Study of Aquifer Tests in Unconfined Aquifer at the Sevilleta Study Site

by

Jingfang He

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Department of Geoscience

New Mexico Institute of Mining and Technology

Socorro, New Mexico

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ABSTRACT

Several pumping test were conducted to estimate the planar anisotropy and other pertinent hydrological parameters of an unconfined alluvial aquifer. Drawdown data were measured at six different wells. Two curve matching methods based on different models were used to analyze the pumping test data. The first model assumes one-dimensional radial flow and includes an empirical delay coefficient to represent the water release process, and to account for the delayed-drawdown region of the time-drawdown graph. This model does not account for the vertical components of flow in the aquifer, thus it can not be used directly to obtain the vertical hydraulic conductivity of the aquifer. The second model assumes two-dimensional radial vertical flow and instantaneous and complete drainage at the water table to account for the delayed-drawdown region of the time-drawdown graph. Sensitivity analysis was performed to better understand the effects on drawdown from interested parameters.

FORTRAN computer programs were written to calculate the theoretical drawdown of both models by numerical inverting the Laplace domain solution. The theoretica drawdown produced by the estimated parameters was compared with measured drawdow data. The comparison of theoretical drawdown and measured drawdown was facilitate by using Matlab software to plot data. The values of transmissivity and elastic storag coefficient obtained from different curve-matching methods were similar, however th value of specific yield were different as it has larger effect on the time-drawdown curve

Using the values of transmissivity from different wells, a system of three linearized simultaneous equations was set up and solved to determine components of the planar anisotropy tensor. The principal value and direction of the planar transmissivity was finally obtained by using Matlab software and the results can be used as input parameters for further modelling study at Sevilleta study site.

INTRODUCTION

Problem and Purpose of Study

The purpose of this study was to estimate the planar anisotropy and other pertinent hydrological parameters in an unconfined alluvial aquifer. Several pumping test were conducted at study site in fall of 1992 and spring of 1993. Drawdown data were measured at six different wells. Two curve matching methods based on different models, Boulton (1954, 1963) and Neuman model (1972, 1973, 1974), were used to analyze the pumping test data. The other pertinent hydrological parameter such as storage coefficient and specific yield were estimated and used to calculate the theoretical drawdown. The theoretical drawdown produced by the estimated parameters was compared with the measured drawdown data to check the accuracy of estimated parameters.

Site Development

The study site is located in the Sevilleta National Wildlife Refuge approximately 25 kilometers north of Socorro, New Mexico as shown in Figure 1. The site lies on the flood plain of the Rio Salado, an ephemeral braided tributary of the Rio Grande, which is dry channel on the average of 320 days per year. The area receives about 20 cm of precipitation per year and the gross annual potential lake evaporation is about 178 cm. The aquifer where the pumping and observation wells are installed consists of Holocene Rio Salado alluvium overlying Pleistocene axial stream deposits of the Sierra Ladrones

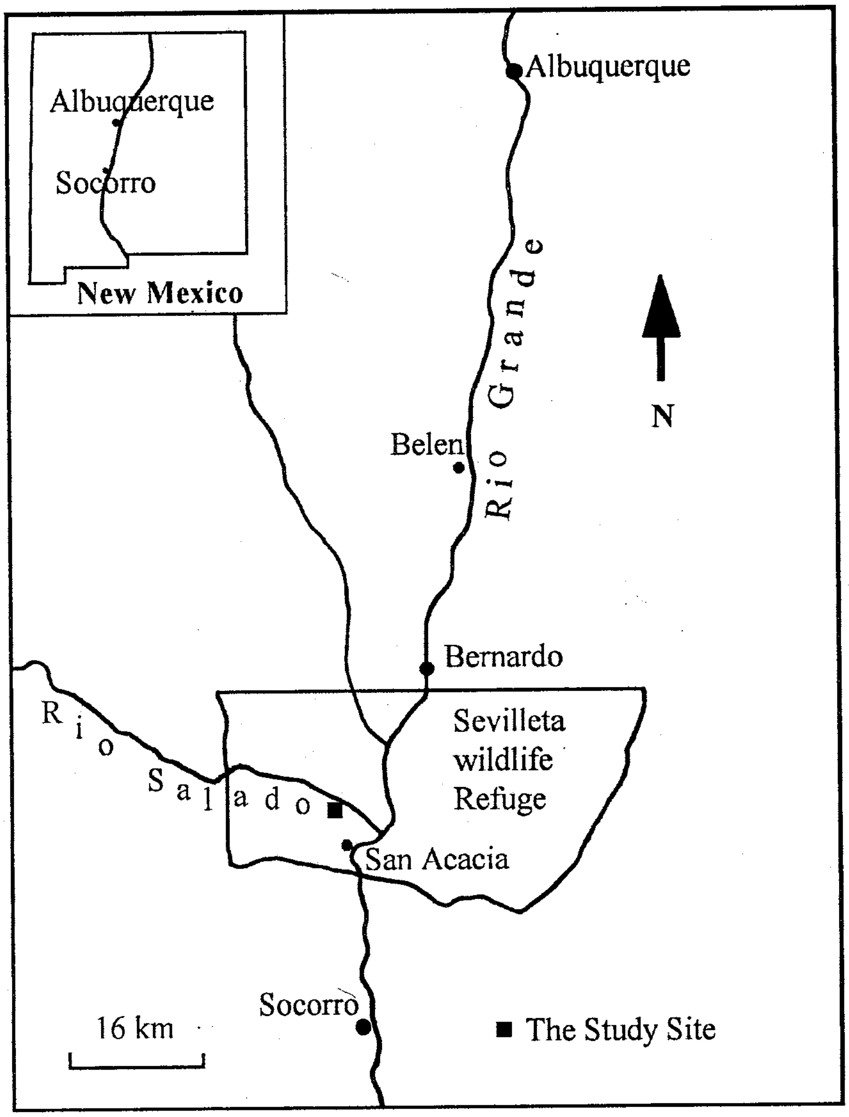


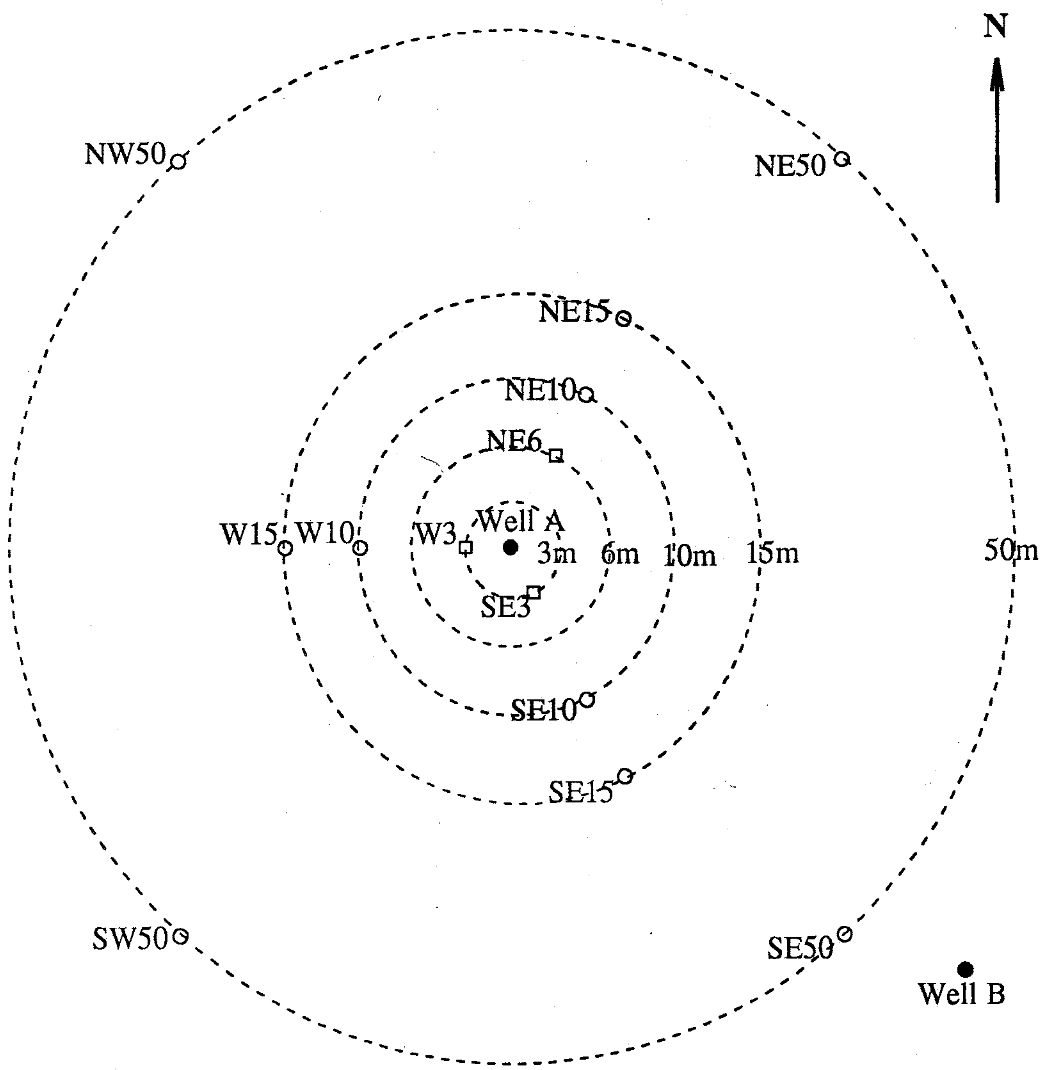
Figure 1: Location of the Sevilleta study site.

formation. The Rio Salado alluvium consists of interbedded sand, gravel, and silt and the axial stream deposits interbedded sand and silt with occasional gravel and clay layers. Split spoon samples taken during the drilling from three boreholes show that the contact between the Rio Salado and axial stream deposits is located between 13.72 m and 19.81 m below the ground surface and the contact depth increases from north to south which indicates that old channel of Rio Salado was located farther south than the present channel. The drilling experience and seismological study suggest that the aquifer is unconfined and more than 100 m thick.

The regional groundwater flow observed in the four outside observation wells was relatively uniform, and the hydraulic gradient was very small about 10^{-3} m/m with direction from NE to SE. The water table is approximately 3 meters below the ground surface.

The detail of configuration of wells at study site is shown in Figure 2. Three multilevel samplers/piezometers (MSLP) and ten fully screened observation wells are located on three rays surrounding Well A. Well A and B are pumping/water supply wells with 15.24 centimeters diameter and cased from the top of well to 6.1 meters in depth and screened from 6.1 to 24.38 meters and 12.19 meters respectively. Only Well A was used as pumping well in this study. The other wells are named by the direction and the distance from Well A. For example, NE10 is the well at 10 meters distance in NE direction from Well A. The ten fully screened observation wells are 5.08 centimeter PVC pipe with 0.8 millimeter machine cut slots.

The well depths and screen intervals of Well A and ten fully screened observatio



● 6-inch pumping/water supply well

□ functional multilevel samplers/piezometers (MLSP)

○ 2-inch fully screened observation well

Figure 2. Configuration of Wells at the Sevilleta Study Site.

wells are shown in detail in Figure 3. All depths are with respect to the top of Well A. The three multilevel samplers/piezometers are used to collect depth-specific groundwater samples and to measure depth-specific drawdown. As this study is emphasize on fully screened (two dimensional) case, MSLP are not described in detail here.

Since the fine sand may settled in wells, the fully screened observation wells were developed periodically to ensure that they are remain free of sediment.

