A CRITICAL EVALUATION OF BOREHOLE PERMEAMETER SOLUTIONS

by

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NOMENCLATURE

SYMBOL	UNITS
A – Length of screened interval	(L)
b – Length before screened interval	(L)
b_1 – Length before screened interval test 1	(L)
b_2 – Length before screened interval test 2	(L)
d_{10} – grain diameter in millimeters such that 10% of the sample are finer and 90% of the sample are coarser	(L)
H – Hydraulic head	(L)
H_1 – Hydraulic head test 1	(L)
H_2 – Hydraulic head test 2	(L)
H_D – Dimensionless head	(L)
K_r – Relative hydraulic conductivity	(L/t)
K_s – Saturated hydraulic conductivity	(L/t)
$K(\Psi)$ – Hydraulic conductivity as a function of Ψ	(L/t)
ln – Natural log	(–)
R – Radial distance from borehole	(L)
R_m – Maximum saturated bulb radius	(L)
r – borehole radius	(L)
S – Sorptivity	($L/\sqrt{t}$)
Q_d – Dimensionless steady flow rate into the borehole	(–)
Q_r – Ratio of Q_{s1} over Q_{s2}	(–)
Q_s – Steady flow rate into the borehole	(L^3/t)
Q_{s1} – Steady flow rate into the borehole test 1	(L^3/t)

SYMBOL	UNITS
Q_{s2} – Steady flow rate into the borehole test 2	(L^3/t)
Z – Distance down from water level in borehole	(L)
Z_m – Maximum saturated bulb depth	(L)
a – α–parameter of the hydraulic conductivity–pressure head relationship	(L^{-1})
a_p – α–parameter of the hydraulic conductivity–pressure head relationship as defined by Philip	(L^{-1})
a_r – α–parameter of the hydraulic conductivity–pressure head relationship as defined by Reynolds	(L^{-1})
a_s – α–parameter of the hydraulic conductivity–pressure head relationship as defined by Stephens	(L^{-1})
ϕ_m – Matric flux potential	(L^2/t)
η – Eccentricity of the saturated bulb	(–)
λ_c – Length of the capillary fringe	(L)
θ – Moisture content	(L^3/L^3)
θ_i – Initial moisture content	(L^3/L^3)
θ_f – Final moisture content	(L^3/L^3)
Ψ – Pressure head	(L)
Ψ_b – Break through pressure head	(L)

ABSTRACT

Determining the hydraulic parameters of the vadose zone is becoming increasingly important in many hydrologic and engineering studies. Currently borehole infiltration tests are the only method available for determining in-situ saturated hydraulic conductivity and related soil parameters at any depth within the vadose zone. There are many steady state models used to determine saturated hydraulic conductivity from constant head borehole infiltration tests. In this study the single head borehole permeameter solutions of Glover (1953), Stephens (1979), Stephens et al. (1987), Reynolds et al. (1986), and Philip (1985) and the duel-head borehole permeameter solution of Reynolds et al. (1986) are evaluated to determine prediction capabilities in the vadose zone.

Computer simulation was used to evaluate the sensitivity of the single head solutions of Glover (1953), Stephens (1979), Stephens et al. (1987), and Philip (1985) and the duel-head solution of Reynolds et al. (1986) to input parameters in determining in-situ saturated hydraulic conductivity in the vadose zone. Field data from 27 borehole infiltration tests conducted in a medium textured, uniform sand was used to further evaluate the solutions.

Statistical evaluation demonstrated that the five single head solutions and the duel-head solution essentially predict the same mean saturated hydraulic conductivity for the Sevilleta site. The mean saturated hydraulic conductivity predicted using borehole permeameter solutions was less than that predicted using other methods.

Use of the single head borehole permeameter solutions are preferred over the duel-head borehole permeameter solution of Reynolds et al. (1986) when determining saturated hydraulic conductivity in the vadose zone. It was concluded from this study that

the variance associated with the duel head borehole permeameter solution of Reynolds et al. (1986) is greater than the variances of the single head borehole permeameter solutions and thus results in a wider range of predicted saturated hydraulic conductivities.

It was concluded that the duel-head solution of Reynolds et al. (1986) does not accurately predict capillary properties of the soil. The Philip solution does not accurately predict the geometry of the saturated bulb surrounding the borehole during borehole permeameter tests. As a consequence of the importance of capillarity on flow rate from the borehole, the Reynolds et al. duel-head solution and the Philip solution may not produce reliable estimates of saturated hydraulic conductivity.

CHAPTER 1

INTRODUCTION

Determining hydraulic parameters of the vadose zone is becoming increasingly necessary when solving many agricultural, hydrological, and environmental problems. Quantifying groundwater recharge and characterizing the movement of pollutants through the vadose zone is of concern to water resource and environmental agencies. Many of the models used to make predictions about water and solute transport require accurate estimation of the hydraulic parameters and coefficients of the porous media. Some of the hydraulic parameters and coefficients of interest in the vadose zone are saturated hydraulic conductivity (K_s), matric flux potential (ϕ_m), soil sorptivity (S), and the α–parameter (α) of the exponential hydraulic conductivity–pressure head relationship (Gardner, 1958). Often K_s is considered the most important of these properties. The borehole permeameter test, or shallow well pump–in method is a procedure for determining hydraulic parameters and coefficients of an insitu porous media, in particular K_s, at any depth, in the absence of a water table.

For the borehole permeameter test an auger hole is drilled to a desired depth. Well screen, but sometimes gravel, is emplaced in the borehole if caving of the borehole is a concern. Water is added at a rate necessary to maintain a constant depth of water (H) in the borehole. The test is complete when a final steady infiltration rate (Q_s) is reached.

The original steady state analytical solutions for borehole permeameter tests are based on free surface theory and include Glover (1953), Nasberg–Terletskaya (1951), Zanger (1953), and Cornwell (1953). The free surface is the outer boundary of the flow field along which the pressure is equal to atmospheric. Within the free surface region flow is radial and downward in response to pressure and gravity gradients. Inside the free surface region the soil is assumed to be completely saturated and outside the free surface region the soil is assumed to be completely dry. Solutions that are based on the existence of a free surface ignore capillarity and do not accurately predict insitu K_s (Philip, 1985).

However, it was recognized that Q_s for borehole permeameter tests depended not only on K_s but also on the capillary properties of the soil. A second generation of steady state analytical solutions for borehole permeameter tests which account for the effects of capillarity were recently developed by Stephens (1979), Stephens et al. (1987), Philip (1985), and Reynolds et al. (1986). These solutions are based on the premise that the flow field is completely saturated only near a small area close to the borehole (Philip, 1968, 1969; Stephens and Neuman, 1982b, c; Stephens et al., 1983a, b). A bulb-shaped region of saturated soil located adjacent to, and extending below, the borehole exists. The geometry of the saturated bulb is directly related to water depth H in the borehole, the radius of the borehole (r), and the capillary properties of the soil. With recognition of the influence of capillarity, more accurate predictions of insitu K_s can be made.

In addition to predicting K_s from borehole permeameter tests, the solution of Reynolds et al. (1986) allows for the determination of α and S by solving simultaneous equations relating a measured Q_s and H with another measured Q_s and H in the same borehole. It is also possible to apply the multi-head approach to determine α using other models such as Stephens (1979) and Stephens et al. (1987). Reynolds et al. (1986) claim their approach yields accurate determination of the capillary properties.

When tensiometers can be used to map the hydraulic head field surrounding a borehole a flow net method allows for determination of capillary properties from a single head borehole permeameter test (Stephens, 1985).

Philip's (1985) solution predicts the shape of the saturated bulb surrounding the borehole during constant head borehole permeameter tests when α is known. To do this Philip proposed simultaneously solving equations relating flow in the saturated bulb to flow in the unsaturated region surrounding the bulb. Currently this is the only solution that can be used to determine the geometry of the saturated bulb surrounding the borehole.

SCOPE OF INVESTIGATION

There are major differences in the approaches used by Stephens (1979), Stephens et al. (1987), Reynolds et al. (1986), and Philip (1985) in accounting for the effects of capillarity in determining K_s. The solutions of Stephens (1979) and Stephens et al. (1987) for K_s are based on numerical simulation of fully saturated–unsaturated flow, coupled with multiple linear regression analysis. The Reynolds et al. solution is analytical, although the C-parameter quantifying flow from the borehole can be determined numerically as well as analytically. The Reynolds et al. solution combines the influence of the inner saturated zone with the outer unsaturated envelope and the initial pressure head in determining an expression for K_s. Philip's solution is quasi-analytical and accounts for both saturated and unsaturated conditions in determining K_s.

The purpose of this study is to critically evaluate the reliability of Glover's (1953) solution, Stephens' (1979) solution, Stephens et al's (1987) solution, Reynolds et al.'s (1986) solution, and Philip's (1985) solution for predicting K_s from borehole permeameter tests. The study will also examine the accuracy and reliability of the Reynolds et al. solution for determining S and α of the porous media and Philip's solution for predicting the geometry of the saturated bulb surrounding the borehole.

Chapter 2 contains detailed descriptions of Glover's (1953), Stephens (1979), Stephens et al. (1987), Philip's (1985), and Reynolds et al.'s (1986) borehole permeameter solutions as well as a discussion of the methodology used in evaluating the solutions. The evaluation of each model requires determining the sensitivity of the solution to borehole geometry and capillary parameters on dimensionless flow rate (Q_d) into the soil. For instance, the sensitivity of Philip's solution α to in determining saturated bulb geometry will be examined. Also, the solution by Reynolds et al. (1986) is examined to identify the sensitivity of Q_s's to K_s, α, and S for a given value of H. A discussion of the results of the sensitivity analysis is presented in Chapter 3. In Chapter 4 K_s will be determined using the four previously mentioned steady-state closed-form analytical expressions with field data obtained from constant head borehole permeameter tests at the Sevilleta National Wildlife Refuge (Watson, 1983). The saturated bulb geometry as determined by Philip's solution will be compared to field measured saturated bulb geometry. The Reynolds et al. solution will be used to determine α and S from field data when possible. Conclusions and recommendations are discussed in Chapters 5 and 6 respectively. The Appendices contain the computer codes used to evaluate the solutions and the computer codes associated outputs.

CHAPTER 2

METHODS OF ANALYSIS

Capillarity is characterized by a relative hydraulic conductivity–pressure head ($K_r - \Psi$) relationship that is unique for each soil. Relative hydraulic conductivity (K_r) is the ratio of hydraulic conductivity at a given pressure head, $K(\Psi)$, to K_s. When Ψ is greater than or equal to atmospheric pressure, K_r equals one. When Ψ is less than atmospheric, as is expected in the vadose zone, K_r ranges from one to perhaps 10^{-6} or less. To account for capillarity in borehole permeameter solutions in a simple manner the $K_r - \Psi$ relationship is often reduced to one or two index parameters. Often the symbol α, representing the exponential hydraulic conductivity–pressure head relationship (Gardner, 1958), is used to denote capillary properties. Different researchers have different methods of computing α. Other characteristic parameters are used to represent capillarity and include sorptivity (S), length of the capillary fringe (λ_c), air entry pressure head (Ψ_b), as well as the parameters of van Genuchten (1980) obtained by a fit to moisture retention data.

The chapter begins with a discussion of the determination of α. Following this the steady–state, analytical, constant–head borehole permeameter solutions of Glover (1953), Stephens (1979), Stephens et al. (1987), Reynolds et al. (1986), and Philip (1985) are described in detail. For simpicity the Reynolds et al. solution will be refered to as the Reynolds solution from this point on.

DETERMINATION OF THE α-PARAMETER OF THE EXPONENTIAL HYDRAULIC CONDUCTIVITY-PRESSURE HEAD RELATIONSHIP

The exponential hydraulic conductivity-pressure head relationship (Gardner, 1958) is an inherent characteristic of soil that reflects the capillary properties of that soil. The a value for a soil is approximately equal to the inverse of the thickness of the capillary fringe (λ_c) of that soil (White and Sully, 1988). In fine soils with strong capillary effects a tends to be small, and in coarse soils with capillary effects less important a tends to be large. White and Sully (1988) and Philip (1985) state that for a wide range of soils it is not unreasonable to suggest that a range between 0.0 and 10.0 m^{-1}, with 5.0 m^{-1} a typical value. However Talsma (1987) states that he is not aware of any reported values of a less than 1.0 m^{-1}. Stroosnijder (1976) listed values of twenty Dutch soils ranging from 1.7 to 22.4 m^{-1}. Scotter et al. (1982) measured a values between 2.0 and 90.0 m^{-1}. For this study the solutions that considered capillarity were evaluated for a between 1.0 and 10.0 m^{-1}. Table 2.1 shows a_s values computed by Stephens et al. (1987) for various soils which are described in a catalog of soils by Mualem (1976).

Although it is universally recognized that a represents a parameter specifying the capillary properties of the soil, the definition of a is not universally agreed upon. Stephens (1979) defines a as the slope of the $\ln(K_r) - \Psi$ curve taken between K_r equals 1.0 and K_r equals 0.5. Stephens' a will be denoted a_s. Reynolds' (1986) defines a as the slope of the $\ln(K_r) - \Psi$ curve and will be denoted a_r. Philip (1985) defines a as the inverse of the area under the $K_r - \Psi$ curve, from Ψ equal zero to infinity, and will be denoted a_p.

Table 2.1: α Values for Various Soils

Soil	Mualem Catalog Number	α_s m^{-1}
Del Monte fine sand	4108	1.2
Yoly light clay	3102	4.0
Silt loam G.E. 3	3310	1.0
Coarse sand	4107	4.6
Gilat loam	3402	1.8
Ida silt loam	3305	198.0

from Stephens et al. (1987)

For a soils, those soils where the $\ln(K_r) - \Psi$ curve is a straight line, a_s, a_r and a_p will be equal. Figure 2.1 shows $\ln(K_r) - \Psi$ and $K_r - \Psi$ curves for an " a soil", approximated from an exponential model, where a_s, a_r and a_p equal 5.0 m^{-1}. Figure 2.2 shows $\ln(K_r) - \Psi$ and $K_r - \Psi$ curves for a "non- a soil", the Sevilleta sand. The Sevilleta sand is typical of most soils in that the $\ln(K_r) - \Psi$ curve is not a straight line. The values of a_s, a_r, and a_p were calculated to be 5.6 m^{-1}, 8.0 m^{-1}, and 9.0 m^{-1} respectively. Figure 2.2 demonstrates that the same soil can be characterized by a_s, a_r, and a_p of different values.

In practice determination of a is not always easy. The slope of the $\ln(K_r) - \Psi$ and $K_r - \Psi$ curves can be quite variable. a could be determined from the slope at the wet end, middle, or dry end of the $\ln(K_r) - \Psi$ curve. Upon linearization of the $\ln(K_r) - \Psi$ curve, K_s may no longer be on the linearized curve; that is, the intercept at Ψ equal zero may not always be at K_r equal to one.

SOLUTIONS TO BOREHOLE PERMEAMETER TESTS

Glover Solution

The solution of Glover (1953) is obtained by superimposing a series of vertically aligned point sources on a gravity flow field. The point source strengths are assumed to increase in strength linearly with depth, simulating the increase of hydrostatic pressure head in the borehole. The effects of capillarity are ignored. For Glover's solution :

$$K_s = \frac{Q_s}{rH}\left[\frac{2\pi(H_D)}{\sinh^{-1}(H_D) - 1}\right]^{-1} \tag{2.1}$$

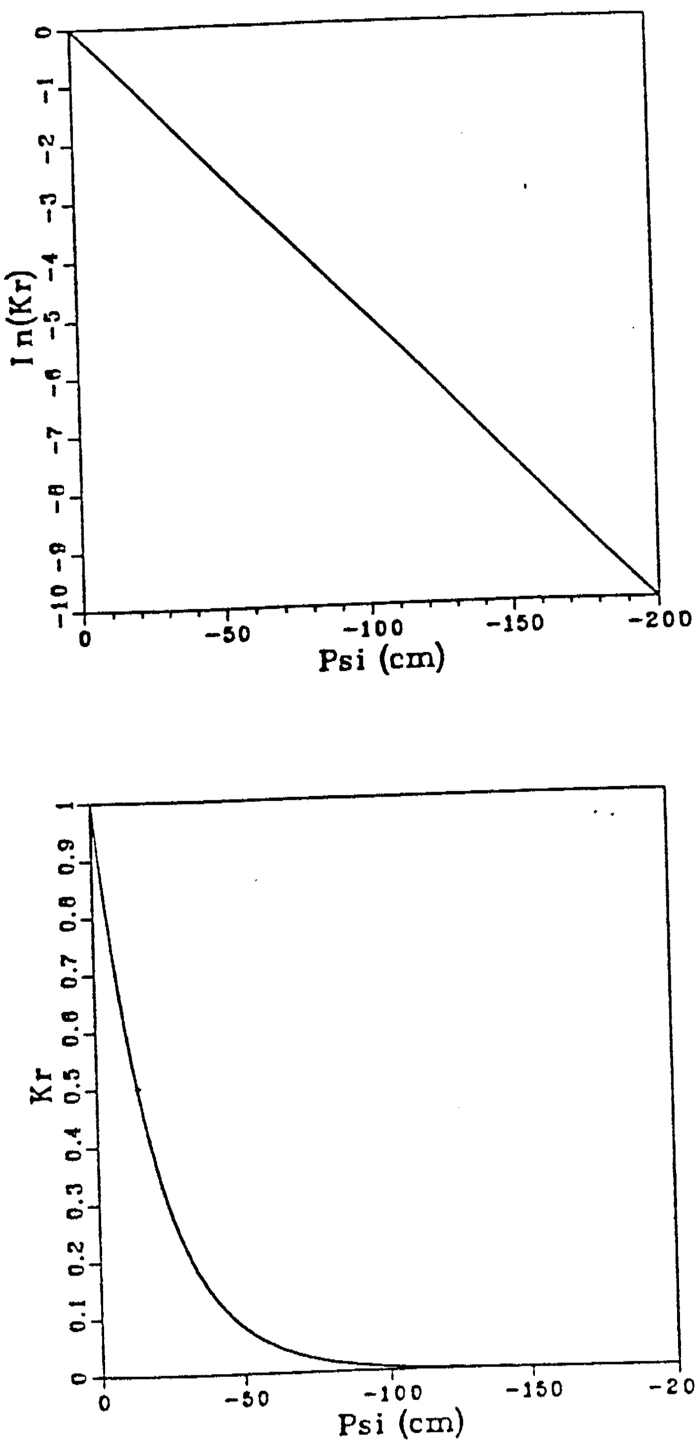

Figure 2.1. $ln(K_r)-\Psi$ and $K_r-\Psi$ curves for an a soil where a_s, a_r, and a_p equal 5.0 m^{-1}.

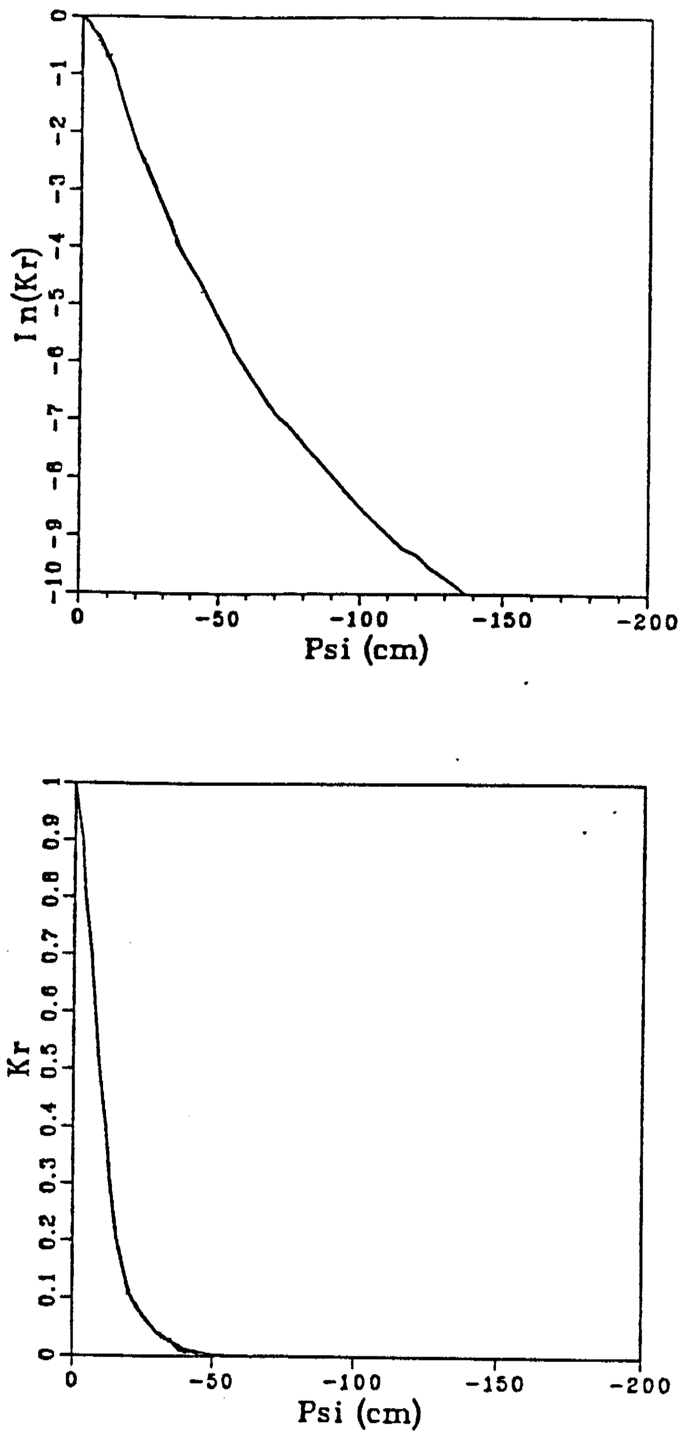

Figure 2.2. $ln(K_r)-\Psi$ and $K_r-\Psi$ curves for an non α soil where α_s equals 5.6 m^{-1}, α_r equals 8.0 m^{-1}, and α_p equal 9.0 m^{-1}.

Interms of dimensionless flow rate, (Q_d), where $Q_d = Q_s/K_s rH$:

$$Q_d = \frac{2\pi(H_D)}{\sinh^{-1}(H_D) - 1} \tag{2.2}$$

where: H_D = dimensionless height of water in the borehole = H/r [L/L]

H = height of water in borehole [L]

r = borehole radius [L]

K_s = saturated hydraulic conductivity [L/t]

Q_s = steady infiltration rate [L^3/t]

Q_d= dimensionless infiltration rate [L^3/L^3]

Note Q_d equals C_u of Stephens (1979), Stephens et al. (1987), and Glover (1953).

For a detailed examination and discussion of this solution see Stephens (1979) or Stephens and Neuman (1982a).

Stephens I and Stephens II Solutions

The Stephens (1979) solution and Stephens et al. (1987) solution, refered to as the Stephens I and Stephens II solutions respectively, are based upon the results of finite element and integrated finite difference computer modeling techniques. Using the results of numerical simulations, a multiple linear regression analysis was applied to derive empirical relationships between the dimensionless flow rate, Q_d, borehole geometry factors H and r, and various α_s values characterizing the $K_r - \Psi$ relationship.

The Stephens I solution is:

$$K_s = \frac{Q_s}{rH} 10^{-[0.658 \log(H_D) - 0.238\sqrt{a_s} - 0.398 \log H + 1.342]} \tag{2.3}$$

and:

$$Q_d = 10^{[0.658 \log(H_D) - 0.238\sqrt{a_s} - 0.398 \log H + 1.342]} \tag{2.4}$$

where a_s is in m^{-1} and H and r are in m.

The 95% confidence limits on the coefficients of the empirical relationship between Q_d, H, r, and a_s are plus or minus 0.029, 0.029, 0.051, and 0.063 respectively (Stephens ,1979). In this work, values of a_s ranged from about 1.0 m^{-1} to 4.6 m^{-1}(Table 2.1).

Stephens et al. (1987) added two more soil types to the multiple linear regression analysis to obtain a more broadly applicable solution than that of equation 2.2. Values of a_s representing the Gilat loam and Ida silt loam are 1.8 $m^{-1}$and 198.0 m^{-1}, respectively (Table 2.1). The Stephens II solution is:

$$K_s = \frac{Q_s}{rH} 10^{-[0.486 \log(H_D) + 0.4 a_s - 0.454 \log H + 0.019\sqrt{H_D} + 0.828]} \qquad R^2 = 0.983 \tag{2.5}$$

and:

$$Q_d = 10^{[0.486 \log(H_D) + 0.4 a_s - 0.454 \log H + 0.019\sqrt{H_D} + 0.828]} \tag{2.6}$$

where a_s is in m^{-1} and H and r are in m.

The 95% confidence limits on the coefficients of the empirical relationship between Q_d, H, r, and a_s are plus or minus 0.114, 0.086, 0.061, 0.073, and 0.008 respectively (Stephens et al., 1979).

For the Stephen I and Stephen II solutions:

H_D = dimensionless height of water in the borehole = H/r [L/L]

H = height of water in borehole [L]

r = borehole radius [L]

K_s = saturated hydraulic conductivity [L/t]

Q_s = steady infiltration rate [L^3/t]

Q_d = dimensionless infiltration rate [L^3/L^3]

a_s = α-parameter of the $ln(K_r) - \Psi$ relationship [1/L]

An important observation concerning Stephens' solution is that a_s must be obtained through some independent measurement.

Stephens et al. (1987) also developed a similar solution to equations 2.3 and 2.5 in terms of the van Genuchten/Mualem parameters a_v and N, which are obtained from $\theta - \Psi$ curves. For a more detailed discussion of the borehole permeameter solutions obtained by numerical simulation and regression analysis, refer to Stephens et al. (1987)

Reynolds et al. Solution

The basis of the Reynolds et al. (1986) solution for constant head borehole permeameter tests is that steady flow out of the borehole can be seperated into pressure and gravity induced fluxes. The affects of gravity and capillary on flow from the borehole are assumed to be additive. This approach contrasts sharply with Stephens (1979) multiple linear regression analysis where pressure and gravity affects are assumed to be inseperable and hence non-additive. A detailed derivation of the Reynolds et al. (1986) solution is contained in Appendix L. To avoid repetition, only the equations derived by Reynolds et al. (1986), for determining K_s , a_r, and S , from single and duel-head borehole permeameter tests, will be presented in this section. For a more detailed discussion of the Reynolds single and duel-head borehole permeameter solutions see Reynolds et al. (1983) and Reynolds et al. (1986).

Reynolds et al. (1986) found for the single head borehole permeameter solution that:

$$K_s = \frac{CQ_s - 2\pi H\phi_m}{2\pi H^2\left[1 + \frac{C}{2}(H_D)^{-2}\right]} \tag{2.7}$$

where:
$$\phi_m = \frac{S^2}{\Delta\theta} \tag{2.8}$$

$$a_r = \frac{2\pi H K_s}{CQ_s - 2H^2K_s\left[1 + \frac{C}{2}(H_D)^{-2}\right]} \tag{2.9}$$

$$S = \frac{\Delta\theta\left[CQ_s - 2\pi H^2K_s\left[1 + \frac{C}{2}(H_D)^{-2}\right]\right]^{\frac{1}{2}}}{\pi H} \tag{2.10}$$

$$C = \frac{H^2\left[\frac{(H-b)}{H}\sinh^{-1}\left[\frac{(H-b)}{r}\right] - \sqrt{(H_D)^{-2} + \left[\frac{(H-b)}{H}\right]^2} + H_D^{-1}\right]}{(H-b)^2} \tag{2.11}$$

Notice that to determine K_s , a_r, and S using the Reynolds single head borehole permeameter solution, ϕ_m must be determined independently.

By maintaining two depths of water, H_1 and H_2, in a borehole of constant r , two steady flow rates, Q_{s1} and Q_{s2}, can be obtained. By assuming K_s , a_r, and S are constant in the region surrounding the borehole the two H's and Q_s's can be used to write two simultaneous equations which can be evaluated to obtain the saturated hydraulic conductivity and capillary parameters. Reynolds et al. (1986) found for the duel–head borehole permeameter test that:

$$K_s = G_2 Q_{s2} - G_1 Q_{s1} \tag{2.12}$$

$$a_r = \frac{M_2 Q_{s2} - M_1 Q_{s1}}{N_2 Q_{s2} - N_1 Q_{s1}} \tag{2.13}$$

$$S = \sqrt{\Delta\theta(J_2 Q_{s2} - J_1 Q_{s1})} \tag{2.14}$$

where:

$$G_1 = \frac{H_2 C_1}{\pi[2H_1H_2(H_2 - H_1) + r^2(H_1C_2 - H_2C_1)]} \tag{2.15}$$

$$G_2 = \frac{H_1 C_2}{\pi[2H_1H_2(H_2 - H_1) + r^2(H_1C_2 - H_2C_1)]} \tag{2.16}$$

$$J_1 = \frac{(2H_2^2 + r^2C_2)C_1}{\pi[2H_1H_2(H_1 - H_2) + r^2(H_2C_1 - H_1C_2)]} \tag{2.17}$$

$$J_2 = \frac{(2H_1^2 + r^2C_1)C_2}{\pi[2H_1H_2(H_1 - H_2) + r^2(H_2C_1 - H_1C_2)]} \tag{2.18}$$

$$M_1 = 2H_2C_1 \tag{2.19}$$

$$M_2 = 2H_1C_2 \tag{2.20}$$

$$N_1 = -(2H_2^2 + r^2C_2)C_1 \tag{2.21}$$

$$N_2 = -(2H_1^2 + r^2C_1)C_2 \tag{2.22}$$

$$C_1 = \frac{H_1^2\left[\frac{(H_1-b_1)}{H_1}\sinh^{-1}\left[\frac{(H_1-b_1)}{r}\right] - \sqrt{(\frac{r}{H_1})^2 + \left[\frac{(H_1-b_1)}{H_1}\right]^2} + \frac{r}{H_1}\right]}{(H_1-b_1)^2} \tag{2.23}$$

$$C_2 = \frac{H_2^2\left[\frac{(H_2-b_2)}{H_2}\sinh^{-1}\left[\frac{(H_2-b_2)}{r}\right] - \sqrt{(\frac{r}{H_2})^2 + \left[\frac{(H_2-b_2)}{H_2}\right]^2} + \frac{r}{H_2}\right]}{(H_2-b_2)^2} \tag{2.24}$$

H_1 = height of water test 1 [L]

H_2 = height of water test 2 [L]

r = borehole radius [L]

b_1 = length of line source test 1 [L]

b_2 = length of line source test 2 [L]

Q_{s1} = steady infiltration rate test 1 [L^3/t]

Q_{s2} = steady infiltration rate test 2 [L^3/t]

K_s = saturated hydraulic conductivity [L/t]

α_r = α-parameter of the exponential $K_r - \Psi$ relationship [1/L]

S = sorptivity [$L/t^{1/2}$]

ϕ_m = matric flux potential [L^2/t]

$\Delta\theta$ = $\theta_f - \theta_i$; θ_f = final volumetric water content [L^3/L^3]

θ_i = initial volumetric water content [L^3/L^3]

Reynolds et al. (1986) state that the multiple linear regression relationship of Stephens and Neuman (1983b) and Stephens et al. (1983a) suggests a dependency of Q_d on H, r, K_s, and α. Equation 2.7 predicts the same dependency.

