ASSESSMENT OF GRAVITY ANOMALIES OVER MINE TUNNELS

by

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ABSTRACT

Underground voids and in particular abandoned mine tunnels are potentially hazardous, thus rendering their detection a benefit to society. To extract mine tunnel anomalies from a set of gravity data, a number of criteria must be satisfied. Among these are: (1) all anomalies of non-interest must be removed or shown not to interfere with mine void anomalies, (2) the magnitude of the mine anomaly must be at least twice the error associated with the reduced data, (3) the station spacing must be less than the half-width of the expected mine anomaly, and (4) the spacing of a series of mine tunnels must be such that gravity anomalies for individual mines are separable. Errors arising in measurements and corrections produce an error associated with the data. If mine parameters can be constrained from auxiliary data sources, then an expected mine anomaly can be calculated, either by a computer program or by a horizontal cylinder approximation. This computed anomaly can then be compared to the expected error in the data. This comparison is critical in determining the feasibility of mine detection with a gravity survey. A study involving detection of abandoned coal mines in Heaton Canyon, Gallup, N.M. is used as an example to illustrate some of the steps necessary in locating abandoned mines. Results of this survey confirm the above statements.

BACKGROUND

Shallow mine voids can produce surface subsidence and thus become a danger to life and property. Mines up to depths of 700 feet have been known to cause surface subsidence. Stoping above subsurface voids can make its way to the surface and create deep near vertical holes overnight. Subsurface caverns in Florida have formed sinkholes which have damaged houses and cars. The detection and location of underground voids by geophysical means could mitigate some problems either by avoiding such locations or by enabling one to incorporate engineering remedies.

Geophysical means of void detection is one of many possible methods. Other methods include drilling and excavation. Possible geophysical methods are gravity, magnetic, seismic, resistivity, electromagnetic and ground penetrating radar. Every method offers advantages and disadvantages. Some advantages to geophysical methods, in general, are that they produce little or no surface and/or subsurface disturbance, and most are less expensive than drilling. Depending upon circumstances under which they are employed, geophysical methods could be accurate in void detection. A major disadvantage is that most geophysical techniques are limited in depth when employed in the search for underground voids.

This study was initiated because of results obtained in a geophysical study of shallow coal mine detection in Heaton Canyon (Map 1), near Gallup, N.M., (Johnpeer et al., 1985, in press). Here, a number of surface subsidence features are associated with shallow coal mines. These features are approximately 10 feet deep and have steep sides. Some are circular while others are elongated in plan view and appear to have level bottoms probably from collecting water from time to time. In Heaton Canyon, the buried El Paso and Transwestern gas pipelines cross a section of the mine close to the subsidence features. Accurate location of voids could pinpoint potential subsidence areas and thus lead to careful monitoring and/or remedial action to insure the integrity of the pipelines.

Part of the town of Gallup also overlies shallow coal mines. Some building foundations within the city have cracked. Location of mine voids could lead to changes in the location of future building sites and/or design features to mitigate the effects of subsidence. Filling the voids could be an alternative remedy.

Gravity, magnetic and seismic surveys were conducted in Heaton Canyon across known shallow mine voids underneath an alluvial cover. Seismic refraction surveys and drilling data constrained the depth of the alluvial cover to less than 35 feet and thus were of considerable aid in interpreting the gravity survey. Estimated void parameters of

10 x 23 feet at depths of approximately 50 feet could not be detected gravimetrically. It is believed that the anomalies from individual mines combined to form a single broad anomaly of low magnitude so that they could not be differentiated from the noise and/or the alluvial valley anomaly. This naturally led to the question of just what could be detected by gravity surveys under assumed and actual conditions, which this paper attempts to answer.

INTRODUCTION

OBJECTIVE

The major objective of this paper is to predict the feasibility of mine detection by gravity methods before a survey is conducted. In order to accomplish this task the major variables involved in underground void detection by the gravity method and their effects will be discussed. Also where appropriate, the discussion will incorporate the Heaton Canyon study as a practical example to illustrate certain points.