Field Set Up of Pumping Test

From analysis of the depth specific pumping test data, a low permeability layer exists at approximately 11 to 13 meters below ground surface. Whenever water is pumped from Well A at any part including the layer below the low permeability layer mentioned above, an anomalous drawdown pattern was noted in the vertically averaged drawdown data take from fully screened observation wells. The drawdowns measured from W15 and SE15 at farther distance from pumping Well A were greater than drawdowns measured from W10 and SE10 at closer distance from pumping Well A respectively. When water was withdrawn from above the pack which was placed at 12.1 to 14.63 meters below the to of Well A, the above anomalous drawdown profile was no observed. It is clearly shown that Well A, W15 and NE15 are deeper than W10, SE10 NE10 and NE15. The existence of a highly permeable zone at about 24 meters below ground surface where Well A, W15 and SE15 reached, provided most portion of water to Well A during pumping. This may be the cause for the anomalous drawdown profile In order to avoid this complication, only upper stratum was used for further study, i.e

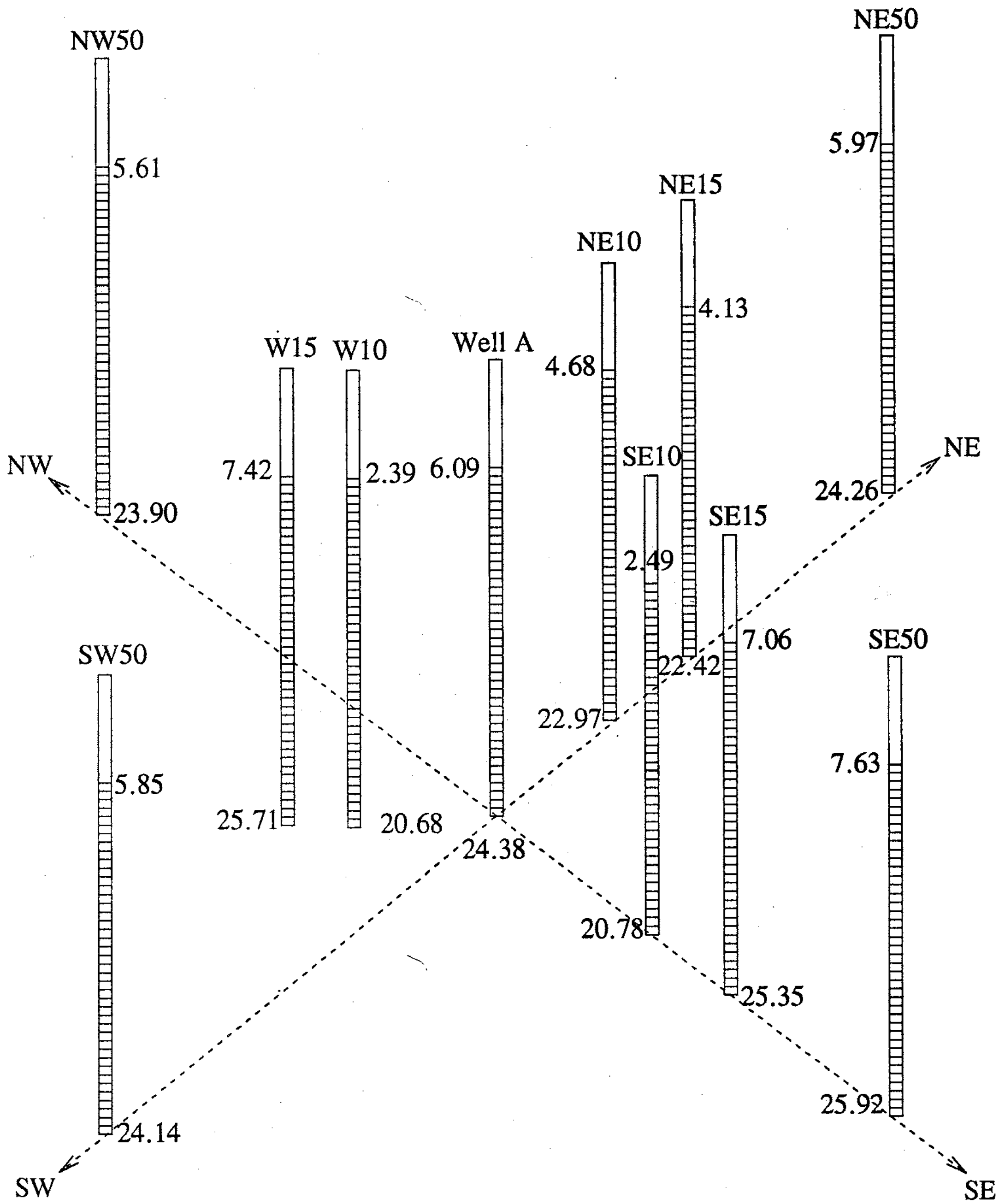


Figure 3. Depths of well screens measured with respect to the top of Well A.

the packer was placed at 12.19 to 14.63 meters below the top of pumping Well A during an aquifer tests.

Because of the lack of diagnostic curvature in the mid-range of time on a plot of drawdown versus time, accurate evaluation of aquifer parameters in unconfined aquifer usually requires long-term pumping. For this study the pumping tests were lasted more 30 hours.

Field Data Collection

The initial groundwater table is about 2.98 meters below the ground surface and is assumed to be horizontal according to monitored regional groundwater movement. The initial saturated thickness therefore is taken as 9 m in the following analysis, since the low permeable layer is located at about 12 meters below ground surface. The verticall averaged drawdowns were collected automatically by a datalogger through pressure transducers in W10, W15, NE10, NE15, SE10, and SE15. When transducers were placed at different depths of the same well, they recorded almost identical drawdow indicating a good vertical mixing inside the fully screened observation wells.

Before conducting pumping test, the pressure transducers were calibrated in orde to insure accurate results. Initially the drawdown data measured in the fully screene observation wells were noisy. Work was done to improve the quality of data. First, a fully screened observation wells and Well A were developed again, in order to be sur each well to be free of sediment. Next, weights were added to the pressure transducer to insure that they remain stable during the pumping test. After these two improvemen were made, the quality of drawdown data improved.

The best data from wells W10 and SE10 were measured during pumping tests on September 17 to 20, 1992 and the data from well NE15 were measured on February 27 to 28, 1993 (Figure 4). These data were selected for analysis. The pumping test conducted on September 17 to 20 was the longer one. About $7*10^4$ seconds after pumping started drawdown began to decrease. The decrease was possibly caused by an unstable pumping rate or recharge boundary being reached. In order to avoid to spend more energy and man power to collect useless data at large time, the second pumping test lasted less than $7*10^4$ seconds and the decrease was not observed. The pumping rate of test on September 17 to 20 was $3.949*10^{-3}$ m^3/s and the pumping rate of test on February 27 to 28 was $6.310*10^{-3}$ m^3/s.

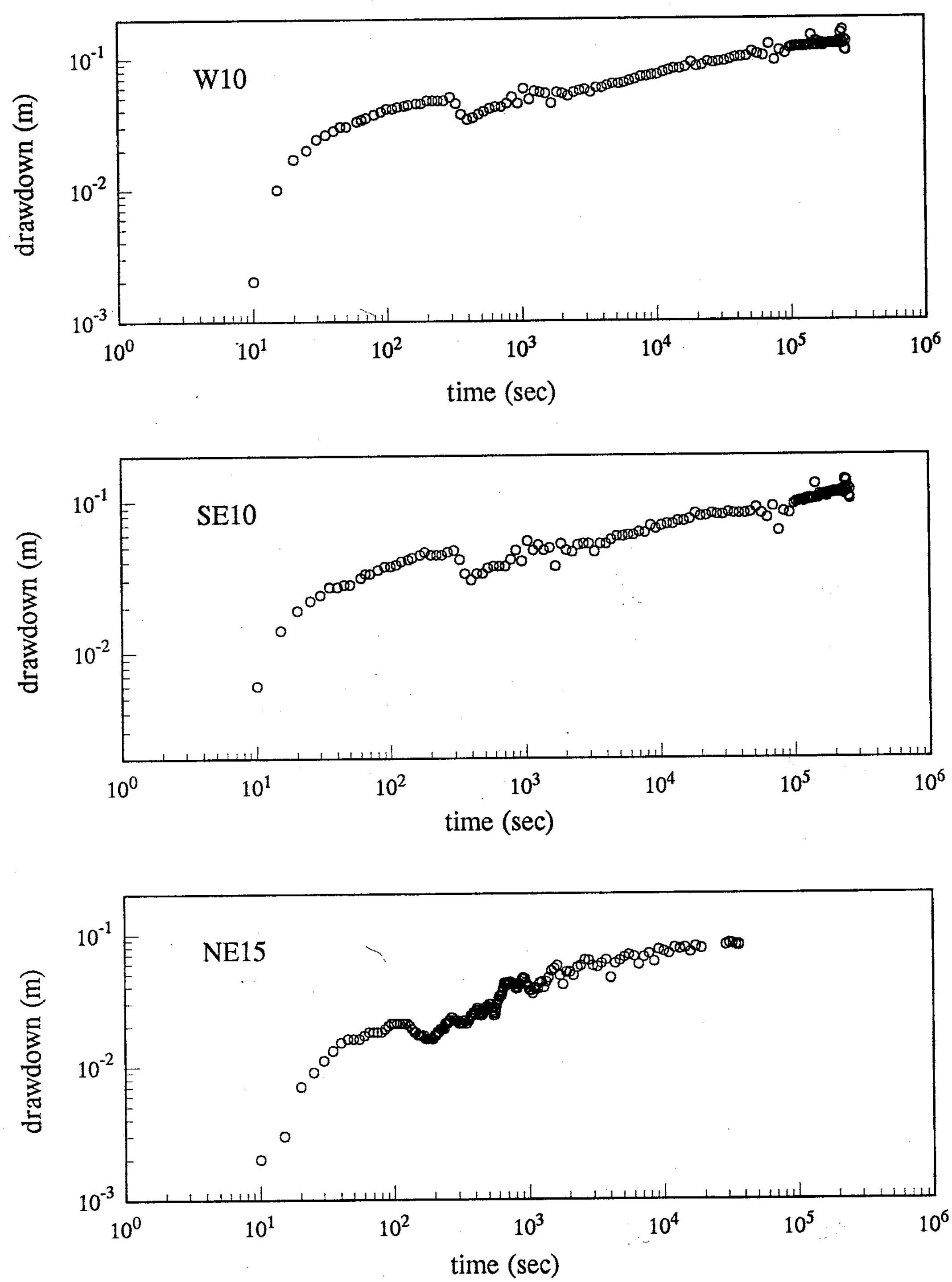


Figure 4: Vertically Averaged Drawdown Data Measured at W10, SE10 and NE15.

METHODOLOGY

Walton (1960) described that the time-drawdown curve measured in an unconfined aquifer with delayed yield can be divided into three distinct segments on logarithmic paper. The first segment, covering a short period after pumping has begun indicates that an unconfined aquifer reacts initially in the same way as does a confined aquifer. Water is released instantaneously from storage by the compaction of the aquifer and by expansion of water itself. Gravity drainage has not yet started. The second segment of the time-drawdown curve shows a decrease in slope because of the replenishment by gravity drainage from the interstices above the cone of depression . During this time, there is marked discrepancy between the observed data curve and the Theis type curve for unsteady state flow. The third segment, which may start from several minutes to several days after pumping has begun, again conforms closely to a second Theis-type curve with respect to specific yield.

Boulton's Curve-Matching Method:

Boulton (1954, 1963) and Prickett (1965) introduced a method of analyzing pumping test data from unconfined aquifers, in which allowance is made for the delayed yield from storage due to slow gravity drainage. The assumptions are list as following The aquifer is unconfined and rests on horizontal impermeable layer; The aquifer is infinite in areal extent; The aquifer is of the same thickness throughout; The aquifer i homogeneous and isotropic; The well has an infinitesimal diameter and penetrates the fully thickness of the formation; The drawdown is very small in comparison to the thickness of the aquifer. The one-dimensional radial flow model for non-equilibrium condition in unconfined aquifers uses the exponential integral in governing equation (1) as a source term to represent the water release process, and to account for the delayed-drawdown region of the time-drawdown graph. An empirical coefficient is included in the source term. Boulton's analytical treatment does not account for vertical components of flow in the aquifer, and the theory can not be used directly to obtain the vertical hydraulic conductivity of the aquifer.