Philip Solution

Philip's (1985) solution takes account of the existence of a bulb–shaped region of saturated soil located adjacent to, and extending below, the borehole during constant head borehole permeameter tests. Figure 2.3 is a schematic representation of the saturated bulb surrounding the borehole. The existence of the saturated bulb is a result of the capillary properties of the soil (Philip 1968,1969; Stephens and Neuman 1982b,c). The dimensions of the saturated bulb remain constant under steady state condition

Philip's procedure is to equate saturated flow inside the bulb with unsaturated flow outside the bulb using Richards equation for steady flow in a homogeneous isotropic soil. Flow inside the saturated bulb takes place under a gradient exceeding that of gravity and pressure alone, because of capillary effects operating outside the saturated bulb. Flow outside the saturated bulb is composed of two discernible parts, one due to gravity and the other due to capillarity. By relating the flow inside the saturated bulb to flow outside the saturated bulb, estimations of saturated hydraulic conductivity and saturated bulb geometry can be made. As indicated by Stephens (1979), Stephens et al. (1987), and Philip (1985), a must be determined by an independent measurement. For Philip's solution:

$$K_s = \frac{Q_s}{r^2}\frac{1}{\pi\sqrt{H_D^2-1}}\left[\frac{(\frac{3}{2})^{(\frac{2}{3})}H_D(1-H_D^{-2})}{\ln(H_D+\sqrt{H_D^2-1})-\sqrt{1-H_D^{-2}}}+\frac{2(\frac{3}{2})^{(\frac{1}{3})}C_p}{(A\ln(H_D+\sqrt{H_D^2-1})}\right]^{-1} \qquad (2.25)$$

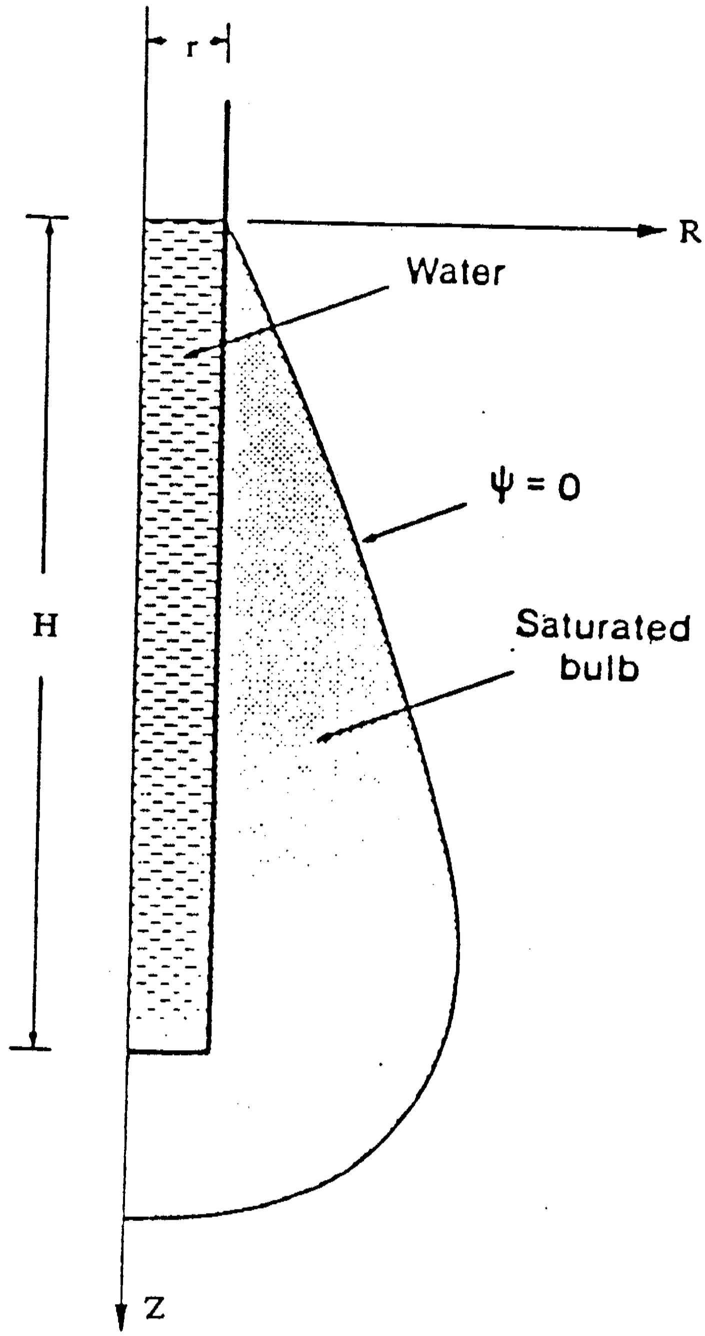

Figure 2.3. Schematic representation of constant head permeameter test in the vadose zone showing the bulb of saturation (from Philip 1985).

and:

$$Q_d = \frac{\pi r\sqrt{H_D^2 - 1}}{H}\left[\frac{(\frac{3}{2})^{(\frac{2}{3})} H_D(1 - H_D^{-2})}{\ln(H_D + \sqrt{H_D^2 - 1}) - \sqrt{1 - H_D^{-2}}} + \frac{2(\frac{3}{2})^{(\frac{1}{3})} C_p}{A\ln(H_D + \sqrt{H_D^2 - 1})}\right] \quad (2.26)$$

where: $C_p = 0.56 + 0.35H_D^{-1}$ (2.27)

$A = 1/2a_p r$ (2.28)

and: H_D = dimensionless height of water in the borehole = H/r [L/L]

H = height of water in borehole [L]

r = borehole radius [L]

K_s = saturated hydraulic conductivity [L/t]

Q_s = steady infiltration rate [L^3/t]

Q_d = dimensionless infiltration rate [L^3/t]

a_p = α-parameter of the $K_r - \Psi$ relationship [1/L]

Philip's solution (eq. 2.26) was coded (Appendix C) for the purpose of making subsequent sensitivity analysis more convienient. The code was verified by comparing its prediction of Q_d vs H_D with results in Philip (1986; Figure 8). The exact agreement of the plots (Figure 2.4) indicates that the computer code produces valid results.

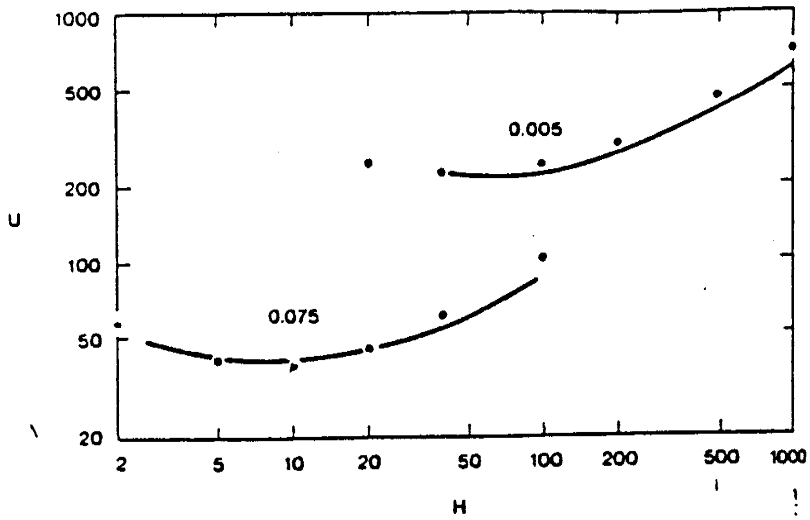

Fig. 8. Comparison with numerical calculations of Stephens and Neuman for "GE3 silt loam." Dots represent points calculated from (32) for $A = 0.005$ and 0.075. Curves show the *Stephens and Neuman* [1982c] results for the same values of A.

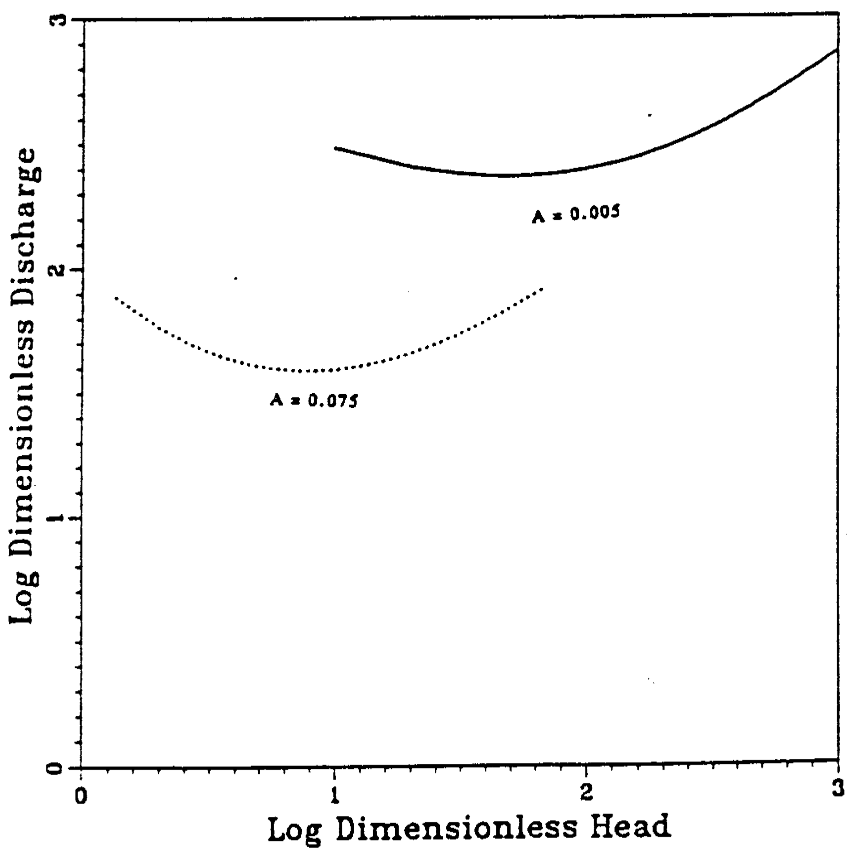

Figure 2.4. Comparison of Figure 8 from Philip (1985) with computer code in Appendix E. Log dimensionless discharge equals U, and log dimensionless head equals H.

The geometry of the saturated bulb is defined by:

$$Z = r\frac{C_p}{A}\left[\frac{\ln[H_D + \sqrt{(H_D^2 - 1)}]}{\ln\coth(\frac{1}{2}\eta) - sech(\eta)}\right]\left[1 - \frac{\ln\coth(\frac{1}{2}\eta)}{\ln[H_D + \sqrt{H_D^2 - 1}]}\right] \tag{2.29}$$

$$R = r\sqrt{\sinh^2\eta\left[(\frac{3}{2})^{\frac{2}{3}}(H_D^2 - 1) - \frac{(\frac{Z}{r})^2}{\cosh^2\eta}\right]} \tag{2.30}$$

where: $C_p = 0.56 + 0.35H_D^{-1}$

$A = 1/2a_p r$

H_D = dimensionless height of water in the borehole = H/r [L/L]

H = height of water in borehole [L]

r = borehole radius [L]

a_p = α–parameter of the exponential $K_r - \Psi$ relationship [1/L]

Z = distance down from water level in borehole [L]

R = radial distance from borehole [L]

η = eccentricity of the saturated bulb [–]

For a fixed H, r, and a_p, by varying η, in equation 2.29, Z values can be determined and substituted into equation 2.30 to determine R for the same η value. This process generates R and Z data pairs that, when connected, represent the saturated bulb surface where Ψ equals zero. Philip's solution for determining saturated bulb geometry (eqs. 2.29 and 2.30) was coded for the purpose of comparing predicted saturated bulb geometry to saturated bulb geometries measured in the field. Figure 2.5 demonstrates the agreement of the saturated bulb geometries generated by the code with the saturated bulbs in Philip (1986; Figure 3)

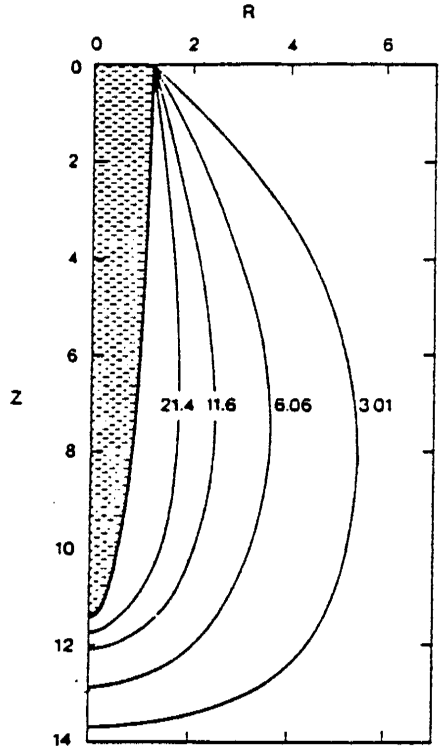

Fig. 3. Configuration of the saturated bulb. Dimensionless bulb cross sections for $H = 10$ and the four indicated values of C/A. Note the systematic decrease in bulb size as C/A increases (i.e., as capillarity grows more dominant).

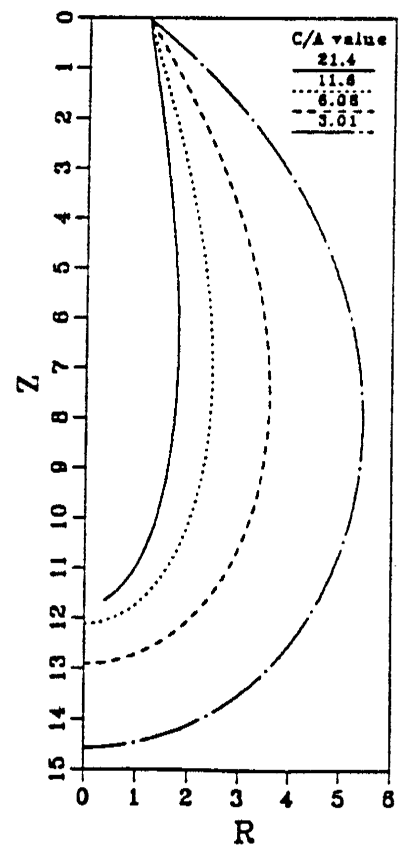

Figure 2.5. Comparison of Figure 3 (from Philip 1985) with computer code in Appendix E. H from Philip = H_D = 10.0

CHAPTER 3

SENSITIVITY ANALYSIS

INTRODUCTION

The main objective of borehole permeameter tests is to obtain an estimate of K_s for the soil surrounding the borehole. When conducting constant head borehole permeameter tests the parameters measured in the field are Q_s, r, H, and sometimes θ. When needed, a can be estimated from $K_r - \psi$ curves. The constant head borehole permeameter solution of Glover (1953) requires input of Q_s, r, and H to estimate K_s. The Stephens (1979) and Stephens et al. (1987) solutions requires values of Q_s, r, H, and a_s to predict K_s. The solution of Philip (1985) requires knowledge of Q_s, r, H, and a_p to estimate K_s and saturated bulb geometry. The Reynolds (1986) single head solution requires values of Q_s, r, H, and ϕ_m to predict K_s. Reynolds' (1986) duel-head solution requires measurements of r, two H's and resulting Q_s's in the same borehole to predict K_s and a_r. When determining S using the Reynolds duel-head solution initial moisture content (θ_i), and final moisture content (θ_f) must be known. Table 3.1 is a summary of the applicability and input requirements needed by the various borehole permeameter solutions to determine K_s, S, a_r, and saturated bulb geometry. The accuracy of the single head and duel-head solutions in predicting K_s is a function of the solution itself, the degree to which the assumptions in the model are satisfied in the field, and the accuracy of the measured input parameters. The sensitivity of the solutions to accurately predict K_s given various combinations of the input parameters needs to be investigated. Philip's (1985) solution also predicts the shape of the saturated bulb given inputs of H, r, and a_p. The predicted saturated bulb shape is

Table 3.1 : Summary of Solutions Required Input Parameters and Applicability

Borehole Permeameter Solution	Number of Required Input Parameters					Solution Determines			
	H	r	a	ϕ_m	Q_s	K_s	S	a	Saturated Bulb Geometry
Glover	1	1	NA	NA	1	Yes	No	No	No
Stephens I	1	1	1	NA	1	Yes	No	No	No
Stephens II	1	1	1	NA	1	Yes	No	No	No
Philip	1	1	1	NA	1	Yes	No	No	Yes
Reynolds	1	1	NA	1	1	Yes	No	No	No
Reynolds	2	1	NA	NA	2	Yes	Yes	Yes	No

NA - not applicable

of interest because it allows for quantification of the size of the flow field sampled, as well as a possible inverse method for obtaining α_p insitu. The sensitivity of Philip's solution to α_p in accurately predicting saturated bulb geometry will be examined. Reynolds' duel–head solution predicts K_s, S, and α_r by solving simultaneous equations relating a measured Q_s and H with another measured Q_s and H in the same borehole. Q_s is a function of the soil characteristics in the region being tested. If the soil is heterogeneous how are the two Q_s's affected for a given H's and how does this impact predictions of K_s, S, and α_r? The sensitivity of Reynolds' duel–head solution to varying Q_s's in predicting K_s, S, and α_r will be explored.

EFFECT OF H_D ON DIMENSIONLESS FLOW OUT OF THE BOREHOLE

The steady discharge rate, Q_s, into a borehole is controlled by K_s, H, r, and α. By holding K_s, r, and α constant while varying H the sensitivity of Q_d to H_D, where $Q_d = Q_s/K_s rH$, and $H_D = H/r$, can be evaluated for solutions developed by Glover (1953) (eq. 2.2), Stephens (1979) (eq. 2.4), Stephens et al. (1987) (eq. 2.6), and Philip (1985) (eq.2.26). Q_d is the same as the Glover (1953), Stephens (1979), and Stephens et al. (1987) C_u parameter. Because Q_d is directly proportional to K_s, the relationship of Q_d to borehole geometry and capillarity is important to quantify. Different H_D values can be obtained by changing H or r, or both H and r. In this study either H or r was kept constant while the other varied. For the sensitivity analysis at constant radius, $r = 0.1\ m$, H ranged between 1.0 and 10.0 m, and at constant head, $H = 1.0\ m$, r ranged between about 0.001 and 0.1 m. With this borehole geometry H_D ranges from 10 to 100 when r is constant, and from 10 to 1000 when H is constant. Four values of α were examined which represent soils having strong capillary properties, $a = 1.0\ m^{-1}$, average capillary properties, $a = 2.5\ m^{-1}$, weak capillary properties, $a = 5.0\ m^{-1}$, and essentially no capillary properties, $a = 10.0\ m^{-1}$. The α–parameter is not used in Glover's solution because capillarity is ignored. The range for H_D was selected because Glover's solution has reasonable validity at H_D greater or equal to 10.0 (Glover, 1953) and the borehole infiltration data collected at the Sevilleta Site is primarily for H_D greater than 10.0. An r value of 0.1 m was used because this approximately corresponds to the radius of an 8–inch–diameter hollow stem auger commonly used in hydrological, environmental, and geotechnical studies. The computer code used to evaluate the solutions sensitivity to H_D in predicting Q_d is contained in Appendix A.

Figure 3.1 is a graphical representation of $\log_{10} Q_d$ vs $\log_{10} H_D$ for the single head borehole permeameter solutions of Glover (1953), Stephens (1979), Stephens et al. (1987), and Philip (1985) when H varies, $r = 0.1\ m$, and $a = 1.0\ m^{-1}$, $2.5\ m^{-1}$, $5.0\ m^{-1}$, and $10.0\ m^{-1}$. The curves in Figure 3.1 were generated using the computer code contained in Appendix A. Each curve in the figure is comprised of 100 data points. The Stephens (1979) and Stephens et al. (1987) solutions will be refered to as the Stephens I and Stephens II solutions respectively, in this, and all subsequent figures. As expected all four solutions show an increase in flow out of the borehole as the H_D ratio increases over the given values of a. Glover's solutions appears to be most sensitive to varying H_D ratios, when r is constant and H varies, in predicting Q_d followed by Philip's solution. Q_d is least sensitive to varying H_D ratios for the Stephens I and Stephens II solutions over the expected range of a values. As H_D increases Glover's solution predicts that the flow into the borehole will increase faster than the other solutions

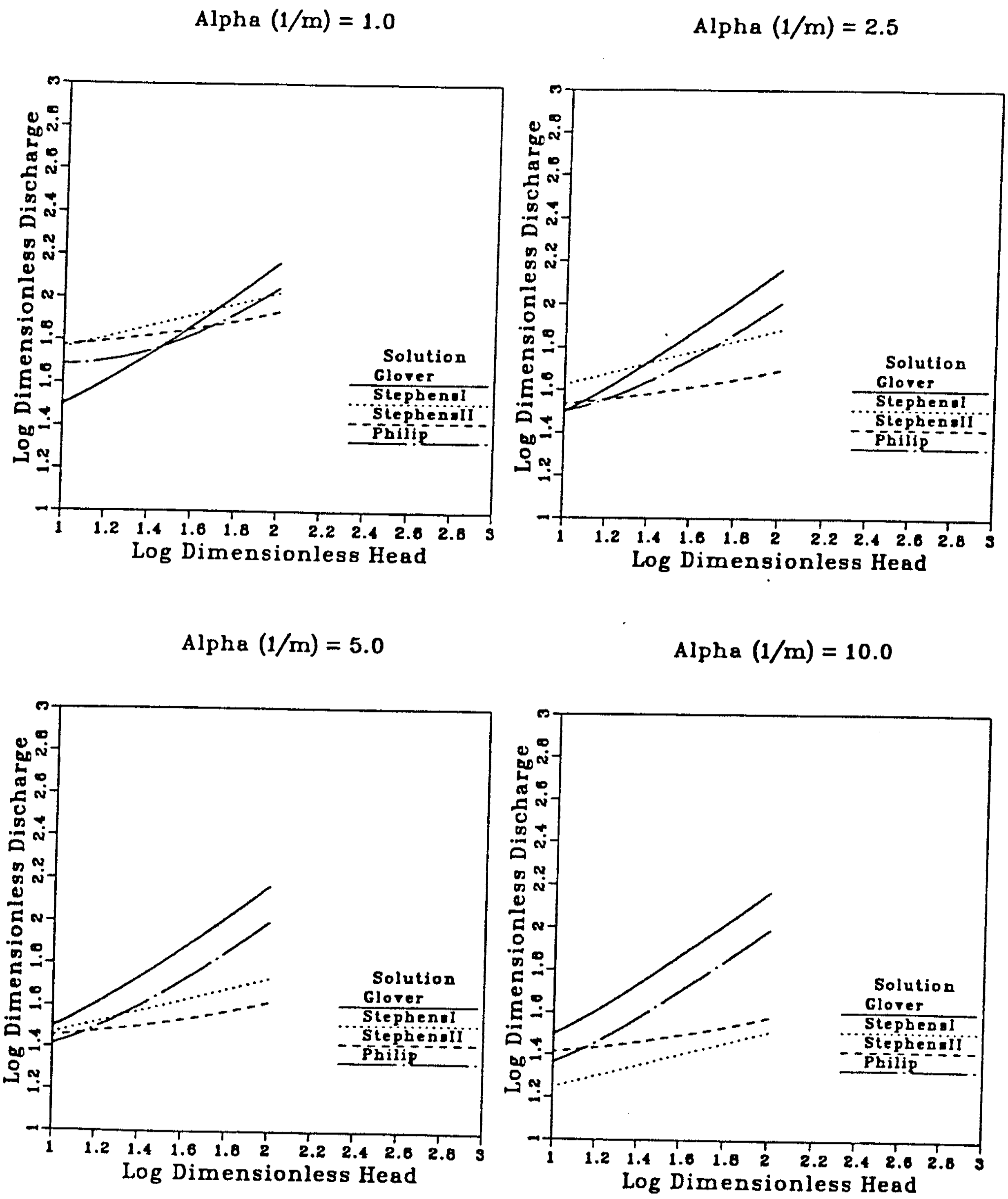

Figure 3.1. Dimensionless discharge vs dimensionless head for the borehole permeameter solutions of Glover, Stephens I, Stephens II, and Philip when H varies and r = 0.1 m.

Figure 3.2 is a graphical representation of $\log_{10} Q_d$ vs $\log_{10} H_D$ for the single head borehole permeameter solutions of Glover (1953), Stephens (1979), Stephens et al. (1987), and Philip (1985) when r varies, $H = 0.1\ m$, and $a = 1.0\ m^{-1}$, $2.5\ m^{-1}$, $5.0\ m^{-1}$, and $10.0\ m^{-1}$. The curves in Figure 3.2 were generated using the computer code contained in Appendix A. Each curve in the figure is comprised of 500 data points. As in Figure 3.1 all four solutions show an increase in flow out of the borehole as the H_D ratio increases over the given values of a. The solutions all appear equally sensitive to varying H_D ratios in predicting Q_d when H is constant and r varies. This suggests that the solutions have the same sensitivity to changing r values, and have different sensitivity to changing H values in predicting Q_d.

Figures 3.3 through 3.5 are a comparison of $\log_{10} Q_d$ vs $\log_{10} H_D$ for the Stephens I, Stephens II, and Philip solutions when r is constant, and when H is constant. As before, the curves in Figures 3.3 through 3.5 were generated using the computer code contained in Appendix A. Figures 3.3 and 3.4 demonstrate that the Stephens I and Stephens II solutions are more sensitive to r than H in predicting Q_d. Figure 3.5 shows that the Philip solution is more sensitive to r than H in predicting Q_d at relatively low H_D values, and becomes equally sensitive to r and H at relatively high H_D values.

EFFECT OF CAPILLARITY ON DIMENSIONLESS FLOW OUT OF THE BOREHOLE

The single head borehole permeameter solutions of Stephens (1979) (eq. 2.4), Stephens et al. (1987) (eq.2.6), Reynolds et al. (1986) (eq. 2.7), and Philip (1985) (eq. 2.26) recognized the effects of capillarity on Q_d. Capillarity is incorporated

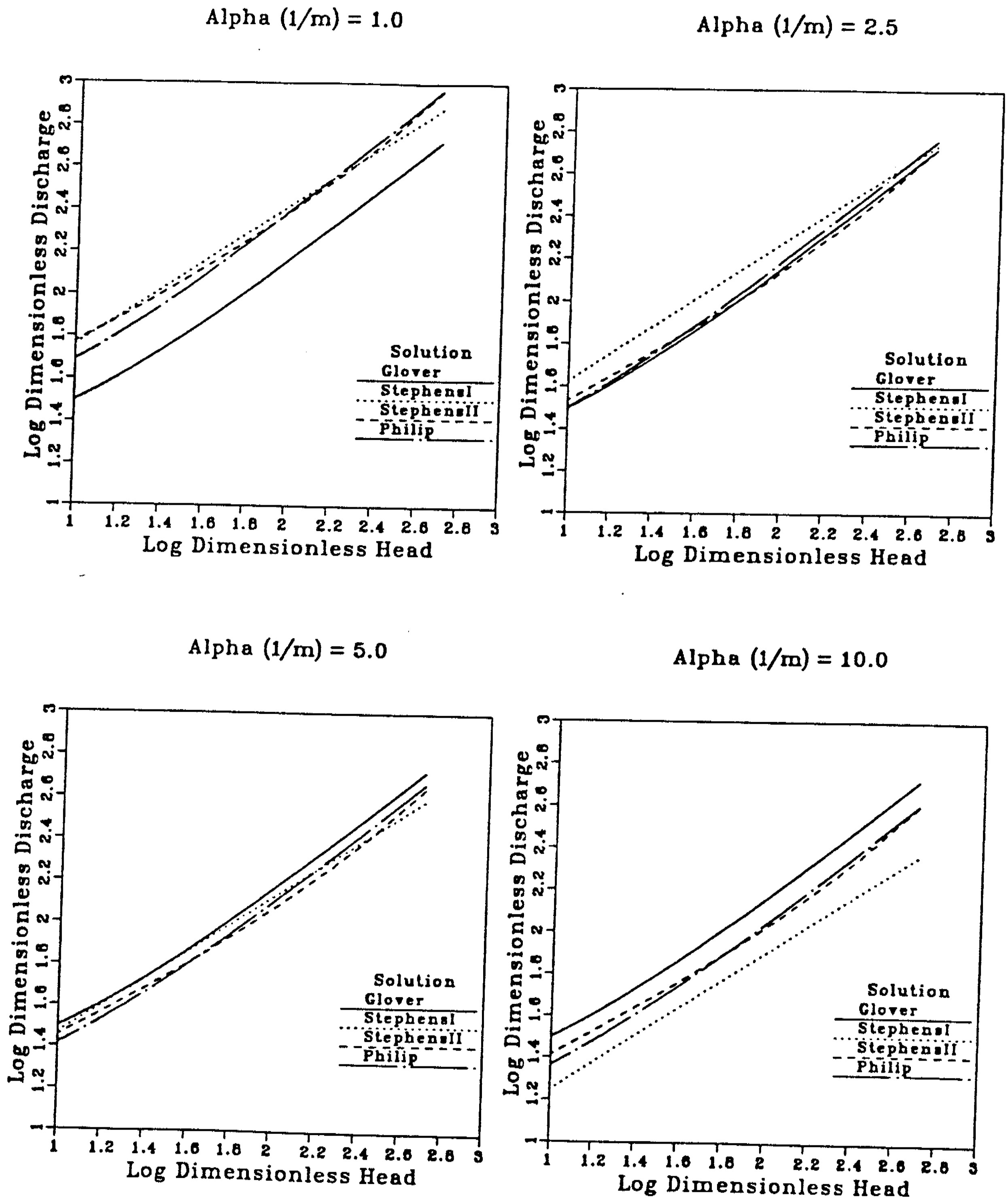

Figure 3.2. Dimensionless discharge vs dimensionless head for the borehole permeameter solutions of Glover, Stephens I, Stephens II, and Philip when r varies and H = 1.0 m.