ANOMALIES

One definition of an anomaly as given by Sherif (1984, p.8) is, "1. A deviation from uniformity in physical properties. Especially a deviation from uniformity of exploration interest, for example, a travel time anomaly, Bouguer anomaly, free-air anomaly.". Gravity surveys are used to detect lateral variations in density. Mine voids exhibit discontinuous lateral density variations which thus produce anomalies. Unfortunately, many other factors also produce anomalies. For example, elevation differences of the data points produce the free-air anomaly. Ideally, all anomalies except those produced by mine voids should be removed.

A major step in this direction is to calculate the Bouguer anomaly. The free-air, Bouguer plate, drift, theoretical gravity and terrain corrections are incorporated into the Bouguer anomaly. All the above reductions attempt to delete unwanted variability in the data by placing the data on a common reference plane. Other anomalies of non-geophysical interest could also interfere with the detection of mine void anomalies. Two likely possibilities to be encountered are regional trends and alluvial-filled valleys. Detecting the factors which affect the data and deleting these unwanted effects is an "art" in geophysics.

ERRORS

A gravity profile models a continuous process with a discontinuous one because of the finite resolution of instruments and measurements and also because of the finite number of data points. Thus the data have an uncertainty or error associated with them because they can vary from the true values. Each correction added to the observed data point also adds an error. A large error in any one correction can influence the total error in the final data. This final error associated with the data is of importance because it sets limits on the detectability of anomalies. Therefore some sort of estimate of its magnitude is critical in understanding the capabilities of the process.

MODELS

The theory of gravitational attraction with its predictive nature is another tool at our disposal. Given such mine parameters as depth, height, width, length and density contrast, the theoretical magnitude and shape of the anomaly produced by the mine can be calculated. The S.K.W. Enterprises Pocket Gopher 1.1 gravity anomaly models program was used in an Apple 2 computer to compute anomalies for a number of models. This program when given a cross section of infinite strike length computes the gravity profile perpendicular to strike using the Talwani (Talwani et al., 1959) computation algorithm in which the coordinates of the corners are used in the calculations. More information regarding this program can be obtained by writing to: S.K.W. Enterprises, Box 3135, Boulder Colorado 80307. Guesses about the subsurface structure can be made and the resulting surface gravity anomaly can be calculated and compared with observed data. Thus by trial and error, the subsurface structure can be modeled, especially if other information constrains some of the subsurface parameters. On the other hand, if subsurface parameters are approximately known, then the magnitude and shape of the anomaly can be calculated. From this information, the chances of detectability can be determined.

CRITERIA FOR DETECTION

Certain requirements need to be satisfied if mines are to be detected with a gravity survey. Variability in the data generated by factors other than mine voids should be removed or shown not to interfere with mine detection. An estimate of the error associated with the data should be available. If limits can be placed on mine parameters, then an expected anomaly can be calculated. Anomalies less then twice the estimated error probably will not be detected. Thus a rough indication on the feasibility of mine detection can be determined. Another factor is the density of the data points. This spacing should not allow any anomaly to be hidden between data points. Also mine spacing/depth geometry should be such that individual mine voids exhibit a detectable anomaly.

HEATON CANYON, GALLUP N.M. EXAMPLE

The observed gravity data from the Heaton Canyon project were reduced to the Bouguer anomaly values, and a regional trend was removed. Although the survey lines crossed an alluvial-filled valley, no attempt was made to eliminate this anomaly as it was felt that the shallow mine tunnel anomalies could be differentiated from the alluvial fill anomaly by a difference in wavelength. This proved to be an incorrect assumption.

An estimate of the associated error was calculated and compared visually to the short wavelength scatter on the gravity profiles. This comparison was favorable except for a number of high-amplitude short-wavelength anomalies. These anomalies were of shallow origin as inferred from their short wavelength and large amplitude. No satisfactory model could be invoked to explain these positive and negative anomalies. Thus these spurious results were the result of some unknown cause. Two likely candidates are incorrect elevations (some elevations were estimated) and errors in instrument reading. These spurious results along with the scatter in the data were deleted by hand-smoothing the profiles.