The mathematical model in cylindrical coordinates are:

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = \frac{S}{T}\frac{\partial h}{\partial t} + \frac{\alpha S_y}{T}\int_o^t \frac{\partial h}{\partial \tau} e^{-\alpha(t-\tau)} d\tau \qquad (1)$$

$$h(r, t=0) = 0$$

$$h(r=\infty, t) = 0$$

$$\lim_{r \to o} r\frac{\partial h}{\partial r} = \frac{-Q}{2\pi T}$$

The terms are defined in the Notation section at the end. The solution [Boulton 1955 Equation 9] is:

$$h = \frac{Q}{4\pi T}\left[\int_o^\alpha (1-e^{-ut}) J_o\left\{r\sqrt{\left(\frac{u}{\alpha}\frac{\eta\alpha-u}{\alpha-u}\right)}\right\}\frac{du}{u} + \int_{\eta\alpha}^\infty (1-e^{-ut}) J_o\left\{r\sqrt{\left(\frac{u}{\alpha}\frac{\eta\alpha-u}{\alpha-u}\right)}\right\}\frac{du}{u}\right] \qquad (2)$$

in which

$$\eta = \frac{S + S_y}{S} = 1 + \frac{S_y}{S} \qquad (3)$$

and μ is dummy variable and J_o denotes the Bessel function of the first kind of zero order.

Because it is difficult to generate appropriate type curves for the full range of the parameters using equation (2), asymptotic solutions at large and small-time were used in the curve-matching method.

Equation (2) can symbolically and in analogy to the Theis equation, can be written as:

$$h = \frac{Q}{4\pi T} W(u_{AB}, \frac{r}{D}) \qquad (4)$$

Where $W(u_{AB}, r/D)$ is called the "well function of Boulton". For the early time, this equation describes the first segment of the time-drawdown curve and equation (4) reduce to

$$h = \frac{Q}{4\pi T} W(u_A, \frac{r}{D}) \qquad (5)$$

where

$$u_A = \frac{r^2 S}{4Tt} \quad (6)$$

For the later time, equation (4) describes the third segment of the time-drawdown curve and reduces to:

$$h = \frac{Q}{4\pi T} W(u_B, \frac{r}{D}) \quad (7)$$

where

$$u_B = \frac{r^2(S_y + S)}{4Tt} \cong \frac{r^2 S_y}{4Tt} \quad (8)$$

The factor η is defined as in equation (3).

The above mentioned formulas are only valid if η tends to infinity, in practice this means that $\eta > 100$. If $10 < \eta < 100$, the second segment of the time-drawdown curve is no longer horizontal, but the Boulton's method still gives a fairly good approximation.

If η tends to infinity, the second segment is described by

$$h = \frac{Q}{2T} k_o(\frac{r}{D}) \quad (9)$$

where K_o (r/D) is the modified Bessel function of the second kind and zero order. The drainage factor D is expressed in meters and defined as:

$$D = \sqrt{\frac{T}{\alpha S_y}} \quad (10)$$

where an empirical constant $1/\alpha$ is called the 'Boulton delay index'.

The family of 'Boulton type curve', which are type A curves ($W(u_A, r/D)$ versus $1/u_A$) and type B curves ($W(u_B, r/D)$ versus $1/u_B$), are constructed for a practical range of values of r/B on double logarithmic paper. The measured drawdown data versus time should be plotted on another sheet of double logarithmic paper with the same scale used for type curves. Superimpose the measured drawdown plot on the type A curves, and adjust by moving and keeping the coordinate axis of the two curves parallel until as much as possible of the early time field data fall on one of the type A curves. Note the value of r/B on type A curve. Select an arbitrary point on the overlapping portion and note the value of h^*_A, t^*_A, $1/u_A$ and $W(u_A, r/D)$ for this point. Substitute these value into equatio (5) and (6). The value of T and S can be calculated. Then superimpose the measured dat plot on type curve B, and move until as much as possible of the later time field data fal on the type B curve with the same value of r/B as noted from type curve A. Select a arbitrary point and note the values of h^*_B, t^{*B}, $1/u_B$ and $W(u_B, r/D)$ for it. Using thes values, T and S_y can be calculated by equation (7) and (8). The transmissivity T calculated by equation (5) and (7) should be approximately the same. According equatio (10), the empirical coefficient can be calculated as following:

$$\alpha = \frac{T\left(\frac{r}{D}\right)^2}{S_y r^2} \tag{11}$$

The factor η should be calculated from equation (3) to check the validation of th method.

Since the Laplace domain solution shown in eq (12) is much simpler in form than real time solution equation (2), drawdowns were calculated using the Stehfest (1970) numerical inversion of Laplace transform solution. Since the Stehfest method is fast, accurate and easy to use, it has been frequently apply to numerically invert Laplace transform solutions of groundwater problems.

$$H = \frac{Q}{2\pi Tp} K_o\left(r\sqrt{\frac{Sp}{T} + \frac{S_y}{T}\frac{p}{p+\alpha}} \right) \tag{12}$$

where K_o is the modified Bessel function of the second kind of zero order. Because k_o is smoothly varying and declines exponentially with increasing value of its argument, Equation (12) is easy to inverse.

Neuman's Curve-Matching Method

Neuman (1972, 1973) gave a three dimensional flow model of unconfined aquifer with instantaneous and complete drainage at the water table assumed. The simplifying assumptions are similar to Boulton model as: the aquifer is unconfined and is infinite areal extent with uniform thickness; aquifer is homogeneous isotropy or anisotropy and water table is horizontal before the pumping start; wells penetrates the entire thickness of the aquifer and diameters are infinitesimal; the influence of unsaturated zone upon the drawdown is negligible.

The mathematical model given below has been adopted from those study. The linearized mathematical model become:

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} + \frac{K_z}{K_h}\frac{\partial^2 h}{\partial z^2} = \frac{S_s}{K_h}\frac{\partial h}{\partial t}, \qquad 0 < z < b$$

$$h(r, z, 0) = 0$$

$$h(\infty, z, t) = 0$$

$$\frac{\partial h}{\partial z}(r, 0, t) = 0$$

$$\frac{\partial h}{\partial z}(r, b, t,) = -\frac{S_y}{K_z}\frac{\partial h}{\partial t}(r, b, t,)$$

$$\lim_{r \to o}\int_o^b r\frac{\partial h}{\partial r}dz = -\frac{Q}{2\pi K_h}$$

Since the Laplace domain solution is much simpler in form than the real time solution, this study uses the Laplace domain solution to calculate the drawdown in FORTRAN program. According to equation A7 (Neuman 1972) Laplace domain solution (Chen et.al. 1993) can be rewritten as

$$H(r,z,t) = \frac{Q}{2\pi T p}K_o\left(r\sqrt{\frac{pS}{T}}\right) - \frac{Q}{2\pi b K_z}\int_o^\infty \frac{aJ_o(ar)}{\frac{K_z}{S_y}\nu^3\sinh(\nu b) + p\nu^2\cosh(\nu b)}\cosh(\nu z)\,da$$

where a is dummy variable and

$$v^2 = \frac{K_h}{K_z}\left(a^2 + \frac{Sp}{bK_h}\right)$$

The averaged drawdown solution in Laplace domain is simply the average over that vertical distance according to Neuman (1974) as following

$$H(r, t) = \frac{1}{b}\int_o^b H(r, z, t)dz$$

$$= \frac{Q}{2\pi Tp} K_o\left(r\sqrt{\frac{pS}{T}}\right) - \frac{Q}{2\pi b^2 K_z}\int_o^\infty \frac{aJ_o(ar)}{\frac{K_z}{S_y}v^4 + p\coth(vb)v^3}da$$

The first term is Theis solution in Laplace domain. Guassian 32 point numerica integration method was used to deal with the integration and IMSL function was used t solve J_o Bessel function in Fortran code.

Neuman (1975) developed curve-matching method base on his theory (1972 1973, 1974), and this method can be used to determine the transmissivity of anisotropi unconfined aquifers from pumping test data.

Neuman (1975) gave the drawdown solution as

$$h = \frac{Q}{4\pi T}W(u_{AB}, \beta) \tag{13}$$

This equation can be reduced to describe the first segment of the time-drawdown curve as

$$h = \frac{Q}{4\pi T} W(u_A, \beta) \tag{14}$$

where

$$u_A = \frac{r^2 S}{4Tt} \tag{15}$$

The late-time time-drawdown curve can be described by reducing equation (13) as

$$h = \frac{Q}{4\pi T} W(u_B, \beta) \tag{16}$$

where

$$u_B = \frac{r^2 S_y}{4Tt} \tag{17}$$

and parameter β is defined as

$$\beta = \frac{r^2 K_z}{b^2 K_h} \tag{18}$$

The curve-matching procedure is very similar to the Boulton's curve-matching method. The family of Neuman type curves should be plotted on the same scale as the measured time-drawdown data. Neuman (1975) provided a series of type curves in practical range. The observed time-drawdown should be superimpose on the type curve

B and match late time observed data on the selected curve as much as possible and note the value of β. Read the values of $W(u_B, \beta)$, $1/u_B$, h^*_B and t^*_B for an arbitrary selected point. Transmissivity T and storage coefficient S can be calculated by equation (14) and (15) by substituting these values with the known values of Q and r. Match the early time drawdown curve on type curve A with the same value of ß, and read the value of $W(u_A, \beta)$, $1/u_A$, h^*_A and t^*_A for an arbitrary point, S_y and T can be obtained by equation (16) and (17). Transmissivity calculated by equation (14) and (16) should be close to each other, and the average value can be taken as final result if the difference is not large. According equation (18), K_z can be calculated as following

$$K_z = \frac{\beta b T}{r^2} \tag{19}$$

After all calculation, the value of S and S_y should be compared to check whether the condition $S_y > 10\,S$ is satisfied.

Type Curves

Boulton (1963) gave asymptotic solution for which η tends to infinity as follows:

$$h = \frac{Q}{4\pi T}\int_o^\infty 2J_o\left(\frac{r}{D}x\right)\left\{1 - \frac{1}{x^2+1}\exp\left(-\frac{\alpha + x^2}{x^2+1}\right) - \epsilon\right\}\frac{dx}{x} \tag{20}$$

where

$$\epsilon = \frac{x^2}{x^2+1}\exp\{-\alpha\eta t(x^2+1)\} \tag{21}$$

For t tends to zero, Equation (20) reduce to

$$h = \frac{Q}{4\pi T}\int_o^\infty 2J_o\left(\frac{r}{D}x\right)\frac{x^2}{x^2+1}\{1-\exp[-\alpha\eta t(x^2+1)]\}\frac{dx}{x} \tag{22}$$

substitute α and η into Equation (22), the type curve A of Boulton model can be calculated by

$$W\left(\frac{r}{D},\frac{1}{u_A}\right)=h\frac{4\pi T}{Q}=\int_o^\infty 2J_o\left(\frac{r}{D}x\right)\frac{x^2}{x^2+1}\left\{1-\exp\left[\frac{1}{4}\left(\frac{r}{D}\right)^2\frac{1}{u_A}(x^2+1)\right]\right\}\frac{dx}{x}$$

where $1/u_A$ and D are defined in Equation (6) and (10) respectively.

