 | 1.10 | 1.11 | 1.09 | 1.06 | 1.03 |
| 100 | 0.00 | 0.93 | 0.87 | 1.05 | 1.10 | 0.80 | 0.91 | 1.03 | 1.14 | 1.24 | 1.30 | 1.34 | 1.35 |
| | 0.50 | 0.92 | 0.88 | 1.06 | 1.12 | 0.85 | 0.90 | 0.97 | 1.06 | 1.14 | 1.21 | 1.28 | 1.34 |
| | 1.00 | 0.92 | 0.89 | 1.06 | 1.13 | 0.86 | 0.90 | 0.97 | 1.05 | 1.13 | 1.20 | 1.27 | 1.32 |
| 190 | 0.00 | 1.00 | 1.06 | 1.02 | 1.04 | 1.08 | 0.96 | 0.92 | 0.92 | 0.95 | 0.98 | 1.02 | 1.05 |
| | 0.50 | 1.01 | 1.08 | 1.00 | 1.03 | 1.13 | 0.99 | 0.93 | 0.91 | 0.90 | 0.91 | 0.92 | 0.94 |
| | 1.00 | 1.01 | 1.08 | 1.00 | 1.03 | 1.14 | 0.99 | 0.94 | 0.91 | 0.90 | 0.90 | 0.91 | 0.93 |
| 270 | 0.00 | 1.17 | 1.38 | 0.95 | 0.90 | 1.53 | 1.17 | 0.96 | 0.84 | 0.78 | 0.75 | 0.73 | 0.73 |
| | 0.50 | 1.21 | 1.38 | 0.94 | 0.87 | 1.53 | 1.22 | 1.03 | 0.91 | 0.82 | 0.76 | 0.72 | 0.69 |
| | 1.00 | 1.22 | 1.38 | 0.94 | 0.86 | 1.53 | 1.22 | 1.04 | 0.92 | 0.83 | 0.76 | 0.72 | 0.69 |

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# Tikhonov Regularization for Determination of Depth Profiles of Electrical Conductivity using Non-Invasive Electromagnetic Induction Measurements

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## Abstract

**Abstract:** Electromagnetic induction is an established non-invasive technique for measuring the apparent electrical conductivity of subsurface soils and contaminants in soils and aquifers. This paper discusses the use of Tikhonov regularization for estimating the electrical conductivity profile of the vadose zone from above ground electromagnetic induction measurements. It was found that this technique is superior to other inverse techniques previously reported in the literature.

## Introduction

The high costs and intrinsic features of invasive sampling techniques such as drilling and cone penetrometer technologies limit their use to a finite number of sampling locations and do not allow complete coverage of the area under consideration. Geostatistical techniques provide ways to interpolate between distinct sampling points and, thus, yield some kind of complete coverage, but the approximate fifty or more sampling locations required for reliable geostatistical interpolations is so high that the expense often prohibits their application outside the research theater. Thus, there is a great need for less expensive sampling techniques. Because the major cost component of invasive techniques is due to the insertion of sampling tools into the vadose zone and underlying aquifers, non-invasive techniques hold the most promise for cost-effective sampling.

Although non-invasive geophysical techniques have been successfully used for many decades by geophysicists studying the earth's crust and hydrologists exploring aquifers, their shallower application in vadose zone hydrology is relatively recent. This is due to improvements in electronics that allow geophysical instruments to detect signals at the short time intervals needed for exploration of the shallow vadose zone (0-6 m). A popular technique for the exploration of the vadose zone and shallow aquifers is the frequency-domain electromagnetic (FEM) induction technique (McNeill 1980) which employs a ground conductivity meter to measure the apparent electrical conductivity of the subsurface. A recent literature search by Hendrickx et al. (1995) revealed that since 1979 the

FEM method has been successfully used for the detection of a large number of different subsurface materials including contaminant plumes migrating from landfills and mines, soil water content (Sheets and Hendrickx, 1995), saline soils (Hendrickx et al., 1992; Sheets et al., 1994), and a variety of industrial waste materials: metal hydroxide, radioactive waste, caustic soda, organics, oils, sulfides, waxes and hydrocarbons resulting from gasoline spills. The majority of the reported surveys deal with the determination of the lateral extent of the subsurface materials, but do not address their distribution with depth. In principle, FEM measurements obtained at different heights above the soil surface can be used to predict the electrical conductivity at different depths. Unfortunately, the prediction is not straightforward since we are faced with an ill-posed inverse problem as explained in the next section.

Previous work on this inverse problem has focused on attempts to find linear combinations of measurements that would predict conductivity over a specified range of depths. Corwin and Rhoades (Rhoades and Corwin, 1981; Corwin and Rhoades, 1984) and Slavich (1990) use multiple linear regression to relate EM measurements to known conductivity profiles. The result is a set of coefficients for each range of depths that can be used to predict the conductivity of the soil within the range. Unfortunately, these coefficients are site specific.

Another approach is based on the fact that the response of ground conductivity meters at salinities below approximately 130 mS/m is roughly a linear function of the apparent soil electrical conductivity at different depths (McNeill

1980; Wait 1982). Corwin and Rhoades (1982) used the linear model to select linear combinations of measurements that can be used to estimate the conductivity within a region of interest. The coefficients are selected so as to maximize the response of the instrument to conductivity in the region of interest. Cook and Walker (1992) improved on this approach by using optimization techniques to select optimal coefficients. However, care must be taken to minimize the size of the coefficients, since large coefficients make the scheme very susceptible to small errors in measurement.