Interpretation then concentrated on the relatively short-wavelength anomalies on the smoothed profiles. Emphasis was placed on anomalies where other evidence suggested the existence of a mine. It was found that the mine structures required to explain the observed anomalies were unrealistic and that realistic mine structures produced anomalies with magnitudes less than the estimated errors. The mine parameters in Heaton Canyon are constrained by drill results and existing mine maps allowing gravity profiles to be calculated across the mines. Thus the Heaton Canyon study is an example of the difficulty of mine detection by the gravity method, and examples from it are used to explain points in the analytical study below.

DATA REDUCTION AND ASSOCIATED ERRORS

Reduction of gravity data involves factors which introduce uncertainties in the final anomaly values. The cumulative error can be found by adding the errors for the individual corrections. The error determination for Heaton Canyon will be used as an example of how the cumulative error can be determined.

G_B - BOUGUER ANOMALY

The Bouguer anomaly is given by the following formula:

$$G_B = G_O - G_{Th} \pm \Delta G_{Fa} \mp \Delta G_{Bp} + \Delta G_T \pm \Delta G_D, \qquad (1)$$

where

G_B = Bouguer anomaly,

G_O = observed gravity,

G_{Th} = theoretical gravity,

ΔG_{Fa} = free-air anomaly,

ΔG_{Bp} = Bouguer plate correction,

ΔG_T = terrain correction,

ΔG_D = drift correction.

An extensive discussion of the Bouguer anomaly can be found in Telford (1976, p.14-24,45-46).

G_O - OBSERVED GRAVITY. - Sensitivities of gravity meters differ. The Worden gravimeter used in the Heaton Canyon survey has a sensitivity of about 0.01 mGal according to the instrument manufacturer. In comparison microgravity instruments may have claimed sensitivities of 0.004-0.006 mGal (Arzi 1975, p.422). However, a more accurate gauge of the error associated with an instrument is probably the repeatability of readings. A number of consecutive readings for the same location were taken with a Worden gravimeter always obtaining the null position from the same direction. The results (Figure 1) of two observers taking ten readings apiece produce a 0.018 mGal standard deviation for all readings. Due to the short time intervals needed to observe the two data sets and the lack of noticable drift (Figure 1), drift corrections were not applied prior to calculating the standard deviation. Because the instrument was not moved and releveled between readings, 0.018 mGal standard deviation probably represents a minimum error associated with the repeatability of readings taken with a Worden gravimeter. It is believed that one possible source for some of the spurious anomalies in the Heaton Canyon study could be errors associated with reading the instrument.

ΔG_D - DRIFT CORRECTION. - Drift corrections are applied to the instrument reading to compensate for instrument drift and tidal effects. The spring