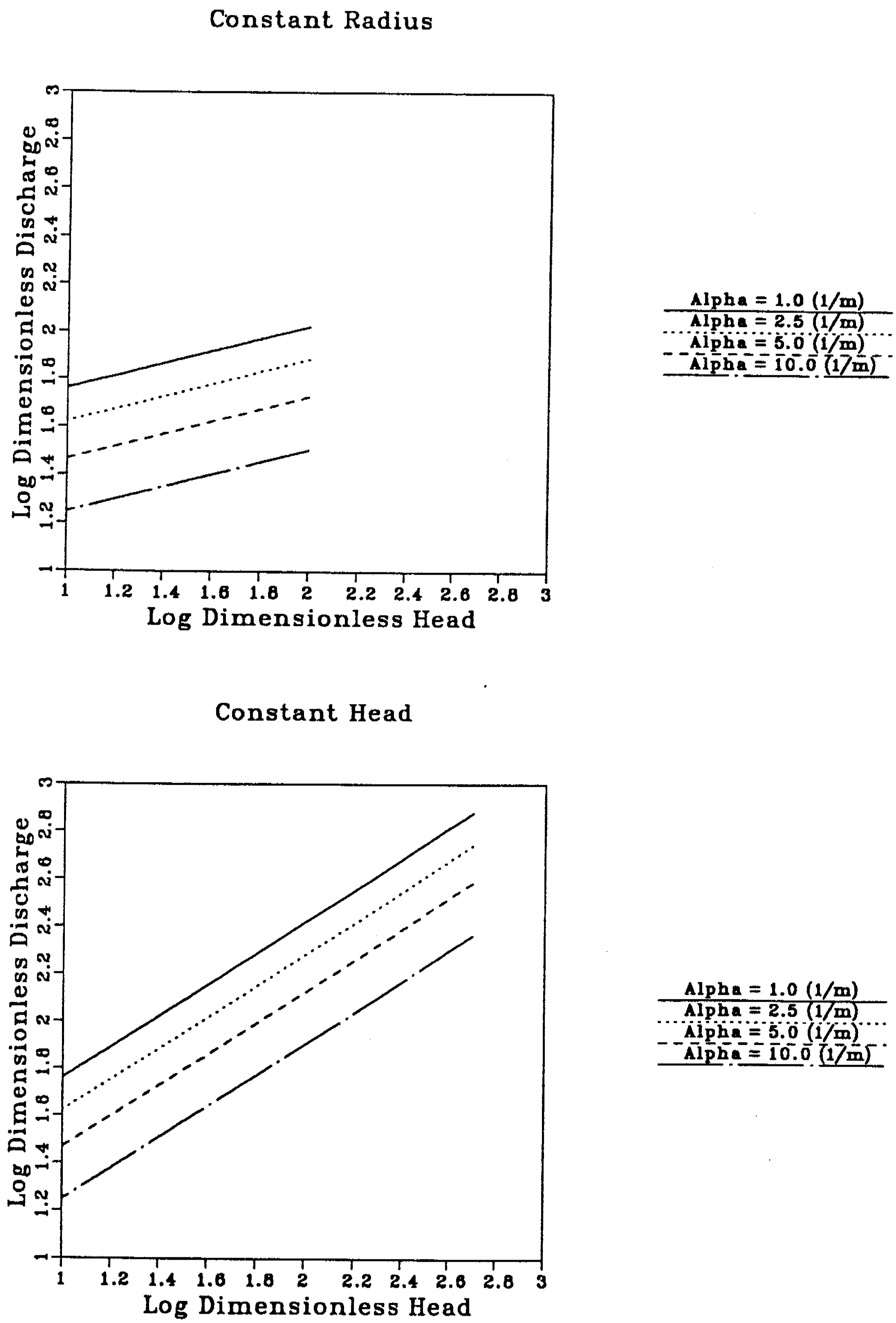

Figure 3.3. Comparison of dimensionless discharge vs dimensionless head for Stephens I solution for constant r = 0.1 m and constant H = 1.0 m.

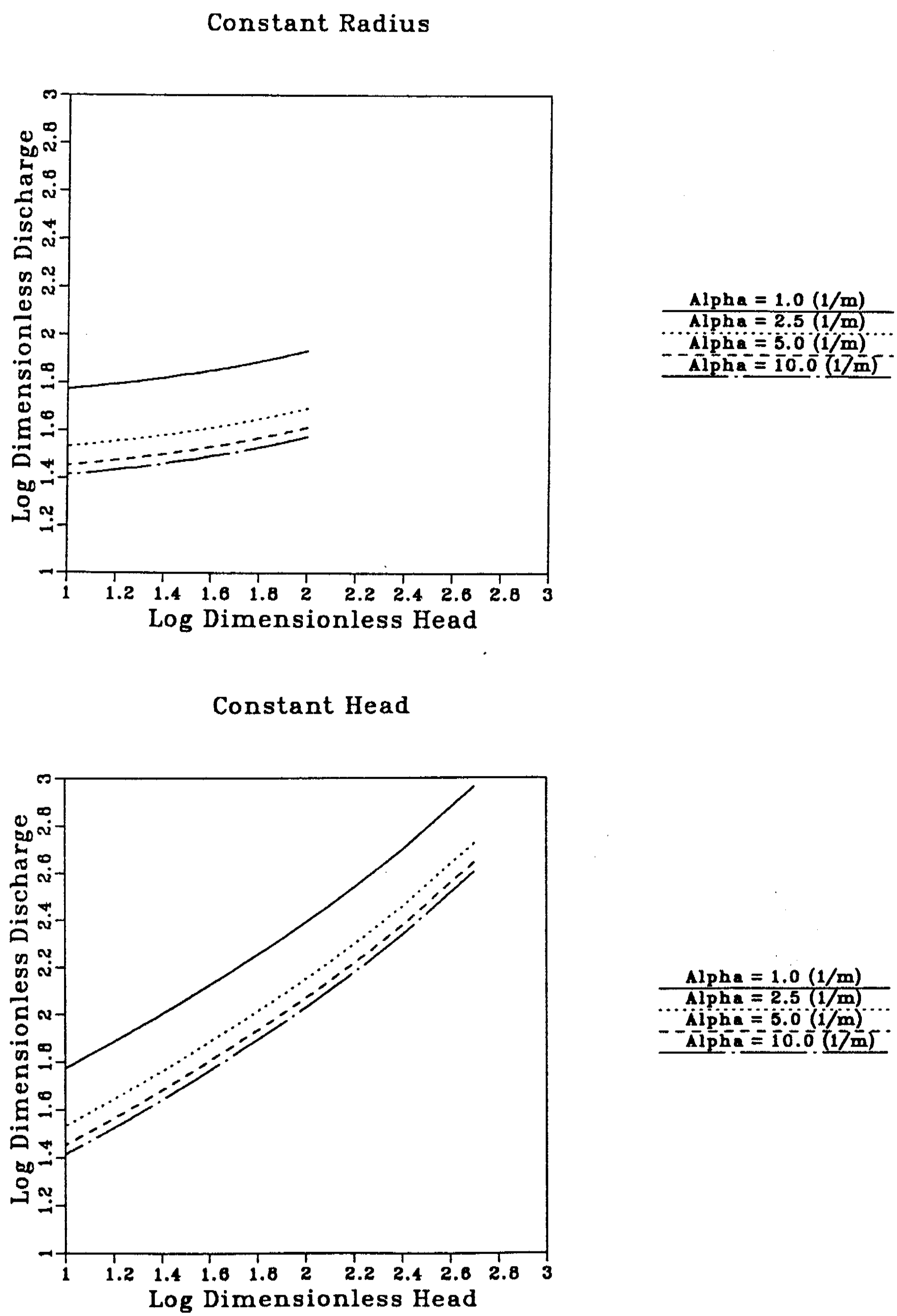

Figure 3.4. Comparison of dimensionless discharge vs dimensionless head for Stephens II solution for constant r = 0.1 m and constant H = 1.0 m.

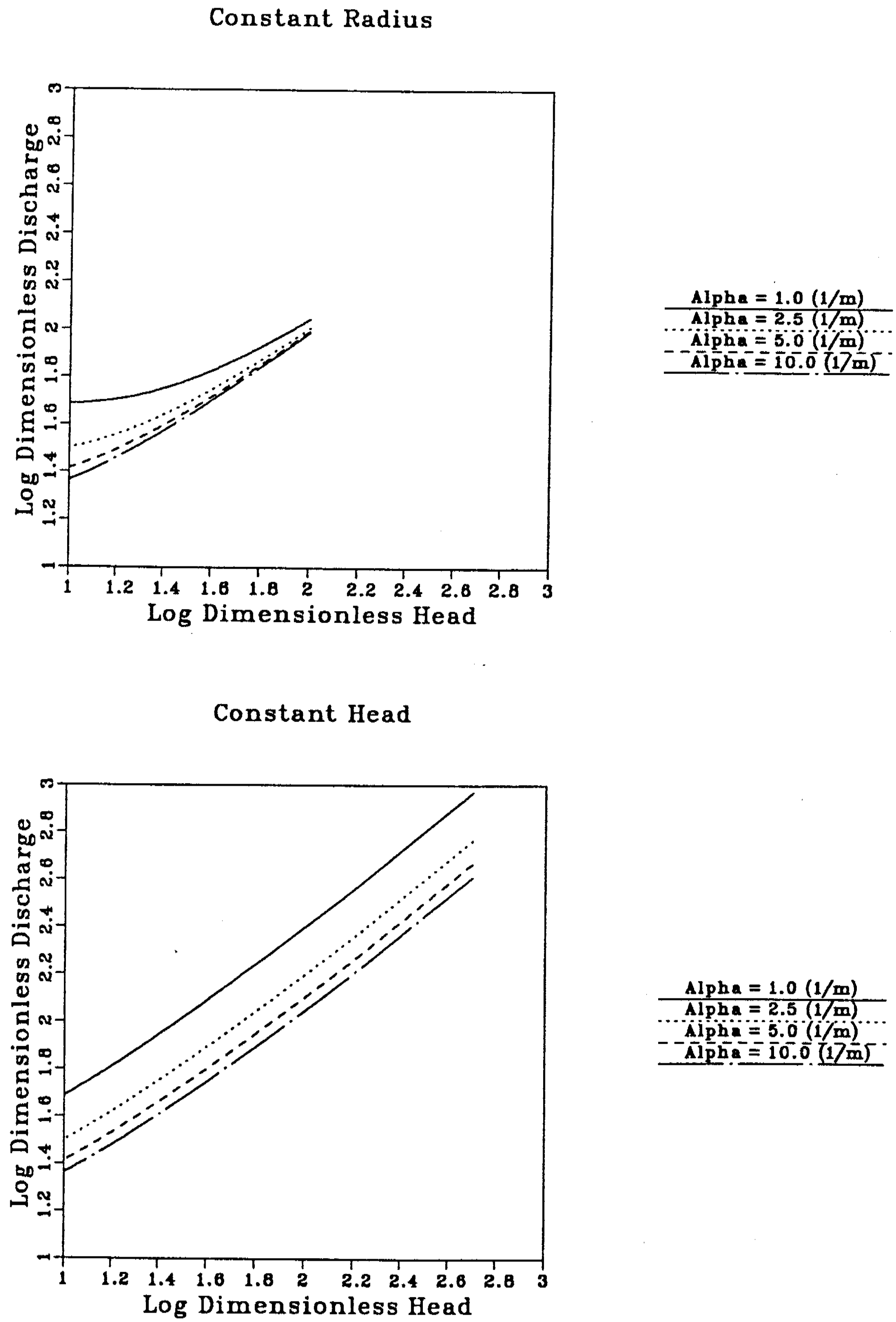

Figure 3.5. Comparison of dimensionless discharge vs dimensionless head for Philip solution for constant r = 0.1 m and constant H = 1.0 m.

into the solutions of Stephens (1979) (eq. 2.4), Stephens et al. (1987) (eq.2.6), and Philip (1985) (eq. 2.26) by use of the α -value of the exponential hydraulic conductivity–pressure head relationship. The Reynolds et al. (1986) (eq. 2.7) solution incorporates capillarity into the solution through use of matric flux potential, ϕ_m. The α–parameter and ϕ_m are determined from $K_r - \psi$ curves obtained through laboratory procedures or regression models.

For each $K_r - \psi$ curve for a soil, there are different methods to represent it by a single parameter α . Because of the uncertainty associated with α , it is important to quantify the sensitivity to this uncertainty. By maintaining r, and H constant the solutions sensitivity to α in predicting Q_d can be evaluated. For the sensitivity analysis $r = 0.1\ m$, and H is assigned values of 1.0 m, 2.0 m, 5.0 m, and 10.0 m. These H values correspond to H_D values of 10, 20, 50, and 100 respectively. The computer code used to evaluate the solutions sensitivity to α in predicting Q_d is contained in Appendix B. The curves, representing the four solutions, in Figures 3.6 and 3.7 are comprised of 50 data points each.

Figure 3.6 is a graphical representation of Q_d vs α when H_D equals 10.0, 20.0, 50.0, and 100.0. Glover's (1953) solution neglects capillarity and does not change with different α values. The solutions of Stephens (1979), Stephens et al. (1987), and Philip (1985) demonstrate Q_d is inversely proportional to α . As α increases, corresponding to diminishing capillary effects, Q_d decreases.

Figure 3.7 demonstrates that the Stephens I, Stephens II, and Philip solutions are sensitive to α , for H_D between 10.0 and 100.0, in predicting Q_d. The Stephens I solution is sensitive to all values of α whereas the Stephens II and Philip solutions, when α is greater than about 2.5 m^{-1}, predicts essentially the same Q_d

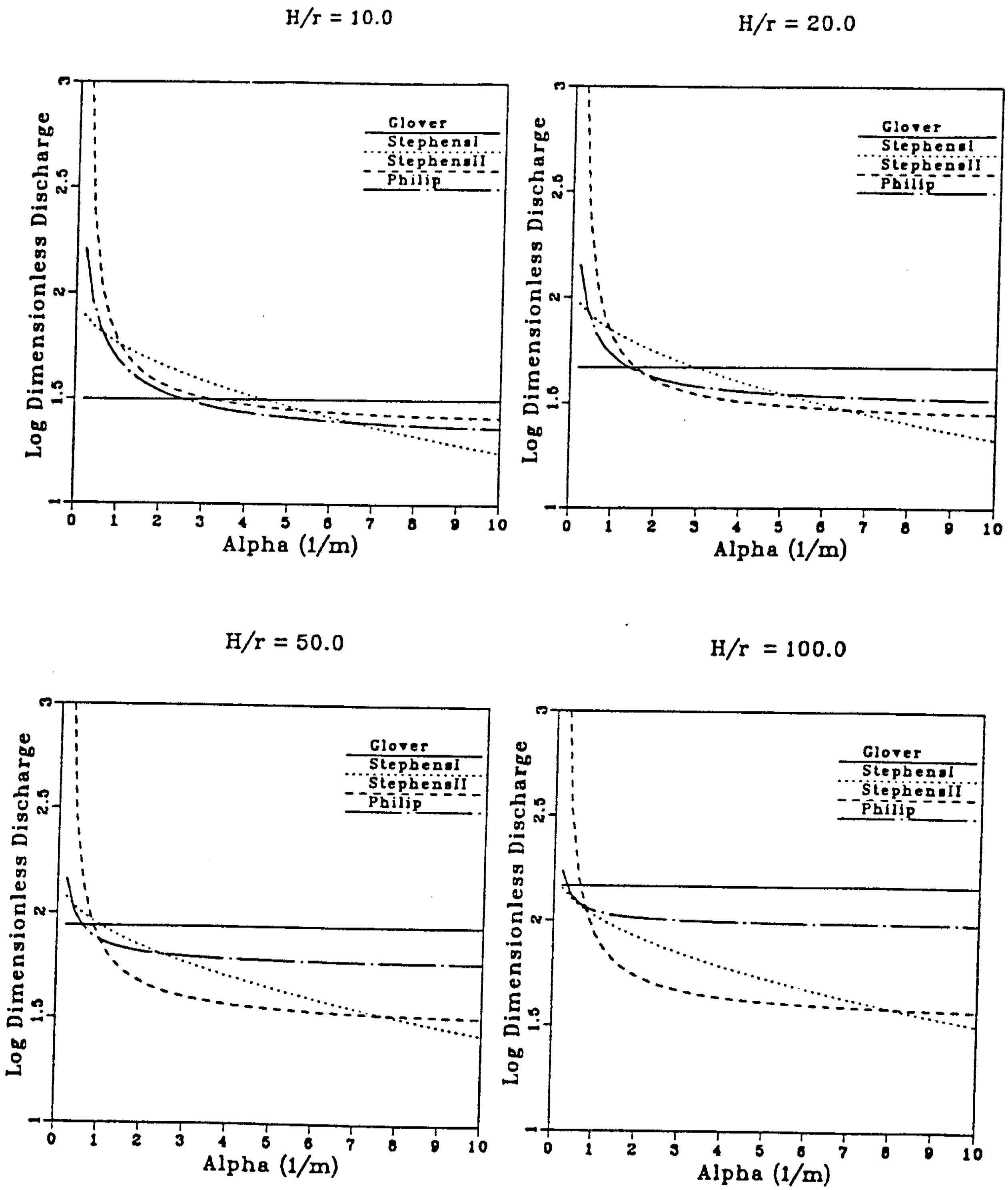

Figure 3.6. Dimensionless discharge vs α for the borehole permeameter solutions of Glover, Stephens I, Stephens II and Philip when $r = 0.1$ m.

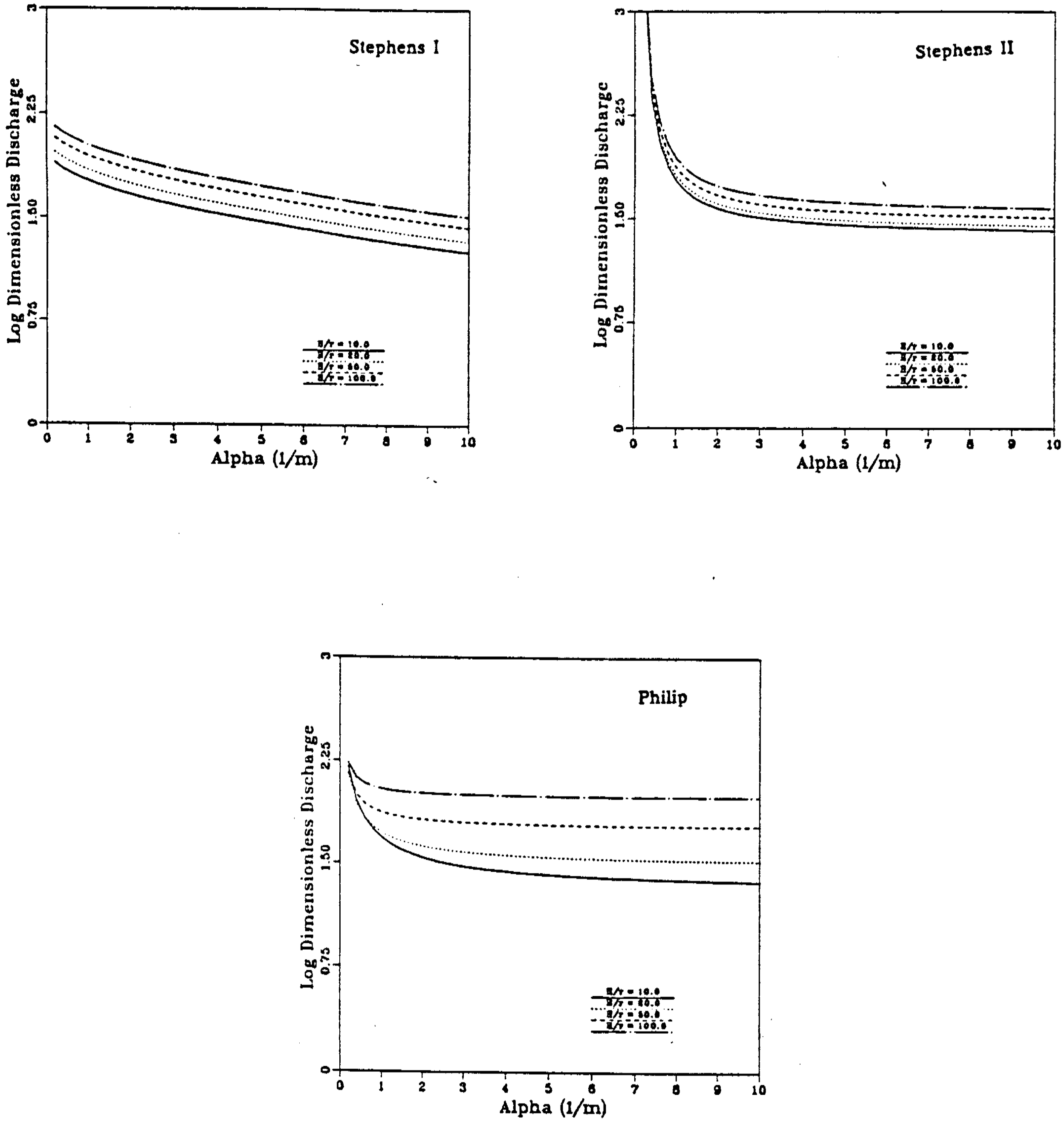

Figure 3.7. Comparison of dimensionless discharge vs α for the borehole permeameter solutions of Stephens I, Stephens II and Philip when $r = 0.1$ m.

APPENDIX B

```
c     ************************************************************
c     al laase, independent study, 6 october 1988
c     ************************************************************
c     this program determines the sensitivity of stephens (1979),
c     and philip's (1985) solutions to alpha in predicting
c     dimmensionless discharge from the borehole.
c     ************************************************************
c     variable dictionary
c
c     h = height of water in borehole (L)
c     r = borehole radius (L)
c     ks = saturated hydraulic conductivity (L/t)
c     qd = dimmensionless discharge ( )
c     alpha = sorptive number (1/L)
c     dalpha = incremental change in alpha (1/L)
c     x1,x2,...,xn = variables to aid in calculation
c     ************************************************************
c     define variables
c
      real h,r,ks,alpha,dalpha
      character *30 alphafile
c     ************************************************************
c     enter data from screen
c
      write(*,*)' enter constant height of water in borehole '
      read(*,*)h
      write(*,*)' enter constant borehole radius '
      read(*,*)r
      write(*,*)' enter saturated hydraulic conductivity '
      read(*,*)ks
      write(*,*)' enter data file name for varing alpha '
      read(*,*)alphafile
c     ************************************************************
c     opening data file
c
      open(unit=21,file=alphafile,status='unknown')
c     ************************************************************
c     glover's solution
c
      alpha=0.0
      dalpha=0.2
      do 5 i=0,250
         alpha=alpha+dalpha
         qd=(2*3.1416*(h/r))/(log((h/r)+sqrt((h/r)**2.0+1.0))-1.0)
         xqd=alog10(qd)
         write(21,11)alpha,xqd
 5    continue
c     ***********************************************************
c     put blank line in data file
c
      write(21,*)
c     ***********************************************************
c     stephens I solution:sensitivity of qd to alpha
c
      alpha=0.0
      dalpha=0.2
      do 10 i=0,250
         alpha=alpha+dalpha
         x1=0.658*alog10(h/r)
         x2=-0.238*sqrt(alpha)
```

```
        x3=-0.398*alog10(h)
        x4=x1+x2+x3+1.342
        qd=10**x4
        xqd=alog10(qd)
        write(21,11)alpha,xqd
 10   continue
c     *********************************************************
c     put blank line in data file
c
      write(21,*)
c     *********************************************************
c     stephens II solution:sensitivity of qd to alpha
c
      alpha=0.0
      dalpha=0.2
      do 15 i=0,250
         alpha=alpha+dalpha
         x1=0.486*alog10(h/r)
         x2=0.4/(alpha)
         x3=-0.454*alog10(h)
         x4=0.019*sqrt(h/r)
         x5=x1+x2+x3+x4+0.828
         qd=10**x5
         xqd=alog10(qd)
         write(21,11)alpha,xqd
 15   continue
c     **********************************************************
c     put blank line in data file
c
      write(21,*)
c     **********************************************************
c     philip's solution:sensitivity of qd to alpha
c
      alpha=0.0
      dalpha=0.2
      do 20 i=0,250
         alpha=alpha+dalpha
         hr=h/r
         x5=(3.1416*r*sqrt(hr**2.0-1.0))/h
         x6=(3.0/2.0)**(2.0/3.0)*hr*(1.0-(hr**(-2.0)))
         x7=(log(hr+sqrt(hr**2.0-1.0))-sqrt(1.0-(hr**(-2.0))))
         x8=2.0*(3.0/2.0)**(1.0/3.0)*(0.56+(0.35/hr))
         x9=0.5*alpha*r*log(hr+sqrt(hr**2.0-1.0))
         qd=x5*((x6/x7)+(x8/x9))
         xqd=alog10(qd)
         write(21,11)alpha,xqd
 20   continue
c     *********************************************************
c     format statement
c
 11   format(f12.8,1x,f16.8)
c     *********************************************************
      stop
      end
```

APPENDIX C

```
c     ***********************************************************
c     al laase, independent study, 16 october 1988
c     ***********************************************************
c     this program uses equations from "approximate analysis of
c     the borehole permeameter in unsaturated soil" j.r. philip
c     water resources research, vol. 21, no. 7, pages 1025-1033,
c     july 1985.  this is a preliminary study designed to
c     determine the relationship between sorptive number and
c     maximumn dimensionless bulb depth and bulb radius.
c     ***********************************************************
c     variable dictionary
c
c     head - dimensionless height of water in borehole
c     borehead - height of water in borehole
c     radius - radius of borehole
c     cdiva - dimensionless constant
c     z - dimensionless depth of the saturated bulb
c     r - dimensionless radius of the saturated bulb
c     eq - entered discharge
c     q - discharge
c     dq - dimensionless discharge
c     dz - dimensioned depth of the saturated bulb
c     dr - dimensioned radius of the saturated bulb
c     nu - an angle related to the saturated bulb
c     dnu - incremental value of nu
c     c - dimensionless constant dependent on head
c     a - dimensionless constant dependent c and z
c     rmax - dimensionless maximum bulb radius
c     rmin - dimensionless minimum bulb radius
c     zmax - dimensionless maximum bulb depth
c     drmax - dimensioned maximum bulb radius
c     drmin - dimensioned minimum bulb radius
c     dzmax - dimensioned maximum bulb depth
c     x1,x2,...,xn - dummy variables used in calculations
c     k - counter variable
c     m - counter variable
c     n - counter variable
c     filename - alpha vs hydraulic conductivity filename
c     plotname - alpha vs bulb depth filename
c     plotfile - alpha vs bulb radius filename
c     outname - alpha vs c/a filename
c     ***********************************************************
c     defining varibles
c
      dimension z(300,2500),r(300,2500),dz(300,2500),dr(300,2500)
      real head,radius,cdiva,nu,dnu,rmax,zmax,a,c
      real ks,eq,ck
      integer k,n,m
      character *30 plotname,plotfile
c     ***********************************************************
c     reading input data
c
      write(*,*)' enter radius of borehole (m) '
      read(*,*)radius
      write(*,*)' enter height of water in borehole (m) '
      read(*,*)borehead
      write(*,*)' enter discharge rate (liters/min) '
      read(*,*)eq
      write(*,*)' enter alpha vs bulb depth output file name '
      read(*,*)plotname
```

```
      write(*,*)' enter alpha vs bulb radius output file name '
      read(*,*)plotfile
c     *****************************************************
c     opening output and plot files
c
      open(unit=22,file=plotname,status='unknown')
      open(unit=23,file=plotfile,status='unknown')
c     *****************************************************
c     calculates the saturated bulb shape using equation 17 and
c     19. calculates alpha using equations 8, 19, and 31. all
c     equations from philip article
c
      head=borehead/radius
      do 5 i=1,20
      zmax=0.0
      rmax=0.0
      alpha=0.5+alpha
      a=0.5*alpha*radius
      c=0.56+(0.35/head)
      cdiva=c/a
         m=0
         n=0
         k=1
         nu=0.0
         dnu=0.002
      do 10 j=1,2500
         nu=nu+dnu
         x1=log(head+sqrt(head**2.0-1.0))
         x2=sqrt(1.0-(head**(-2.0)))
         x3=log((1+cosh(nu))/sinh(nu))
         x4=1.0/cosh(nu)
         z(i,j)=cdiva*((x1-x2)/(x3-x4))*(1.0-(x3/x1))
         dz(i,j)=z(i,j)*radius
         if(z(i,j).gt.0.0)then
            x5=((3.0/2.0)**(2.0/3.0)*((head**2.0)-1.0))
            x6=(z(i,j)**2.0)/(cosh(nu)**2.)
            x7=(sinh(nu))**2.0
            x8=x5-x6
            if(x8.lt.2.0.and.m.eq.0)then
               dnu=dnu/5.0
               m=1
            end if
            if(x8.lt.0.5.and.m.eq.1)then
               dnu=dnu/10.0
               m=2
            end if
            if(x8.lt.0.1.and.m.eq.2)then
               dnu=dnu/10.0
               m=3
            end if
            if(x8.lt.0.0001.and.m.eq.3)then
               dnu=dnu/100.0
               m=4
            end if
            if(x6.gt.x5)goto 500
            r(i,j)=sqrt((x5-x6)*x7)
            dr(i,j)=r(i,j)*radius
            if(z(i,j).gt.zmax)then
               zmax=z(i,j)
            end if
```

```
            if(r(i,j).gt.rmax)then
               rmax=r(i,j)
            end if
            k=k+1
       end if
       n=n+1
 10    continue
 500   continue
       x9=sqrt((head**2.0)-1.0)
       x10=log(head+(sqrt((head**2.0)-1.0)))
       x11=4.117*head*(1.0-(head**(-2.0)))
       dq=x9*((x11/(x10-x2))+(cdiva*(7.192/x1)))
       q=(dq*ks*((radius*100.0)**2.0))*(60.0/1000.0)
       ck=(eq/(dq*((radius*100.0)**2.0)))*(1000.0/60.0)
       dzmax=zmax*radius
       drmax=rmax*radius
       xck=log10(ck)
       write(*,*)
       write(*,*)' number of data points     = ',k
       write(*,*)' c/a                        = ',cdiva
       write(*,*)' alpha                       ',alpha
       write(*,*)' hydraulic conductivity (cm/s) = ',ck
       write(*,*)' log hydraulic conductivity    = ',xck
       write(*,*)' maximum bulb depth (m)     = ',dzmax
       write(*,*)' maximum bulb radius (m)     ',drmax
       write(22,11)zmax,alpha
       write(23,11)rmax,alpha
 5     continue
c      ****************************************************
 11    format(1x,f12.8,3x,f12.8)
       stop
       end
```