The objective of this study is to explore the performance of second order Tikhonov regularization, a method that has not previously been applied in hydrogeology or soil science to the problem of inverting above-ground finite-domain electromagnetic induction measurements.

## Theory

In this section, we summarize the linear model of electromagnetic induction developed by Wait (1982). This model is based on a number of assumptions that may not hold true in all cases. For the Geonics EM-38 device used in this study, the linear model is appropriate when the conductivity of the soil is relatively low. However, the model breaks down and errors become significant as the average soil conductivity increases. For example, with the Geonics EM-38 in vertical mode, the measurement predicted by the linear model and the measurement predicted by a more accurate nonlinear model differ by as much as 5% for a model earth with a constant conductivity profile of 40 mS/meter (Schlue, 1992).

In the following, all distances are normalized with respect to the distance between the two coils. For example, a depth of 2.0 indicates a depth of twice the intercoil spacing. Conductivity is measured in mS/meter. We denote the conductivity at depth $z$ by $\sigma(z)$. In general, the superscript $V$ will be used for measurements taken with the coils in the vertical orientation, and the superscript $H$ will be used for measurements taken in the horizontal mode. We denote the measurement made at a height $h$ above the ground by $m^H(h)$ or $m^V(h)$.

The sensitivity of the instrument to conductivity at depth $z$ is given by the functions $\phi^V$ and $\phi^H$, where

$$\phi^V(z) = \frac{4z}{(4z^2+1)^{3/2}} \tag{1}$$

and

$$\phi^H(z) = 2 - \frac{4z}{(4z^2+1)^{1/2}}. \tag{2}$$

These sensitivity functions are shown in Figure 1. With the coils in the horizontal mode, the instrument is most sensitive to the conductivity of soil near the surface, while with the coils in the vertical mode, the instrument is most sensitive to the conductivity of the soil at a depth of about 0.4 times the distance between the coils. If we hold the instrument above the ground at a

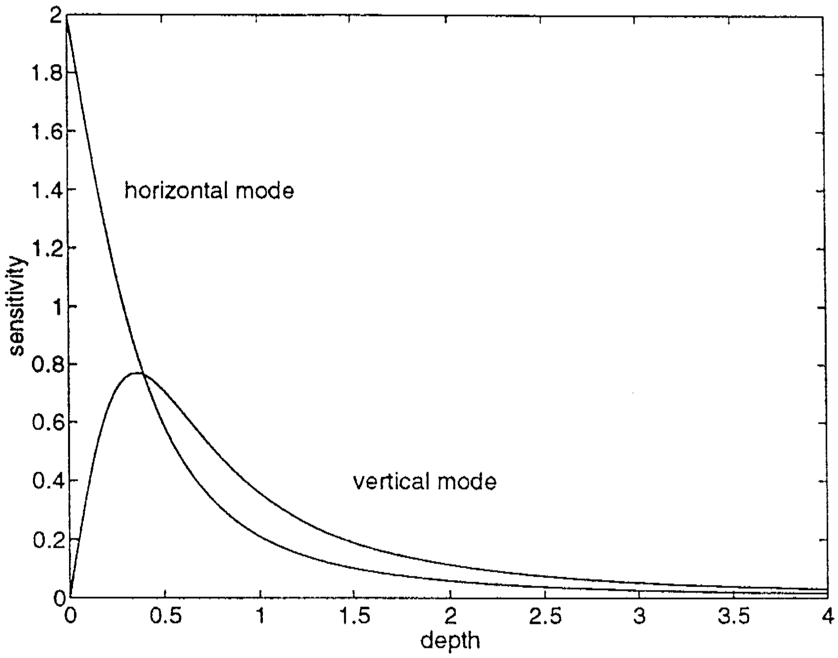


Figure 1: The Sensitivity Functions $\phi^V$ and $\phi^H$.

height $h$, the sensitivity functions become $\phi^V(z+h)$ and $\phi^H(z+h)$.

The linear model consists of a system of two Fredholm integral equations of the first kind.

$$m^H(h) = \int_0^\infty \phi^H(z+h)\sigma(z)dz \tag{3}$$

$$m^V(h) = \int_0^\infty \phi^V(z+h)\sigma(z)dz \tag{4}$$

We are interested in solving for the unknown $\sigma(z)$, given the measurements $m^V$ and $m^H$.

There are a number of survey papers and books that discuss Fredholm integral equations of the first kind and the associated inverse problems (Bertero et al., 1985; Bertero et al., 1988; Wing, 1991; Delves and Mohamed, 1985; Tikhonov and Arsenin, 1977; Hansen, 1992b; Varah, 1979). In this paper, we will discuss second order Tikhonov regularization, a method that has not previously been applied to the problem of inverting electromagnetic induction measurements.