When t >> 0, the function ϵ defined in Equation (21) vanishes. Equation (20) reduces to

$$h=\frac{Q}{4\pi T}\int_o^\infty 2J_o\left(\frac{r}{D}x\right)\left\{1-\frac{1}{x^2+1}\exp\left(\frac{\alpha t x^2}{x^2+1}\right)\right\}\frac{dx}{x} \tag{23}$$

again substitute α and rearrange them, the type curve B of Boulton model can be calculated by

$$W\left(\frac{r}{D},\frac{1}{u_B}\right)=h\frac{Q}{4\pi T}=\int_o^\infty 2J_o\left(\frac{r}{D}x\right)\left\{1-\frac{1}{x^2+1}\exp\left(\frac{1}{4}\left(\frac{r}{D}\right)^2\frac{1}{u_B}\frac{x^2}{x^2+1}\right)\right\}\frac{dx}{x}$$

where $1/u_B$ and D are defined in Equation (8) and (10) respectively.

According to Laplace transform solution provided by Neuman, Moench (1993) gave Laplace transform solution in dimensionless term. Again for two dimensional radial vertical flow, the averaged dimensionless drawdown should be integrated as follows

(Neuman 1974):

$$H_D(p_D,\ \beta,\ \sigma)\ =\ \int_o^1 H_D(p_D,\ \beta,\ z_D,\ \sigma)dz_D$$

therefore

$$H_D(p_D,\ \beta,\ \sigma)\ =\ \frac{2K_o(\sqrt{p_D})}{p_D}+\frac{2}{\beta}\int_0^\infty \frac{-\sinh(1)xJo(x)dx}{\psi^2\sinh(\psi)[\sigma\beta\psi+p_D\coth(\psi)]} \tag{24}$$

where

$$p_D\ =\ \frac{u_{AB}}{4},\qquad \sigma\ =\ \frac{S}{S_y},\qquad \psi\ =\ \frac{x^2+p_D}{\beta} \tag{25}$$

and β is the same as defined in Equation (18). In order to reduce independent dimensionless parameters from three to two a convenient way is to consider the case in which σ approaches zero (Neuman 1972). Two asymptotic form can be obtained for calculation of type curve A and B. For type curve A, t tends to zero, thus $p_D >> 1$ asymptotic form of Equation (24) for type curve A becomes

$$W\left(\frac{1}{4u_A},\ \beta\right)=L^{-1}\{H_D(p_{DA},\ \beta)\}\ =\ L^{-1}\left\{\frac{\sqrt{2\pi}}{(p_{DA})^{5/4}\exp(\sqrt{p_{DA}})}\ +\ \frac{4.7}{\beta}\int_o^\infty \frac{xJ_o(x)}{\psi^2 p_{DA}\ e^\psi}\ dx\right\}$$

in which $1/u_A$, β, and ψ are defined in Equation (15), (18) and (25) respectively, and p_D is inversely related to 1/4 of $1/u_A$.

When $t >> 1$, then p_D tends to zero. asymptotic form of Equation (24) for typ curve B can be written as

in which $1/u_B$, β, and ψ are defined in Equation (17), (18) and (25) respectively, and p_D

$$W\left(\frac{1}{4u_B},\beta\right)=L^{-1}\{H_D(p_{DB},\beta)\}=L^{-1}\left\{\frac{-2\ln\sqrt{p_{DB}}}{p_{DB}}-\frac{2.35}{\beta}\int_o^{\infty}\frac{xJ_o(x)}{\frac{x^4}{\beta^2}\sinh\left(\frac{x^2}{\beta}\right)p_{DB}\coth\left(\frac{x^2}{\beta}\right)}dx\right\}$$

is inversely related to 1/4 of $1/u_B$.

Parameter Sensitivity Analysis

In order to better understand the effect of interested parameters, theoretical time-drawdown curves for several conditions were calculated using both the Boulton and Neuman models. All effects were observed as parameters were varied.

The effects of changing each of the pertinent hydrological parameters of Boulton's are shown in Figure 5. In each diagram, one parameter was varied, while the other parameters remain fixed. The actual (or close to actual) pumping rate, Q=3.14*10-3 m3/s, radius, r=10m, from the field test, Sy=0.1, T=0.01 m^2/s and α=0.001 1/s were selected as basic data for the simulations. The elastic storage coefficient was varied from 0.0001 to 0.01, while the other parameters remained constant. Clearly, the elastic storage coefficient only affects the early time drawdown and has negligible effect on late time drawdown. As S increased, the drawdown become smaller at early time (Figure 5). The values of Specific yield Sy tested ranging from 0.05 to 0.3. As can been seen it has influence both on the mid-time and large-time drawdown curve. A clear relationship appears to exit between the mid-time and large-time drawdown curve resulting from varying the S_y parameter values. As the value of S_y increased, the mid-time and large-time drawdown decreases. Increasing transmissivity T from 0.0005 to

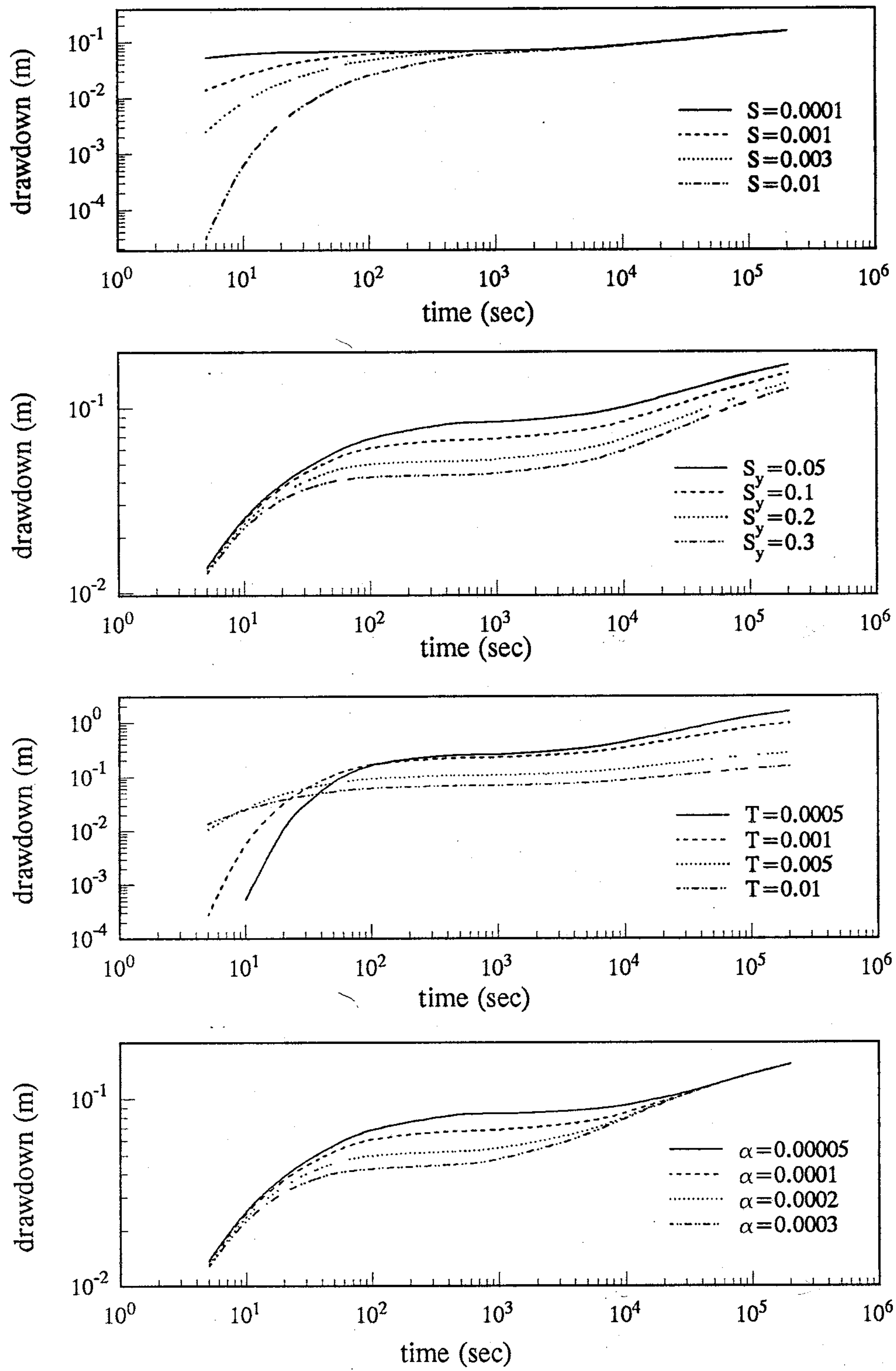


Figure 5: Boulton's Model Parameter Sensitivity.

0.01 affected the whole time-drawdown curve but did not reveal a clear trend i.e. the effects of permutations of the T parameter were not constant. The effects of variations in the empirical coefficient on drawdown curve were evaluated for changes of α from 0.00005 to 0.0003. The empirical coefficient (reciprocal of delay index) only affects the delayed drawdown curve region. The smaller value of α, the smaller the drawdown at delayed draw own region, i.e. the mid-time drawdown curve region.

Sensitivity of parameter S, Sy, T and Kz are presented in Figure 6 for Neuman's model. The actual (or very close to actual) pumping rate, $Q=3.14*10^{-3}$ m^3/s, radius, r=10m, and initial saturated thickness, b=10m, in the field test were selected for the simulations. The values of S, S_y and T were changed in the same range as Boulton's model, and S and T own the same effects on the time-drawdown curve. Unlike Boulton's model, specific yield shown less effect on the mid-time drawdown. Vertical hydraulic conductivity K_z increased two order magnitude from $5*10^{-6}$ to $1*10^{-4}$, and it is the only parameter to control delayed drawdown region and shows very similar effect as α in Boulton's model.

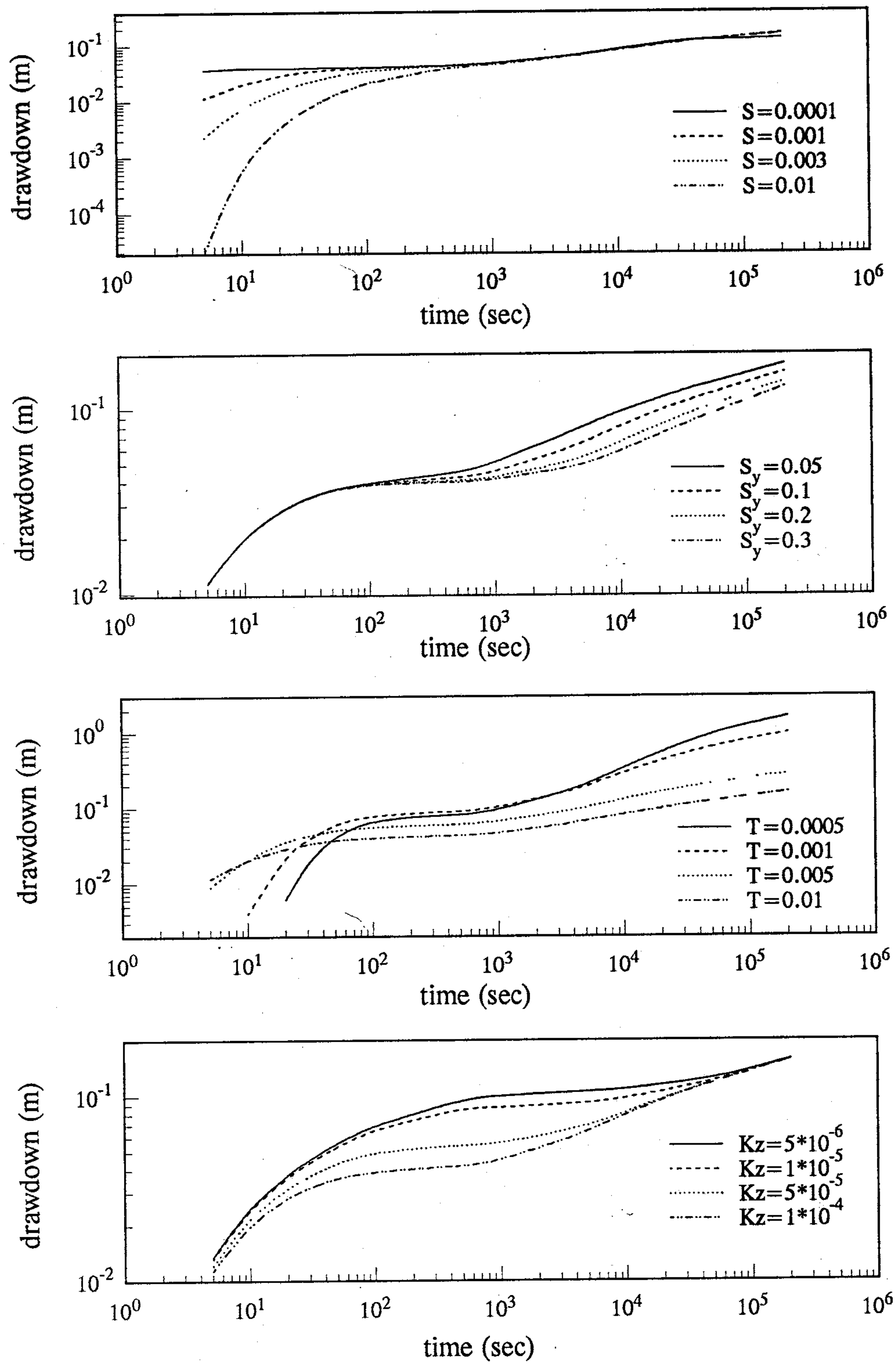


Figure 6: Neuman's Model Parameter Sensitivity.