APPENDIX D

```
c     ******************************************************************
c     al laase, independent study, 6 october 1988
c     ******************************************************************
c     this program uses equations from "the constant head well permeameter:
c     effect of unsaturated flow" w.d. reynolds, d.e. elrick, and b.e.
c     clothier, soil science, vol. 139, no. 2, pages 172-180, february 1986.
      making use of two constant head values in the same borehole saturated
c     field hydraulic conductivity, sorptivity, and alpha parameter.
c     ******************************************************************
c     variable dictionary
c
c     h1 - constant head value associated with first borehole test
c     h2 - constant head value associated with second borehole test
c     q1 - flow rate into borehole associated with first borehole test
c     q2 - flow rate into borehole associated with second borehole test
c     dq - change in flow rate q2
c     radius - radius of borehole
c     depth - depth of borehole
c     b1 - length along borehole before line source test 1
c     b2 - length along borehole before line source test 2
c     open1 - open length of borehole test 1
c     open2 - open length of borehole test 2
c     kfs - field saturated hydraulic conductivity
c     lkfs - log field saturated hydraulic conductivity
c     s - sorptivity
c     s1 - used in calculating sorptivity
c     alpha - alpha number
c     theta - moisture content
c     dtheta - change in moisture content
c     stheta - saturated moisture content
c     pi - as related to circle
c     c1,c2 - used to determine kfs,s,alpha
c     g1,g2 - used to determine kfs
      j1,j2 - used to determine s
c     m1,m2 - used to determine alpha
c     n1,n2 - used to determine alpha
c     w1,w2,...,wn - dummy variables used in calculations
c     x1,x2,...,xn - dummy variables used in calculations
c     y1,y2,...,yn - dummy variables used in calculations
c     z1,z2,...,zn - dummy variables used in calculations
c     ksfilename - plotfile ks vs qr
c     sfilename - plotfile s vs qr
c     alphafilename - plotfile alpha vs qr
c     n - counter variable
c     ******************************************************************
c     defining variables
c
      real h1,h2,q1,q2,radius,b1,b2,kfs,s,alpha,dtheta,c1,c2,g1,g2,j1,j2
      real m1,m2,n1,n2,theta,pi,depth,stheta,dq,lkfs
      integer n
      character *30 ksfilename,sfilename,alphafilename
      pi=3.14159
c     ******************************************************************
c     reading input data
c
      write(*,*)' enter radius of borehole (m) '
      read(*,*)radius
      write(*,*)' enter depth of borehole (m) '
      read(*,*)depth
      write(*,*)' enter length along borehole before screen test 1 (m) '
```

```
      read(*,*)b1
      write(*,*)' enter constant head for test 1 (m) '
      read(*,*)h1
      write(*,*)' enter length along borehole before screen test 2 (m) '
      read(*,*)b2
      write(*,*)' enter constant head for test 2 (m) '
      read(*,*)h2
      write(*,*)' enter initial moisture content of soil '
      read(*,*)theta
      write(*,*)' enter saturated moisture content of soil '
      read(*,*)stheta
      write(*,*)' enter Q1/Q2 vs Ks output file name '
      read(*,*)ksfilename
      write(*,*)' enter Q1/Q2 vs S outout file name '
      read(*,*)sfilename
      write(*,*)' enter Q1/Q2 vs alpha output file name '
      read(*,*)alphafilename
c     ***********************************************************************
c     opening output file
c
      open(unit=21,file=ksfilename,status='unknown')
      open(unit=22,file=sfilename,status='unknown')
      open(unit=23,file=alphafilename,status='unknown')
c     ***********************************************************************
c     calculate c1 using equation 29
c
      x1=h1**2.0
      x2=(h1-b1)/h1
      x3=(h1-b1)/radius
      x4=(radius/h1)**2.0
      x5=x2**2.0
      x6=radius/h1
      x7=(h1-b1)**2.0
      x8=log(x3+sqrt((x3**2.0)+1.0))
      c1=(x1*((x2*x8)-sqrt(x4+x5)+x6))/x7
c     ***********************************************************************
c     calculate c2 using equation 29
c
      y1=h2**2.0
      y2=(h2-b2)/h2
      y3=(h2-b2)/radius
      y4=(radius/h2)**2.0
      y5=y2**2.0
      y6=radius/h2
      y7=(h2-b2)**2.0
      y8=log(y3+sqrt((y3**2.0)+1.0))
      c2=(y1*((y2*y8)-sqrt(y4+y5)+y6))/y7
c     ***********************************************************************
c     calculate g1 using equation 38
c
      z1=h2*c1
      z2=2*h1*h2*(h2-h1)
      z3=((radius)**2.0)*((h1*c2)-(h2*c1))
      g1=z1/(pi*(z2+z3))
c     ***********************************************************************
c     calculate g2 using equation 38
c
      z4=h1*c2
      z5=2*h1*h2*(h2-h1)
      z6=(radius**2.0)*((h1*c2)-(h2*c1))
```

```
      g2=z4/(pi*(z5+z6))
c     *******************************************************************
c     calculate j1 using equation 39
c
      w1=((2.0*(h2**2.0))+((radius**2.0)*c2))*c1
      w2=2.0*h1*h2*(h1-h2)
      w3=((radius)**2.0)*((h2*c1)-(h1*c2))
      j1=w1/(pi*(w2+w3))
c     *******************************************************************
c     calculate j2 using equation 39
c
      w4=((2.0*(h1**2.0))+((radius**2.0)*c1))*c2
      j2=w4/(pi*(w2+w3))
c     *******************************************************************
c     calculate m1 and m2 using equation 40
c
      m1=2.0*h2*c1
      m2=2.0*h1*c2
c     *******************************************************************
c     calculate n1 and n2
c
      n1=-c1*((2.0*(h2**2.0))+((radius**2.0)*c2))
      n2=-c2*((2.0*(h1**2.0))+((radius**2.0)*c1))
c     *******************************************************************
c     calculate kfs,s, and alpha using equations 38,39,40
c
      q2=100.00
      q1=1.0
      dq=1.0
      n=0
      dtheta=stheta-theta
      do 10 i=1,2000
         kfs=((g2*q2)-(g1*q1))*(1000.0/60.0)*(1/10000.0)
         lkfs=alog10(kfs)
         s1=(dtheta*((j2*q2*(1000.0/6000.0))-(j1*q1*(1000.0/6000.0))))
         s=sqrt(s1)
         alpha=((m2*q2)-(m1*q1))/((n2*q2)-(n1*q1))
         qr=q1/q2
         if(qr.gt.1.0)goto 500
         write(21,11)lkfs,qr
         write(22,11)s,qr
         write(23,11)alpha,qr
         q2=q2-dq
         if(qr.gt.0.30.and.n.eq.0)then
            dq=0.01
            n=1
         end if
         if(qr.gt.0.40.and.n.eq.1)then
            dq=0.005
            n=2
         end if
         if(qr.gt.0.50.and.n.eq.2)then
            dq=0.001
            n=3
         end if
         if(qr.gt.0.60.and.n.eq.3)then
            dq=0.0005
            n=4
         end if
         if(qr.gt.0.70.and.n.eq.4)then
```

```
            dq=0.00001
            n=5
         end if
         if(qr.gt.0.701178.and.n.eq.5)then
            dq=0.000001
            n=6
         end if
         if(qr.gt.0.701186.and.n.eq.6)then
            dq=0.0000001
            n=7
         end if
         if(qr.gt.0.701187.and.n.eq.7)then
            dq=0.00000008
            n=8
         end if
         if(qr.gt.0.701187950.and.n.eq.8)then
            dq=0.00000001
            n=9
         end if
 10    continue
 500   continue
c     ********************************************************************
c     format statement
c
 11   format(f12.6,1x,f18.12)
c     ********************************************************************
      stop
      end
```

```
c       *************************************************************************
c       al laase, independent study, 6 october 1988
c       *************************************************************************
c       this program uses equations from "the constant head well permeameter:
c       effect of unsaturated flow" w.d. reynolds, d.e. elrick, and b.e.
        clothier, soil science, vol. 139, no. 2, pages 172-180, february 1986.
c       making use of two constant head values in the same borehole saturated
c       field hydraulic conductivity, sorptivity, and alpha parameter.
c       *************************************************************************
c       variable dictionary
c
c       h1 - constant head value associated with first borehole test
c       h2 - constant head value associated with second borehole test
c       q1 - flow rate into borehole associated with first borehole test
c       q2 - flow rate into borehole associated with second borehole test
c       dq - change in flow rate q2
c       radius - radius of borehole
c       depth - depth of borehole
c       b1 - length along borehole before line source test 1
c       b2 - length along borehole before line source test 2
c       open1 - open length of borehole test 1
c       open2 - open length of borehole test 2
c       kfs - field saturated hydraulic conductivity
c       lkfs - log field saturated hydraulic conductivity
c       s - sorptivity
c       s1 - used in calculating sorptivity
c       alpha - alpha number
c       theta - moisture content
c       dtheta - change in moisture content
c       stheta - saturated moisture content
c       pi - as related to circle
c       c1,c2 - used to determine kfs,s,alpha
c       g1,g2 - used to determine kfs
c       j1,j2 - used to determine s
c       m1,m2 - used to determine alpha
c       n1,n2 - used to determine alpha
c       w1,w2,...,wn - dummy variables used in calculations
c       x1,x2,...,xn - dummy variables used in calculations
c       y1,y2,...,yn - dummy variables used in calculations
c       z1,z2,...,zn - dummy variables used in calculations
c       ksfilename - plotfile ks vs qr
c       sfilename - plotfile s vs qr
c       alphafilename - plotfile alpha vs qr
c       n - counter variable
c       *************************************************************************
c       defining variables
c
        real h1,h2,q1,q2,radius,b1,b2,kfs,s,alpha,dtheta,c1,c2,g1,g2,j1,j2
        real m1,m2,n1,n2,theta,pi,depth,stheta,dq,lkfs
        integer n
        character *30 ksfilename,sfilename,alphafilename
        pi=3.14159
c       *************************************************************************
c       reading input data
c
        write(*,*)' enter radius of borehole (m) '
        read(*,*)radius
        write(*,*)' enter depth of borehole (m) '
        read(*,*)depth
        write(*,*)' enter length along borehole before screen test 1 (m) '
```

```
      read(*,*)b1
      write(*,*)' enter constant head for test 1 (m) '
      read(*,*)h1
      write(*,*)' enter length along borehole before screen test 2 (m) '
      read(*,*)b2
      write(*,*)' enter constant head for test 2 (m) '
      read(*,*)h2
      write(*,*)' enter initial moisture content of soil '
      read(*,*)theta
      write(*,*)' enter saturated moisture content of soil '
      read(*,*)stheta
      write(*,*)' enter Q1/Q2 vs Ks output file name '
      read(*,*)ksfilename
      write(*,*)' enter Q1/Q2 vs S outout file name '
      read(*,*)sfilename
      write(*,*)' enter Q1/Q2 vs alpha output file name '
      read(*,*)alphafilename
c     ************************************************************************
c     opening output file
c
      open(unit=21,file=ksfilename,status='unknown')
      open(unit=22,file=sfilename,status='unknown')
      open(unit=23,file=alphafilename,status='unknown')
c     ************************************************************************
c     calculate c1 using equation 29
c
      x1=h1**2.0
      x2=(h1-b1)/h1
      x3=(h1-b1)/radius
      x4=(radius/h1)**2.0
      x5=x2**2.0
      x6=radius/h1
      x7=(h1-b1)**2.0
      x8=log(x3+sqrt((x3**2.0)+1.0))
      c1=(x1*((x2*x8)-sqrt(x4+x5)+x6))/x7
c     ************************************************************************
c     calculate c2 using equation 29
c
      y1=h2**2.0
      y2=(h2-b2)/h2
      y3=(h2-b2)/radius
      y4=(radius/h2)**2.0
      y5=y2**2.0
      y6=radius/h2
      y7=(h2-b2)**2.0
      y8=log(y3+sqrt((y3**2.0)+1.0))
      c2=(y1*((y2*y8)-sqrt(y4+y5)+y6))/y7
c     ************************************************************************
c     calculate g1 using equation 38
c
      z1=h2*c1
      z2=2*h1*h2*(h2-h1)
      z3=((radius)**2.0)*((h1*c2)-(h2*c1))
      g1=z1/(pi*(z2+z3))
c     ************************************************************************
c     calculate g2 using equation 38
c
      z4=h1*c2
      z5=2*h1*h2*(h2-h1)
      z6=(radius**2.0)*((h1*c2)-(h2*c1))
```

```
      g2=z4/(pi*(z5+z6))
c     ********************************************************************
c     calculate j1 using equation 39
c
      w1=((2.0*(h2**2.0))+((radius**2.0)*c2))*c1
      w2=2.0*h1*h2*(h1-h2)
      w3=((radius)**2.0)*((h2*c1)-(h1*c2))
      j1=w1/(pi*(w2+w3))
c     ********************************************************************
c     calculate j2 using equation 39
c
      w4=((2.0*(h1**2.0))+((radius**2.0)*c1))*c2
      j2=w4/(pi*(w2+w3))
c     ********************************************************************
c     calculate m1 and m2 using equation 40
c
      m1=2.0*h2*c1
      m2=2.0*h1*c2
c     ********************************************************************
c     calculate n1 and n2
c
      n1=-c1*((2.0*(h2**2.0))+((radius**2.0)*c2))
      n2=-c2*((2.0*(h1**2.0))+((radius**2.0)*c1))
c     ********************************************************************
c     calculate kfs,s, and alpha using equations 38,39,40
c
      dqr=0.000001
      qr=0.353975
      n=0
      dtheta=stheta-theta
      do 10 i=1,2000
         q1=1.0
         q2=q1/qr
         qr=qr+dqr
         kfs=((g2*q2)-(g1*q1))*(1000.0/60.0)*(1/10000.0)
         lkfs=alog10(kfs)
         s1=(dtheta*((j2*q2*(1000.0/6000.0))-(j1*q1*(1000.0/6000.0))))
         s=sqrt(s1)
         alpha=((m2*q2)-(m1*q1))/((n2*q2)-(n1*q1))
         if(qr.gt.1.0)goto 500
         if(qr.gt.0.35398.and.n.eq.0)then
            dqr=0.0001
            n=1
         end if
         if(qr.gt.0.354.and.n.eq.1)then
            dqr=0.01
            n=2
         end if
         write(22,11)s,qr
         write(23,11)alpha,qr
  10   continue
 500   continue
c     ********************************************************************
c     format statement
c
 11   format(f12.6,1x,f18.12)
c     ********************************************************************
      stop
      end
```

APPENDIX E

```
c     ***********************************************************
c     al laase, independent study, 31 january 1989
c     ***********************************************************
c     this program uses equations from glover(1953), stephens(1979)
c     stephens et al.(1987), philip(1985), and reynolds et al.(1986)
c     to determine saturated hydraulic conductivity from single head
c     borehole permeameter tests.
c     ***********************************************************
c     variable dictionary
c
c     head - dimensionless height of water in borehole
c     borehead - height of water in borehole
c     radius - radius of borehole
c     cdiva - dimensionless constant
c     z - dimensionless depth of the saturated bulb
c     r - dimensionless radius of the saturated bulb
c     eq - entered discharge
c     dq - dimensionless discharge
c     qd - dimensionless discharge
c     ks - saturated hydraulic conductivity
c     pck - philip's calculated saturated hydraulic conductivity
c     sck1 - stephens1' calculated saturated hydraulic conductivity
c     sck2 - stephens2' calculated saturated hydraulic conductivity
c     gck - glover's calculated saturated hydraulic conductivity
c     rck - reynolds calculated saturated hydraulic conductivity
c     rmfp - matric flux potential
c     cr - constant used in reynolds solution
c     b - length along screen from top of water level to screen
c     cu - dimensionless discharge divided by dimensionless head
c     hr - head in borehole divided by radius of borehole
c     dz - dimensioned depth of the saturated bulb
c     dr - dimensioned radius of the saturated bulb
c     nu - an angle related to the saturated bulb
c     dnu - incremental value of nu
c     c - dimensionless constant dependent on head
c     a - dimensionless constant dependent c and z
c     ealpha - philip's entered sorptive number
c     salpha - stephens' entered sorptive number
c     rmax - dimensionless maximum bulb radius
c     rmin - dimensionless minimum bulb radius
c     zmax - dimensionless maximum bulb depth
c     zrmax - dimensionless depth of maximum radius
c     perzrmax - dimensionless % total borehole depth of zrmax location
c     drmax - dimensioned maximum bulb radius
c     drmin - dimensioned minimum bulb radius
c     dzmax - dimensioned maximum bulb depth
c     dzrmax - dimensioned depth of maximum radius
c     dperzrmax - dimensioned % total borehole depth of zrmax location
c     x1,x2,...,xn - dummy variables used in calculations
c     k - counter variable
c     m - counter variable
c     n - counter variable
c     filename - output filename
c     plotfile - dimensioned plot filename
c     testname - borehole permeameter test name
c     ***********************************************************
c     defining varibles
c
      dimension z(2500),r(2500),dz(2500),dr(2500)
      real head,depth,radius,cdiva,nu,dnu,rmax,zmax,zrmax,a,c
```

```fortran
      real perzrmax,hr,eq,pck,sck1,sck2,gck,rck,b,cr
      real ealpha,salpha,dq,qd,rmfp
      integer k,n,m
      character *30 filename,plotfile,testname
      pi=3.14159
c     **********************************************************
c     reading input data
c
      write(*,*)' enter radius of borehole (m) '
      read(*,*)radius
      write(*,*)' enter depth of borehole (m) '
      read(*,*)depth
      write(*,*)' enter height of water in borehole (m) '
      read(*,*)borehead
      write(*,*)' enter length along borehole before screen '
      read(*,*)b
      write(*,*)' enter discharge rate (liters/min) '
      read(*,*)eq
      write(*,*)' enter stephens sorptive number (1/m) '
      read(*,*)salpha
      write(*,*)' enter philips sorptive number (1/m) '
      read(*,*)ealpha
      write(*,*)' enter reynolds matric flux potential (cm**2/s) '
      read(*,*)rmfp
      write(*,*)' enter output file name '
      read(*,*)filename
      write(*,*)' enter bulb shape plotfile name '
      read(*,*)plotfile
      write(*,*)' enter borehole test name '
      read(*,*)testname
c     **********************************************************
c     opening output and plot files
c
      open(unit=21,file=filename,status='unknown')
      open(unit=23,file=plotfile,status='unknown')
c     **********************************************************
c     calculates the saturated bulb shape using equation 17 and
c     19 from philip article.
c
      k=1
      m=0
      n=0
      zmax=0.0
      rmax=0.0
      zrmax=0.0
      perzrmax=0.0
      head=borehead/radius
      a=0.5*ealpha*radius
      c=0.56+(0.35/head)
      cdiva=c/a
      dnu=0.002
      write(*,*)
      write(*,*)' calculating bulb shape '
      write(*,*)
      write(*,*)' dnu = ',dnu
      do 10 i=1,2500
         nu=nu+dnu
         x1=log(head+sqrt(head**2.0-1.0))
         x2=sqrt(1.0-(head**(-2.0)))
         x3=log((1+cosh(nu))/sinh(nu))
```

```
         x4=1.0/cosh(nu)
         z(i)=cdiva*((x1-x2)/(x3-x4))*(1.0-(x3/x1))
         dz(i)=z(i)*radius
         if(z(i).gt.0.0)then
            x5=((3.0/2.0)**(2.0/3.0)*((head**2.0)-1.0))
            x6=(z(i)**2.0)/(cosh(nu)**2.)
            x7=(sinh(nu))**2.0
            x8=x5-x6
            if(x8.lt.2.0.and.m.eq.0)then
               dnu=dnu/5.0
               m=1
               write(*,*)' dnu = ',dnu
            end if
            if(x8.lt.0.5.and.m.eq.1)then
               dnu=dnu/10.0
               m=2
               write(*,*)' dnu = ',dnu
            end if
            if(x8.lt.0.1.and.m.eq.2)then
               dnu=dnu/10.0
               m=3
               write(*,*)' dnu = ',dnu
            end if
            if(x8.lt.0.0001.and.m.eq.3)then
               dnu=dnu/100.0
               m=4
               write(*,*)' dnu = ',dnu
            end if
            if(x6.gt.x5)goto 500
            r(i)=sqrt((x5-x6)*x7)
            dr(i)=r(i)*radius
            if(z(i).gt.zmax)then
               zmax=z(i)
            end if
            if(r(i).gt.rmax)then
               rmax=r(i)
               zrmax=z(i)
            end if
            if(r(i).lt.r(i-1))then
               rmin=r(i)
            end if
            k=k+1
      end if
      n=n+1
 10   continue
 500  continue
      perzrmax=(zrmax/zmax)*100.0
c     ********************************************************
c     calculate saturated hydraulic conductivity and cu and h/r
c     values using equations 18, and 33 from philip article.
c
      x9=sqrt((head**2.0)-1.0)
      x10=log(head+(sqrt((head**2.0)-1.0)))
      x11=4.117*head*(1.0-(head**(-2.0)))
      dq=x9*((x11/(x10-x2))+(cdiva*(7.192/x1)))
      pck=(eq/(dq*((radius*100.0)**2.0)))*(1000.0/60.0)
      cu=dq/head
      hr=borehead/radius
c     ********************************************************
c     calculate saturated hydraulic conductivity using glover's
```

```
c     (1953) borehole permeameter solution.
c
      qd=(2*pi*(hr))/(log((hr)+sqrt((hr)**2.0+1.0))-1.0)
      gck=(eq*(1000.0/60.0))/(qd*borehead*100.0*radius*100.0)
      **********************************************************
c     calculate saturated hydraulic conductivity using stephens'
c     (1979) borehole permeameter solution. (stephens I)
c
      x12=0.658*alog10(hr)
      x13=-0.238*sqrt(salpha)
      x14=-0.398*alog10(borehead)
      x15=x12+x13+x14+1.342
      qd1=10.0**x15
      sck1=(eq*(1000.0/60.0))/(qd1*borehead*100.0*radius*100.0)
c     **********************************************************
c     calculate saturated hydraulic conductivity using stephens'
c     (1987) borehole permeameter solution. (stephens II)
c
      x16=0.486*alog10(hr)
      x17=0.004/(salpha/100.0)
      x18=-0.454*alog10(borehead)
      x19=0.019*sqrt(hr)
      x20=x16+x17+x18+x19+0.828
      qd2=10**x20
      sck2=(eq*(1000.0/60.0))/(qd2*borehead*100.0*radius*100.0)
c     **********************************************************
c     calculate saturated hydraulic conductivity using reynolds
c     et al.'s (1986) borehole permeameter solution.
c
      x21=borehead**2.0
      x22=(borehead-b)/borehead
      x23=(borehead-b)/radius
      x24=(radius/borehead)**2.0
      x25=x22**2.0
      x26=radius/borehead
      x27=(borehead-b)**2.0
      x28=log(x23+sqrt((x23**2.0 + 1.0)))
      cr=(x21*((x22*x28)-sqrt(x24+x25)+x26))/x27
      x29=(cr*(eq*1000.0/60.0))-(2.0*pi*borehead*100.0*rmfp)
      x30=2.0*pi*((borehead*100.0)**2.0)
      x31=(1+((cr/2.0)*(((radius*100.0)/(borehead*100.0))**2.0)))
      rck=x29/(x30*x31)
c     **********************************************************
c     convert dimensionless values to dimensioned values
c
      dzmax=zmax*radius
      drmax=rmax*radius
      drmin=rmin*radius
      dzrmax=zrmax*radius
c     **********************************************************
c     write bulb shape to plotting file
c
      do 40 i=1,n
         if(z(i).gt.0.0)then
           write(23,16)dr(i),dz(i)
         end if
 40   continue
c     **********************************************************
c     determine coordinates for plotting borehole
c
```

```
      write(23,*)
      write(23,16)0.0,0.0
      write(23,16)radius,0.0
      write(23,16)radius,borehead
      write(23,16)0.0,borehead
c     ***********************************************************
c     print out results
c
      write(21,11)
      write(21,12)
      write(21,17)
      write(21,13)
      write(21,14)
      write(21,18)
      write(21,*)' borehole permeameter test identification = ',testname
      write(21,*)
      write(21,*)' output file name                          = ',filename
      write(21,*)' bulb shape plotfile                       = ',plotfile
      write(21,*)
      write(21,*)' *** borehole parameters *** '
      write(21,*)
      write(21,*)' radius of borehole (m)                    = ',radius
      write(21,*)' depth of borehole (m)                     = ',depth
      write(21,*)' constant head in borehole (m)             = ',borehead
      write(21,*)' h/r                                       = ',hr
      write(21,*)' discharge into borehole (liters/min)      = ',eq
      write(21,*)
      write(21,*)' *** soil parameters *** '
      write(21,*)
      write(21,*)' stephens sorptive number (1/m)            = ',salpha
      write(21,*)' philips sorptive number (1/m)             = ',ealpha
      write(21,*)' reynolds matric flux potential (cm**2/s) = ',rmfp
      write(21,*)
      write(21,*)' *** calculated hydraulic conductivity *** '
      write(21,*)
      write(21,*)' glovers solution (cm/s)                   = ',gck
      write(21,*)' stephens I solution (cm/s)                = ',sck1
      write(21,*)' stephens II solution (cm/s)               = ',sck2
      write(21,*)' philips solution (cm/s)                   = ',pck
      write(21,*)' reynolds solution (cm/s)                  = ',rck
      write(21,*)
      write(21,*)' *** bulb shape *** '
      write(21,*)
      write(21,*)' maximum bulb depth (m)                    = ',dzmax
      write(21,*)' maximum bulb radius (m)                   = ',drmax
      write(21,*)' depth of maximum bulb radius (m)          = ',dzrmax
      write(21,*)' % of total depth max bulb radius          = ',perzrmax
      write(21,*)' minimum bulb radius (m)                   = ',drmin
      write(21,*)
      write(21,*)' *** information on data points for bulb shape *** '
      write(21,*)
      write(21,*)' total number of data points               = ',k
c     ***********************************************************
c     format statements
c
 11   format(/,3x,'al laase, independent study, borehole research',/)
 12   format(3x,'equations from glover(1953), stephens(1979),')
 17   format(5x,'stephens(1987),philip(1985), and reynolds(1986)')
 13   format(/,3x,'borehole input data and results',/)
 14   format(3x,'sorptive numbers, matric flux potential,')
```

```
 18    format(5x,'radius, head, and discharge are known',/)
 16    format(1x,f12.8,1x,f12.8)
c      ******************************************************
       stop
       end
```

APPENDIX F

al laase, independent study, borehole research

equations from glover(1953), stephens(1979),
stephens(1987),philip(1985), and reynolds(1986)

borehole input data and results

sorptive numbers, matric flux potential,
radius, head, and discharge are known

borehole permeameter test identification	=	s1t1
output file name	=	t1.out
bulb shape plotfile	=	t1.plot

*** borehole parameters ***

radius of borehole (m)	=	3.20000e-02
depth of borehole (m)	=	3.000000
constant head in borehole (m)	=	1.130000
h/r	=	35.31250
discharge into borehole (liters/min)	=	3.100000

*** soil parameters ***

stephens sorptive number (1/m)	=	5.600000
philips sorptive number (1/m)	=	9.000000
reynolds matric flux potential (cm**2/s)	=	2.00000e-02

*** calculated hydraulic conductivity ***

glovers solution (cm/s)	=	2.09783e-03
stephens I solution (cm/s)	=	2.39211e-03
stephens II solution (cm/s)	=	2.59688e-03
philips solution (cm/s)	=	2.78751e-03
reynolds solution (cm/s)	=	1.93626e-03

*** bulb shape ***

maximum bulb depth (m)	=	1.810837
maximum bulb radius (m)	=	0.750192
depth of maximum bulb radius (m)	=	0.994725
% of total depth max bulb radius	=	54.93177
minimum bulb radius (m)	=	7.50655e-02

*** information on data points for bulb shape ***

total number of data points	=	422

```
al laase, independent study, borehole research

equations from glover(1953), stephens(1979),
  stephens(1987),philip(1985), and reynolds(1986)

borehole input data and results

sorptive numbers, matric flux potential,
  radius, head, and discharge are known

borehole permeameter test identification =   s2t1

output file name                         =   t2.out
bulb shape plotfile                      =   t2.plot

*** borehole parameters ***

radius of borehole (m)                   =     4.70000e-02
depth of borehole (m)                    =    3.000000
constant head in borehole (m)            =    0.602000
h/r                                      =    12.80851
discharge into borehole (liters/min)     =    2.200000

*** soil parameters ***

stephens sorptive number (1/m)           =    5.600000
philips sorptive number (1/m)            =    9.000000
reynolds matric flux potential (cm**2/s) =     2.00000e-02

*** calculated hydraulic conductivity ***

glovers solution (cm/s)                  =     3.61470e-03
stephens I solution (cm/s)               =     3.29104e-03
stephens II solution (cm/s)              =     3.21249e-03
philips solution (cm/s)                  =     4.40506e-03
reynolds solution (cm/s)                 =     3.37940e-03

*** bulb shape ***

maximum bulb depth (m)                   =    0.909325
maximum bulb radius (m)                  =    0.360226
depth of maximum bulb radius (m)         =    0.504306
% of total depth max bulb radius         =    55.45940
minimum bulb radius (m)                  =     6.07872e-02

*** information on data points for bulb shape ***

total number of data points              =   358
```

al laase, independent study, borehole research

equations from glover(1953), stephens(1979), stephens(1987),philip(1985), and reynolds(1986)

borehole input data and results

sorptive numbers, matric flux potential, radius, head, and discharge are known

borehole permeameter test identification	=	s2t2
output file name	=	t3.out
bulb shape plotfile	=	t3.plot

*** borehole parameters ***

radius of borehole (m)	=	5.10000e-02
depth of borehole (m)	=	3.000000
constant head in borehole (m)	=	0.914000
h/r	=	17.92157
discharge into borehole (liters/min)	=	5.100000

*** soil parameters ***

stephens sorptive number (1/m)	=	5.600000
philips sorptive number (1/m)	=	9.000000
reynolds matric flux potential (cm**2/s)	=	2.00000e-02

*** calculated hydraulic conductivity ***

glovers solution (cm/s)	=	4.17787e-03
stephens I solution (cm/s)	=	4.38378e-03
stephens II solution (cm/s)	=	4.50992e-03
philips solution (cm/s)	=	5.43793e-03
reynolds solution (cm/s)	=	4.03036e-03

*** bulb shape ***

maximum bulb depth (m)	=	1.487253
maximum bulb radius (m)	=	0.632947
depth of maximum bulb radius (m)	=	0.799940
% of total depth max bulb radius	=	53.78644
minimum bulb radius (m)	=	4.37774e-02

*** information on data points for bulb shape ***

total number of data points	=	420

al laase, independent study, borehole research

equations from glover(1953), stephens(1979),
stephens(1987),philip(1985), and reynolds(1986)

borehole input data and results

sorptive numbers, matric flux potential,
radius, head, and discharge are known

borehole permeameter test identification	=	s2t3
output file name	=	t4.out
bulb shape plotfile	=	t4.plot

*** borehole parameters ***

radius of borehole (m)	=	4.70000e-02
depth of borehole (m)	=	3.000000
constant head in borehole (m)	=	1.030000
h/r	=	21.91489
discharge into borehole (liters/min)	=	3.400000

*** soil parameters ***

stephens sorptive number (1/m)	=	5.600000
philips sorptive number (1/m)	=	9.000000
reynolds matric flux potential (cm**2/s)	=	2.00000e-02