It is possible to show that if a solution to the inverse problem exists, then the solution is unique. However, the inverse problem is ill-posed in the sense that an arbitrarily small error in measurement, $e_n^\sigma(z)$, can lead to a large error in $\sigma(z)$. To see this, let

$$e_n^\sigma(z) = \sin nz \tag{5}$$

For large $n$, this is a very rapidly oscillating function. Consider

$$e_n^H(h) = \int_0^\infty \phi^H(z+h)e_n^\sigma(z)dz \tag{6}$$

and

$$e_n^V(h) = \int_0^\infty \phi^V(z+h)e_n^\sigma(z)dz. \tag{7}$$

As $n$ goes to infinity, $||e_n^H||$ and $||e_n^V||$ go to zero because of the oscillatory nature of the integrand. Since the integral equations are linear,

$$m^H(h) + e_n^H(h) = \int_0^\infty \phi^H(z+h)(\sigma(z) + e_n^\sigma(z))dz \tag{8}$$

and

$$m^V(h) + e_n^V(h) = \int_0^\infty \phi^V(z+h)(\sigma(z) + e_n^\sigma(z))dz. \tag{9}$$

In the limit as $n$ goes to infinity, the error terms on the left hand side become negligible, while the corresponding error in $\sigma(z)$ remains large. Thus small errors in the measurements can result in relatively large errors in the solution $\sigma(z)$. This property is typical of integral equations of the first kind, and it makes the inverse problem difficult to solve in practice.

If we assume that $\sigma(z)$ is constant within specified layers, and we only take measurements at a finite set of heights, we can discretize the integral equations (3) and (4). Let $l_j$ and $u_j$ be the lower and upper limits of the $j$th layer. Let $\sigma_j$ be the conductivity of the $j$th layer. Let $h_i$ be the height at which the $i$th measurement is taken and let $m_i^H$ and $m_i^V$ be the measurements at height $h_i$. Then the system of two integral equations becomes a system of linear algebraic equations

$$m^H = H\sigma \tag{10}$$

and

$$m^V = V\sigma. \tag{11}$$

The matrices $H$ and $V$ are defined by

$$H_{i,j} = \int_{u_j}^{l_j} \phi^H(z+h_i)dz \tag{12}$$

and

$$V_{i,j} = \int_{u_j}^{l_j} \phi^V(z + h_i)dz. \tag{13}$$

For convenience, we will rewrite this system of equations as

$$K\sigma = d \tag{14}$$

where

$$K = \begin{bmatrix} V \\ H \end{bmatrix} \tag{15}$$

and

$$d = \begin{bmatrix} m^V \\ m^H \end{bmatrix}. \tag{16}$$

The discrete version of the linear inverse problem is to find a vector $\sigma$ that satisfies or comes close to satisfying $K\sigma = d$. A method for the solution of this discrete linear inverse problem is discussed in the next section.

## Tikhonov Regularization

One obvious approach to the discrete version of the inverse problem would be to find $\sigma$ that minimizes $||K\sigma - d||$. This is a linear least squares problem that can easily be solved with the help of the singular value decomposition (Lawson and Hanson, 1974). Unfortunately, because the matrix $K$ is ill-conditioned, the solution to the least squares problem is very sensitive to small errors in the data $d$.

Regularization is one way to get around this sensitivity to errors in $d$. In Tikhonov regularization (Tikhonov and Arsenin, 1977) we minimize the sensitivity to errors in the data by finding a solution to the minimization problem

$$\min ||K\sigma - d||^2 + \lambda^2 ||L\sigma||^2. \tag{17}$$

In the extreme case, $\lambda = 0$, we are simply solving the least squares problem. For larger values of $\lambda$, we find a solution that balances the property measured by $L$ with the norm of the residual. If $L$ and $\lambda$ are chosen carefully, this problem will not be overly sensitive to errors in the data.

In Tikhonov regularization, $||L\sigma||$ is used to measure some desirable property of the solution while the factor $\lambda$ controls the trade-off between minimizing the norm of the residual and minimizing $||L\sigma||$. A common choice of $L$ is the identity matrix $L = I$. In this case, we are minimizing a weighted combination of the norm of the residual and the norm of $\sigma$. This helps to prevent solutions that involve very large conductivities. Another common choice is to make $L$ the discrete approximation of a derivative operator, such as

$$L = \begin{bmatrix} -1 & 2 & 1 & & & \\ & -1 & 2 & 1 & & \\ & & & & & \\ & & -1 & 2 & 1 & \\ & & & -1 & 2 & 1 \end{bmatrix} \tag{18}$$

In this case, we are minimizing a weighted norm of the residual and an approximation to the norm of the second derivative of $\sigma$. We could also use a second or third derivative operator $L$. Tikhonov regularization with $L = I$ is commonly referred to as zeroth order regression, while regularization with $L$ approximating the $k$th derivative operator is referred to as $k$th order regularization.

The minimization problem is a damped least squares problem that can easily be solved by means of the generalized singular value decomposition (Loan, 1976; Hansen, 1992b; Hansen, 1993). Since conductivity is always nonnegative, we may want to add the constraint $\sigma \geq 0$. In this case, we can no longer use the generalized singular value decomposition to solve the minimization problem. However, standard techniques for nonnegative least squares problems can be used.

A major issue in Tikhonov regularization is the choice of the parameter $\lambda$. If we plot $\log ||K\sigma - d||$ versus $\log ||L\sigma||$ for different values of $\lambda$, a characteristic L–curve appears. For large values of $\lambda$, no improvement in $||L\sigma||$ is possible, while for small values of $\lambda$, we have effectively solved the least squares problem, and no further improvement in $||K\sigma - d||$ is possible. See Figure 2.