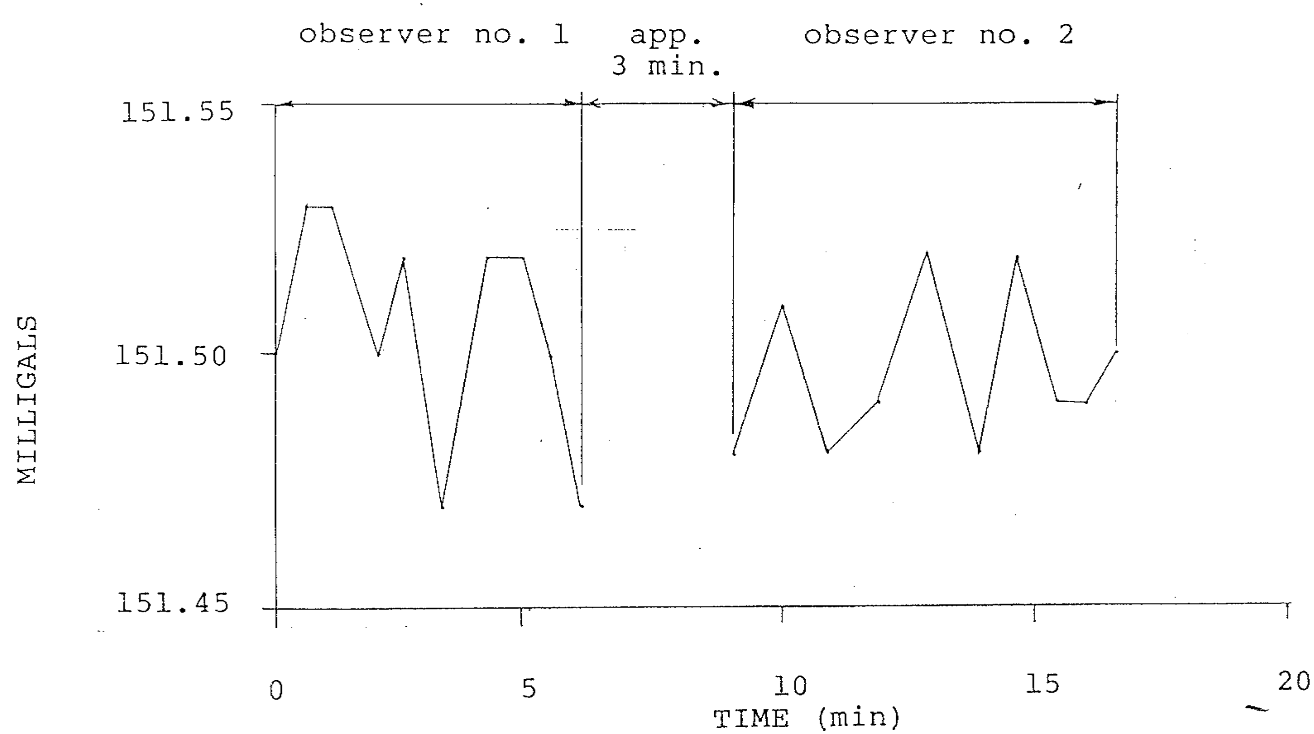

FIGURE 1. The repeatability of readings in mGal with a Worden gravimeter (serial no. 771590; scale constant 0.0839 mGal/div). The two sets of readings by two different observers give a combined standard deviation of 0.018 mGal.

constants inside the instrument may vary slowly due to temperature changes and/or creep. The relative position of the moon and sun with respect to the observation position can also affect the gravity reading on a predictable temporal basis. Therefore one way of removing the tidal effects is with theoretical computations which use readings from an astronomical almanac. However, a simpler method, that also incorporates instrument drift, is to repeat observations every hour or so at a specified base station. Thus a drift curve can be established by linearly connecting these observations or fitting them with a polynomial curve. By eliminating the drift another error is introduced. The uncertainty in the measurements of the base station lead to an uncertainty in the algebraic manipulations used in deleting the drift. This produces a one standard deviation error of 0.018 mGal. The difference between a linear versus polynomial curve is 0.0037 mGal as measured on a typical drift curve (Dobrin 1979, Figure 11.25, p.393). Thus a standard deviation of 0.0217 mGal can be associated with the drift correction.

G_{Th} - THEORETICAL GRAVITY. - Given an ellipsoid of rotation as an idealized shape for the earth, the angular velocity and the assumption that no lateral variations in density exist, the theoretical gravity at sea level for any location on the earth can be computed from the following formula (Stacy 1977, p.75) adopted by the International Association of Geodesy in 1967:

$$G_{Th} = 978.0318 \, [\, 1+0.003024 \, (\sin\phi)^2 + 0.0000059 \, (\, \sin 2\phi)^2 \,], \qquad (2)$$

where

ϕ = latitude,

G_{Th} = gravity at sea level.

This equation gives the variance of gravity with latitude, and therefore is sometimes referred to as the latitude correction. Gravity increases poleward from the equator with the maximum rate of change occurring at 45 degrees latitude where it is 0.01 mGal/40 feet. This variation in gravity is approximately linear over distances of one mile. Thus for small scale surveys a linear approach is used to correct all stations to the same latitude. Usually a particular station is picked as the reference to which all other stations are corrected. Here, the uncertainty in the surveying determines the error.

The Heaton Canyon survey, performed with a theodolite, stadia rod and tape measure extended over distances of 600 feet. Horizontal surveying uncertainties of 1.0 foot were estimated for the data points. Differentiating equation 2 with respect to latitude and then substituting 35 degrees for the latitude and 1.0 foot for the N-S horizontal uncertainty, a possible standard deviation of 0.000134 mGal is introduced with the latitude correction.