*** calculated hydraulic conductivity ***

glovers solution (cm/s)	=	2.36401e-03
stephens I solution (cm/s)	=	2.58527e-03
stephens II solution (cm/s)	=	2.71799e-03
philips solution (cm/s)	=	3.11984e-03
reynolds solution (cm/s)	=	2.20126e-03

*** bulb shape ***

maximum bulb depth (m)	=	1.695290
maximum bulb radius (m)	=	0.725893
depth of maximum bulb radius (m)	=	0.912615
% of total depth max bulb radius	=	53.83236
minimum bulb radius (m)	=	1.00571e-03

*** information on data points for bulb shape ***

total number of data points	=	440

al laase, independent study, borehole research

equations from glover(1953), stephens(1979), stephens(1987),philip(1985), and reynolds(1986)

borehole input data and results

sorptive numbers, matric flux potential, radius, head, and discharge are known

borehole permeameter test identification	=	s2t4
output file name	=	t5.out
bulb shape plotfile	=	t5.plot

*** borehole parameters ***

radius of borehole (m)	=	5.70000e-02
depth of borehole (m)	=	3.000000
constant head in borehole (m)	=	0.920000
h/r	=	16.14035
discharge into borehole (liters/min)	=	4.000000

*** soil parameters ***

stephens sorptive number (1/m)	=	5.600000
philips sorptive number (1/m)	=	9.000000
reynolds matric flux potential (cm**2/s)	=	2.00000e-02

*** calculated hydraulic conductivity ***

glovers solution (cm/s)	=	3.10316e-03
stephens I solution (cm/s)	=	3.28275e-03
stephens II solution (cm/s)	=	3.34965e-03
philips solution (cm/s)	=	4.05183e-03
reynolds solution (cm/s)	=	2.94669e-03

*** bulb shape ***

maximum bulb depth (m)	=	1.513471
maximum bulb radius (m)	=	0.658534
depth of maximum bulb radius (m)	=	0.813677
% of total depth max bulb radius	=	53.76232
minimum bulb radius (m)	=	9.67339e-02

*** information on data points for bulb shape ***

total number of data points	=	426

```
al laase, independent study, borehole research

equations from glover(1953), stephens(1979),
  stephens(1987),philip(1985), and reynolds(1986)

borehole input data and results

sorptive numbers, matric flux potential,
  radius, head, and discharge are known

borehole permeameter test identification =   s2t5

output file name                          =   t6.out
bulb shape plotfile                       =   t6.plot

*** borehole parameters ***

radius of borehole (m)                    =     8.90000e-02
depth of borehole (m)                     =    3.000000
constant head in borehole (m)             =    0.940000
h/r                                       =   10.561797
discharge into borehole (liters/min)      =    4.200000

*** soil parameters ***

stephens sorptive number (1/m)            =    5.600000
philips sorptive number (1/m)             =    9.000000
reynolds matric flux potential (cm**2/s) =     2.00000e-02

*** calculated hydraulic conductivity ***

glovers solution (cm/s)                   =     2.58805e-03
stephens I solution (cm/s)                =     2.88056e-03
stephens II solution (cm/s)               =     2.82922e-03
philips solution (cm/s)                   =     3.43222e-03
reynolds solution (cm/s)                  =     2.46534e-03

*** bulb shape ***

maximum bulb depth (m)                    =    1.659729
maximum bulb radius (m)                   =    0.771170
depth of maximum bulb radius (m)          =    0.856575
% of total depth max bulb radius          =    51.60931
minimum bulb radius (m)                   =     5.78976e-03

*** information on data points for bulb shape ***

total number of data points               =   497
```

al laase, independent study, borehole research

equations from glover(1953), stephens(1979), stephens(1987),philip(1985), and reynolds(1986)

borehole input data and results

sorptive numbers, matric flux potential, radius, head, and discharge are known

borehole permeameter test identification	=	s3t1
output file name	=	t7.out
bulb shape plotfile	=	t7.plot

*** borehole parameters ***

radius of borehole (m)	=	6.00000e-02
depth of borehole (m)	=	3.000000
constant head in borehole (m)	=	0.900000
h/r	=	15.00000
discharge into borehole (liters/min)	=	5.700000

*** soil parameters ***

stephens sorptive number (1/m)	=	5.600000
philips sorptive number (1/m)	=	9.000000
reynolds matric flux potential (cm**2/s)	=	2.00000e-02

*** calculated hydraulic conductivity ***

glovers solution (cm/s)	=	4.48423e-03
stephens I solution (cm/s)	=	4.72565e-03
stephens II solution (cm/s)	=	4.78586e-03
philips solution (cm/s)	=	5.84566e-03
reynolds solution (cm/s)	=	4.35841e-03

*** bulb shape ***

maximum bulb depth (m)	=	1.492653
maximum bulb radius (m)	=	0.649425
depth of maximum bulb radius (m)	=	0.795115
% of total depth max bulb radius	=	53.26855
minimum bulb radius (m)	=	2.28839e-03

*** information on data points for bulb shape ***

total number of data points	=	448

```
al laase, independent study, borehole research

equations from glover(1953), stephens(1979),
  stephens(1987),philip(1985), and reynolds(1986)

borehole input data and results

sorptive numbers, matric flux potential,
  radius, head, and discharge are known

borehole permeameter test identification =   s3t2

output file name                         =   t8.out
bulb shape plotfile                      =   t8.plot

*** borehole parameters ***

radius of borehole (m)                   =     8.90000e-02
depth of borehole (m)                    =    3.000000
constant head in borehole (m)            =    0.965000
h/r                                      =    10.84270
discharge into borehole (liters/min)     =    5.800000

*** soil parameters ***

stephens sorptive number (1/m)           =    5.600000
philips sorptive number (1/m)            =    9.000000
reynolds matric flux potential (cm**2/s) =     2.00000e-02

*** calculated hydraulic conductivity ***

glovers solution (cm/s)                  =     3.43437e-03
stephens I solution (cm/s)               =     3.84850e-03
stephens II solution (cm/s)              =     3.79547e-03
philips solution (cm/s)                  =     4.56696e-03
reynolds solution (cm/s)                 =     3.34168e-03

*** bulb shape ***

maximum bulb depth (m)                   =    1.710781
maximum bulb radius (m)                  =    0.796816
depth of maximum bulb radius (m)         =    0.881509
% of total depth max bulb radius         =    51.52670
minimum bulb radius (m)                  =     5.46304e-03

*** information on data points for bulb shape ***

total number of data points              =   495
```

al laase, independent study, borehole research

equations from glover(1953), stephens(1979),
  stephens(1987),philip(1985), and reynolds(1986)

borehole input data and results

sorptive numbers, matric flux potential,
  radius, head, and discharge are known

```
borehole permeameter test identification =   s3t3

output file name                         =   t9.out
bulb shape plotfile                      =   t9.plot

*** borehole parameters ***

radius of borehole (m)                   =     8.90000e-02
depth of borehole (m)                    =    3.000000
constant head in borehole (m)            =    0.965000
h/r                                      =    10.84270
discharge into borehole (liters/min)     =    8.500000

*** soil parameters ***

stephens sorptive number (1/m)           =    5.600000
philips sorptive number (1/m)            =    9.000000
reynolds matric flux potential (cm**2/s) =     2.00000e-02

*** calculated hydraulic conductivity ***

glovers solution (cm/s)                  =     5.03312e-03
stephens I solution (cm/s)               =     5.64005e-03
stephens II solution (cm/s)              =     5.56233e-03
philips solution (cm/s)                  =     6.69296e-03
reynolds solution (cm/s)                 =     5.24090e-03

*** bulb shape ***

maximum bulb depth (m)                   =    1.710781
maximum bulb radius (m)                  =    0.796816
depth of maximum bulb radius (m)         =    0.881509
% of total depth max bulb radius         =    51.52670
minimum bulb radius (m)                  =     5.46304e-03

*** information on data points for bulb shape ***

total number of data points              =   495
```

```
al laase, independent study, borehole research

equations from glover(1953), stephens(1979),
  stephens(1987),philip(1985), and reynolds(1986)

borehole input data and results

sorptive numbers, matric flux potential,
  radius, head, and discharge are known

borehole permeameter test identification =   s3t4

output file name                         =   t10.out
bulb shape plotfile                      =   t10.plot

*** borehole parameters ***

radius of borehole (m)                   =     8.90000e-02
depth of borehole (m)                    =    3.000000
constant head in borehole (m)            =    0.965000
h/r                                      =    10.84270
discharge into borehole (liters/min)     =    6.700000

*** soil parameters ***

stephens sorptive number (1/m)           =    5.600000
philips sorptive number (1/m)            =    9.000000
reynolds matric flux potential (cm**2/s) =     2.00000e-02

*** calculated hydraulic conductivity ***

glovers solution (cm/s)                  =     3.96729e-03
stephens I solution (cm/s)               =     4.44568e-03
stephens II solution (cm/s)              =     4.38442e-03
philips solution (cm/s)                  =     5.27562e-03
reynolds solution (cm/s)                 =     4.08759e-03

*** bulb shape ***

maximum bulb depth (m)                   =    1.710781
maximum bulb radius (m)                  =    0.796816
depth of maximum bulb radius (m)         =    0.881509
% of total depth max bulb radius         =    51.52670
minimum bulb radius (m)                  =     5.46304e-03

*** information on data points for bulb shape ***

total number of data points              =   495
```

al laase, independent study, borehole research

equations from glover(1953), stephens(1979),
stephens(1987),philip(1985), and reynolds(1986)

borehole input data and results

sorptive numbers, matric flux potential,
radius, head, and discharge are known

borehole permeameter test identification	=	s3t6
output file name	=	t11.out
bulb shape plotfile	=	t11.plot

*** borehole parameters ***

radius of borehole (m)	=	8.90000e-02
depth of borehole (m)	=	3.000000
constant head in borehole (m)	=	0.965000
h/r	=	10.84270
discharge into borehole (liters/min)	=	15.20000

*** soil parameters ***

stephens sorptive number (1/m)	=	5.600000
philips sorptive number (1/m)	=	9.000000
reynolds matric flux potential (cm**2/s)	=	2.00000e-02

*** calculated hydraulic conductivity ***

glovers solution (cm/s)	=	9.00041e-03
stephens I solution (cm/s)	=	1.00857e-02
stephens II solution (cm/s)	=	9.94676e-03
philips solution (cm/s)	=	1.19686e-02
reynolds solution (cm/s)	=	9.53376e-03

*** bulb shape ***

maximum bulb depth (m)	=	1.710781
maximum bulb radius (m)	=	0.796816
depth of maximum bulb radius (m)	=	0.881509
% of total depth max bulb radius	=	51.52670
minimum bulb radius (m)	=	5.46304e-03

*** information on data points for bulb shape ***

total number of data points	=	495

al laase, independent study, borehole research

equations from glover(1953), stephens(1979),
stephens(1987),philip(1985), and reynolds(1986)

borehole input data and results

sorptive numbers, matric flux potential,
radius, head, and discharge are known

borehole permeameter test identification = s3t7

output file name	=	t12.out
bulb shape plotfile	=	t12.plot

*** borehole parameters ***

radius of borehole (m)	=	8.90000e-02
depth of borehole (m)	=	3.000000
constant head in borehole (m)	=	0.965000
h/r	=	10.84270
discharge into borehole (liters/min)	=	9.400000

*** soil parameters ***

stephens sorptive number (1/m)	=	5.600000
philips sorptive number (1/m)	=	9.000000
reynolds matric flux potential (cm**2/s)	=	2.00000e-02

*** calculated hydraulic conductivity ***

glovers solution (cm/s)	=	5.56604e-03
stephens I solution (cm/s)	=	6.23723e-03
stephens II solution (cm/s)	=	6.15128e-03
philips solution (cm/s)	=	7.40162e-03
reynolds solution (cm/s)	=	5.81755e-03

*** bulb shape ***

maximum bulb depth (m)	=	1.710781
maximum bulb radius (m)	=	0.796816
depth of maximum bulb radius (m)	=	0.881509
% of total depth max bulb radius	=	51.52670
minimum bulb radius (m)	=	5.46304e-03

*** information on data points for bulb shape ***

total number of data points = 495

al laase, independent study, borehole research

equations from glover(1953), stephens(1979),
stephens(1987),philip(1985), and reynolds(1986)

borehole input data and results

sorptive numbers, matric flux potential,
radius, head, and discharge are known

borehole permeameter test identification	=	s4t2
output file name	=	t13.out
bulb shape plotfile	=	t13.plot

*** borehole parameters ***

radius of borehole (m)	=	5.80000e-02
depth of borehole (m)	=	3.000000
constant head in borehole (m)	=	0.914000
h/r	=	15.75862
discharge into borehole (liters/min)	=	6.900000

*** soil parameters ***

stephens sorptive number (1/m)	=	5.600000
philips sorptive number (1/m)	=	9.000000
reynolds matric flux potential (cm**2/s)	=	2.00000e-02

*** calculated hydraulic conductivity ***

glovers solution (cm/s)	=	5.37112e-03
stephens I solution (cm/s)	=	5.67576e-03
stephens II solution (cm/s)	=	5.77758e-03
philips solution (cm/s)	=	7.01002e-03
reynolds solution (cm/s)	=	5.26031e-03

*** bulb shape ***

maximum bulb depth (m)	=	1.506568
maximum bulb radius (m)	=	0.656166
depth of maximum bulb radius (m)	=	0.809560
% of total depth max bulb radius	=	53.73535
minimum bulb radius (m)	=	8.83857e-02

*** information on data points for bulb shape ***

total number of data points	=	427

al laase, independent study, borehole research

equations from glover(1953), stephens(1979),
stephens(1987),philip(1985), and reynolds(1986)

borehole input data and results

sorptive numbers, matric flux potential,
radius, head, and discharge are known

borehole permeameter test identification = s4t3

output file name = t14.out
bulb shape plotfile = t14.plot

*** borehole parameters ***

radius of borehole (m)	=	5.80000e-02
depth of borehole (m)	=	3.000000
constant head in borehole (m)	=	0.914000
h/r	=	15.75862
discharge into borehole (liters/min)	=	1.300000

*** soil parameters ***

stephens sorptive number (1/m)	=	5.600000
philips sorptive number (1/m)	=	9.000000
reynolds matric flux potential (cm**2/s)	=	2.00000e-02

*** calculated hydraulic conductivity ***

glovers solution (cm/s)	=	1.01195e-03
stephens I solution (cm/s)	=	1.06935e-03
stephens II solution (cm/s)	=	1.08853e-03
philips solution (cm/s)	=	1.32073e-03
reynolds solution (cm/s)	=	8.14375e-04

*** bulb shape ***

maximum bulb depth (m)	=	1.506568
maximum bulb radius (m)	=	0.656166
depth of maximum bulb radius (m)	=	0.809560
% of total depth max bulb radius	=	53.73535
minimum bulb radius (m)	=	8.83857e-02

*** information on data points for bulb shape ***

total number of data points = 427

al laase, independent study, borehole research

equations from glover(1953), stephens(1979),
stephens(1987),philip(1985), and reynolds(1986)

borehole input data and results

sorptive numbers, matric flux potential,
radius, head, and discharge are known

borehole permeameter test identification	=	s4t4
output file name	=	t15.out
bulb shape plotfile	=	t15.plot

*** borehole parameters ***

radius of borehole (m)	=	5.80000e-02
depth of borehole (m)	=	3.000000
constant head in borehole (m)	=	0.229000
h/r	=	3.948276
discharge into borehole (liters/min)	=	0.300000

*** soil parameters ***

stephens sorptive number (1/m)	=	5.600000
philips sorptive number (1/m)	=	9.000000
reynolds matric flux potential (cm**2/s)	=	2.00000e-02

*** calculated hydraulic conductivity ***

glovers solution (cm/s)	=	1.64204e-03
stephens I solution (cm/s)	=	1.41155e-03
stephens II solution (cm/s)	=	1.14297e-03
philips solution (cm/s)	=	1.70894e-03
reynolds solution (cm/s)	=	1.06074e-03

*** bulb shape ***

maximum bulb depth (m)	=	0.332533
maximum bulb radius (m)	=	0.136996
depth of maximum bulb radius (m)	=	0.167236
% of total depth max bulb radius	=	50.29157
minimum bulb radius (m)	=	5.12972e-04

*** information on data points for bulb shape ***

total number of data points	=	550

```
al laase, independent study, borehole research

equations from glover(1953), stephens(1979),
  stephens(1987),philip(1985), and reynolds(1986)

borehole input data and results

sorptive numbers, matric flux potential,
  radius, head, and discharge are known

borehole permeameter test identification =   s5t3

output file name                         =   t16.out
bulb shape plotfile                      =   t16.plot

*** borehole parameters ***

radius of borehole (m)                   =     5.80000e-02
depth of borehole (m)                    =    3.000000
constant head in borehole (m)            =    0.762000
h/r                                      =    13.13793
discharge into borehole (liters/min)     =    8.000000

*** soil parameters ***

stephens sorptive number (1/m)           =    5.600000
philips sorptive number (1/m)            =    9.000000
reynolds matric flux potential (cm**2/s) =     2.00000e-02

*** calculated hydraulic conductivity ***

glovers solution (cm/s)                  =     8.29647e-03
stephens I solution (cm/s)               =     8.27549e-03
stephens II solution (cm/s)              =     8.20470e-03
philips solution (cm/s)                  =     1.05657e-02
reynolds solution (cm/s)                 =     8.24564e-03

*** bulb shape ***

maximum bulb depth (m)                   =    1.227242
maximum bulb radius (m)                  =    0.519354
depth of maximum bulb radius (m)         =    0.659753
% of total depth max bulb radius         =    53.75897
minimum bulb radius (m)                  =     3.77416e-03

*** information on data points for bulb shape ***

total number of data points              =   419
```

al laase, independent study, borehole research

equations from glover(1953), stephens(1979),
stephens(1987),philip(1985), and reynolds(1986)

borehole input data and results

sorptive numbers, matric flux potential,
radius, head, and discharge are known

borehole permeameter test identification = s5t4

output file name	=	t17.out
bulb shape plotfile	=	t17.plot

*** borehole parameters ***

radius of borehole (m)	=	5.80000e-02
depth of borehole (m)	=	3.000000
constant head in borehole (m)	=	0.762000
h/r	=	13.13793
discharge into borehole (liters/min)	=	4.900000

*** soil parameters ***

stephens sorptive number (1/m)	=	5.600000
philips sorptive number (1/m)	=	9.000000
reynolds matric flux potential (cm**2/s)	=	2.00000e-02

*** calculated hydraulic conductivity ***

glovers solution (cm/s)	=	5.08159e-03
stephens I solution (cm/s)	=	5.06874e-03
stephens II solution (cm/s)	=	5.02538e-03
philips solution (cm/s)	=	6.47152e-03
reynolds solution (cm/s)	=	4.94943e-03

*** bulb shape ***

maximum bulb depth (m)	=	1.227242
maximum bulb radius (m)	=	0.519354
depth of maximum bulb radius (m)	=	0.659753
% of total depth max bulb radius	=	53.75897
minimum bulb radius (m)	=	3.77416e-03

*** information on data points for bulb shape ***

total number of data points = 419

al laase, independent study, borehole research

equations from glover(1953), stephens(1979),
stephens(1987),philip(1985), and reynolds(1986)

borehole input data and results

sorptive numbers, matric flux potential,
radius, head, and discharge are known

borehole permeameter test identification	=	s5t5
output file name	=	t18.out
bulb shape plotfile	=	t18.plot

*** borehole parameters ***

radius of borehole (m)	=	5.80000e-02
depth of borehole (m)	=	3.000000
constant head in borehole (m)	=	0.762000
h/r	=	13.13793
discharge into borehole (liters/min)	=	6.600000

*** soil parameters ***

stephens sorptive number (1/m)	=	5.600000
philips sorptive number (1/m)	=	9.000000
reynolds matric flux potential (cm**2/s)	=	2.00000e-02

*** calculated hydraulic conductivity ***

glovers solution (cm/s)	=	6.84459e-03
stephens I solution (cm/s)	=	6.82728e-03
stephens II solution (cm/s)	=	6.76888e-03
philips solution (cm/s)	=	8.71674e-03
reynolds solution (cm/s)	=	6.75703e-03

*** bulb shape ***

maximum bulb depth (m)	=	1.227242
maximum bulb radius (m)	=	0.519354
depth of maximum bulb radius (m)	=	0.659753
% of total depth max bulb radius	=	53.75897
minimum bulb radius (m)	=	3.77416e-03

*** information on data points for bulb shape ***

total number of data points	=	419

al laase, independent study, borehole research

equations from glover(1953), stephens(1979),
stephens(1987),philip(1985), and reynolds(1986)

borehole input data and results

sorptive numbers, matric flux potential,
radius, head, and discharge are known

borehole permeameter test identification	=	s5t6
output file name	=	t19.out
bulb shape plotfile	=	t19.plot

*** borehole parameters ***

radius of borehole (m)	=	5.80000e-02
depth of borehole (m)	=	3.000000
constant head in borehole (m)	=	0.762000
h/r	=	13.13793
discharge into borehole (liters/min)	=	6.800000

*** soil parameters ***

stephens sorptive number (1/m)	=	5.600000
philips sorptive number (1/m)	=	9.000000
reynolds matric flux potential (cm**2/s)	=	2.00000e-02

*** calculated hydraulic conductivity ***

glovers solution (cm/s)	=	7.05200e-03
stephens I solution (cm/s)	=	7.03417e-03
stephens II solution (cm/s)	=	6.97399e-03
philips solution (cm/s)	=	8.98089e-03
reynolds solution (cm/s)	=	6.96969e-03

*** bulb shape ***

maximum bulb depth (m)	=	1.227242
maximum bulb radius (m)	=	0.519354
depth of maximum bulb radius (m)	=	0.659753
% of total depth max bulb radius	=	53.75897
minimum bulb radius (m)	=	3.77416e-03

*** information on data points for bulb shape ***

total number of data points	=	419

al laase, independent study, borehole research

equations from glover(1953), stephens(1979),
stephens(1987),philip(1985), and reynolds(1986)

borehole input data and results

sorptive numbers, matric flux potential,
radius, head, and discharge are known

borehole permeameter test identification	=	s5t7
output file name	=	t20.out
bulb shape plotfile	=	t20.plot

*** borehole parameters ***

radius of borehole (m)	=	5.80000e-02
depth of borehole (m)	=	3.000000
constant head in borehole (m)	=	0.762000
h/r	=	13.13793
discharge into borehole (liters/min)	=	6.900000

*** soil parameters ***

stephens sorptive number (1/m)	=	5.600000
philips sorptive number (1/m)	=	9.000000
reynolds matric flux potential (cm**2/s)	=	2.00000e-02

*** calculated hydraulic conductivity ***

glovers solution (cm/s)	=	7.15571e-03
stephens I solution (cm/s)	=	7.13761e-03
stephens II solution (cm/s)	=	7.07655e-03
philips solution (cm/s)	=	9.11296e-03
reynolds solution (cm/s)	=	7.07602e-03

*** bulb shape ***

maximum bulb depth (m)	=	1.227242
maximum bulb radius (m)	=	0.519354
depth of maximum bulb radius (m)	=	0.659753
% of total depth max bulb radius	=	53.75897
minimum bulb radius (m)	=	3.77416e-03

*** information on data points for bulb shape ***

total number of data points	=	419

al laase, independent study, borehole research

equations from glover(1953), stephens(1979),
stephens(1987),philip(1985), and reynolds(1986)

borehole input data and results

sorptive numbers, matric flux potential,
radius, head, and discharge are known

borehole permeameter test identification = s5t8

output file name = t21.out
bulb shape plotfile = t21.plot

*** borehole parameters ***

radius of borehole (m)	=	5.80000e-02
depth of borehole (m)	=	3.000000
constant head in borehole (m)	=	0.762000
h/r	=	13.13793
discharge into borehole (liters/min)	=	9.000000

*** soil parameters ***

stephens sorptive number (1/m)	=	5.600000
philips sorptive number (1/m)	=	9.000000
reynolds matric flux potential (cm**2/s)	=	2.00000e-02

*** calculated hydraulic conductivity ***

glovers solution (cm/s)	=	9.33353e-03
stephens I solution (cm/s)	=	9.30993e-03
stephens II solution (cm/s)	=	9.23029e-03
philips solution (cm/s)	=	1.18865e-02
reynolds solution (cm/s)	=	9.30893e-03

*** bulb shape ***

maximum bulb depth (m)	=	1.227242
maximum bulb radius (m)	=	0.519354
depth of maximum bulb radius (m)	=	0.659753
% of total depth max bulb radius	=	53.75897
minimum bulb radius (m)	=	3.77416e-03

*** information on data points for bulb shape ***

total number of data points = 419

al laase, independent study, borehole research

equations from glover(1953), stephens(1979),
stephens(1987),philip(1985), and reynolds(1986)

borehole input data and results

sorptive numbers, matric flux potential,
radius, head, and discharge are known

borehole permeameter test identification = s6t1

output file name = t22.out
bulb shape plotfile = t22.plot

*** borehole parameters ***

radius of borehole (m) = 5.80000e-02
depth of borehole (m) = 3.000000
constant head in borehole (m) = 1.550000
h/r = 26.72414
discharge into borehole (liters/min) = 18.30000

*** soil parameters ***

stephens sorptive number (1/m) = 5.600000
philips sorptive number (1/m) = 9.000000
reynolds matric flux potential (cm**2/s) = 2.00000e-02

*** calculated hydraulic conductivity ***

glovers solution (cm/s) = 6.01917e-03
stephens I solution (cm/s) = 7.73750e-03
stephens II solution (cm/s) = 8.42996e-03
philips solution (cm/s) = 8.31100e-03
reynolds solution (cm/s) = 7.00605e-03

*** bulb shape ***

maximum bulb depth (m) = 2.816760
maximum bulb radius (m) = 1.324852
depth of maximum bulb radius (m) = 1.474992
% of total depth max bulb radius = 52.36486
minimum bulb radius (m) = 0.177056

*** information on data points for bulb shape ***

total number of data points = 504

al laase, independent study, borehole research

equations from glover(1953), stephens(1979),
stephens(1987),philip(1985), and reynolds(1986)

borehole input data and results

sorptive numbers, matric flux potential,
radius, head, and discharge are known

borehole permeameter test identification	=	s6t3
output file name	=	t23.out
bulb shape plotfile	=	t23.plot

*** borehole parameters ***

radius of borehole (m)	=	5.80000e-02
depth of borehole (m)	=	3.000000
constant head in borehole (m)	=	1.210000
h/r	=	20.86207
discharge into borehole (liters/min)	=	9.100000

*** soil parameters ***

stephens sorptive number (1/m)	=	5.600000
philips sorptive number (1/m)	=	9.000000
reynolds matric flux potential (cm**2/s)	=	2.00000e-02

*** calculated hydraulic conductivity ***

glovers solution (cm/s)	=	4.50366e-03
stephens I solution (cm/s)	=	5.25653e-03
stephens II solution (cm/s)	=	5.55703e-03
philips solution (cm/s)	=	6.07235e-03
reynolds solution (cm/s)	=	4.40146e-03

*** bulb shape ***

maximum bulb depth (m)	=	2.090127
maximum bulb radius (m)	=	0.949572
depth of maximum bulb radius (m)	=	1.111053
% of total depth max bulb radius	=	53.15722
minimum bulb radius (m)	=	0.149899

*** information on data points for bulb shape ***

total number of data points	=	467

```
al laase, independent study, borehole research

equations from glover(1953), stephens(1979),
  stephens(1987),philip(1985), and reynolds(1986)

borehole input data and results

sorptive numbers, matric flux potential,
  radius, head, and discharge are known

borehole permeameter test identification =   s6t4

output file name                         =   t24.out
bulb shape plotfile                      =   t24.plot

*** borehole parameters ***

radius of borehole (m)                   =     5.80000e-02
depth of borehole (m)                    =    3.000000
constant head in borehole (m)            =    0.580000
h/r                                      =   10.000000
discharge into borehole (liters/min)     =    0.800000

*** soil parameters ***

stephens sorptive number (1/m)           =    5.600000
philips sorptive number (1/m)            =    9.000000
reynolds matric flux potential (cm**2/s) =     2.00000e-02

*** calculated hydraulic conductivity ***

glovers solution (cm/s)                  =     1.26051e-03
stephens I solution (cm/s)               =     1.16718e-03
stephens II solution (cm/s)              =     1.10960e-03
philips solution (cm/s)                  =     1.53954e-03
reynolds solution (cm/s)                 =     9.65515e-04

*** bulb shape ***

maximum bulb depth (m)                   =    0.901912
maximum bulb radius (m)                  =    0.369806
depth of maximum bulb radius (m)         =    0.486490
% of total depth max bulb radius         =   53.93987
minimum bulb radius (m)                  =     2.39190e-03

*** information on data points for bulb shape ***

total number of data points              =   417
```

```
al laase, independent study, borehole research

equations from glover(1953), stephens(1979),
  stephens(1987),philip(1985), and reynolds(1986)

borehole input data and results

sorptive numbers, matric flux potential,
  radius, head, and discharge are known

borehole permeameter test identification =   s6t5

output file name                         =   t25.out
bulb shape plotfile                      =   t25.plot

*** borehole parameters ***

radius of borehole (m)                   =     5.80000e-02
depth of borehole (m)                    =    3.000000
constant head in borehole (m)            =    0.570000
h/r                                      =    9.827586
discharge into borehole (liters/min)     =    3.000000

*** soil parameters ***

stephens sorptive number (1/m)           =    5.600000
philips sorptive number (1/m)            =    9.000000
reynolds matric flux potential (cm**2/s) =     2.00000e-02

*** calculated hydraulic conductivity ***

glovers solution (cm/s)                  =     4.85185e-03
stephens I solution (cm/s)               =     4.47390e-03
stephens II solution (cm/s)              =     4.24144e-03
philips solution (cm/s)                  =     5.90892e-03
reynolds solution (cm/s)                 =     4.68714e-03

*** bulb shape ***

maximum bulb depth (m)                   =    0.884599
maximum bulb radius (m)                  =    0.362084
depth of maximum bulb radius (m)         =    0.477507
% of total depth max bulb radius         =    53.98005
minimum bulb radius (m)                  =     1.93967e-03

*** information on data points for bulb shape ***

total number of data points              =   401
```

al laase, independent study, borehole research

equations from glover(1953), stephens(1979),
stephens(1987),philip(1985), and reynolds(1986)

borehole input data and results

sorptive numbers, matric flux potential,
radius, head, and discharge are known

borehole permeameter test identification	=	s6t6
output file name	=	t26.out
bulb shape plotfile	=	t26.plot

*** borehole parameters ***

radius of borehole (m)	=	5.80000e-02
depth of borehole (m)	=	3.000000
constant head in borehole (m)	=	0.914000
h/r	=	15.75862
discharge into borehole (liters/min)	=	3.000000