We would like to pick a value of $\lambda$ that causes the solution to lie somewhere near the corner of this curve. Under the L–curve criterion (Hansen, 1992a), we simply pick the value of $\lambda$ which corresponds to the corner of the curve. This can easily be done manually by examining a plot of the L-curve, or an automated procedure can be used to find the point on the curve where the curvature is maximized. Another approach to this problem is the discrepancy principle (Morozov, 1984), in which we compute the expected value of the norm

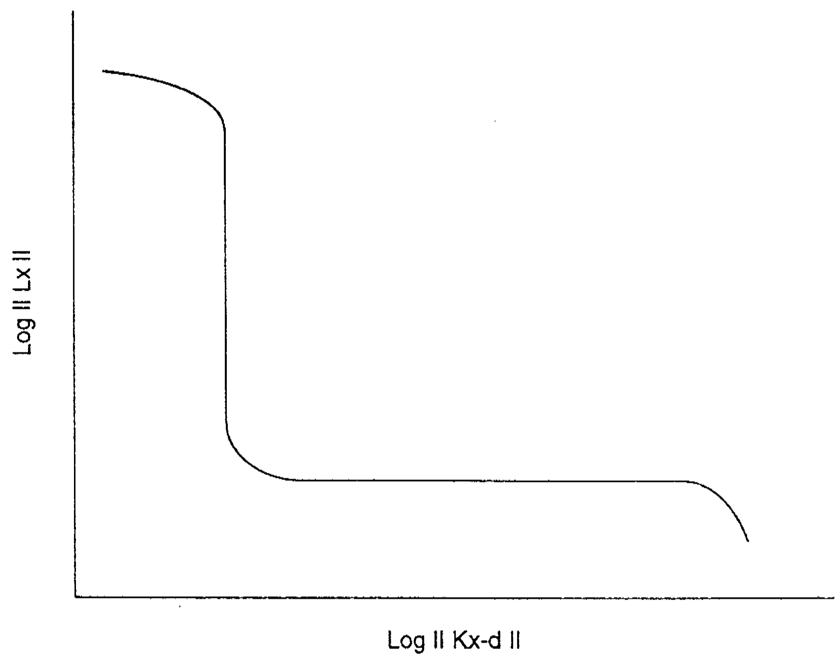


Figure 2: The L–curve.

of the error in the measured data and pick $\lambda$ so that the norm of the residual is less than this expected value.

We have implemented second order Tikhonov regularization on a 486 PC using Matlab. After a set of observations has been recorded, the software can be used to compute the $K$ and $L$ matrices, plot the L-curve, select a regularization parameter $\lambda$, and plot a solution. The entire process takes no more than five minutes.

## Methods and Materials

We tested the inversion technique described in the previous section by making measurements with a Geonics EM–38, estimating the conductivity profile, and comparing it with measurements obtained by direct measurements of the conductivity of soil samples.

The results presented in this section are based on two test pits that were dug during the summer of 1994 in a sand soil at the Bosque Del Apache wildlife refuge approximately 20 km south of Socorro, NM. At each site, we took conductivity readings with the EM–38 in vertical and horizontal modes with the instrument at heights of 0, 10, 20, ..., 100, and 120 centimeters. We then dug out a pit, taking measurements of the apparent electrical conductivity every 10 centimeters with a TDR instrument following the method of Nadler et al. (1991). In the first pit, we were able to measure the conductivity profile down to 90 centimeters, at which point we reached a hard caliche layer. In the second pit, we were able to measure the profile down to a depth of 130 centimeters.

To analyze these data, we constructed a layered model with 25 layers, from 0 to 10 cm, 10 to 20 cm, ..., 230 to 240 cm, and from 240 cm down. This model was selected to give as much information as possible about the upper 1.5 meters of the profile, while still ensuring that the model was not overly sensitive to conductivity in the bottom layer.

## Results and Discussion

Figure 3 shows the L-curve for the first test pit. There is a sharp corner at the point where $||K\sigma - d||$ is 3.4, and $||L\sigma||$ is 25. With $\lambda = 0.05$, we obtained a solution with $||K\sigma - d|| = 3.42$ and $||L\sigma|| = 22.76$. This solution is shown in figure 4, along with the measured conductivity profile.

For comparison, we used the Cook and Walker code to generate coefficients for use in estimating the conductivity in the layer from 0 to 0.5 meters below the surface. Using $\lambda = 10^{-3}$, we obtained a set of coefficients with $F = 91.2\%$ and $\sum a_i^2 = 8.82$. For the layer from 0.5 to 1.5 meters below the surface we obtained a set of coefficients with $F = 25.3\%$ and $\sum a_i^2 = 5.86$ using $\lambda = 10^{-3}$. The Cook and Walker estimates are also shown in figure 4. Clearly, the method of Cook and Walker has underestimated the actual conductivity.

Figure 5 shows the L-curve for the second test pit. Again, there is a sharp corner. With $\lambda = 0.3$, we obtained a solution with $||K\sigma - d|| = 3.06$ and $||L\sigma|| = 7.36$. This solution is shown in figure 6, along with the measured conductivity profile and the Cook and Walker estimates.

## Summary and Conclusions

In this paper we have described a linear model of electromagnetic induction that can be used to relate electromagnetic induction measurements to the soil conductivity profile. We have described an approach, second order Tikhonov regularization, to inverting electromagnetic induction measurements to obtain an estimate of the soil conductivity profile. This approach has been applied to measurements using the Geonics EM-38 at various heights above the ground. The estimates provided by this technique were superior to estimates by the earlier approach of Cook and Walker.