Estimation of Planar Anisotropy

Horizontally isotropy is the assumption for both Boulton and Neuman model which are applied above, but both solutions has been adapted to anisotropic aquifers by using coordinated transformation given by Papadopulos (1965). Anisotropic aquifers were further studied by Hantush (1965,1966), Way S.C. and C.R. McKee (1982), and Neuman et. al. (1984). The two dimensional anisotropy tensor can be expressed as:

$$\underline{\underline{T}} = \begin{bmatrix} T_{xx} & T_{xy} \\ T_{yx} & T_{yy} \end{bmatrix}$$

where $T_{xy}=T_{yx}$, and T_{xx}, T_{xy} and T_{yy} are the components of the transmissivity tensor. In the coordinate transformation, the term r and Tr are expressed as

$$r^2 = \frac{T_{yy}x^2+T_{xx}y^2-2T_{xy}xy}{T_e} \tag{26}$$

and

$$T_e^2 = T_{xx}T_{yy} - T_{xy}^2 \tag{27}$$

substitute equation (26) and (27) into equation (6), (8), (15), or (17) and using the dat from three pair of matching point. Three simultaneous equation can be made as:

$$u_i = \frac{S}{4t_i^*}\left(\frac{T_{xx}y_i^2+T_{yy}x_i^2-2T_{xy}x_iy_i}{T_e^2}\right)$$

where i=1, 2, 3 for each matching point. In order to linearize the three simultaneou equations, T_e is taken as:

$$T_e = \frac{1}{3}\,(T_1+T_2+T_3)$$

in which T_1, T_2 and T_3 are the value obtained by the curve-matching method for each set of measured data. This approximation made three simultaneous equations linearized and solved without difficulty. After storage coefficient lumped with transmissivity, three equations can be written as:

$$\begin{bmatrix} y_1^2 & x_1^2 & -2x_1y_1 \\ y_2^2 & x_2^2 & -2x_2y_2 \\ y_3^2 & x_3^2 & -2x_3y_3 \end{bmatrix} \begin{bmatrix} \Gamma_{xx} \\ \Gamma_{yy} \\ \Gamma_{xy} \end{bmatrix} = \begin{bmatrix} 4t_1u_1T_e^2 \\ 4t_2u_2T_e^2 \\ 4t_3u_3T_e^2 \end{bmatrix}$$

where

$$\begin{bmatrix} \Gamma_{xx} \\ \Gamma_{yy} \\ \Gamma_{xy} \end{bmatrix} = S \begin{bmatrix} T_{xx} \\ T_{yy} \\ T_{xy} \end{bmatrix} \qquad \textbf{(28)}$$

After Γ_{xx}, Γ_{yy} and Γ_{xy} obtained by solving three simultaneous equation, storage coefficien can be calculated according to equation (27) and (28):

$$S = \frac{\sqrt{\Gamma_{xx}\Gamma_{yy} - \Gamma_{xy}^{\ 2}}}{T_e}$$

substitute S into equation (28), then the components of transmissivity are:

$$\begin{bmatrix} T_{xx} \\ T_{yy} \\ T_{xy} \end{bmatrix} = \frac{1}{S} \begin{bmatrix} \Gamma_{xx} \\ \Gamma_{yy} \\ \Gamma_{xy} \end{bmatrix}$$

Once the symmetrical components of transmissivity tensor are known, associated principal values (eigenvalues) and principal directions (eigenvectors) can be easily calculated using Matlab software.

RESULTS AND DISCUSSIONS

The results from Boulton and Neuman curve-matching method were list in table (1) in the following. It was checked that $S_y > 10$ S was true for all three set of data in Neuman's curve matching methods. Only one set of data from observation well NE15, $\eta = 7.43$ which was less than 10 in Boulton's curve method. In order to check the accuracy of parameter obtained by both method, the comparison of measured drawdown and theoretical drawdown calculated by estimated parameters were plotted in Figure 7 and Figure 8. This procedure was important to adjust the accuracy of graphical method, since error can be easily caused by eye matching and reading from logarithm scale.

Table 1. The results obtained by curve-matching methods.

	Boulton model				Neuman model			
	T (m^2/s)	S	S_y	α(1/s)	T(m^2/s)	S	S_y	K_z(m/s)
W10	0.01	0.0038	0.25	0.00015	0.01	0.0034	0.29	0.000094
SE10	0.015	0.0036	0.15	0.00016	0.013	0.0035	0.30	0.000064
NE15	0.028	0.0056	0.036	0.0011	0.025	0.0049	0.047	0.0002

"Since the values of T, Sy, and S are determined from the early and lat

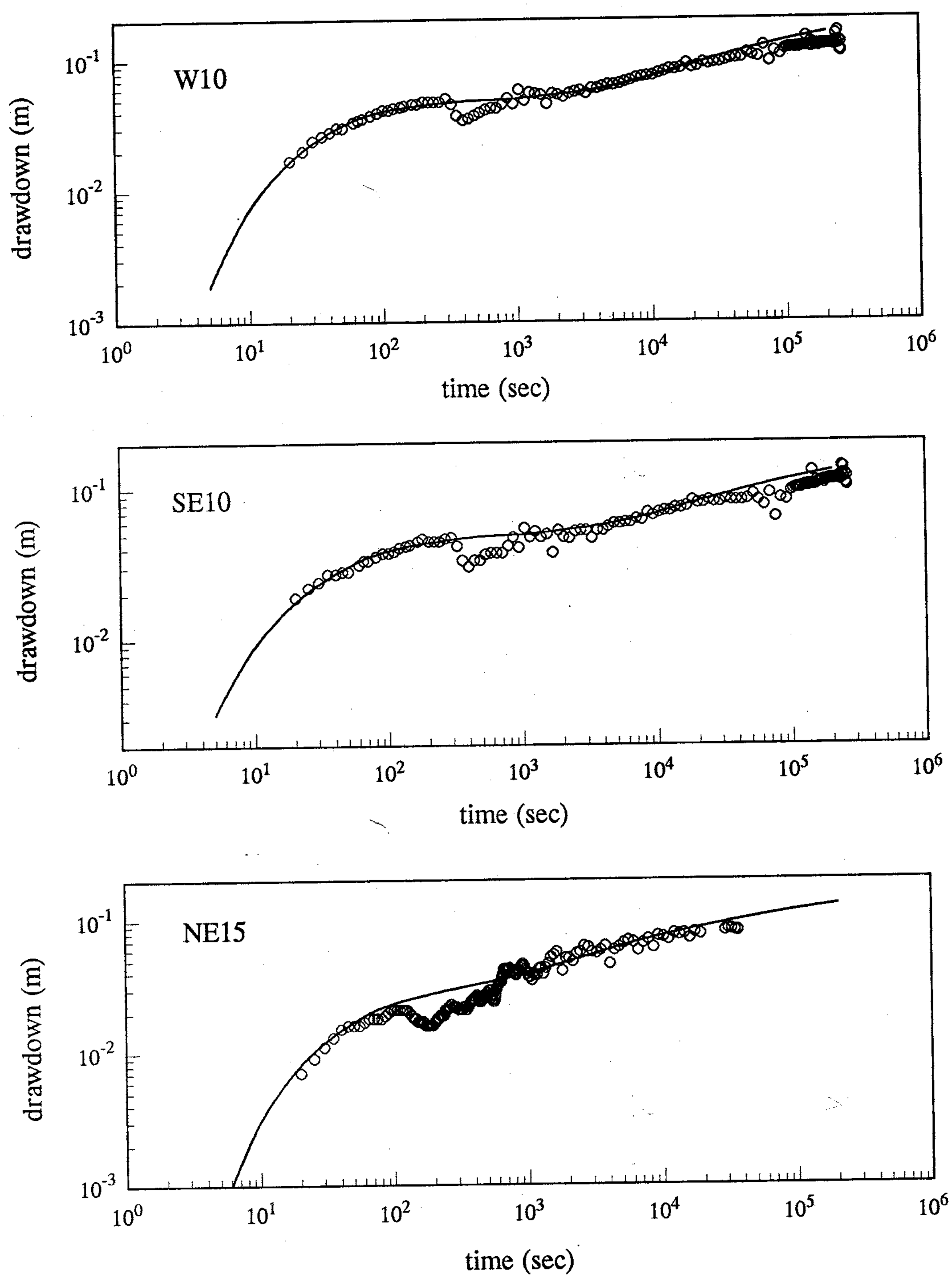


Figure 7: Comparison of measured drawdown with theoritic drawdown calculated by Boulton's model using estimated parameters.