*** soil parameters ***

stephens sorptive number (1/m)	=	5.600000
philips sorptive number (1/m)	=	9.000000
reynolds matric flux potential (cm**2/s)	=	2.00000e-02

*** calculated hydraulic conductivity ***

glovers solution (cm/s)	=	2.33527e-03
stephens I solution (cm/s)	=	2.46772e-03
stephens II solution (cm/s)	=	2.51199e-03
philips solution (cm/s)	=	3.04783e-03
reynolds solution (cm/s)	=	2.26817e-03

*** bulb shape ***

maximum bulb depth (m)	=	1.506568
maximum bulb radius (m)	=	0.656166
depth of maximum bulb radius (m)	=	0.809560
% of total depth max bulb radius	=	53.73535
minimum bulb radius (m)	=	8.83857e-02

*** information on data points for bulb shape ***

total number of data points	=	427

```
al laase, independent study, borehole research

equations from glover(1953), stephens(1979),
  stephens(1987),philip(1985), and reynolds(1986)

borehole input data and results

sorptive numbers, matric flux potential,
  radius, head, and discharge are known

borehole permeameter test identification =   s7t1

output file name                          =   t27.out
bulb shape plotfile                       =   t27.plot

*** borehole parameters ***

radius of borehole (m)                    =    0.152000
depth of borehole (m)                     =    3.000000
constant head in borehole (m)             =    0.914000
h/r                                       =    6.013158
discharge into borehole (liters/min)      =    15.00000

*** soil parameters ***

stephens sorptive number (1/m)            =    5.600000
philips sorptive number (1/m)             =    9.000000
reynolds matric flux potential (cm**2/s) =     2.00000e-02

*** calculated hydraulic conductivity ***

glovers solution (cm/s)                   =     7.11544e-03
stephens I solution (cm/s)                =     8.87499e-03
stephens II solution (cm/s)               =     8.18005e-03
philips solution (cm/s)                   =     9.74183e-03
reynolds solution (cm/s)                  =     7.45357e-03

*** bulb shape ***

maximum bulb depth (m)                    =    1.764188
maximum bulb radius (m)                   =    0.887896
depth of maximum bulb radius (m)          =    0.868222
% of total depth max bulb radius          =    49.21370
minimum bulb radius (m)                   =     5.20889e-03

*** information on data points for bulb shape ***

total number of data points               =   643
```

APPENDIX G

```
c     *******************************************************************
c     al laase, independent study, 26 august 1988
c     *******************************************************************
c     this program uses equations from "the constant head well permeameter:
c     effect of unsaturated flow" w.d. reynolds, d.e. elrick, and b.e.
c     clothier, soil science, vol. 139, no. 2, pages 172-180, february 1986.
c     making use of two constant head values in the same borehole saturated
c     field hydraulic conductivity, sorptivity, and alpha parameter.
c     *******************************************************************
c     variable dictionary
c
c     h1 - constant head value associated with first borehole test
c     h2 - constant head value associated with second borehole test
c     h3,h4,h5 - dimensionless heads
c     hr1,hr2 - head/radius
c     q1 - flow rate into borehole associated with first borehole test
c     q2 - flow rate into borehole associated with second borehole test
c     q3,q4,q5 - dimensionless discharge
c     radius - radius of borehole
c     depth - depth of borehole
c     b1 - length along borehole before line source test 1
c     b2 - length along borehole before line source test 2
c     open1 - open length of borehole test 1
c     open2 - open length of borehole test 2
c     kfs - field saturated hydraulic conductivity
c     s - sorptivity
c     s1 - used in calculating sorptivity
c     alpha - alpha number
c     theta - moisture content
c     dtheta - change in moisture content
c     stheta - saturated moisture content
c     pi - as related to circle
c     c1,c2 - used to determine kfs,s,alpha
c     g1,g2 - used to determine kfs
c     j1,j2 - used to determine s
c     m1,m2 - used to determine alpha
c     n1,n2 - used to determine alpha
c     w1,w2,...,wn - dummy variables used in calculations
c     x1,x2,...,xn - dummy variables used in calculations
c     y1,y2,...,yn - dummy variables used in calculations
c     z1,z2,...,zn - dummy variables used in calculations
c     filename - output filename
c     test1 - identification for borehole test 1
c     test2 - identification for borehole test 2
c     *******************************************************************
c     defining variables
c
      real h1,h2,q1,q2,radius,b1,b2,kfs,s,alpha,dtheta,c1,c2,g1,g2,j1,j2
      real m1,m2,n1,n2,theta,pi,open1,open2,depth,stheta,h3,h4,h5
      real hr1,hr2,q3,q4,q5,s1
      character *30 filename,test1,test2
      pi=3.14159
c     *******************************************************************
c     reading input data
c
      write(*,*)' enter radius of borehole (m) '
      read(*,*)radius
      write(*,*)' enter depth of borehole (m) '
      read(*,*)depth
      write(*,*)' enter test 1 identification number '
```

```
      read(*,*)test1
      write(*,*)' enter length along borehole before screen test 1 (m) '
      read(*,*)b1
      write(*,*)' enter constant head for test 1 (m) '
      read(*,*)h1
      write(*,*)' enter discharge rate test 1 (liters/min) '
      read(*,*)q1
      write(*,*)' enter test 2 identification number '
      read(*,*)test2
      write(*,*)' enter length along borehole before screen test 2 (m) '
      read(*,*)b2
      write(*,*)' enter constant head for test 2 (m) '
      read(*,*)h2
      write(*,*)' enter discharge rate test 2 (liters/min) '
      read(*,*)q2
      write(*,*)' enter initial moisture content of soil '
      read(*,*)theta
      write(*,*)' enter saturated moisture content of soil '
      read(*,*)stheta
      write(*,*)' enter output file name '
      read(*,*)filename
c     ***********************************************************************
c     opening output file
c
      open(unit=21,file=filename,status='unknown')
c     ***********************************************************************
c     calculate open area for each test
c
      open1=h1-b1
      open2=h2-b2
c     ***********************************************************************
c     calculate c1 using equation 29
c
      x1=h1**2.0
      x2=(h1-b1)/h1
      x3=(h1-b1)/radius
      x4=(radius/h1)**2.0
      x5=x2**2.0
      x6=radius/h1
      x7=(h1-b1)**2.0
      x8=log(x3+sqrt((x3**2.0)+1.0))
      c1=(x1*((x2*x8)-sqrt(x4+x5)+x6))/x7
c     ***********************************************************************
c     calculate c2 using equation 29
c
      y1=h2**2.0
      y2=(h2-b2)/h2
      y3=(h2-b2)/radius
      y4=(radius/h2)**2.0
      y5=y2**2.0
      y6=radius/h2
      y7=(h2-b2)**2.0
      y8=log(y3+sqrt((y3**2.0)+1.0))
      c2=(y1*((y2*y8)-sqrt(y4+y5)+y6))/y7
c     ***********************************************************************
c     calculate g1 using equation 38
c
      z1=h2*c1
      z2=2*h1*h2*(h2-h1)
      z3=((radius)**2.0)*((h1*c2)-(h2*c1))
```

```
      g1=z1/(pi*(z2+z3))
c     ************************************************************************
c     calculate g2 using equation 38
c
      z4=h1*c2
      z5=2*h1*h2*(h2-h1)
      z6=(radius**2.0)*((h1*c2)-(h2*c1))
      g2=z4/(pi*(z5+z6))
c     ************************************************************************
c     calculate j1 using equation 39
c
      w1=((2.0*(h2**2.0))+((radius**2.0)*c2))*c1
      w2=2.0*h1*h2*(h1-h2)
      w3=((radius)**2.0)*((h2*c1)-(h1*c2))
      j1=w1/(pi*(w2+w3))
c     ************************************************************************
c     calculate j2 using equation 39
c
      w4=((2.0*(h1**2.0))+((radius**2.0)*c1))*c2
      j2=w4/(pi*(w2+w3))
c     ************************************************************************
c     calculate m1 and m2 using equation 40
c
      m1=2.0*h2*c1
      m2=2.0*h1*c2
c     ************************************************************************
c     calculate n1 and n2
c
      n1=-c1*((2.0*(h2**2.0))+((radius**2.0)*c2))
      n2=-c2*((2.0*(h1**2.0))+((radius**2.0)*c1))
c     ************************************************************************
c     calculate kfs,s, and alpha using equations 38,39,40
c
      dtheta=stheta-theta
      kfs=((g2*q2)-(g1*q1))*(1000.0/60.0)*(1/10000.0)
      s1=(dtheta*((j2*q2*(1000.0/6000.0))-(j1*q1*(1000.0/6000.0))))
      s=sqrt(s1)
      alpha=((m2*q2)-(m1*q1))/((n2*q2)-(n1*q1))
c     ************************************************************************
c     determine possible parameters of interest
c
      h3=h1/h2
      h4=(h2-h1)/h2
      h5=(h2-h1)/radius
      hr1=h1/radius
      hr2=h2/radius
      q3=q1/q2
      q4=(q2-q1)/q2
      q5=((q2-q1)/(h2-h1))*radius
c     ************************************************************************
c     print out results
c
      write(21,11)
      write(21,12)
      write(21,13)
      write(21,14)
      write(21,15)
      write(21,16)
      write(21,17)
      write(21,*)
```

```
      write(21,*)' output file name                              = ',filename
      write(21,*)
      write(21,*)' *** borehole parameters *** '
      write(21,*)
      write(21,*)' radius of borehole (m)                        = ',radius
      write(21,*)' depth of borehole (m)                         = ',depth
      write(21,*)
      write(21,*)' *** test 1 parameters *** '
      write(21,*)
      write(21,*)' borehole test identification                  = ',test1
      write(21,*)' constant head in borehole test 1 (m)          = ',h1
      write(21,*)' discharge test 1 (liters/min)                 = ',q1
      write(21,*)' open interval of borehole test 1 (m)          = ',open1
      write(21,*)
      write(21,*)' *** test 2 parameters *** '
      write(21,*)
      write(21,*)' borehole test identification                  = ',test2
      write(21,*)' constant head in borehole test 2 (m)          = ',h2
      write(21,*)' discharge test 2 (liters/min)                 = ',q2
      write(21,*)' open interval of borehole test 2 (m)          = ',open2
      write(21,*)
      write(21,*)' *** soil parameters *** '
      write(21,*)
      write(21,*)' initial moisture content before test          = ',theta
      write(21,*)' final moisture content after test             = ',stheta
      write(21,*)
      write(21,*)' *** calculated soil parameters *** '
      write(21,*)
      write(21,*)' saturated hydraulic conductivity (cm/s) = ',kfs
      write(21,*)' sorptivity (1/s**0.5)                         = ',s
      write(21,*)' before taking square root for s               = ',s1
      write(21,*)' alpha (1/m)                                   = ',alpha
      write(21,*)
      write(21,*)' *** calculated constants *** '
      write(21,*)
      write(21,*)' c1                                            = ',c1
      write(21,*)' c2                                            = ',c2
      write(21,*)' g1 (1/m**2)                                   = ',g1
      write(21,*)' g2 (1/m**2)                                   = ',g2
      write(21,*)' j1 (1/m)                                      = ',j1
      write(21,*)' j2 (1/m)                                      = ',j2
      write(21,*)' m1 (m)                                        = ',m1
      write(21,*)' m2 (m)                                        = ',m2
      write(21,*)' n1 (m**2)                                     = ',n1
      write(21,*)' n2 (m**2)                                     = ',n2
      write(21,*)' Q1/Q2                                         = ',q3
      write(21,*)' H1/H2                                         = ',h3
      write(21,18)
      write(21,*)'                                                  page 2'
      write(21,*)
      write(21,11)
      write(21,*)
      write(21,*)' output file name                              = ',filename
      write(21,*)
      write(21,*)' *** calculated parameters *** '
      write(21,*)
      write(21,*)' h1/h2                                         = ',h3
      write(21,*)' (h2-h1)/h2                                    = ',h4
      write(21,*)' (h2-h1)/radius                                = ',h5
      write(21,*)' h1/radius                                     = ',hr1
```

```
      write(21,*)' h2/radius                                      = ',hr2
      write(21,*)' q1/q2                                          = ',q3
      write(21,*)' (q2-q1)/q2                                     = ',q4
      write(21,*)' radius*(q2-q1)/(h2-h1) (liters/min)            = ',q5
c     ***************************************************************************
c     format statement
c
 11   format(/,3x,'al laase, independent study, borehole research',/)
 12   format(3x,'equations from reynolds, elrick, and clothier')
 13   format(3x,'soil science, vol 139, no. 2, 1986',/)
 14   format(3x,'borehole input data and results',/)
 15   format(3x,'two seperate tests in same borehole')
 16   format(3x,'heads, discharge rates, initial moisture content')
 17   format(3x,'and final moisture content known ')
 18   format(/,/)
c     ***************************************************************************
      stop
      end
```

APPENDIX H

al laase, independent study, borehole research

equations from reynolds, elrick, and clothier
soil science, vol 139, no. 2, 1986

borehole input data and results

two seperate tests in same borehole
heads, discharge rates, initial moisture content
and final moisture content known

output file name = r1.out

*** borehole parameters ***

radius of borehole (m)	=	5.80000e-02
depth of borehole (m)	=	3.000000

*** test 1 parameters ***

borehole test identification	=	s4t4
constant head in borehole test 1 (m)	=	0.229000
discharge test 1 (liters/min)	=	0.300000
open interval of borehole test 1 (m)	=	0.229000

*** test 2 parameters ***

borehole test identification	=	s4t3
constant head in borehole test 2 (m)	=	0.914000
discharge test 2 (liters/min)	=	1.300000
open interval of borehole test 2 (m)	=	0.914000

*** soil parameters ***

initial moisture content before test	=	0.125000
final moisture content after test	=	0.350000

*** calculated soil parameters ***

saturated hydraulic conductivity (cm/s)	=	7.27945e-04
sorptivity (1/s**0.5)	=	0.112129
before taking square root for s	=	1.25729e-02
alpha (1/m)	=	2.605416

*** calculated constants ***

c1	=	1.303790
c2	=	2.512986
g1 (1/m**2)	=	1.332456
g2 (1/m**2)	=	0.643465
j1 (1/m)	=	-2.448054
j2 (1/m)	=	-0.307031
m1 (m)	=	2.383328
m2 (m)	=	1.150948
n1 (m**2)	=	-2.189384
n2 (m**2)	=	-0.274589
Q1/Q2	=	0.230769
H1/H2	=	0.250547

al laase, independent study, borehole research

equations from reynolds, elrick, and clothier
soil science, vol 139, no. 2, 1986

borehole input data and results

two seperate tests in same borehole
heads, discharge rates, initial moisture content
and final moisture content known

```
output file name                              =   r2.out

*** borehole parameters ***

radius of borehole (m)                        =    5.80000e-02
depth of borehole (m)                         =    3.000000

*** test 1 parameters ***

borehole test identification                  =   s4t4
constant head in borehole test 1 (m)          =    0.229000
discharge test 1 (liters/min)                 =    0.300000
open interval of borehole test 1 (m)          =    0.229000

*** test 2 parameters ***

borehole test identification                  =   s4t2
constant head in borehole test 2 (m)          =    0.914000
discharge test 2 (liters/min)                 =    6.900000
open interval of borehole test 2 (m)          =    0.914000

*** soil parameters ***

initial moisture content before test          =    0.125000
final moisture content after test             =    0.350000

*** calculated soil parameters ***

saturated hydraulic conductivity (cm/s)       =    6.73361e-03
sorptivity (1/s**0.5)                         =   NaN
before taking square root for s               =   -5.19036e-02
alpha (1/m)                                   =   -5.837988

*** calculated constants ***

c1                                            =    1.303790
c2                                            =    2.512986
g1 (1/m**2)                                   =    1.332456
g2 (1/m**2)                                   =    0.643465
j1 (1/m)                                      =   -2.448054
j2 (1/m)                                      =   -0.307031
m1 (m)                                        =    2.383328
m2 (m)                                        =    1.150948
n1 (m**2)                                     =   -2.189384
n2 (m**2)                                     =   -0.274589
Q1/Q2                                         =    4.34783e-02
H1/H2                                         =    0.250547
```

al laase, independent study, borehole research

equations from reynolds, elrick, and clothier
soil science, vol 139, no. 2, 1986

borehole input data and results

two seperate tests in same borehole
heads, discharge rates, initial moisture content
and final moisture content known

```
output file name                                  =   r3.out

*** borehole parameters ***

radius of borehole (m)                            =     5.80000e-02
depth of borehole (m)                             =    3.000000

*** test 1 parameters ***

borehole test identification                      =   s5t5
constant head in borehole test 1 (m)              =    0.762000
discharge test 1 (liters/min)                     =    6.600000
open interval of borehole test 1 (m)              =    0.762000

*** test 2 parameters ***

borehole test identification                      =   s5t7
constant head in borehole test 2 (m)              =    0.762000
discharge test 2 (liters/min)                     =    6.900000
open interval of borehole test 2 (m)              =    0.762000

*** soil parameters ***

initial moisture content before test              =    0.130000
final moisture content after test                 =    0.350000

*** calculated soil parameters ***

saturated hydraulic conductivity (cm/s)           =  NaN(1)
sorptivity (1/s**0.5)                             =  NaN(1)
before taking square root for s                   =  NaN(1)
alpha (1/m)                                       =   -1.303487

*** calculated constants ***

c1                                                =    2.343319
c2                                                =    2.343319
g1 (1/m**2)                                       =  Inf
g2 (1/m**2)                                       =  Inf
j1 (1/m)                                          =  Inf
j2 (1/m)                                          =  Inf
m1 (m)                                            =    3.571218
m2 (m)                                            =    3.571218
n1 (m**2)                                         =   -2.739740
n2 (m**2)                                         =   -2.739740
Q1/Q2                                             =    0.956522
H1/H2                                             =    1.000000
```

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equations from reynolds, elrick, and clothier
soil science, vol 139, no. 2, 1986

borehole input data and results

two seperate tests in same borehole
heads, discharge rates, initial moisture content
and final moisture content known

output file name	=	r4.out

*** borehole parameters ***

radius of borehole (m)	=	5.80000e-02
depth of borehole (m)	=	3.000000

*** test 1 parameters ***

borehole test identification	=	s6t3
constant head in borehole test 1 (m)	=	1.210000
discharge test 1 (liters/min)	=	9.100000
open interval of borehole test 1 (m)	=	1.210000

*** test 2 parameters ***

borehole test identification	=	s6t1
constant head in borehole test 2 (m)	=	1.550000
discharge test 2 (liters/min)	=	18.30000
open interval of borehole test 2 (m)	=	1.220000

*** soil parameters ***

initial moisture content before test	=	9.00000e-02
final moisture content after test	=	0.350000

*** calculated soil parameters ***

saturated hydraulic conductivity (cm/s)	=	1.63055e-02
sorptivity (1/s**0.5)	=	NaN
before taking square root for s	=	-0.740993
alpha (1/m)	=	-1.144256

*** calculated constants ***

c1	=	2.778440
c2	=	3.539954
g1 (1/m**2)	=	1.074939
g2 (1/m**2)	=	1.069139
j1 (1/m)	=	-3.340570
j2 (1/m)	=	-2.595576
m1 (m)	=	8.613162
m2 (m)	=	8.566689
n1 (m**2)	=	-13.38349
n2 (m**2)	=	-10.398782
Q1/Q2	=	0.497268
H1/H2	=	0.780645

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equations from reynolds, elrick, and clothier
soil science, vol 139, no. 2, 1986

borehole input data and results

two seperate tests in same borehole
heads, discharge rates, initial moisture content
and final moisture content known

output file name = r5.out

*** borehole parameters ***

radius of borehole (m)	=	5.80000e-02
depth of borehole (m)	=	3.000000

*** test 1 parameters ***

borehole test identification	=	s6t4
constant head in borehole test 1 (m)	=	0.580000
discharge test 1 (liters/min)	=	0.800000
open interval of borehole test 1 (m)	=	0.580000

*** test 2 parameters ***

borehole test identification	=	s6t1
constant head in borehole test 2 (m)	=	1.550000
discharge test 2 (liters/min)	=	18.30000
open interval of borehole test 2 (m)	=	1.220000

*** soil parameters ***

initial moisture content before test	=	9.00000e-02
final moisture content after test	=	0.350000

*** calculated soil parameters ***

saturated hydraulic conductivity (cm/s)	=	1.06641e-02
sorptivity (1/s**0.5)	=	NaN
before taking square root for s	=	-0.285172
alpha (1/m)	=	-1.944565

*** calculated constants ***

c1	=	2.093235
c2	=	3.539954
g1 (1/m**2)	=	0.593524
g2 (1/m**2)	=	0.375590
j1 (1/m)	=	-1.844483
j2 (1/m)	=	-0.440244
m1 (m)	=	6.489029
m2 (m)	=	4.106347
n1 (m**2)	=	-10.082922
n2 (m**2)	=	-2.406608
Q1/Q2	=	4.37158e-02
H1/H2	=	0.374194

```
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equations from reynolds, elrick, and clothier
soil science, vol 139, no. 2, 1986

borehole input data and results

two seperate tests in same borehole
heads, discharge rates, initial moisture content
and final moisture content known

output file name                              =   r6.out

*** borehole parameters ***

radius of borehole (m)                        =     5.80000e-02
depth of borehole (m)                         =    3.000000

*** test 1 parameters ***

borehole test identification                  =   s6t4
constant head in borehole test 1 (m)          =    0.580000
discharge test 1 (liters/min)                 =    0.800000
open interval of borehole test 1 (m)          =    0.580000

*** test 2 parameters ***

borehole test identification                  =   s6t3
constant head in borehole test 2 (m)          =    1.210000
discharge test 2 (liters/min)                 =    9.100000
open interval of borehole test 2 (m)          =    1.210000

*** soil parameters ***

initial moisture content before test          =     9.00000e-02
final moisture content after test             =    0.350000

*** calculated soil parameters ***

saturated hydraulic conductivity (cm/s) =     7.60905e-03
sorptivity (1/s**0.5)                         =   NaN
before taking square root for s               =   -0.192066
alpha (1/m)                                   =   -2.060075

*** calculated constants ***

c1                                            =    2.093235
c2                                            =    2.778440
g1 (1/m**2)                                   =    0.914945
g2 (1/m**2)                                   =    0.582130
j1 (1/m)                                      =   -2.221234
j2 (1/m)                                      =   -0.682339
m1 (m)                                        =    5.065629
m2 (m)                                        =    3.222990
n1 (m**2)                                     =   -6.148977
n2 (m**2)                                     =   -1.888899
Q1/Q2                                         =     8.79121e-02
H1/H2                                         =    0.479339
```

```
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soil science, vol 139, no. 2, 1986

borehole input data and results

two seperate tests in same borehole
heads, discharge rates, initial moisture content
and final moisture content known

output file name                              =   r7.out

*** borehole parameters ***

radius of borehole (m)                        =     5.80000e-02
depth of borehole (m)                         =    3.000000

*** test 1 parameters ***

borehole test identification                  =   s6t4
constant head in borehole test 1 (m)          =    0.580000
discharge test 1 (liters/min)                 =    0.800000
open interval of borehole test 1 (m)          =    0.580000

*** test 2 parameters ***

borehole test identification                  =   s6t6
constant head in borehole test 2 (m)          =    0.914000
discharge test 2 (liters/min)                 =    3.000000
open interval of borehole test 2 (m)          =    0.853000

*** soil parameters ***

initial moisture content before test          =     9.00000e-02
final moisture content after test             =    0.350000

*** calculated soil parameters ***

saturated hydraulic conductivity (cm/s)       =     4.56247e-03
sorptivity (1/s**0.5)                         =  NaN
before taking square root for s               =    -9.92196e-02
alpha (1/m)                                   =   -2.391145

*** calculated constants ***

c1                                            =    2.093235
c2                                            =    2.623388
g1 (1/m**2)                                   =    1.726167
g2 (1/m**2)                                   =    1.372806
j1 (1/m)                                      =   -3.172101
j2 (1/m)                                      =   -1.609122
m1 (m)                                        =    3.826434
m2 (m)                                        =    3.043129
n1 (m**2)                                     =   -3.515833
n2 (m**2)                                     =   -1.783488
Q1/Q2                                         =    0.266667
H1/H2                                         =    0.634573
```

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equations from reynolds, elrick, and clothier
soil science, vol 139, no. 2, 1986

borehole input data and results

two seperate tests in same borehole
heads, discharge rates, initial moisture content
and final moisture content known

output file name	=	r8.out

*** borehole parameters ***

radius of borehole (m)	=	5.80000e-02
depth of borehole (m)	=	3.000000

*** test 1 parameters ***

borehole test identification	=	s6t5
constant head in borehole test 1 (m)	=	0.570000
discharge test 1 (liters/min)	=	3.000000
open interval of borehole test 1 (m)	=	0.570000

*** test 2 parameters ***

borehole test identification	=	s6t1
constant head in borehole test 2 (m)	=	1.550000
discharge test 2 (liters/min)	=	18.30000
open interval of borehole test 2 (m)	=	1.220000

*** soil parameters ***

initial moisture content before test	=	9.00000e-02
final moisture content after test	=	0.350000

*** calculated soil parameters ***

saturated hydraulic conductivity (cm/s)	=	8.37250e-03
sorptivity (1/s**0.5)	=	NaN
before taking square root for s	=	-0.100009
alpha (1/m)	=	-4.353313

*** calculated constants ***

c1	=	2.077510
c2	=	3.539954
g1 (1/m**2)	=	0.593305
g2 (1/m**2)	=	0.371771
j1 (1/m)	=	-1.843805
j2 (1/m)	=	-0.428378
m1 (m)	=	6.440279
m2 (m)	=	4.035548
n1 (m**2)	=	-10.007174
n2 (m**2)	=	-2.325002
Q1/Q2	=	0.163934
H1/H2	=	0.367742

```
al laase, independent study, borehole research

equations from reynolds, elrick, and clothier
soil science, vol 139, no. 2, 1986

borehole input data and results

two seperate tests in same borehole
heads, discharge rates, initial moisture content
and final moisture content known

output file name                              =   r9.out

*** borehole parameters ***

radius of borehole (m)                        =     5.80000e-02
depth of borehole (m)                         =    3.000000

*** test 1 parameters ***

borehole test identification                  =   s6t5
constant head in borehole test 1 (m)          =    0.570000
discharge test 1 (liters/min)                 =    3.000000
open interval of borehole test 1 (m)          =    0.570000

*** test 2 parameters ***

borehole test identification                  =   s6t3
constant head in borehole test 2 (m)          =    1.210000
discharge test 2 (liters/min)                 =    9.100000
open interval of borehole test 2 (m)          =    1.210000

*** soil parameters ***

initial moisture content before test          =     9.00000e-02
final moisture content after test             =    0.350000

*** calculated soil parameters ***

saturated hydraulic conductivity (cm/s) =     4.14337e-03
sorptivity (1/s**0.5)                         =    0.163373
before taking square root for s               =     2.66909e-02
alpha (1/m)                                   =    8.072240

*** calculated constants ***

c1                                            =    2.077510
c2                                            =    2.778440
g1 (1/m**2)                                   =    0.909600
g2 (1/m**2)                                   =    0.573057
j1 (1/m)                                      =   -2.208259
j2 (1/m)                                      =   -0.660311
m1 (m)                                        =    5.027574
m2 (m)                                        =    3.167421
n1 (m**2)                                     =   -6.102782
n2 (m**2)                                     =   -1.824848
Q1/Q2                                         =    0.329670
H1/H2                                         =    0.471074
```

```
al laase, independent study, borehole research

equations from reynolds, elrick, and clothier
soil science, vol 139, no. 2, 1986

borehole input data and results

two seperate tests in same borehole
heads, discharge rates, initial moisture content
and final moisture content known

output file name                              =   r10.out

*** borehole parameters ***

radius of borehole (m)                        =     5.80000e-02
depth of borehole (m)                         =    3.000000

*** test 1 parameters ***

borehole test identification                  =   s6t5
constant head in borehole test 1 (m)          =    0.570000
discharge test 1 (liters/min)                 =    3.000000
open interval of borehole test 1 (m)          =    0.570000

*** test 2 parameters ***

borehole test identification                  =   s6t4
constant head in borehole test 2 (m)          =    0.580000
discharge test 2 (liters/min)                 =    0.800000
open interval of borehole test 2 (m)          =    0.580000

*** soil parameters ***

initial moisture content before test          =     9.00000e-02
final moisture content after test             =    0.350000

*** calculated soil parameters ***

saturated hydraulic conductivity (cm/s) =   -0.214745
sorptivity (1/s**0.5)                         =    2.565995
before taking square root for s               =    6.584327
alpha (1/m)                                   =   -1.695959

*** calculated constants ***

c1                                            =    2.077510
c2                                            =    2.093235
g1 (1/m**2)                                   =    58.35886
g2 (1/m**2)                                   =    57.78680
j1 (1/m)                                      =   -68.40480
j2 (1/m)                                      =   -66.58548
m1 (m)                                        =    2.409911
m2 (m)                                        =    2.386288
n1 (m**2)                                     =   -1.412377
n2 (m**2)                                     =   -1.374813
Q1/Q2                                         =    3.750000
H1/H2                                         =    0.982759
```

al laase, independent study, borehole research

equations from reynolds, elrick, and clothier
soil science, vol 139, no. 2, 1986

borehole input data and results

two seperate tests in same borehole
heads, discharge rates, initial moisture content
and final moisture content known

output file name = rl1.out

*** borehole parameters ***

radius of borehole (m)	=	5.80000e-02
depth of borehole (m)	=	3.000000

*** test 1 parameters ***

borehole test identification	=	s6t5
constant head in borehole test 1 (m)	=	0.570000
discharge test 1 (liters/min)	=	3.000000
open interval of borehole test 1 (m)	=	0.570000

*** test 2 parameters ***

borehole test identification	=	s6t6
constant head in borehole test 2 (m)	=	0.914000
discharge test 2 (liters/min)	=	3.000000
open interval of borehole test 2 (m)	=	0.853000

*** soil parameters ***

initial moisture content before test	=	9.00000e-02
final moisture content after test	=	0.350000

*** calculated soil parameters ***

saturated hydraulic conductivity (cm/s)	=	-1.79852e-03
sorptivity (1/s**0.5)	=	0.452441
before taking square root for s	=	0.204703
alpha (1/m)	=	-0.456874