However, there are several issues that need to be considered in future research. First, the linear model used in this paper is only appropriate for soils of relatively low conductivity. An area in which further research is needed is the development of a more accurate nonlinear model that is appropriate for soils of relatively high conductivity. Second, our method has only been tested with EM-38 measurements and only to depths of about one meter. Tests should also be conducted with other electromagnetic induction instruments such as the EM-31 and EM-34 and at greater depths. Finally, second order Tikhonov regularization implicitly searches for "smooth" solutions. This is typical for many field solute breakthrough curves. However, it is not clear that this method would be appropriate for conductivity profiles with large discontinuities.

## Acknowledgements

We would like to thank Peter Cook for providing his Fortran code to compute coefficients for the inversion of geomagnetic induction data.

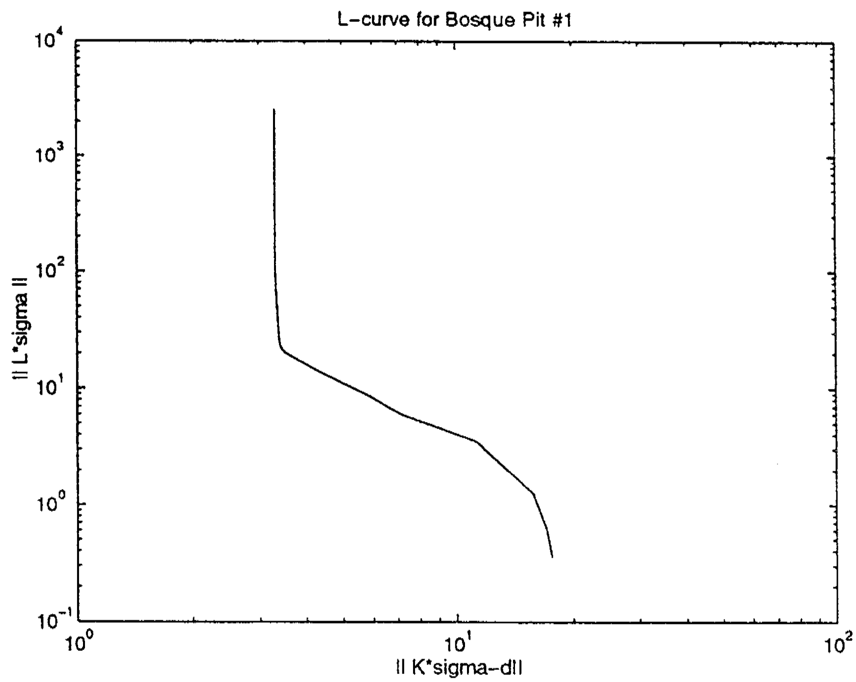


Figure 3: L-curve for Bosque Pit #1.