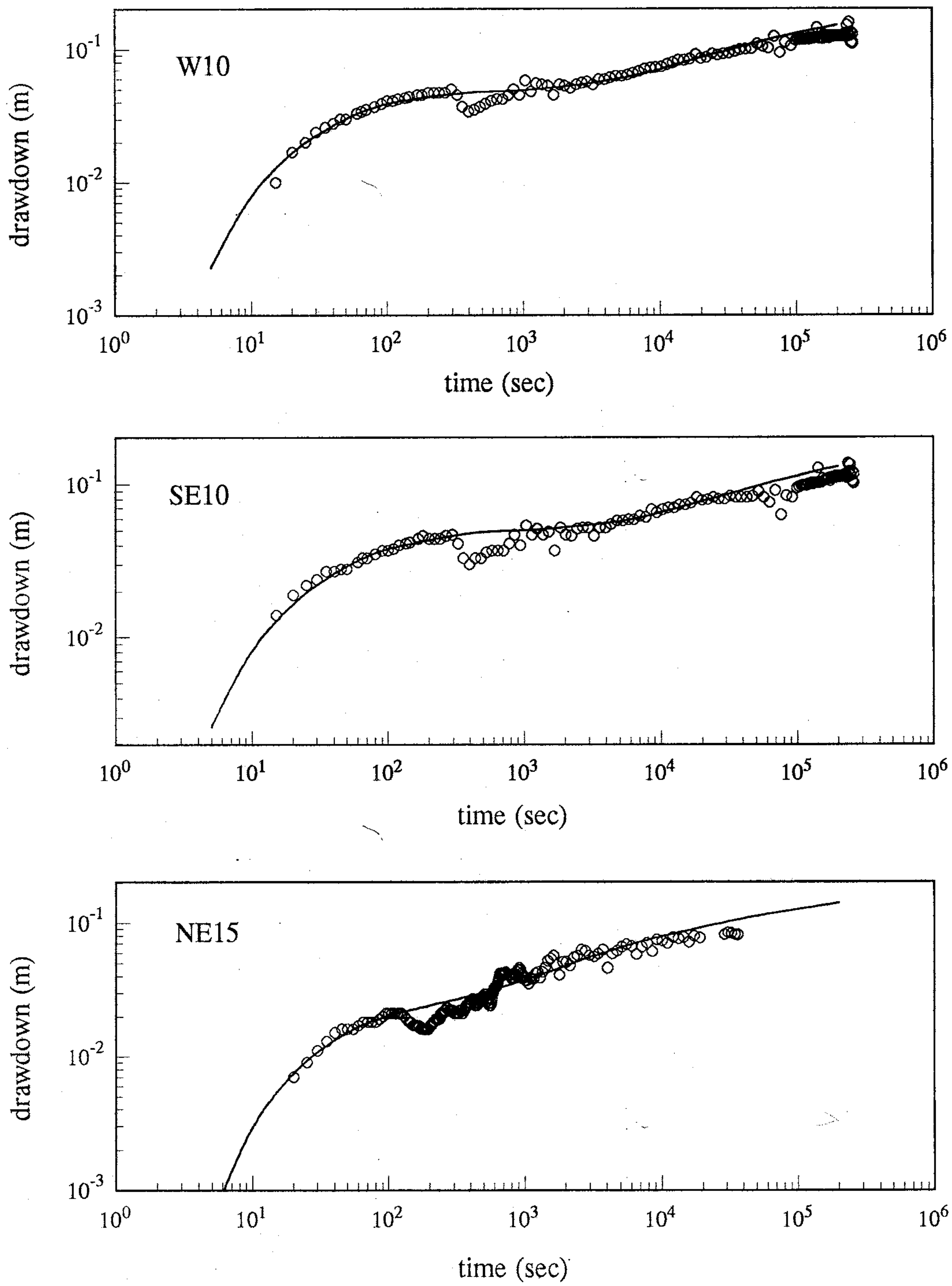


Figure 8: Comparison of measured drawdown with theoritic drawdown calculated by Neuman model using estimated parameters.

drawdown data that fit the early and the late Theis curves, respectively, the results obtained with the aid of Boulton's theory will be practically identical to those obtained with the methods described in this paper" Neuman (1975). It was not seen in this study. Comparing the results obtained by two methods, only the values of T were very close to each other and S were close enough. But the values of S_y were different. Recall the simulation of both model parameter sensitivity, S_y showed the different effect on time-drawdown curve. That may be a reason for the cause of difference. The main drawback of Boulton's model was the empirical constant - delayed index ($1/\alpha$) which was not clear to any physical definition. Overall the results obtained by Neuman's curve-matching and the early time matching point data was used to estimate the planar anisotropy of transmissivity. It was found that two orthogonal horizontal principal transmissivity were 0.021 meter square per second in the direction N20.83E and 0.012 meter square per second in the direction of N69.13W. The ratio of maximum and minimum transmissivity at Sevilleta study site was 1.75. The graphic presentation of anisotropy was shown in Figure 9. Comparison of measured drawdown with theoretic drawdown calculated by estimated planar anisotropy are shown in Figure 10, and they show fair agreement. The inaccuracy may be caused by linearizing the equation, i.e. taking averaged transmissivity as effective transmissivity T_e. This could be avoided by solving three non-linear equations directly.

In this study, it was found that choosing appropriate curve (i.e. appropriate r/B in Boulton's type curve , appropriate ß in Neuman's type curve) to matching was crucial in matching procedure. Otherwise the results may get into the range out of physic

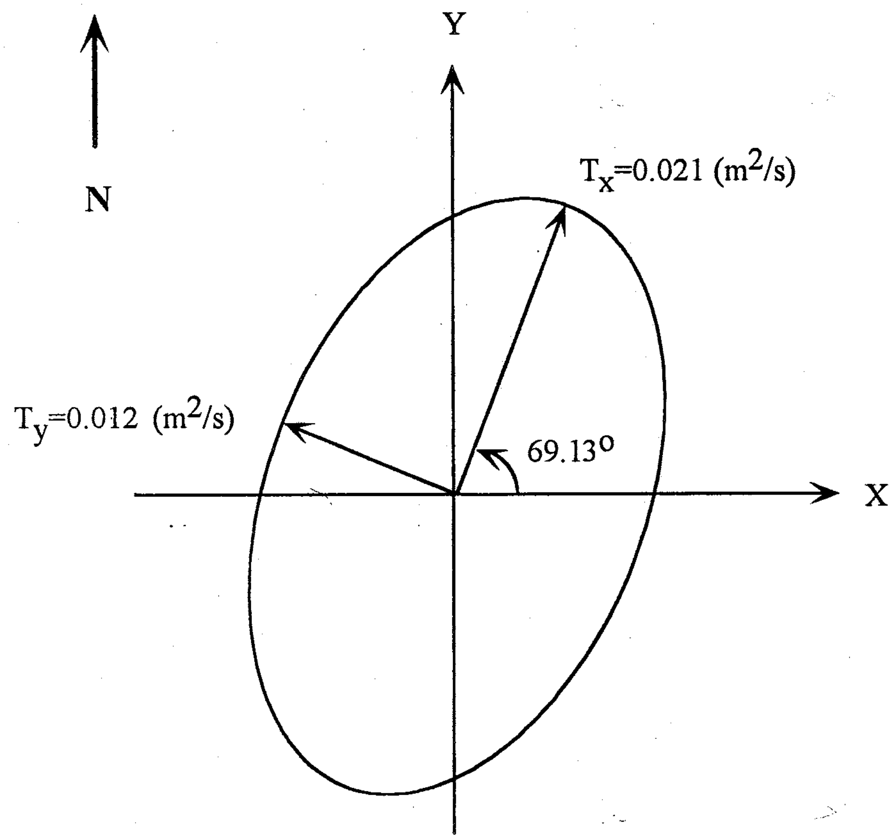


Figure 9. The planar anisotropy ellipse of transmissivity at Sevilleta study site.

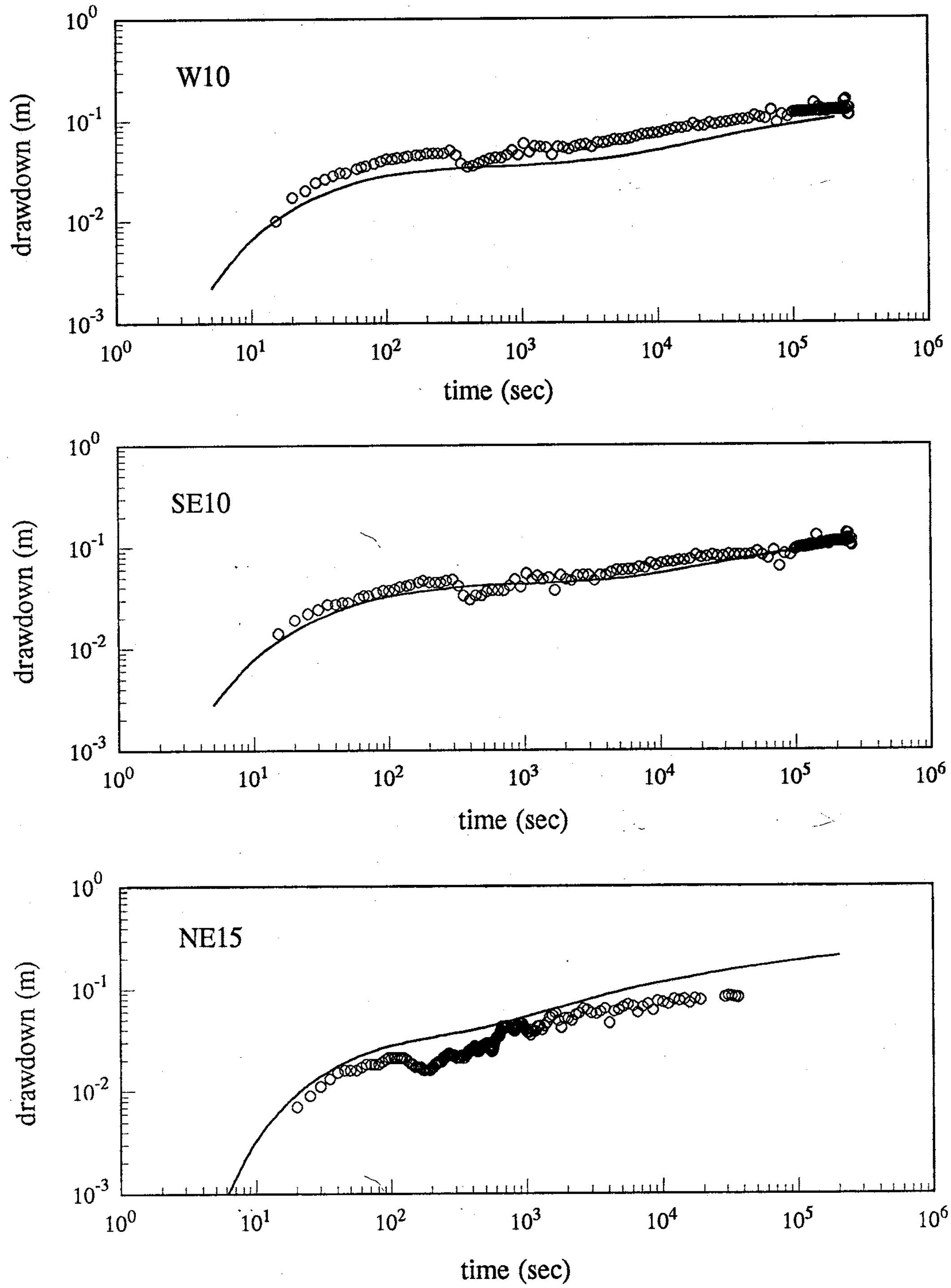


Figure 10: Comparison of measured drawdown with theoritic drawdown calculated by Neuman model using estimated planar anisotropy.

meaning such as $S_y > 1$. This mistake was made at the first matching, since a too big value of r/D or ß type curve was chosen for trying to fit drawdown data within 20 seconds too. Those drawdown were neglected later on. As the pumping rate can not reach the constant right after the pumping and the time lap in responding, drawdown data within 20 seconds were smaller than they should be. Matching the late time drawdown curve first may avoid this problem as in Neuman's curve-matching method (Neuman 1975).

SUMMARY AND CONCLUSIONS

The major work of this study is using Boulton's and Neuman's curve matching method to determine transmissivity in three different directions, setting three simultaneous equations to obtain the components of the planar transmissivity tensor, and finally calculating the principal value and direction of the planar transmissivity tensor. The two curve matching methods based on different theory and assumption provided similar results. All estimated parameters were verified by comparing measured drawdowns with analytical drawdowns which were calculated by estimated parameters.

Using pumping test data, aquifer properties can be estimated by nonlinear least-square analysis (Kashyap et al. 1988; Johns et al. 1992). The advantage of nonlinea least-square method is that all test data both early and late time are matched together. Bu it still can not replace graphical method. The results obtained by graphical method cai provide preliminary estimation of aquifer parameters for nonlinear least-square method As the time of this study is limited, nonlinear least-square method may be applied fo further study.