*** calculated constants ***

c1	=	2.077510
c2	=	2.623388
g1 (1/m**2)	=	1.692693
g2 (1/m**2)	=	1.332988
j1 (1/m)	=	-3.110585
j2 (1/m)	=	-1.535950
m1 (m)	=	3.797688
m2 (m)	=	2.990662
n1 (m**2)	=	-3.489420
n2 (m**2)	=	-1.723011
Q1/Q2	=	1.000000
H1/H2	=	0.623632

```
al laase, independent study, borehole research

equations from reynolds, elrick, and clothier
soil science, vol 139, no. 2, 1986

borehole input data and results

two seperate tests in same borehole
heads, discharge rates, initial moisture content
and final moisture content known

output file name                              =   r12.out

*** borehole parameters ***

radius of borehole (m)                        =     5.80000e-02
depth of borehole (m)                         =    3.000000

*** test 1 parameters ***

borehole test identification                  =   s6t6
constant head in borehole test 1 (m)          =    0.914000
discharge test 1 (liters/min)                 =    3.000000
open interval of borehole test 1 (m)          =    0.853000

*** test 2 parameters ***

borehole test identification                  =   s6t1
constant head in borehole test 2 (m)          =    1.550000
discharge test 2 (liters/min)                 =    18.30000
open interval of borehole test 2 (m)          =    1.220000

*** soil parameters ***

initial moisture content before test          =     9.00000e-02
final moisture content after test             =    0.350000

*** calculated soil parameters ***

saturated hydraulic conductivity (cm/s) =     1.38615e-02
sorptivity (1/s**0.5)                         =  NaN
before taking square root for s               =   -0.543518
alpha (1/m)                                   =   -1.326171

*** calculated constants ***

c1                                            =    2.623388
c2                                            =    3.539954
g1 (1/m**2)                                   =    0.719372
g2 (1/m**2)                                   =    0.572405
j1 (1/m)                                      =   -2.235581
j2 (1/m)                                      =   -1.051883
m1 (m)                                        =    8.132502
m2 (m)                                        =    6.471036
n1 (m**2)                                     =   -12.63662
n2 (m**2)                                     =   -5.945767
Q1/Q2                                         =    0.163934
H1/H2                                         =    0.589677
```

```
al laase, independent study, borehole research

equations from reynolds, elrick, and clothier
soil science, vol 139, no. 2, 1986

borehole input data and results

two seperate tests in same borehole
heads, discharge rates, initial moisture content
and final moisture content known

output file name                             =   rl3.out

*** borehole parameters ***

radius of borehole (m)                       =     5.80000e-02
depth of borehole (m)                        =    3.000000

*** test 1 parameters ***

borehole test identification                 =   s6t6
constant head in borehole test 1 (m)         =    0.914000
discharge test 1 (liters/min)                =    3.000000
open interval of borehole test 1 (m)         =    0.853000

*** test 2 parameters ***

borehole test identification                 =   s6t3
constant head in borehole test 2 (m)         =    1.210000
discharge test 2 (liters/min)                =    9.100000
open interval of borehole test 2 (m)         =    1.210000

*** soil parameters ***

initial moisture content before test         =     9.00000e-02
final moisture content after test            =    0.350000

*** calculated soil parameters ***

saturated hydraulic conductivity (cm/s) =     1.10452e-02
sorptivity (1/s**0.5)                        =  NaN
before taking square root for s              =   -0.408956
alpha (1/m)                                  =   -1.404425

*** calculated constants ***

c1                                           =    2.623388
c2                                           =    2.778440
g1 (1/m**2)                                  =    1.548331
g2 (1/m**2)                                  =    1.238691
j1 (1/m)                                     =   -3.758922
j2 (1/m)                                     =   -2.276288
m1 (m)                                       =    6.348598
m2 (m)                                       =    5.078987
n1 (m**2)                                    =   -7.706324
n2 (m**2)                                    =   -4.666714
Q1/Q2                                        =    0.329670
H1/H2                                        =    0.755372
```

APPENDIX I

```
c       ********************************************************
c       al laase, independent study, 1 february 1989
c       ********************************************************
c       this program determines t-values for t-tests
        ********************************************************
c       variable dictionary
c
c       solution1 - first solution
c       solution2 - second solution
c       mean1 - mean of first solution
c       mean2 - mean of second solution
c       var1 - variance of first solution
c       var2 - variance of second solution
c       size1 - sample size of first solution
c       size2 - sample size of second solution
c       varall - combined variance
c       se - standard error of mean
c       dfree - degrees of freedom
c       ********************************************************
c       define variables
c
        real mean1,mean2,var1,var2,size1,size2,varall,se,dfree
        character *30 solution1,solution2,output
c       ********************************************************
c       read input data
c
        write(*,*)' enter output file name '
        read(*,*)output
        write(*,*)' enter solution 1 name '
        read(*,*)solution1
        write(*,*)' enter mean of solution 1 '
        read(*,*)mean1
        write(*,*)' enter variance of solution 1 '
        read(*,*)var1
        write(*,*)' enter sample size for solution 1 '
        read(*,*)size1
        write(*,*)' enter solution 2 name '
        read(*,*)solution2
        write(*,*)' enter mean of solution 2 '
        read(*,*)mean2
        write(*,*)' enter variance of solution 2 '
        read(*,*)var2
        write(*,*)' enter sample size for solution 2 '
        read(*,*)size2
c       ********************************************************
c       open output file
c
        open(unit=21,file=output,status='unknown')
c       ********************************************************
c       calculate t value
c

        dfree=size1+size2-2.0
        varall=(((size1-1.0)*var1)+((size2-1.0)*var2))/dfree
        se=sqrt(varall)*sqrt(((1.0/size1)+(1.0/size2)))
        t=(mean1-mean2)/se
c       ********************************************************
c       print out results

        write(21,*)
```

```
      write(21,*)
      write(21,*)
      write(21,*)' al laase, independent study '
      write(21,*)
      write(21,*)' comparison of calculated mean hydraulic conductivity'
      write(21,*)
      write(21,*)' t-test analysis '
      write(21,*)
      write(21,*)solution1
      write(21,*)' vs '
      write(21,*)solution2
      write(21,*)
      write(21,*)'      borehole permeameter solution 1 '
      write(21,*)
      write(21,*)' mean hydraulic conductivity 10**(-3) cm/s) = ',mean1
      write(21,*)' variance                                   = ',var1
      write(21,*)' sample size                                = ',size1
      write(21,*)
      write(21,*)'      borehole permeameter solution 2 '
      write(21,*)
      write(21,*)' mean hydraulic conductivity 10**(-3) cm/s) = ',mean2
      write(21,*)' variance                                   = ',var2
      write(21,*)' sample size                                = ',size2
      write(21,*)
      write(21,*)'      t-test results '
      write(21,*)
      write(21,*)' t-value                                    = ',t
      write(21,*)' degrees of freedom                         = ',dfree
c     *****************************************************************
      stop
      end
```

al laase, independent study

comparison of calculated mean hydraulic conductivity

t-test analysis

glover
vs
stephensI

borehole permeameter solution 1

```
mean hydraulic conductivity 10**(-3) cm/s) =    4.720000
variance                                   =    5.440000
sample size                                =    27.00000
```

borehole permeameter solution 2

```
mean hydraulic conductivity 10**(-3) cm/s) =    5.020000
variance                                   =    6.420000
sample size                                =    27.00000
```

t-test results

```
t-value                                    =   -0.452648
degrees of freedom                         =    52.00000
```

al laase, independent study

comparison of calculated mean hydraulic conductivity

t-test analysis

glover
vs
stephensII

borehole permeameter solution 1

mean hydraulic conductivity 10**(-3) cm/s)	=	4.720000
variance	=	5.440000
sample size	=	27.00000

borehole permeameter solution 2

mean hydraulic conductivity 10**(-3) cm/s)	=	5.010000
variance	=	6.320000
sample size	=	27.00000

t-test results

t-value	=	-0.439417
degrees of freedom	=	52.00000

al laase, independent study

comparison of calculated mean hydraulic conductivity

t-test analysis

glover
vs
philip

borehole permeameter solution 1

mean hydraulic conductivity 10**(-3) cm/s)	=	4.720000
variance	=	5.440000
sample size	=	27.00000

borehole permeameter solution 2

mean hydraulic conductivity 10**(-3) cm/s)	=	6.120000
variance	=	9.350000
sample size	=	27.00000

t-test results

t-value	=	-1.891585
degrees of freedom	=	52.00000

al laase, independent study

comparison of calculated mean hydraulic conductivity

t-test analysis

glover
vs
reynolds

borehole permeameter solution 1

mean hydraulic conductivity 10**(-3) cm/s)	=	4.720000
variance	=	5.440000
sample size	=	27.00000

borehole permeameter solution 2

mean hydraulic conductivity 10**(-3) cm/s)	=	4.960000
variance	=	6.030000
sample size	=	27.00000

t-test results

t-value	=	-0.368224
degrees of freedom	=	52.00000

al laase, independent study

comparison of calculated mean hydraulic conductivity

t-test analysis

stephensI
vs
stephensII

borehole permeameter solution 1

mean hydraulic conductivity 10**(-3) cm/s)	=	5.020000
variance	=	6.420000
sample size	=	27.00000

borehole permeameter solution 2

mean hydraulic conductivity 10**(-3) cm/s)	=	5.010000
variance	=	6.320000
sample size	=	27.00000

t-test results

t-value	=	1.45575e-02
degrees of freedom	=	52.00000

al laase, independent study

comparison of calculated mean hydraulic conductivity

t-test analysis

stephensI
vs
philip

borehole permeameter solution 1

mean hydraulic conductivity 10**(-3) cm/s)	=	5.020000
variance	=	6.420000
sample size	=	27.00000

borehole permeameter solution 2

mean hydraulic conductivity 10**(-3) cm/s)	=	6.120000
variance	=	9.350000
sample size	=	27.00000

t-test results

t-value	=	-1.439324
degrees of freedom	=	52.00000

al laase, independent study

comparison of calculated mean hydraulic conductivity

t-test analysis

stephensI
vs
reynolds

borehole permeameter solution 1

```
mean hydraulic conductivity 10**(-3) cm/s) =    5.020000
variance                                   =    6.420000
sample size                                =    27.00000
```

borehole permeameter solution 2

```
mean hydraulic conductivity 10**(-3) cm/s) =    4.960000
variance                                   =    6.030000
sample size                                =    27.00000
```

t-test results

```
t-value                                    =     8.83584e-02
degrees of freedom                         =    52.00000
```

al laase, independent study

comparison of calculated mean hydraulic conductivity

t-test analysis

stephensI
vs
reynolds 2 head

borehole permeameter solution 1

mean hydraulic conductivity 10**(-3) cm/s)	=	5.020000
variance	=	6.430000
sample size	=	27.00000

borehole permeameter solution 2

mean hydraulic conductivity 10**(-3) cm/s)	=	4.800000
variance	=	9.540000
sample size	=	4.000000

t-test results

t-value	=	0.158032
degrees of freedom	=	29.00000

al laase, independent study

comparison of calculated mean hydraulic conductivity

t-test analysis

sterphensII
vs
philip

borehole permeameter solution 1

mean hydraulic conductivity 10**(-3) cm/s)	=	5.010000
variance	=	6.320000
sample size	=	27.00000

borehole permeameter solution 2

mean hydraulic conductivity 10**(-3) cm/s)	=	6.120000
variance	=	9.350000
sample size	=	27.00000

t-test results

t-value	=	-1.457036
degrees of freedom	=	52.00000

al laase, independent study

comparison of calculated mean hydraulic conductivity

t-test analysis

stephensII
vs
reynolds

borehole permeameter solution 1

mean hydraulic conductivity 10**(-3) cm/s)	=	5.010000
variance	=	6.320000
sample size	=	27.00000

borehole permeameter solution 2

mean hydraulic conductivity 10**(-3) cm/s)	=	4.960000
variance	=	6.030000
sample size	=	27.00000

t-test results

t-value	=	7.39299e-02
degrees of freedom	=	52.00000

al laase, independent study

comparison of calculated mean hydraulic conductivity

t-test analysis

stephensII
vs
reynolds 2 head

borehole permeameter solution 1

mean hydraulic conductivity 10**(-3) cm/s)	=	5.010000
variance	=	6.320000
sample size	=	27.00000

borehole permeameter solution 2

mean hydraulic conductivity 10**(-3) cm/s)	=	4.800000
variance	=	9.540000
sample size	=	4.000000

t-test results

t-value	=	0.151963
degrees of freedom	=	29.00000

al laase, independent study

comparison of calculated mean hydraulic conductivity

t-test analysis

philip
vs
reynolds

borehole permeameter solution 1

mean hydraulic conductivity 10**(-3) cm/s)	=	6.120000
variance	=	9.350000
sample size	=	27.00000

borehole permeameter solution 2

mean hydraulic conductivity 10**(-3) cm/s)	=	4.960000
variance	=	6.030000
sample size	=	27.00000

t-test results

t-value	=	1.536957
degrees of freedom	=	52.00000

al laase, independent study

comparison of calculated mean hydraulic conductivity

t-test analysis

philip
vs
reynolds 2 head

borehole permeameter solution 1

mean hydraulic conductivity 10**(-3) cm/s)	=	6.130000
variance	=	9.350000
sample size	=	27.00000

borehole permeameter solution 2

mean hydraulic conductivity 10**(-3) cm/s)	=	4.800000
variance	=	9.540000
sample size	=	4.000000

t-test results

t-value	=	0.811000
degrees of freedom	=	29.00000

al laase, independent study

comparison of calculated mean hydraulic conductivity

t-test analysis

reynolds 1 head
vs
reynolds 2 head

borehole permeameter solution 1

mean hydraulic conductivity 10**(-3) cm/s)	=	4.960000
variance	=	6.030000
sample size	=	27.00000

borehole permeameter solution 2

mean hydraulic conductivity 10**(-3) cm/s)	=	4.800000
variance	=	9.540000
sample size	=	4.000000

t-test results

t-value	=	0.118112
degrees of freedom	=	29.00000

APPENDIX J

```
c     *********************************************************
c     al laase, independent study, 1 february 1989
c     *********************************************************
c     this program determines u values for u-tests
      *********************************************************
c     variable dictionary
c
c     u - u-value
c     *********************************************************
c     define variables
c
      dimension value1(100),value2(100),value(100),type(100),rank(100)
      integer n,m
      character *30 filename1,filename2,output
      character *30 solution1,solution2,flag
c     *********************************************************
c     input data
c
      write(*,*)'enter borehole permeameter solution 1'
      read(*,*)solution1
      write(*,*)' enter data filename for solution 1'
      read(*,*)filename1
      write(*,*)'enter borehole permeameter solution 2'
      read(*,*)solution2
      write(*,*)' enter data filename for solution 2'
      read(*,*)filename2
      write(*,*)' enter output file name'
      read(*,*)output
c     *********************************************************
c     open data files
c
      open(unit=21,file=filename1,status='unknown')
      open(unit=22,file=filename2,status='unknown')
      open(unit=23,file=output,status='unknown')
c     *********************************************************
c     read datafile for solution 1
c
      n=0
      size1=0.0
      do 10 i=1,100
         read(21,*,end=89)value1(i)
         n=n+1
         size1=size1+1.0
 10   continue
 89   continue
c     *********************************************************
c     read datafile for solution 2
      size2=0.0
      m=0
      do 20 i=1,100
         read(22,*,end=99)value2(i)
         m=m+1
         size2=size2+1.0
 20   continue
 99   continue
c     *********************************************************
c     combine data files
c
      do 30 j=1,n
         value(j)=value1(j)
```

```
         type(j)=1.0
         rank(j)=j
 30   continue
      do 40 j=1+n,m+n
         value(j)=value2(j-n)
         type(j)=2.0
         rank(j)=j
 40   continue
c     ***************************************************
c     sort values from low to high
c
      do 50 i=n+m,2,-1
         flag='off'
         do 60 j=1,n+m-1
            if(value(j).gt.value(j+1))then
               temp=value(j)
               hold=type(j)
               value(j)=value(j+1)
               type(j)=type(j+1)
               value(j+1)=temp
               type(j+1)=hold
               flag='on'
            end if
 60      continue
         if(flag.eq.'off')goto 500
 50   continue
 500  continue
c     ***************************************************
c     average ranks
c
      ii=1
      do 90 i=1,n+m
         obser=value(ii)
         kount=1
         sum=rank(ii)
         do 100 j=ii+1,n+m
            if(value(j).eq.obser)then
            kount=kount+1
            sum=sum+rank(j)
         else
            goto 110
         end if
 100     continue
 110     if(kount.gt.1)then
            do 120 jj=ii,j-1
               rank(jj)=sum/kount
 120        continue
         end if
         ii=j
         if(ii.eq.n+m)goto 140
 90      continue
 140     continue
c     ***************************************************
c     determine u value
c
      sum=0.0
      do 150 j=1,n+m
        if(type(j).eq.1.0)then
           sum=sum+rank(j)
        end if
```

```
 150   continue
       u=size1*size2+((size1*(size1+1))/2.0)-sum
       xmean=0.5*size1*size2
       var=(size1*size2*(size1+size2+1.0))/12
       std=sqrt(var)
       z=(u-xmean)/std
       dfree=size1+size2-2.0
c      ****************************************************

c      print out results
c
       write(23,11)
       write(23,*)' u-test results '
       write(23,*)
       write(23,*)solution1,' (solution 1) '
       write(23,*)'  vs '
       write(23,*)solution2,' (solution 2) '
       write(23,*)
       write(23,*)'             *** values *** '
       write(23,*)
       write(23,*)' u-value                                    = ',u
       write(23,*)' mean                                       = ',xmean
       write(23,*)' variance                                   = ',var
       write(23,*)' r1                                         = ',sum
       write(23,*)' z-value                                    = ',z
       write(23,*)
       write(23,*)' rank  solution   hydraulic conductivity '
       write(23,*)'                      10**(-3) cm/s '
       write(23,*)
       do 160 j=1,n+m
          write(23,12)rank(j),type(j),value(j)
 160   continue
c      ****************************************************
c      format statements
c
 11    format(/,/,/,3x,'al laase, independent study',/)
 12    format(3x,f5.2,4x,f3.1,9x,f5.2)
c      ****************************************************
       stop
       end
```

al laase, independent study

u-test results

test1 (solution 1)
vs
test2 (solution 2)

*** values ***

u-value	=	68.50000
mean	=	45.00000
variance	=	150.0000
r1	=	66.50000
z-value	=	1.918767

rank	solution	hydraulic conductivity 10**(-3) cm/s
1.00	1.0	0.10
3.00	1.0	0.20
3.00	1.0	0.20
3.00	2.0	0.20
5.00	1.0	0.40
6.00	1.0	0.60
7.50	2.0	0.70
7.50	2.0	0.70
9.00	1.0	0.80
10.50	1.0	0.90
10.50	2.0	0.90
12.00	1.0	1.30
13.00	2.0	1.50
14.00	2.0	1.70
15.00	2.0	2.60
16.00	2.0	3.50
17.00	1.0	5.10
18.00	2.0	7.80
19.00	2.0	15.30

al laase, independent study

u-test results

glover (solution 1)
vs
stephens I (solution 2)

*** values ***

u-value	=	386.5000
mean	=	364.5000
variance	=	3341.250
rl	=	720.5000
z-value	=	0.380599

rank	solution	hydraulic conductivity 10**(-3) cm/s
1.00	1.0	1.01
2.00	2.0	1.07
3.00	2.0	1.17
4.00	1.0	1.26
5.00	2.0	1.41
6.00	1.0	1.64
7.00	1.0	2.10
8.00	1.0	2.34
9.00	1.0	2.36
10.00	2.0	2.39
11.00	2.0	2.47
12.50	1.0	2.59
12.50	2.0	2.59
14.00	2.0	2.88
15.00	1.0	3.10
16.00	2.0	3.28
17.00	2.0	3.29
18.00	1.0	3.43
19.00	1.0	3.62
20.00	2.0	3.85
21.00	1.0	3.97
22.00	1.0	4.18
23.00	2.0	4.38
24.00	2.0	4.45
25.00	2.0	4.47
26.00	1.0	4.49
27.00	1.0	4.50
28.00	2.0	4.73
29.00	1.0	4.85
30.00	1.0	5.03
31.00	2.0	5.07
32.00	1.0	5.08
33.00	2.0	5.26
34.00	1.0	5.37
35.00	1.0	5.57
36.00	2.0	5.64
37.00	2.0	5.68
38.00	1.0	6.02

39.00	2.0	6.24
40.00	2.0	6.82
41.00	1.0	6.85
42.00	2.0	7.03
43.00	1.0	7.05
44.00	1.0	7.12
45.00	2.0	7.14
46.00	1.0	7.16
47.00	2.0	7.74
48.00	2.0	8.28
49.00	1.0	8.30
50.00	2.0	8.88
51.00	1.0	9.00
52.00	2.0	9.31
53.00	1.0	9.33
54.00	2.0	10.09

al laase, independent study

u-test results

glover (solution 1)
vs
stephensII (solution 2)

*** values ***

u-value	=	387.5000
mean	=	364.5000
variance	=	3341.250
r1	=	719.5000
z-value	=	0.397899

rank	solution	hydraulic conductivity 10**(-3) cm/s
1.00	1.0	1.01
2.00	2.0	1.09
3.00	2.0	1.11
4.00	2.0	1.14
5.00	1.0	1.26
6.00	1.0	1.64
7.00	1.0	2.10
8.00	1.0	2.34
9.00	1.0	2.36
10.00	2.0	2.51
11.00	1.0	2.59
12.00	2.0	2.60
13.00	2.0	2.72
14.00	2.0	2.83
15.00	1.0	3.10
16.00	2.0	3.21
17.00	2.0	3.35
18.00	1.0	3.43
19.00	1.0	3.62
20.00	2.0	3.80
21.00	1.0	3.97
22.00	1.0	4.18
23.00	2.0	4.24
24.00	2.0	4.38
25.00	1.0	4.49
26.00	1.0	4.50
27.00	2.0	4.51
28.00	2.0	4.79
29.00	1.0	4.85
30.50	1.0	5.03
30.50	2.0	5.03
32.00	1.0	5.08
33.00	1.0	5.37
34.50	2.0	5.56
34.50	2.0	5.56
36.00	1.0	5.57
37.00	2.0	5.78
38.00	1.0	6.02

39.00	2.0	6.15
40.00	2.0	6.77
41.00	1.0	6.85
42.00	2.0	6.97
43.00	1.0	7.05
44.00	2.0	7.08
45.00	1.0	7.12
46.00	1.0	7.16
47.00	2.0	8.18
48.00	2.0	8.20
49.00	1.0	8.30
50.00	2.0	8.43
51.00	1.0	9.00
52.00	2.0	9.23
53.00	1.0	9.33
54.00	2.0	9.95

al laase, independent study

u-test results

glover (solution 1)
vs
philip (solution 2)

*** values ***

u-value	=	461.5000
mean	=	364.5000
variance	=	3341.250
r1	=	645.5000
z-value	=	1.678098

rank	solution	hydraulic conductivity 10**(-3) cm/s
1.00	1.0	1.01
2.00	1.0	1.26
3.00	2.0	1.32
4.00	2.0	1.54
5.00	1.0	1.64
6.00	2.0	1.71
7.00	1.0	2.10
8.00	1.0	2.34
9.00	1.0	2.36
10.00	1.0	2.59
11.00	2.0	2.79
12.00	2.0	3.05
13.00	1.0	3.10
14.00	2.0	3.12
15.50	1.0	3.43
15.50	2.0	3.43
17.00	1.0	3.62
18.00	1.0	3.97
19.00	2.0	4.05
20.00	1.0	4.18
21.00	2.0	4.41
22.00	1.0	4.49
23.00	1.0	4.50
24.00	2.0	4.57
25.00	1.0	4.85
26.00	1.0	5.03
27.00	1.0	5.08
28.00	2.0	5.28
29.00	1.0	5.37
30.00	2.0	5.44
31.00	1.0	5.57
32.00	2.0	5.85
33.00	2.0	5.91
34.00	1.0	6.02
35.00	2.0	6.07
36.00	2.0	6.47
37.00	2.0	6.69
38.00	1.0	6.85

39.00	2.0	7.01
40.00	1.0	7.05
41.00	1.0	7.12
42.00	1.0	7.16
43.00	2.0	7.40
44.00	1.0	8.30
45.00	2.0	8.31
46.00	2.0	8.72
47.00	2.0	8.98
48.00	1.0	9.00
49.00	2.0	9.11
50.00	1.0	9.33
51.00	2.0	9.74
52.00	2.0	10.57
53.00	2.0	11.89
54.00	2.0	11.97

al laase, independent study

u-test results

glover (solution 1)
vs
reynolds (solution 2)

*** values ***

u-value	=	378.0000
mean	=	364.5000
variance	=	3341.250
rl	=	729.0000
z-value	=	0.233550

rank	solution	hydraulic conductivity 10**(-3) cm/s
1.00	2.0	0.97
2.00	1.0	1.01
3.00	2.0	1.06
4.00	1.0	1.26
5.00	1.0	1.64
6.00	2.0	1.94
7.00	1.0	2.10
8.00	2.0	2.20
9.00	2.0	2.27
10.00	1.0	2.34
11.00	1.0	2.36
12.00	2.0	2.47
13.00	1.0	2.59
14.00	2.0	2.95
15.00	1.0	3.10
16.00	2.0	3.34
17.00	2.0	3.38
18.00	1.0	3.43
19.00	1.0	3.62
20.00	1.0	3.97
21.00	2.0	4.03
22.00	2.0	4.09
23.00	1.0	4.18
24.00	2.0	4.36
25.00	2.0	4.40
26.00	1.0	4.49
27.00	1.0	4.50
28.00	2.0	4.69
29.00	1.0	4.85
30.00	2.0	4.95
31.00	1.0	5.03
32.00	1.0	5.08
33.00	2.0	5.24
34.00	2.0	5.26
35.00	1.0	5.37
36.00	1.0	5.57
37.00	2.0	5.82
38.00	1.0	6.02

39.00	2.0	6.76
40.00	1.0	6.85
41.00	2.0	6.97
42.00	2.0	7.01
43.00	1.0	7.05
44.00	2.0	7.08
45.00	1.0	7.12
46.00	1.0	7.16
47.00	2.0	7.45
48.00	2.0	8.14
49.00	2.0	8.25
50.00	1.0	8.30
51.00	1.0	9.00
52.00	2.0	9.31
53.00	1.0	9.33
54.00	2.0	9.53

al laase, independent study

u-test results

stephens I (solution 1)
vs
stephens II (solution 2)

*** values ***

u-value	=	361.5000
mean	=	364.5000
variance	=	3341.250
r1	=	745.5000
z-value	=	-5.18999e-02

rank	solution	hydraulic conductivity 10**(-3) cm/s
1.00	1.0	1.07
2.00	2.0	1.09
3.00	2.0	1.11
4.00	2.0	1.14
5.00	1.0	1.17
6.00	1.0	1.41
7.00	1.0	2.39
8.00	1.0	2.47
9.00	2.0	2.51
10.00	1.0	2.59
11.00	2.0	2.60
12.00	2.0	2.72
13.00	2.0	2.83
14.00	1.0	2.88
15.00	2.0	3.21
16.00	1.0	3.28
17.00	1.0	3.29
18.00	2.0	3.35
19.00	2.0	3.80
20.00	1.0	3.85
21.00	2.0	4.24
22.50	1.0	4.38
22.50	2.0	4.38
24.00	1.0	4.45
25.00	1.0	4.47
26.00	2.0	4.51
27.00	1.0	4.73
28.00	2.0	4.79
29.00	2.0	5.03
30.00	1.0	5.07
31.00	1.0	5.26
32.50	2.0	5.56
32.50	2.0	5.56
34.00	1.0	5.64
35.00	1.0	5.68
36.00	2.0	5.78
37.00	2.0	6.15
38.00	1.0	6.24

39.00	2.0	6.77
40.00	1.0	6.82
41.00	2.0	6.97
42.00	1.0	7.03
43.00	2.0	7.08
44.00	1.0	7.14
45.00	1.0	7.74
46.00	2.0	8.18
47.00	2.0	8.20
48.00	1.0	8.28
49.00	2.0	8.43
50.00	1.0	8.88
51.00	2.0	9.23
52.00	1.0	9.31
53.00	2.0	9.95
54.00	1.0	10.09

al laase, independent study

u-test results

stephens I (solution 1)
vs
philip (solution 2)

*** values ***

u-value	=	442.0000
mean	=	364.5000
variance	=	3341.250
r1	=	665.0000
z-value	=	1.340748

rank	solution	hydraulic conductivity 10**(-3) cm/s
1.00	1.0	1.07
2.00	1.0	1.17
3.00	2.0	1.32
4.00	1.0	1.41
5.00	2.0	1.54
6.00	2.0	1.71
7.00	1.0	2.39
8.00	1.0	2.47
9.00	1.0	2.59
10.00	2.0	2.79
11.00	1.0	2.88
12.00	2.0	3.05
13.00	2.0	3.12
14.00	1.0	3.28
15.00	1.0	3.29
16.00	2.0	3.43
17.00	1.0	3.85
18.00	2.0	4.05
19.00	1.0	4.38
20.00	2.0	4.41
21.00	1.0	4.45
22.00	1.0	4.47
23.00	2.0	4.57
24.00	1.0	4.73
25.00	1.0	5.07
26.00	1.0	5.26
27.00	2.0	5.28
28.00	2.0	5.44
29.00	1.0	5.64
30.00	1.0	5.68
31.00	2.0	5.85
32.00	2.0	5.91
33.00	2.0	6.07
34.00	1.0	6.24
35.00	2.0	6.47
36.00	2.0	6.69
37.00	1.0	6.82
38.00	2.0	7.01

39.00	1.0	7.03
40.00	1.0	7.14
41.00	2.0	7.40
42.00	1.0	7.74
43.00	1.0	8.28
44.00	2.0	8.31
45.00	2.0	8.72
46.00	1.0	8.88
47.00	2.0	8.98
48.00	2.0	9.11
49.00	1.0	9.31
50.00	2.0	9.74
51.00	1.0	10.09
52.00	2.0	10.57
53.00	2.0	11.89
54.00	2.0	11.97

al laase, independent study

u-test results

stephens I (solution 1)
vs
reynolds (solution 2)

*** values ***

u-value	=	354.5000
mean	=	364.5000
variance	=	3341.250
r1	=	752.5000
z-value	=	-0.173000

rank	solution	hydraulic conductivity 10**(-3) cm/s
1.00	2.0	0.97
2.00	2.0	1.06
3.00	1.0	1.07
4.00	1.0	1.17
5.00	1.0	1.41
6.00	2.0	1.94
7.00	2.0	2.20
8.00	2.0	2.27
9.00	1.0	2.39
10.50	1.0	2.47
10.50	2.0	2.47
12.00	1.0	2.59
13.00	1.0	2.88
14.00	2.0	2.95
15.00	1.0	3.28
16.00	1.0	3.29
17.00	2.0	3.34
18.00	2.0	3.38
19.00	1.0	3.85
20.00	2.0	4.03
21.00	2.0	4.09
22.00	2.0	4.36
23.00	1.0	4.38
24.00	2.0	4.40
25.00	1.0	4.45
26.00	1.0	4.47
27.00	2.0	4.69
28.00	1.0	4.73
29.00	2.0	4.95
30.00	1.0	5.07
31.00	2.0	5.24
32.50	1.0	5.26
32.50	2.0	5.26
34.00	1.0	5.64
35.00	1.0	5.68
36.00	2.0	5.82
37.00	1.0	6.24
38.00	2.0	6.76