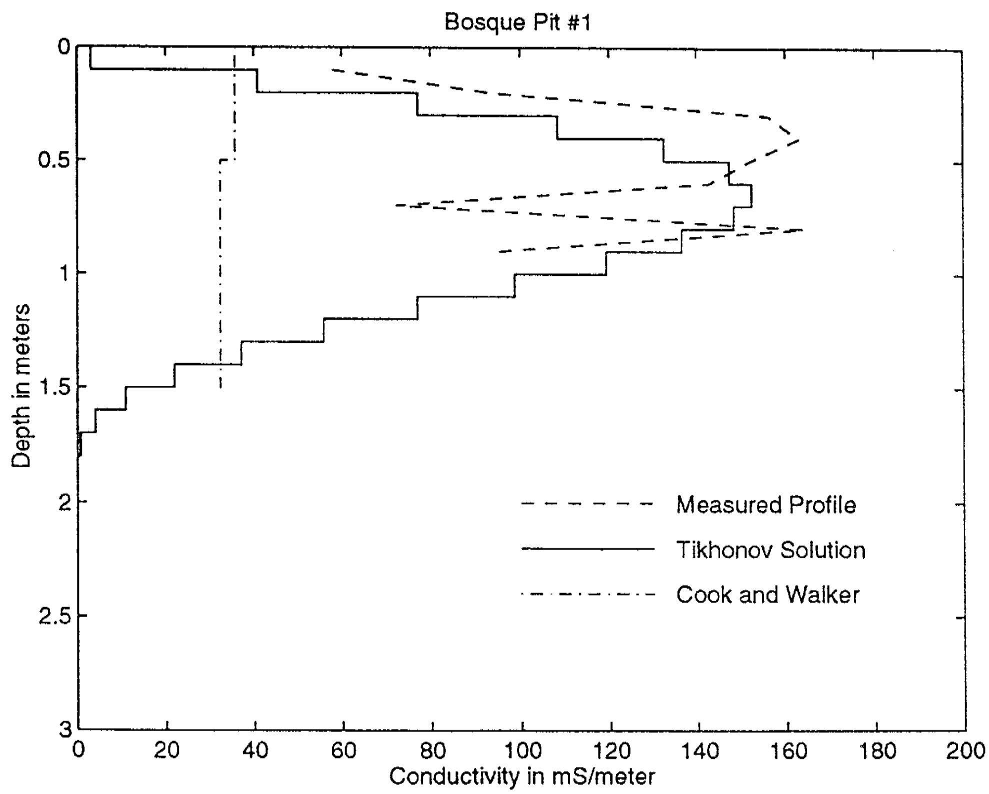


Figure 4: Conductivity Profile for Bosque Pit #1

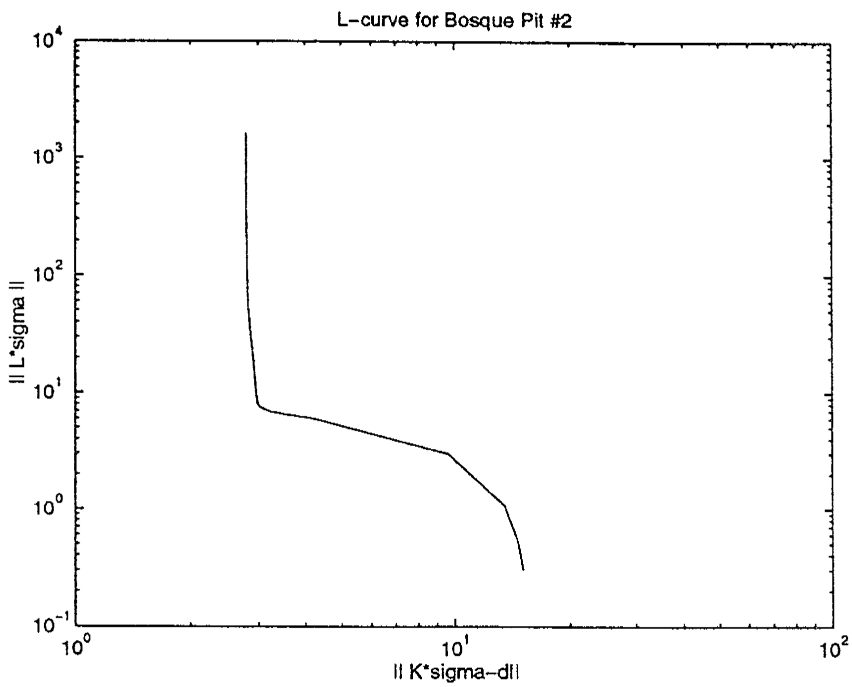


Figure 5: L-curve for Bosque Pit #2.