NOTATION

b	initial saturated aquifer thickness [L].
D	drainage factor $D=(T/\alpha S_y)^{1/2}$ [L].
h	drawdown [L].
h^*_A, h^*_B	arbitrary drawdown matching point [L].
H	Laplace domain variable inversely related with drawdown h.
K_z	vertical hydraulic conductivity [L/T].
K_h	horizontal hydraulic conductivity [L/T].
p	Laplace domain variable inversely related with time t.
Q	pumping rate [L^3/T].
r	radial distance from pumping well (L).
S	elastic storage coefficient.
S_y	specific yield.
t	elapsed time [T].
t^*_A, t^*_B	arbitrary time matching point [T].
T	transmissivity [L^2/T]
$\underline{\underline{T}}$	transmissivity tensor
Txx, Tyy, Txy	components of transmissivity [L^2/T]
Tx, Ty	principal transmissivity [L^2/T]
x, y, z	space coordinates [L]

α	reciprocal of delay index [1/T]
Γxx, Γyy, Γxy	ST_{xx}, ST_{yy}, ST_{xy} respectively [L^2/T].

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APPENDIX

Table 2. The data of matching points

	Boulton model					Neuman model				
	h^*_A(m)	t^*_A(s)	h^*_B(m)	t^*_B (s)	r/D	h^*_A(m)	t^*_A(s)	h^*_B(m)	t^*_B (s)	β
W10	0.03	9	0.03	600	0.6	0.03	8	0.03	700	0.1
SE10	0.02	5	0.023	250	0.4	0.025	7	0.025	600	0.6
NE15	0.016	12	0.02	60	0.6	0.021	11	0.021	110	0.2

Table 3. Diary of calculation of Eigenvectors and Eigenvalues in MATLAB.

```
>> t =[0.0132    0.00306
       0.00306   0.0201]  % input the component of anisotropy tensor

t =

   0.0132   0.0031

   0.0031   0.0201

>> [v,d] = eig(t)  % calculate the eigenvector and eigenvalue of t
```

v =

0.9349	0.3549
-0.3549	0.9349

d =

0.0120	0
0	0.0213

Table 4. FORTRAN program for calculating the theoretical drawdown for Boulton model.

```
******************************************************************
*  The program is used to calculate  drawdowns in unconfined
*  of Boulton's model by inversing Laplace domain solution
*  using  Stehfest's numerical inverse method.
******************************************************************
      program main
      implicit double precision (a-h,o-z)
      dimension w(18),t(200),h(200)
      dimension tm(200),hm(200),th(200),cb(200),cm(200)
      character*12 file,infile,outfile,plotfile
      integer exten

      data w/     .000049603174603175,
```

```
*         -0.609573412698412809,
*         274.594047619047614717,
*      -26306.956746031748480164,
*      957257.201388888875953853,
*   -17358694.845833331346511841,
*   182421222.647222220897674561,
* -1218533288.309126853942871094,
*  5491680025.283035278320312500,
* -17362131115.206844329833984375,
*  39455096903.527381896972656250,
* -65266516985.175003051757812500,
*  78730068328.220825195312500000,
* -68556444196.120834350585937500,
*  41984343475.053573608398437500,
* -17160934711.839284896850585938,
*  4204550039.102678775787353516,
*  -467172226.566964268684387207/

  pi=4.0d0*datan(1.0d0)

  write(*,*)'Please enter data file name (no extension name):'
  read (*,*) file
```

```
exten=index(file,' ')
infile  (1:exten)=file
infile  (exten:exten+3)='.dat'
outfile (1:exten)=file
outfile (exten:exten+3)='.out'
plotfile(1:exten)=file
plotfile(exten:exten+1)='.m'

open (5,file=infile)
open (7,file=outfile)
open (9,file=plotfile)

read(5,*) mchoice
if (mchoice .eq. 1) then
   read(5,*) n,q,x,y,ha,ta,hb,tb,rd
   read(5,*)
   t1=q/(4.0*pi*ha)
   r=dsqrt(x**2+y**2)
   s=4.0*t1*ta/r**2
   t2=q/(4.0*pi*hb)
   sy=4.0*t2*tb/r**2
   tr=0.5*(t1+t2)
```

```
   alp=rd**2/4./tb
   write(*,5)'   T   = ',tr
   write(*,5)'   Sy  = ',sy
   write(*,5)'   S   = ',s
   write(*,6)' alpha = ',alp
 elseif (mchoice .eq. 2) then
   read(5,*) n,q,x,y,tr,s,sy,alp
   r=dsqrt(y**2+x**2)
 else
   read(5,*) n,q,x,y,txx,tyy,txy,s,sy,alp
   tr=dsqrt(txx*tyy-txy**2)
   r =dsqrt((txx*y**2+tyy*x**2-2.0d0*txy*x*y)/tr)
 endif
5 format(a,f7.4)
6 format(a,e9.4)


 do i=1,n
   sum=0
   tsum=0
   read(5,*) t(i)
   p=dlog(2.0d0)/t(i)
   do j=1,18
```

```
      pj=p*j
      x=r*dsqrt(s*pj/tr+sy*alp*pj/(pj+alp)/tr)
      y=r*dsqrt(s*pj/tr)
      h(i)=q/(2.0d0*pi*tr*pj)*dbsk0(x)
      th(i)=q/(2.0d0*pi*tr*pj)*dbsk0(y)
      sum=sum+w(j)*h(i)
      tsum=tsum+w(j)*th(i)
    enddo
    h(i)=p*sum
    th(i)=p*tsum
    cb(i)=th(i)-h(i)
    cm(i)=th(i)-hm(i)
  enddo

  write(7,*)'                    ',outfile,"    Boulton model"
  write(7,*)
  write(7,300)' Q    =',q
  if (mchoice .eq. 3) then
    write(7,400)' x    =',x
    write(7,400)' y    =',y
    write(7,500)' Txx  =',txx
    write(7,500)' Tyy  =',tyy
```

```
      write(7,500)' Txy =',txy
    else
      write(7,600)' r   =',r
      write(7,600)' Tr  =',tr
    endif
    write(7,600)' S   =',s
    write(7,500)' Sy  =',sy
    write(7,600)' alp =',alp
    write(7,*)
300 format(16x,a,e7.4)
400 format(16x,a,2x,f7.1)
500 format(16x,a,f7.3)
600 format(16x,a,e10.2)
700 format(16x,a,2x,f5.1)


    write(7,100)'time','h-bton'
    sum=0.
    write(9,*)'clear'
    read (5,*) m
    do i=1,m
      read (5,*) tm(i), hm(i)
      write(9,10) 'tm(',i,')=',tm(i),';  hm(',i,')=',hm(i),';'
```

```
      enddo
 10   format(a,i3,a,f8.0,a,i3,a,f10.5,a)
      do i=1,n
        write(7,120) t(i),h(i)
        write(9,10)'tc(',i,')=',t(i),';  hc(',i,')=',h(i),';'
        sum=sum+abs(hm(i)-h(i))
      enddo
      write(9,*)"loglog(tm,hm,'o',tc,hc,'-')"
c     write(9,*)"title('Boulton type curve method - ",outfile,"')"
c     write(9,*)"xlabel('time (sec)')"
c     write(9,*)"ylabel('drawdown (m)')"
 100  format(17x,a,5x,a,5x,a,4x,a)
 110  format(11x,e12.5,6(3x,e12.5))
 120  format(11x,f10.0,3(3x,f8.5))
 130  format(15x,a,f8.2)

      write(*,*)'  The results are in the file : ',outfile
      write(*,*)'  The Matlab for plot is      : ',plotfile
      close(5)
      close(7)
      close(9)
      end
```