39.00	1.0	6.82
40.00	2.0	6.97
41.00	2.0	7.01
42.00	1.0	7.03
43.00	2.0	7.08
44.00	1.0	7.14
45.00	2.0	7.45
46.00	1.0	7.74
47.00	2.0	8.14
48.00	2.0	8.25
49.00	1.0	8.28
50.00	1.0	8.88
51.50	1.0	9.31
51.50	2.0	9.31
53.00	2.0	9.53
54.00	1.0	10.09

al laase, independent study

u-test results

stephens II (solution 1)
vs
philip (solution 2)

*** values ***

u-value	=	444.0000
mean	=	364.5000
variance	=	3341.250
r1	=	663.0000
z-value	=	1.375348

rank	solution	hydraulic conductivity 10**(-3) cm/s
1.00	1.0	1.09
2.00	1.0	1.11
3.00	1.0	1.14
4.00	2.0	1.32
5.00	2.0	1.54
6.00	2.0	1.71
7.00	1.0	2.51
8.00	1.0	2.60
9.00	1.0	2.72
10.00	2.0	2.79
11.00	1.0	2.83
12.00	2.0	3.05
13.00	2.0	3.12
14.00	1.0	3.21
15.00	1.0	3.35
16.00	2.0	3.43
17.00	1.0	3.80
18.00	2.0	4.05
19.00	1.0	4.24
20.00	1.0	4.38
21.00	2.0	4.41
22.00	1.0	4.51
23.00	2.0	4.57
24.00	1.0	4.79
25.00	1.0	5.03
26.00	2.0	5.28
27.00	2.0	5.44
28.50	1.0	5.56
28.50	1.0	5.56
30.00	1.0	5.78
31.00	2.0	5.85
32.00	2.0	5.91
33.00	2.0	6.07
34.00	1.0	6.15
35.00	2.0	6.47
36.00	2.0	6.69
37.00	1.0	6.77
38.00	1.0	6.97

39.00	2.0	7.01
40.00	1.0	7.08
41.00	2.0	7.40
42.00	1.0	8.18
43.00	1.0	8.20
44.00	2.0	8.31
45.00	1.0	8.43
46.00	2.0	8.72
47.00	2.0	8.98
48.00	2.0	9.11
49.00	1.0	9.23
50.00	2.0	9.74
51.00	1.0	9.95
52.00	2.0	10.57
53.00	2.0	11.89
54.00	2.0	11.97

al laase, independent study

u-test results

stephens II (solution 1)
vs
reynolds (solution 2)

*** values ***

u-value	=	356.0000
mean	=	364.5000
variance	=	3341.250
r1	=	751.0000
z-value	=	-0.147050

rank	solution	hydraulic conductivity 10**(-3) cm/s
1.00	2.0	0.97
2.00	2.0	1.06
3.00	1.0	1.09
4.00	1.0	1.11
5.00	1.0	1.14
6.00	2.0	1.94
7.00	2.0	2.20
8.00	2.0	2.27
9.00	2.0	2.47
10.00	1.0	2.51
11.00	1.0	2.60
12.00	1.0	2.72
13.00	1.0	2.83
14.00	2.0	2.95
15.00	1.0	3.21
16.00	2.0	3.34
17.00	1.0	3.35
18.00	2.0	3.38
19.00	1.0	3.80
20.00	2.0	4.03
21.00	2.0	4.09
22.00	1.0	4.24
23.00	2.0	4.36
24.00	1.0	4.38
25.00	2.0	4.40
26.00	1.0	4.51
27.00	2.0	4.69
28.00	1.0	4.79
29.00	2.0	4.95
30.00	1.0	5.03
31.00	2.0	5.24
32.00	2.0	5.26
33.50	1.0	5.56
33.50	1.0	5.56
35.00	1.0	5.78
36.00	2.0	5.82
37.00	1.0	6.15
38.00	2.0	6.76

39.00	1.0	6.77
40.50	1.0	6.97
40.50	2.0	6.97
42.00	2.0	7.01
43.50	1.0	7.08
43.50	2.0	7.08
45.00	2.0	7.45
46.00	2.0	8.14
47.00	1.0	8.18
48.00	1.0	8.20
49.00	2.0	8.25
50.00	1.0	8.43
51.00	1.0	9.23
52.00	2.0	9.31
53.00	2.0	9.53
54.00	1.0	9.95

al laase, independent study

u-test results

philip (solution 1)
vs
reynolds (solution 2)

*** values ***

u-value	=	283.5000
mean	=	364.5000
variance	=	3341.250
r1	=	823.5000
z-value	=	-1.401298

rank	solution	hydraulic conductivity 10**(-3) cm/s
1.00	2.0	0.97
2.00	2.0	1.06
3.00	1.0	1.32
4.00	1.0	1.54
5.00	1.0	1.71
6.00	2.0	1.94
7.00	2.0	2.20
8.00	2.0	2.27
9.00	2.0	2.47
10.00	1.0	2.79
11.00	2.0	2.95
12.00	1.0	3.05
13.00	1.0	3.12
14.00	2.0	3.34
15.00	2.0	3.38
16.00	1.0	3.43
17.00	2.0	4.03
18.00	1.0	4.05
19.00	2.0	4.09
20.00	2.0	4.36
21.00	2.0	4.40
22.00	1.0	4.41
23.00	1.0	4.57
24.00	2.0	4.69
25.00	2.0	4.95
26.00	2.0	5.24
27.00	2.0	5.26
28.00	1.0	5.28
29.00	1.0	5.44
30.00	2.0	5.82
31.00	1.0	5.85
32.00	1.0	5.91
33.00	1.0	6.07
34.00	1.0	6.47
35.00	1.0	6.69
36.00	2.0	6.76
37.00	2.0	6.97
38.50	1.0	7.01

38.50	2.0	7.01
40.00	2.0	7.08
41.00	1.0	7.40
42.00	2.0	7.45
43.00	2.0	8.14
44.00	2.0	8.25
45.00	1.0	8.31
46.00	1.0	8.72
47.00	1.0	8.98
48.00	1.0	9.11
49.00	2.0	9.31
50.00	2.0	9.53
51.00	1.0	9.74
52.00	1.0	10.57
53.00	1.0	11.89
54.00	1.0	11.97

APPENDIX K

```
c     **********************************************************
c     al laase, independent study, 1 february 1989
c     **********************************************************
c     this program determines the mean, variance, standard
      deviation, range, high, and low of a group of values.
c     **********************************************************
c     variable dictionary
c
c     mean - mean of group
c     var - variance of group
c     std - standard deviation of group
c     range - range of group
c     high - high of group
c     low - low of group
c     size - sample size
c     sum - sum of values
c     value - number value
c     **********************************************************
c     define variables
c
      dimension value(100)
      real mean,var,std,range,high,low,size,sum
      integer n
      character *30 filename,output,solution,flag
c     **********************************************************
c     input data
c
      write(*,*)'enter borehole permeameter solution'
      read(*,*)solution
      write(*,*)' enter data filename'
      read(*,*)filename
      write(*,*)' enter output file name'
      read(*,*)output
c     **********************************************************
c     open data files
c
      open(unit=21,file=filename,status='unknown')
      open(unit=22,file=output,status='unknown')
c     **********************************************************
c     read datafile
c
      n=0
      size=0.0
      do 10 i=1,100
         read(21,*,end=99)value(i)
         n=n+1
         size=size+1.0
 10   continue
 99   continue
c     **********************************************************
c     sort values from low to high
c
      do 20 i=n,2,-1
         flag='off'
         do 30 j=1,n-1
            if(value(j).gt.value(j+1))then
               temp=value(j)
               value(j)=value(j+1)
               value(j+1)=temp
               flag='on'
```

```
            end if
 30      continue
         if(flag.eq.'off')goto 500
 20   continue
500   continue
c     ******************************************************
c     determine mean, variance, standard deviation, range
c     high and low of values
c
      sumdiff=0.0
      diff=0.0
      sum=0.0
      do 40 j=1,n
         sum=value(j)+sum
 40   continue
      mean=sum/size
      do 50 j=1,n
         diff=(value(j)-mean)**2.0
         sumdiff=sumdiff+diff
 50   continue
      var=sumdiff/(size-1.0)
      std=sqrt(var)
      high=value(n)
      low=value(1)
      range=high-low
c     ******************************************************
c     print out results
c
      write(22,11)
      write(22,*)' borehole permeameter solution = ',solution
      write(22,*)
      write(22,*)'         *** statistical evaluation *** '
      write(22,*)'         *** hydraulic conductivity *** '
      write(22,*)'         ***     10**(-3) cm/s      *** '
      write(22,*)
      write(22,*)' mean                            = ',mean
      write(22,*)' variance                        = ',var
      write(22,*)' standard deviation              = ',std
      write(22,*)' range                           = ',range
      write(22,*)' high                            = ',high
      write(22,*)' low                             = ',low
      write(22,*)' sample size                     = ',size
      write(22,*)
      write(22,*)'                *** values *** '
      write(22,*)
      write(22,*)'     rank      hydraulic conductivity '
      write(22,*)
      k=1
      do 60 j=1,n
         write(22,12)k,value(j)
         k=k+1
 60   continue
c     ******************************************************
c     format statements
c
 11   format(/,/,/,3x,'al laase, independent study',/)
 12   format(7x,i2,12x,f12.8)
c     ******************************************************
      stop
      end
```

al laase, independent study

borehole permeameter solution = glover

*** statistical evaluation ***
*** hydraulic conductivity ***
*** 10**(-3) cm/s ***

mean	=	4.715556
variance	=	5.440564
standard deviation	=	2.332502
range	=	8.320000
high	=	9.330000
low	=	1.010000
sample size	=	27.00000

*** values ***

rank	hydraulic conductivity
1	1.00999999
2	1.25999999
3	1.63999999
4	2.09999991
5	2.33999991
6	2.35999990
7	2.58999991
8	3.09999991
9	3.43000007
10	3.61999989
11	3.97000003
12	4.17999983
13	4.48999977
14	4.50000000
15	4.84999991
16	5.03000021
17	5.07999992
18	5.36999989
19	5.57000017
20	6.01999998
21	6.84999991
22	7.05000019
23	7.11999989
24	7.15999985
25	8.30000019
26	9.00000000
27	9.32999992

al laase, independent study

borehole permeameter solution = stephensI

*** statistical evaluation ***
*** hydraulic conductivity ***
*** 10**(-3) cm/s ***

mean	=	5.022592
variance	=	6.424958
standard deviation	=	2.534750
range	=	9.020000
high	=	10.090000
low	=	1.070000
sample size	=	27.00000

*** values ***

rank	hydraulic conductivity
1	1.07000005
2	1.16999996
3	1.40999997
4	2.39000011
5	2.47000003
6	2.58999991
7	2.88000012
8	3.27999997
9	3.28999996
10	3.84999991
11	4.38000012
12	4.44999981
13	4.46999979
14	4.73000002
15	5.07000017
16	5.26000023
17	5.63999987
18	5.67999983
19	6.23999977
20	6.82000017
21	7.03000021
22	7.13999987
23	7.73999977
24	8.27999973
25	8.88000011
26	9.31000042
27	10.09000020

al laase, independent study

borehole permeameter solution = stephensII

*** statistical evaluation ***
*** hydraulic conductivity ***
*** 10**(-3) cm/s ***

mean	=	5.006296
variance	=	6.315801
standard deviation	=	2.513126
range	=	8.860000
high	=	9.950000
low	=	1.090000
sample size	=	27.00000

*** values ***

rank	hydraulic conductivity
1	1.09000003
2	1.11000001
3	1.13999999
4	2.50999999
5	2.59999991
6	2.72000003
7	2.82999992
8	3.21000004
9	3.34999991
10	3.79999995
11	4.23999977
12	4.38000012
13	4.51000023
14	4.78999996
15	5.03000021
16	5.55999994
17	5.55999994
18	5.78000021
19	6.15000010
20	6.76999998
21	6.96999979
22	7.07999992
23	8.18000030
24	8.19999981
25	8.43000030
26	9.22999954
27	9.94999981

al laase, independent study

borehole permeameter solution = philip

*** statistical evaluation ***
*** hydraulic conductivity ***
*** 10**(-3) cm/s ***

mean	=	6.125926
variance	=	9.349555
standard deviation	=	3.057704
range	=	10.65000
high	=	11.97000
low	=	1.320000
sample size	=	27.00000

*** values ***

rank	hydraulic conductivity
1	1.32000005
2	1.53999996
3	1.71000004
4	2.78999996
5	3.04999995
6	3.11999989
7	3.43000007
8	4.05000019
9	4.40999985
10	4.57000017
11	5.28000021
12	5.44000006
13	5.84999991
14	5.90999985
15	6.07000017
16	6.46999979
17	6.69000006
18	7.01000023
19	7.40000010
20	8.31000042
21	8.72000026
22	8.97999954
23	9.10999965
24	9.73999977
25	10.56999970
26	11.89000030
27	11.97000030

al laase, independent study

borehole permeameter solution = reynolds 1 head

*** statistical evaluation ***
*** hydraulic conductivity ***
*** 10**(-3) cm/s ***

mean	=	4.960001
variance	=	6.025484
standard deviation	=	2.454686
range	=	8.559999
high	=	9.530000
low	=	0.970000
sample size	=	27.00000

*** values ***

rank	hydraulic conductivity
1	0.97000003
2	1.05999994
3	1.94000006
4	2.20000005
5	2.26999998
6	2.47000003
7	2.95000005
8	3.33999991
9	3.38000012
10	4.03000021
11	4.09000015
12	4.36000013
13	4.40000010
14	4.69000006
15	4.94999981
16	5.23999977
17	5.26000023
18	5.82000017
19	6.76000023
20	6.96999979
21	7.01000023
22	7.07999992
23	7.44999981
24	8.14000034
25	8.25000000
26	9.31000042
27	9.52999973

al laase, independent study

borehole permeameter solution = reynolds 2 head

*** statistical evaluation ***
*** hydraulic conductivity ***
*** 10**(-3) cm/s ***

mean	=	4.802500
variance	=	9.540492
standard deviation	=	3.088769
range	=	6.880000
high	=	7.610000
low	=	0.730000
sample size	=	4.000000

*** values ***

rank	hydraulic conductivity
1	0.73000002
2	4.13999987
3	6.73000002
4	7.61000013

APPENDIX L

Derivation of Reynolds et al's (1986) borehole permeameter solution

Description of steady flow out of a well

Steady flow out of a well can be described in terms of pressure and gravity–induced fluxes. Water flows from the well by; radial pressure–induced flux ($\bar{v}_{rp}$), through the wall; vertical pressure induced flux ($\bar{v}_{sp}$), through the base; and vertical gravity–induced flux($\bar{v}_g$), through the base. The above fluxes can be described in terms of the Darcy–Buckingham relationships of the form:

$$\bar{v}_{rp} = -K(\Psi)\frac{\partial \Psi}{\partial r}|r = 0 \quad \hat{r} \tag{1}$$

$$\bar{v}_{sp} = -K(\Psi)\frac{\partial \Psi}{\partial z}|z = 0 \quad \hat{k} \tag{2}$$

and

$$\bar{v}_g = -K(\Psi)\frac{\partial \Psi_z}{\partial z}|z = 0 \quad \hat{k} = -K_s\hat{k} \tag{3}$$

where Ψ_z is the elevation head defined relative to the base of the borehole. r and z (positive up) are radial and vertical coordinate directions respectively. $\hat{r}$ and $\hat{k}$ are unit vectors in positive r and z directions. Equations (1) and (2) represent the combined influence of the inner field–saturated zone, the outer unsaturated envelope, and the initial pressure head. K_s replaces $K(\Psi)$ in eq.(3) because $\bar{v}_g$ at z = 0 is dependent only on K at z = 0.

Equations (1) and (2) can be linearized using the flux potential ϕ defined by (Gardner, 1958)

$$\phi = \int_{\Psi_i}^{\Psi} K(\Psi)d\Psi \tag{4}$$

Thus

$$\bar{v}_{rp} = \frac{\partial \phi}{\partial r}|r = r_b \quad \hat{r} \tag{5}$$

and

$$\bar{v}_{sp} = \frac{\partial \phi}{\partial z}|z = 0 \quad \hat{k} \tag{6}$$

respectively

Multiplying the fluxes by the area across which they flow gives the discharge rate from the borehole.

$$Q_t = 2\pi H^2 \left[-\frac{r_b}{H^2} \int_0^H \frac{\partial \phi}{\partial r} |_{r=r_b} dz + \frac{1}{H^2} \int_0^{r_b} \frac{\partial \phi}{\partial z} |_{z=0} r dr + \frac{K_s}{2} \left(\frac{r_b}{H} \right)^2 \right] \tag{7}$$

Equation (7) can be rewritten

$$Q_t = 2\pi H^2 \left[\frac{1}{C^*} + \frac{K_s}{2} \left(\frac{r_b}{H} \right)^2 \right] \tag{8}$$

$$C^* = \left[-\frac{r_b}{H^2} \int_0^H \frac{\partial \phi}{\partial r} |_{r=r_b} dz + \frac{1}{H^2} \int_0^{r_b} \frac{\partial \phi}{\partial z} |_{z=0} r dr \right]^{-1} \tag{9}$$

If gravity flow is ignored, the last term on the RHS of Eq.(7) is dropped, and Eq.(8) simplifies to

$$Q = \frac{2\pi H^2}{C^*} \tag{10}$$

where Q represents only pressure-induced flow. Estimates of C^* can be obtained by numerical or analytical means.

Analytical evaluation of C^*

Combined saturated-unsaturated flow in a homogeneous isotropic porous medium may be described by Richards equation:

$$\nabla \cdot [K(\Psi)\nabla\Psi] = 0 \tag{11}$$

in symemetric, spherical coordinates Richards equation is:

$$\frac{d}{d\varrho} \left[\varrho^2 K(\Psi) \frac{d\Psi}{d\varrho} \right] = 0 \tag{12}$$

where ϱ is the radius from the origin. Substituting Eq.(4) into Eq.(12) yields:

$$\frac{d}{d\varrho} \left[\varrho^2 \frac{d\phi}{d\varrho} \right] = 0 \tag{13}$$

For a point source of strength q [L^3/t] where ϕ approaches 0 as ϱ approaches ∞, it can be shown that:

$$\phi = \frac{q}{4\pi\varrho} \tag{14}$$

In symmetrical cylindrical coordinates Eq.(14) is:

where h equals the linear strength distribution.

$$\phi = \frac{q}{4\pi\sqrt{r^2 + (z-h)^2}} \tag{15}$$

Eq.(15) can be used to represent a well by integrating a series of point sources along the well axis to produce a point source. The source strength gradient of the line source can be described by:

$$dq = \frac{2Q[(H-b)-h]}{(H-b)^2}dh \tag{16}$$

Substituting Eq.(16) into the differential form of Eq.(15) and integrating between h equals 0, and $h = (H-b)$ produces,

$$Q = \frac{2\pi(H-b)^2\phi}{[z-(H-b)]\left[\sinh^{-1}\left[\frac{z-(H-b)}{r}\right] - \sinh^{-1}\left(\frac{z}{r}\right)\right] - \sqrt{r^2 + [z-(H-b)]^2} + \sqrt{r^2+z^2}} \tag{17}$$

Because the boundry conditions at the well are

$$\Psi = H \quad at \quad z = 0, \quad 0 \le r \le r_b \tag{18}$$

$$\Psi = (H-z) \quad at \quad r = 0, \quad 0 \le z \le H \tag{19}$$

the finite line source representation of the borehole (Eq.17) can be exact at only one point. This point is arbitrarily chosen to be

$$\Psi = H \quad at \quad r = r_b, \quad z = 0 \tag{20}$$

Therefore

$$Q = \frac{2\pi H^2(H-b)^2\phi_{r_b,0}}{H^2\left\{(H-b)\sinh^{-1}\left[\frac{(H-b)}{r_b}\right] - \sqrt{r_b^2 + (H-b)^2} + r_b\right\}} \tag{21}$$

where $\phi_{r_b,0}$ is the flux potential at point ($r_b, 0$).

An expression for C^* can now be obtained by comparing Eq.(10) and Eq.(21) yielding

$$C^* = \frac{H^2\left\{(H-b)\sinh^{-1}\left[\frac{(H-b)}{r_b}\right] - \sqrt{r_b^2 + (H-b)^2} + r_b\right\}}{(H-b)^2\phi_{r_b,0}} \qquad (22)$$

Along the borehole wall ϕ may be expressed as

$$\phi_{r_b,z} = \int_{\Psi_i}^{\Psi_{r_b,z}} K(\Psi)d\Psi = K_s\Psi_{r_b,z} + \int_{\Psi_i}^{0} K(\Psi)d\Psi \qquad (23)$$

where $\phi_{r_b,z}$ and $\Psi_{r_b,z}$ are the flux potential and pressure head respectively, for $0 \leq z \leq H$.

Equation (23) can be rewritten in the form

$$\phi_{r_b,z} = \phi_p + \phi_m \qquad (24)$$

where

$$\text{pressure flux potential} = \phi_p = K_s\Psi_{r_b,0} \qquad (25)$$

$$\text{matric flux potential} = \phi_m = \int_{\Psi_i}^{0} K(\Psi)d\Psi \qquad (26)$$

At point (0, r_b) Eq.(24) becomes

$$\phi_{r_b,0} = K_s\Psi_{r_b,0} + \phi_m = K_sH + \phi_m \qquad (27)$$

Equations (8) and (10) can now be solved in terms of the physically well-defined parameters in Eq.(27).

Extended constant head well permeameter (CHWP) theory

When Eq.(27) is substituted into Eq.(22)

$$C^* = \frac{HC}{K_sH + \phi_m} \qquad (28)$$

where

$$C = \frac{H^2\left[\frac{(H-b)}{H}\sinh^{-1}\left[\frac{(H-b)}{r_b}\right] - \sqrt{\left(\frac{r_b}{H}\right)^2 + \left[\frac{(H-b)}{H}\right]^2} + \frac{r_b}{H}\right]}{(H-b)^2} \tag{29}$$

Substituiting Eq.(28) into Eq.(8) produces the case of combined pressure and gravity flow.

$$K_s = \frac{CQ_t - 2\pi H\phi_m}{2\pi H^2\left[1 + \frac{C}{2}\left(\frac{r_b}{H}\right)^2\right]} \tag{30}$$

For pressure only flow.

$$K_s = \frac{CQ - 2\pi H\phi_m}{2\pi H^2} \tag{31}$$

Equations (26), (29), (30), and (31) constitute an extended CHWP theory that accounts for the combined effects of field-saturated and unsaturated flow around a well. The effects of unsaturated flow are embodied in the ϕ_m term defined in Eq.(26).

Estimation of sorptivity

Sorptivity can be defined in terms of ϕ_m and an assumed diffusivity-water content function.

$$\phi_m = \int_{\Psi_i}^{0} K(\Psi)d\Psi = \int_{\theta_i}^{\theta_f} D(\theta)d\theta = \frac{S^2}{2(\theta_f - \theta_i)} = \frac{S^2}{2\Delta\theta} \tag{32}$$

where θ_f is the final volumetric water content, θ_i is the initial water content, and $\Delta\theta$ is the difference between the final and initial volumetric water content.

Equation (30) can therefore be expressed

$$S = \frac{\Delta\theta\left[CQ_t - 2\pi H^2K_s\left[1 + \frac{C}{2}\left(\frac{r_b}{H}\right)^2\right]\right]^{\frac{1}{2}}}{\pi H} \tag{33}$$

Estimation of α

The extended CHWP theory is completely general with respect to the $K(\Psi)$ relationship and can be therefore be used to estimate the α-parameter of the exponential $K(\Psi)$ function.

$$K(\Psi) = K_s e^{(\alpha\Psi)} \qquad (34$$

where α is constant and depends on soil properties.

Substituting Eq.(34) into Eq.(26) produces

$$\phi_m = \int_{\Psi_i}^{0} K(\Psi)d\Psi = \frac{K_s}{\alpha}[1 - e^{\alpha\Psi_i}] \qquad (35)$$

For many soils initially at "field capacity" or drier, $e^{(\alpha\Psi_i)} << 1$, and thus Eq.(35) reduces to

$$\phi_m = \frac{K_s}{\alpha} \qquad (36)$$

Substituting Eq.(36) into Eq.(30) and solving for α produces

$$\alpha = \frac{2\pi H K_s}{CQ_t - 2H^2K_s\left[1 + \frac{C}{2}\left(\frac{r_b}{H}\right)^2\right]} \qquad (37)$$

Application of extended CHWP theory

Equations (30), (31), (33), and (37) all contain two unknowns and can not be solved directly. By using two water levels in a single borehole two simulataneous equations can be obtained. Two steady flow rates Q_{s1} and Q_{s2} are obtained by successively ponding two depths of water, H_1 and H_2, in a single well. Because K_s, ϕ_m, S, and α are constant the two Q_s and H values can be used to write simultaneous equations from each of Equations (30), (33), and (37).

For calculating K_s, ϕ_m is eliminated from the simultaneous equations based on Eq.(30) producing

$$K_s = G_2Q_{s2} - G_1Q_{s1} \qquad (38)$$

where:

$$G_1 = \frac{H_2C_1}{\pi[2H_1H_2(H_2 - H_1) + r^2(H_1C_2 - H_2C_1)]}$$

$$G_2 = \frac{H_1C_2}{\pi[2H_1H_2(H_2 - H_1) + r^2(H_1C_2 - H_2C_1)]}$$

and

$$C_1 = \frac{H_1^2\left[\frac{(H_1-b_1)}{H_1}\sinh^{-1}\left[\frac{(H_1-b_1)}{r}\right] - \sqrt{(\frac{r}{H_1})^2 + \left[\frac{(H_1-b_1)}{H_1}\right]^2} + \frac{r}{H_1}\right]}{(H_1-b_1)^2}$$

$$C_2 = \frac{H_2^2\left[\frac{(H_2-b_2)}{H_2}\sinh^{-1}\left[\frac{(H_2-b_2)}{r}\right] - \sqrt{(\frac{r}{H_2})^2 + \left[\frac{(H_2-b_2)}{H_2}\right]^2} + \frac{r}{H_2}\right]}{(H_2-b_2)^2}$$

Sorptivity is estimated by eliminating K_s from the two simultaneous equations based on Eq.(33)

$$S = \sqrt{\Delta\theta(J_2 Q_{s2} - J_1 Q_{s1})} \qquad (39)$$

where

$$J_1 = \frac{(2H_2^2 + r^2 C_2)C_1}{\pi[2H_1H_2(H_1 - H_2) + r^2(H_2C_1 - H_1C_2)]}$$

$$J_2 = \frac{(2H_1^2 + r^2 C_1)C_2}{\pi[2H_1H_2(H_1 - H_2) + r^2(H_2C_1 - H_1C_2)]}$$

The required $\Delta\theta$ value in Eq.(39) must be estimated or measured.

The α-parameter is obtained by eliminating K_s from the two simultaneous equations based on Eq.(37)

$$\alpha_r = \frac{M_2 Q_{s2} - M_1 Q_{s1}}{N_2 Q_{s2} - N_1 Q_{s1}} \qquad (40)$$

where

$$M_1 = 2H_2C_1$$

$$M_2 = 2H_1C_2$$

$$N_1 = -(2H_2^2 + r_b^2 C_2)C_1$$

$$N_2 = -(2H_1^2 + r_b^2 C_1)C_2$$

APPENDIX M

```
al laase, independent study, borehole research

equations from reynolds, elrick, and clothier
soil science, vol 139, no. 2, 1986

borehole input data and results

two seperate tests in same borehole
heads, discharge rates, initial moisture content
and final moisture content known

output file name                                  =   sample1.out

*** borehole parameters ***

radius of borehole (m)                            =    0.100000
depth of borehole (m)                             =    3.000000

*** test 1 parameters ***

borehole test identification                      =   sample
constant head in borehole test 1 (m)              =    1.000000
discharge test 1 (liters/min)                     =    1.000000
open interval of borehole test 1 (m)              =    0.500000

*** test 2 parameters ***

borehole test identification                      =   sample
constant head in borehole test 2 (m)              =    2.000000
discharge test 2 (liters/min)                     =    1.670000
open interval of borehole test 2 (m)              =    1.000000

*** soil parameters ***

initial moisture content before test              =   0.
final moisture content after test                 =    0.250000

*** calculated soil parameters ***

saturated hydraulic conductivity (cm/s)           =     1.36003e-04
sorptivity (l/s**0.5)                             =    0.180809
before taking square root for s                   =     3.26917e-02
alpha (1/m)                                       =    0.208009

*** calculated constants ***

c1                                                =    2.985269
c2                                                =    4.186471
g1 (1/m**2)                                       =    0.477249
g2 (1/m**2)                                       =    0.334642
j1 (1/m)                                          =   -1.918987
j2 (1/m)                                          =   -0.679273
m1 (m)                                            =    11.94108
m2 (m)                                            =    8.372941
n1 (m**2)                                         =   -24.00713
n2 (m**2)                                         =   -8.497918
Q1/Q2                                             =    0.598802
H1/H2                                             =    0.500000
```

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al laase, independent study, borehole research

equations from reynolds, elrick, and clothier
soil science, vol 139, no. 2, 1986

borehole input data and results

two seperate tests in same borehole
heads, discharge rates, initial moisture content
and final moisture content known

output file name                              =   sample2.out

*** borehole parameters ***

radius of borehole (m)                        =    0.100000
depth of borehole (m)                         =    3.000000

*** test 1 parameters ***

borehole test identification                  =   sample
constant head in borehole test 1 (m)          =    1.000000
discharge test 1 (liters/min)                 =    1.000000
open interval of borehole test 1 (m)          =    0.500000

*** test 2 parameters ***

borehole test identification                  =   sample
constant head in borehole test 2 (m)          =    2.000000
discharge test 2 (liters/min)                 =    1.430000
open interval of borehole test 2 (m)          =    1.000000

*** soil parameters ***

initial moisture content before test          =   0.
final moisture content after test             =    0.250000

*** calculated soil parameters ***

saturated hydraulic conductivity (cm/s)       =     2.14676e-06
sorptivity (1/s**0.5)                         =    0.198707
before taking square root for s               =     3.94845e-02
alpha (1/m)                                   =     2.71845e-03

*** calculated constants ***

c1                                            =    2.985269
c2                                            =    4.186471
g1 (1/m**2)                                   =    0.477249
g2 (1/m**2)                                   =    0.334642
j1 (1/m)                                      =   -1.918987
j2 (1/m)                                      =   -0.679273
m1 (m)                                        =    11.94108
m2 (m)                                        =    8.372941
n1 (m**2)                                     =   -24.00713
n2 (m**2)                                     =   -8.497918
Q1/Q2                                         =    0.699301
H1/H2                                         =    0.500000
```