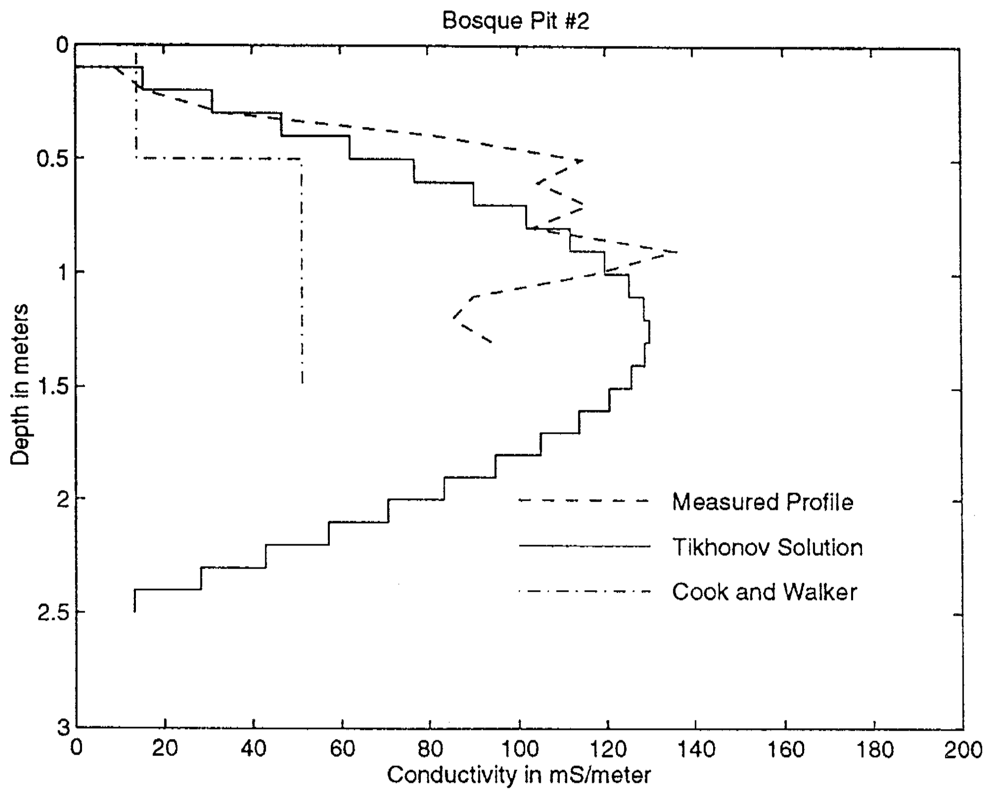


Figure 6: Conductivity profile for Bosque Pit #2

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# APPENDIX A

**RAW DATA**
**EM-31, EM-38 and TDR**
**SAVIETTA and BOSQUE DEL APACHE**

## Savietta Sand Dunes - Pit #1

Above Ground Conductivity Measurements -- EM-38

| Height (cm) | N/S Vertical | N/S Horizontal | E/W Vertical | E/W Horizontal | Average Vertical | Average Horizontal |
|---|---|---|---|---|---|---|
| 0 | 2.5 | 3.1 | 2.5 | 3.2 | 2.50 | 3.15 |
| 10 | 5.0 | 3.8 | 5.5 | 4.1 | 5.25 | 3.95 |
| 20 | 8.2 | 2.7 | 8.3 | 2.4 | 8.25 | 2.55 |
| 30 | 11.0 | 2.0 | 11.2 | 1.4 | 11.10 | 1.70 |
| 40 | 12.7 | 1.1 | 12.8 | 0.6 | 12.75 | 0.85 |
| 50 | 13.4 | 0.0 | 13.6 | 0.0 | 13.50 | 0.00 |
| 60 | 14.2 | -3.2 | 13.8 | -1.7 | 14.00 | -2.45 |
| 70 | 13.7 | -7.5 | 13.4 | -2.1 | 13.55 | -4.80 |
| 80 | 11.6 | | 12.2 | | 11.90 | |
| 90 | 11.2 | | 11.7 | | 11.45 | |
| 100 | 11.2 | | 11.5 | | 11.35 | |
| 120 | 10.9 | | 11.0 | | 10.95 | |

Below Ground Conductivity Measurements -- TDR

| Depth (cm) | 1 | 2 | 3 | 4 | Average | ECa (mS/m) |
|---|---|---|---|---|---|---|
| 10 | 595.0 | 592.5 | 594.4 | | 593.97 | 7.00 |
| 20 | 556.0 | 564.0 | 566.0 | | 562.00 | 7.40 |
| 30 | 492.0 | 496.0 | 497.0 | | 495.00 | 8.40 |
| 40 | 435.0 | 454.0 | 446.0 | | 445.00 | 9.35 |
| 50 | 439.5 | 403.0 | 432.0 | | 424.83 | 9.79 |
| 60 | 455.0 | 474.0 | 472.0 | | 467.00 | 8.91 |
| 70 | 465.0 | 497.0 | 459.0 | 468.0 | 472.25 | 8.81 |
| 80 | 494.0 | 460.0 | 466.0 | | 473.33 | 8.79 |
| 90 | 465.0 | 470.0 | 450.0 | 468.0 | 463.25 | 8.98 |
| 100 | 472.0 | 473.0 | 491.0 | 486.0 | 480.50 | 8.66 |

## Bosque Pit #1

Above Ground Conductivity Measurements -- EM-38

| Height (cm) | N/S Vertical | N/S Horizontal | E/W Vertical | E/W Horizontal | Average Vertical | Average Horizontal |
|---|---|---|---|---|---|---|
| 0 | 72.9 | 63.5 | 70.8 | 62.6 | 71.85 | 63.05 |
| 10 | 65.8 | 51.4 | 65.4 | 51.3 | 65.60 | 51.35 |
| 20 | 58.4 | 41.3 | 58.0 | 41.5 | 58.20 | 41.40 |
| 30 | 51.8 | 33.8 | 51.5 | 34.0 | 51.65 | 33.90 |
| 40 | 45.7 | 28.1 | 45.5 | 28.1 | 45.60 | 28.10 |
| 50 | 40.0 | 23.5 | 39.8 | 23.8 | 39.90 | 23.65 |
| 60 | 35.4 | 19.4 | 35.2 | 19.9 | 35.30 | 19.65 |
| 70 | 31.3 | 16.5 | 31.0 | 16.2 | 31.15 | 16.35 |
| 80 | 27.9 | 14.0 | 27.7 | 13.8 | 27.80 | 13.90 |
| 90 | 25.0 | 12.1 | 24.8 | 12.2 | 24.90 | 12.15 |
| 100 | 22.4 | 10.4 | 22.3 | 10.1 | 22.35 | 10.25 |
| 120 | 18.5 | 7.9 | 18.3 | 7.6 | 18.40 | 7.75 |

Below Ground Conductivity Measurements -- TDR

| Depth (cm) | 1 | 2 | 3 | 4 | Average | ECa (mS/m) |
|---|---|---|---|---|---|---|
| 10 | 82.5 | 61.7 | 80.8 | 61.8 | 71.70 | 58.02 |
| 20 | 44.4 | 36.5 | 44.3 | | 44.35 | 93.80 |
| 30 | 49.1 | 26.8 | 26.5 | | 26.65 | 156.10 |
| 40 | 25.7 | 27.6 | 27.1 | 21.6 | 25.50 | 163.14 |
| 50 | 20.5 | 28.4 | 33.3 | | 27.40 | 151.82 |
| 60 | 23.4 | 26.1 | 38.0 | | 29.17 | 142.63 |
| 70 | 23.1 | 45.9 | 66.4 | 60.7 | 57.67 | 72.14 |
| 80 | 25.3 | 23.9 | 26.7 | | 25.30 | 164.43 |
| 90 | 42.8 | 43.1 | 45.5 | | 43.80 | 94.98 |