Table 5. FORTRAN program for calculating the theoretical drawdown for Neuman model.

```
*******************************************************************
*    This program calculate the drawdwon in unconfined aquifer of
*    Neuman model by numerical inverting Laplace transform solution
*    using Stehfest method.
*******************************************************************

      implicit double precision (a-h,k-z)
      dimension ht(500),hc(500),t(500),count(500)
      dimension tm(500),hm(500)
      dimension wi(18),x0(40),c(16),w(16)
      dimension psum(52)
      common /a1/r,tr,s,kz,b,qpi2,fhc
      common /a2/rs,ry,kd,p
      common /a3/wi,x0,c,w
      integer i,j,k,n,nres,numrl2,count,kk,exten
      character*12 file,infile,outfile,plotfile

      data wi/    .000049603174603175,
     *          -0.609573412698412809,
     *         274.594047619047614717,
     *      -26306.956746031748480164,
```

```
     *     957257.201388888875953853,
     *   -17358694.845833331346511841,
     *   182421222.647222220897674561,
     * -1218533288.309126853942871094,
     *  5491680025.283035278320312500,
     * -17362131115.206844329833984375,
     *  39455096903.527381896972656250,
     * -65266516985.175003051757812500,
     *  78730068328.220825195312500000,
     * -68556444196.120834350585937500,
     *  41984343475.053573608398437500,
     * -17160934711.839284896850585938,
     *   4204550039.102678775787353516,
     *   -467172226.566964268684387207/

      data x0/2.4048256d0,   5.5200781d0,   8.6537279d0,  11.7915344d0,
     *       14.9309177d0,  18.0710640d0,  21.2116366d0,  24.3524715d0,
     *       27.4934791d0,  30.6346065d0,  33.7758202d0,  36.9170984d0,
     *       40.0584258d0,  43.1997917d0,  46.3411884d0,  49.4826099d0,
     *       52.6240518d0,  55.7655108d0,  58.9069839d0,  62.0404692d0,
     *       65.1899648d0,  68.3314693d0,  71.4729816d0,  74.6145006d0,
     *       77.7560256d0,  80.8975559d0,  84.0390908d0,  87.1806298d0,
```

```
*     90.3221726d0,  93.4637188d0,  96.6052680d0,  99.7468199d0,
*     102.8883743d0, 106.0299309d0, 109.1714896d0, 112.3130503d0,
*     115.4546127d0, 118.5961766d0, 121.7377421d0, 124.8793089d0/

 data c / .49863193092474078D0,  .49280575577263417D0,
*        .48238112779375322D0,  .46745303796886984D0,
*        .44816057788302606D0,  .42468380686628499D0,
*        .39724189798397120D0,  .36609105937014484D0,
*        .33152213346510760D0,  .29385787862038116D0,
*        .25344995446611470D0,  .21067563806531767D0,
*        .16593430114106382D0,  .11964368112606854D0,
*        .7223598079139825D-1,  .24153832843869158D-1/

 data w / .35093050047350483D-2, .8137197365452835D-2,
*        .12696032654631030D-1, .17136931456510717D-1,
*        .21417949011113340D-1, .25499029631188088D-1,
*        .29342046739267774D-1, .32911111388180923D-1,
*        .36172897054424253D-1, .39096947893535153D-1,
*        .41655962113473378D-1, .43826046502201906D-1,
*        .45586939347881942D-1, .46922199540402283D-1,
*        .47819360039637430D-1, .48270044257363900D-1/
```

```
kk=5
pi=4.0d0*datan(1.0d0)
log2 = dlog(2.0d0)

write(*,*)'    please give data file name (no extension name):'
read (*,*) file

exten=index(file,' ')
infile  (1:exten)=file
infile  (exten:exten+3)='.dat'
outfile (1:exten)=file
outfile (exten:exten+3)='.out'
plotfile(1:exten)=file
plotfile(exten:exten+1)='.m'

open (5,file=infile)
open (7,file=outfile)
open (9,file=plotfile)

read(5,*)mchoice
if (mchoice .eq. 1) then
   read(5,*) n,q,x,y,b,ha,ta,hb,tb,gama
```

```
  t1=q/(4.0d0*pi*ha)
  r=dsqrt(x**2+y**2)
  s=4.0d0*t1*ta/r**2
  t2=q/(4.0d0*pi*hb)
  sy=4.0d0*t2*tb/r**2
  tr=0.5d0*(t1+t2)
  kz=gama*tr*b/r**2
  write(*,5)'   T  = ',tr
  write(*,5)'   S  = ',s
  write(*,6)'   Sy = ',sy
  write(*,6)'   Kz = ',kz
elseif(mchoice .eq. 2) then
  read (5,*) n,q,x,y,tr,s,sy,kz,b
  r=dsqrt(x**2+y**2)
else
  read(5,*) n,q,x,y,txx,tyy,txy,s,sy,kz,b
  tr=dsqrt(txx*tyy-txy**2)
  r =dsqrt((txx*y**2+tyy*x**2-2.0d0*txy*x*y)/tr)
  write(7,*)'          ',outfile,"     Neuman's model"
  write(7,*)
  write(7,300)' Q  =',q
  write(7,400)' x  =',x
```

```
      write(7,400)' y   =',y
      write(7,400)' Txx=',txx
      write(7,400)' Tyy=',tyy
      write(7,500)' Txy=',txy
      write(7,600)' S   =',s
      write(7,500)' Sy  =',sy
      write(7,600)' Kz  =',kz
      write(7,700)' b   =',b
    endif
    if (mchoice .eq. 3) goto 7
      write(7,300)' Q   =',q
      write(7,400)' r   =',r
      write(7,500)' Tr=',tr
      write(7,600)' S   =',s
      write(7,500)' Sy  =',sy
      write(7,600)' Kz  =',kz
      write(7,700)' b   =',b

5   format(a,f7.4)
6   format(a,f9.6)
7   qpi2 = q/2.0d0/pi
    rs=tr/s
```

```
ry=kz/sy
kd=kz*b/tr
fhc=qpi2/b**2/kz/r**2

do i=1,n
   read (5,*) t(i)
   write(*,*) 'runing at...time: ',t(i)
   p=log2/t(i)
   call stehfest(0,1,f)
   ht(i)=f
   xl=0
   yy=0
   nres=0
   numrl2=0
   presult=0
   count(i)=0
   do j=1,kk
      count(i)=j
      xu=x0(j)
      a=0.5d0*(xu+xl)
      bb=xu-xl
      y=0.d0
```

```
            do k=1,16
              ptl=a-c(k)*bb
              ptr=a+c(k)*bb
              y=y+w(k)*(gfunc(ptl)+gfunc(ptr))
            enddo
            y=y*bb
            yy=yy+y
            if (abs(y) .lt. 1e-12) go to 20
            xl=xu
            if (j .lt. 2) go to 10
            numrl2=numrl2+1
            psum(numrl2)=yy
            if (numrl2.lt.3) go to 10
            call dqelg(numrl2,psum,yy,abseps,res3la,nres)
            if (abseps .lt. 1.0d-12) go to 20
10        enddo
20        hc(i)=yy
        enddo

        write(7,*)
        write(7,100)'time','h-cul'
        write(9,*)'clear'
```

```
      read (5,*) m
      do i=1,m
        read (5,*) tm(i), hm(i)
        write(9,9) 'tm(',i,')=',tm(i),';  hm(',i,')=',hm(i),';'
      enddo
 9    format(a,i3,a,f8.0,a,i3,a,f10.5,a)


      sum=0.
      t(1)=5.0
      do i=1,n
c         write(7,200)t(i),ht(i)-hc(i),ht(i),hc(i),count(i)
        write(7,222) t(i),ht(i)-hc(i)
        write(9,9) 'tc(',i,')=',t(i),';  hc(',i,')=',ht(i)-hc(i),';'
        er=hm(i)-ht(i)+hc(i)
        sum=sum+abs(er)
      enddo
      write(9,*)"loglog(tm,hm,'o',tc,hc,'-')"
c      write(9,*)"title('Neuman type curve method - ",outfile,"')"
c      write(9,*)"xlabel('time (sec)')"
c      write(9,*)"ylabel('drawdown (m)')"
 100  format(15x,a,7x,a,6x,a,6x,a)
 200  format(11x,e8.2,3(3x,f12.2),6x,i3)
```

```
222 format(11x,f9.0,3(3x,f10.5))
300 format(16x,a,f7.3)
400 format(16x,a,2x,f6.0)
500 format(16x,a,f7.3)
600 format(16x,a,e10.2)
700 format(16x,a,2x,f5.1)
800 format(16x,a)
      write(*,*)'  The results are in the file : ',outfile
      write(*,*)'  The the Matlab file for plot: ',plotfile
      close(5)
      close(7)
      close(9)
      end
*************************************************************************
*
      subroutine stehfest(x,i,h)
      implicit double precision (a-h,k-z)
      dimension wi(18),x0(40),c(16),w(16)
      common /a1/r,tr,s,kz,b,qpi2,fhc
      common /a2/rs,ry,kd,p
      common /a3/wi,x0,c,w
      external gfunc,lintg
```

```
      sum=0
      pj=0
      if (i .eq. 1) then
        do j=1,18
          pj=pj+p
          xx=r*dsqrt(s*pj/tr)
          th=qpi2*dbsk0(xx)/tr/pj
          sum=sum+wi(j)*th
        enddo
      else
        do j=1,18
          pj=pj+p
          sum=sum+wi(j)*lintg(x,pj)
        enddo
      endif
      h=p*sum
      return
      end

*************************************************************************

c  integrand function
      function gfunc(x)
```

```
      implicit double precision (a-h,k-z)
      dimension wi(18),x0(40),c(16),w(16)
      common /a1/r,tr,s,kz,b,qpi2,fhc
      common /a2/rs,ry,kd,p
      common /a3/wi,x0,c,w

      call stehfest(x,3,f)
      gfunc=fhc*x*dbsj0(x)*f
      return
      end
**************************************************************************
c  the part of integrand in Laplace form
      function lintg(x,pj)
      implicit double precision (a-h,k-z)
      dimension wi(18),x0(40),c(16),w(16)
      common /a1/r,tr,s,kz,b,qpi2,fhc
      common /a2/rs,ry,kd,p
      common /a3/wi,x0,c,w

      n=dsqrt((x/r)**2/kd+pj/rs/kd)
      lintg=1.0d0/(ry*n**4+pj/dtanh(n*b)*n**3)
      return
```

```
      end
***********************************************************************

      subroutine dqelg(n,epstab,result,abserr,res3la,nres)
      double precision abserr,dabs,delta1,delta2,delta3,dmax1,d1mach,
     * epmach,epsinf,epstab,error,err1,err2,err3,e0,e1,e1abs,e2,e3,
     * oflow,res,result,res3la,ss,tol1,tol2,tol3
      integer i,ib,ib2,ie,indx,k1,k2,k3,limexp,n,newelm,nres,num
      dimension epstab(52),res3la(3)

      epmach = d1mach(4)
      oflow = d1mach(2)
      nres = nres+1
      abserr = oflow
      result = epstab(n)
      if(n.lt.3) go to 100
      limexp = 50
      epstab(n+2) = epstab(n)
      newelm = (n-1)/2
      epstab(n) = oflow
      num = n
      k1 = n
```

```
      do 40 i = 1,newelm
        k2 = k1-1
        k3 = k1-2
        res = epstab(k1+2)
        e0 = epstab(k3)
        e1 = epstab(k2)
        e2 = res
        e1abs = dabs(e1)
        delta2 = e2-e1
        err2 = dabs(delta2)
        tol2 = dmax1(dabs(e2),e1abs)*epmach
        delta3 = e1-e0
        err3 = dabs(delta3)
        tol3 = dmax1(e1abs,dabs(e0))*epmach
        if(err2.gt.tol2.or.err3.gt.tol3) go to 10
        result = res
        abserr = err2+err3
        go to 100
10      e3 = epstab(k1)
        epstab(k1) = e1
        delta1 = e1-e3
        err1 = dabs(delta1)
```

```
      toll = dmax1(e1abs,dabs(e3))*epmach
      if(err1.le.toll.or.err2.le.tol2.or.err3.le.tol3) go to 20
      ss = 0.1d+01/delta1+0.1d+01/delta2-0.1d+01/delta3
      epsinf = dabs(ss*e1)
      if(epsinf.gt.0.1d-03) go to 30
20     n = i+i-1
      go to 50
30     res = e1+0.1d+01/ss
      epstab(k1) = res
      k1 = k1-2
      error = err2+dabs(res-e2)+err3
      if(error.gt.abserr) go to 40
      abserr = error
      result = res
40   continue
50   if(n.eq.limexp) n = 2*(limexp/2)-1
    ib = 1
    if((num/2)*2.eq.num) ib = 2
    ie = newelm+1
    do 60 i=1,ie
      ib2 = ib+2
      epstab(ib) = epstab(ib2)
```

```
      ib = ib2
60   continue
     if(num.eq.n) go to 80
     indx = num-n+1
     do 70 i = 1,n
        epstab(i)= epstab(indx)
        indx = indx+1
70   continue
80   if(nres.ge.4) go to 90
     res3la(nres) = result
     abserr = oflow
     go to 100
90   abserr = dabs(result-res3la(3))+dabs(result-res3la(2))
    *     +dabs(result-res3la(1))
     res3la(1) = res3la(2)
     res3la(2) = res3la(3)
     res3la(3) = result
100  abserr = dmax1(abserr,0.5d+01*epmach*dabs(result))
     return
     end
************************************************************************
```

```
DOUBLE PRECISION FUNCTION D1MACH(I)

INTEGER SMALL(2)
INTEGER LARGE(2)
INTEGER RIGHT(2)
INTEGER DIVER(2)
INTEGER LOG10(2)
INTEGER SC
DOUBLE PRECISION DMACH(5)

EQUIVALENCE (DMACH(1),SMALL(1))
EQUIVALENCE (DMACH(2),LARGE(1))
EQUIVALENCE (DMACH(3),RIGHT(1))
EQUIVALENCE (DMACH(4),DIVER(1))
EQUIVALENCE (DMACH(5),LOG10(1))

DATA SMALL(1),SMALL(2) /    1048576,          0 /
DATA LARGE(1),LARGE(2) / 2146435071,         -1 /
DATA RIGHT(1),RIGHT(2) / 1017118720,          0 /
DATA DIVER(1),DIVER(2) / 1018167296,          0 /
DATA LOG10(1),LOG10(2) / 1070810131, 1352628735 /, SC/987/
```

```
      IF (SC .NE. 987) STOP 779
      IF (DMACH(4) .GE. 1.0D0) STOP 778
      IF (I .LT. 1  .OR.  I .GT. 5) GOTO 999
      D1MACH = DMACH(I)
      RETURN
  999 WRITE(*,1999) I
 1999 FORMAT(' D1MACH - I OUT OF BOUNDS',I10)
      STOP
      END
      INTEGER FUNCTION I1MACH(I)
      INTEGER IMACH(16),OUTPUT,SANITY
      EQUIVALENCE (IMACH(4),OUTPUT)

      DATA IMACH( 1) /    5 /
      DATA IMACH( 2) /    6 /
      DATA IMACH( 3) /    7 /
      DATA IMACH( 4) /    6 /
      DATA IMACH( 5) /   32 /
      DATA IMACH( 6) /    4 /
      DATA IMACH( 7) /    2 /
      DATA IMACH( 8) /   31 /
      DATA IMACH( 9) / 2147483647 /
```

```
      DATA IMACH(10) /    2 /
      DATA IMACH(11) /   24 /
      DATA IMACH(12) / -125 /
      DATA IMACH(13) /  128 /
      DATA IMACH(14) /   53 /
      DATA IMACH(15) / -1021 /
      DATA IMACH(16) /  1024 /, SANITY/987/
      IF (SANITY .NE. 987) STOP 777
      IF (I .LT. 1  .OR.  I .GT. 16) GO TO 10
      I1MACH = IMACH(I)
      IF(I.EQ.6) I1MACH=1
      RETURN
   10 WRITE(OUTPUT,1999) I
 1999 FORMAT(' I1MACH - I OUT OF BOUNDS',I10)
      STOP
      END
```