## Bosque Pit #2

Above Ground Conductivity Measurements -- EM-38

| Height (cm) | N/S Vertical | N/S Horizontal | E/W Vertical | E/W Horizontal | Average Vertical | Average Horizontal |
|---|---|---|---|---|---|---|
| 0 | 67.1 | 48.7 | 66.0 | 43.3 | 66.55 | 46.00 |
| 10 | 59.4 | 41.4 | 58.9 | 36.9 | 59.15 | 39.15 |
| 20 | 52.5 | 34.9 | 52.3 | 31.2 | 52.40 | 33.05 |
| 30 | 47.0 | 29.9 | 46.9 | 26.5 | 46.95 | 28.20 |
| 40 | 42.0 | 25.5 | 42.1 | 22.8 | 42.05 | 24.15 |
| 50 | 37.6 | 22.2 | 37.7 | 19.7 | 37.65 | 20.95 |
| 60 | 33.9 | 19.2 | 34.1 | 17.2 | 34.00 | 18.20 |
| 70 | 30.4 | 16.8 | 30.9 | 15.1 | 30.65 | 15.95 |
| 80 | 27.7 | 15.0 | 28.1 | 13.1 | 27.90 | 14.05 |
| 90 | 25.2 | 13.0 | 25.5 | 11.3 | 25.35 | 12.15 |
| 100 | 23.1 | 11.7 | 23.6 | 10.0 | 23.35 | 10.85 |
| 120 | 19.6 | 9.2 | 20.1 | 7.5 | 19.85 | 8.35 |

Below Ground Conductivity Measurements -- TDR

| Depth (cm) | 1 | 2 | 3 | 4 | 5 | Average | ECa | ECa (mS/m) |
|---|---|---|---|---|---|---|---|---|
| 10 | 605.5 | 486.0 | 407.0 | 383.5 | 488.0 | 474.00 | 0.09 | 8.78 |
| 20 | 300.0 | 298.0 | 199.0 | 278.0 | 278.4 | 270.68 | 0.15 | 15.37 |
| 30 | 117.0 | 82.0 | 73.2 | 192.5 | 156.7 | 124.28 | 0.33 | 33.47 |
| 40 | 58.3 | 70.6 | 36.0 | 40.2 | 47.3 | 50.48 | 0.82 | 82.41 |
| 50 | 34.5 | 39.5 | 25.9 | 39.0 | 42.6 | 36.30 | 1.15 | 114.60 |
| 60 | 39.5 | 25.1 | 42.8 | 48.5 | 43.0 | 39.78 | 1.05 | 104.58 |
| 70 | 48.5 | 36.8 | 39.0 | 22.4 | 32.6 | 35.86 | 1.16 | 116.01 |
| 80 | 39.2 | 38.6 | 44.3 | 41.1 | 38.5 | 40.34 | 1.03 | 103.12 |
| 90 | 33.3 | 25.1 | 25.7 | 37.0 | 31.9 | 30.60 | 1.36 | 135.95 |
| 100 | 29.0 | 32.3 | 40.7 | 35.0 | 40.7 | 35.54 | 1.17 | 117.05 |
| 110 | 43.5 | 39.0 | 46.1 | 56.4 | 46.0 | 46.20 | 0.90 | 90.04 |
| 120 | 53.3 | 52.6 | 42.8 | 55.6 | 39.3 | 48.72 | 0.85 | 85.39 |
| 130 | 49.2 | 41.6 | 36.7 | 56.7 | 37.1 | 44.26 | 0.94 | 93.99 |

## Savietta - Pit #2

Above Ground Conductivity Measurements -- EM-38

| Height | N/S | | E/W | | Average | Average |
|---|---|---|---|---|---|---|
| (cm) | Vertical | Horizontal | Vertical | Horizontal | Vertical | Horizontal |
| 0 | 10.9 | 7.7 | 10.5 | 7.8 | 10.70 | 7.75 |
| 10 | 11.7 | 6.8 | 12.3 | 7.4 | 12.00 | 7.10 |
| 20 | 13.0 | 5.5 | 13.2 | 5.9 | 13.10 | 5.70 |
| 30 | 14.0 | 4.2 | 13.8 | 4.6 | 13.90 | 4.40 |
| 40 | 14.3 | 3.4 | 14.0 | 4.1 | 14.15 | 3.75 |
| 50 | 14.0 | 3.0 | 13.8 | 3.2 | 13.90 | 3.10 |
| 60 | 13.5 | 2.4 | 13.4 | 2.4 | 13.45 | 2.40 |
| 70 | 12.9 | 1.5 | 12.8 | 2.2 | 12.85 | 1.85 |
| 80 | 12.2 | 1.2 | 12.3 | 1.6 | 12.25 | 1.40 |
| 90 | 11.7 | 0.9 | 11.7 | 1.0 | 11.70 | 0.95 |
| 100 | 11.2 | 0.4 | 11.2 | 0.5 | 11.20 | 0.45 |
| 120 | 10.4 | 0.0 | 10.3 | 0.0 | 10.35 | 0.00 |

Below Ground Conductivity Measurements -- TDR

| Depth (cm) | 1 | 2 | 3 | 4 | Average | ECa (mS/m) |
|---|---|---|---|---|---|---|
| 10 | 571.0 | 593.0 | 579.0 | | 581.00 | 7.16 |
| 20 | 431.0 | 455.0 | 418.0 | | 434.67 | 9.57 |
| 30 | 405.0 | 505.0 | 329.0 | 343.0 | 395.50 | 10.52 |
| 40 | 481.0 | 409.0 | | | 445.00 | 9.35 |
| 50 | 324.0 | 340.0 | 312.0 | | 325.33 | 12.79 |
| 60 | 357.0 | 290.0 | 335.0 | 305.0 | 321.75 | 12.93 |
| 70 | 295.0 | 324.0 | | | 309.50 | 13.44 |