ΔG_{Fa} - FREE-AIR CORRECTION. - Elevation affects the magnitude of a gravity observation. Ignoring second and higher order effects, such as the changes in centrifugal acceleration associated with changes in elevation, the free-air correction is given by:

$$\Delta G_{Fa} = 0.09406 \text{ mGal/ft} \times \text{elevation.} \qquad (3)$$

Large scale surveys correct all readings to sea level. However, in small scale surveys some convenient elevation, such as highest, lowest or average elevation encountered, is often chosen for the datum. The major uncertainty involved with this correction is the elevation uncertainty.

For Heaton Canyon a standard deviation of 0.1 vertical foot was estimated which introduced a 0.009406 mGal error. However, in flat traverses some data point elevations were estimated by linear interpolation and thus could have a 0.5 foot error which could give a 0.047 mGal standard deviation. This could be another factor which contributed to the spurious gravity anomalies.

ΔG_{Bp} - BOUGUER PLATE CORRECTION. - When a datum elevation for the free-air correction is chosen the attraction or lack thereof due to the mass gain or loss associated within this elevation change should be considered. For example, in correcting to a lower elevation, the vertical component of the gravitational attraction of the mass between these two elevations must be subtracted. This is known as the Bouguer plate correction. This correction is approximated by a plate of infinite lateral extent whose thickness is equal to the elevation change. More explicitly it is given by:

$$\Delta G_{Bp} = 2\pi G \rho h, \tag{4}$$

where

G = gravitational constant,

ρ = density of the plate,

h = elevation change.

Thus the elevation uncertainty plays a part in determining the error associated with this correction.

Also the density used in the plate correction must be estimated. If a gravity profile is run across a non-structural high, then the density which graphs the smoothest Bouguer anomaly across this structure would be the optimal density to use (Vajk 1956, p.1010). However, this is time consuming and other techniques are usually employed. The rock types exposed can be identified and typical densities found in tables. The densities of exposed formations can be measured in the laboratory, or a density log can be run down a borehole. However, even then one observational point might be located on competent sandstone while another might be on alluvium or weathered sandstone. This density difference might vary by up to 0.5 gm/cm^3 in extreme cases. Because in a typical survey, a single density is usually used for all the Bouguer plate corrections the above density difference could result in an anomaly which under extreme topographic and geologic conditions could be mistaken for an underground void. It is advisable to keep the survey confined to a certain rock type wherever possible. Even then density variations in this surface plate are next to impossible to monitor, and its effects are ignored by using some "average" density for all the corrections. Thus the estimate of the total error should incorporate the deviation from average density along with the uncertainty in elevation.

The standard deviation of the Heaton Canyon error for the Bouguer plate correction is 0.0034 mGal and is based only on the uncertainty in elevation. The density used was 2.0 gm/cm^3, and density variations were ignored in this estimate. Thus this error represents a minimum error.

ΔG_T - TERRAIN CORRECTIONS. - Terrain corrections contribute the most uncertainty to the data and are dependent upon both density and elevation. Terrain corrections eliminate the anomaly produced by lateral changes in elevation or topography. Different stations are affected to different degrees, and therefore each observation point is corrected individually. In essence mass in mountains above a station elevation has an upward vertical gravity component and a deficiency of mass below the station in valleys lacks a downward component. Therefore terrain corrections for both hills and valleys are added to the station reading.

Only the Hammer method (Hammer 1939, p.184-194) for terrain corrections will be considered because it is the most popular and also the most accurate. Here again the uncertainty as to which density should be used leads to the use of an "average" density for all terrain corrections. Density errors affect the corrections directly and thus small errors in density due to extreme topographic and geologic conditions could affect the overall accuracy of the terrain corrections (Jackson et al., 1983, p.211).

Terrain corrections are very time consuming, and the correction for a single station may take a few hours. An overlay consisting of concentric circles centered on the observation station divides the surrounding area into 12 zones which are further subdivided into compartments. The B zone extends from 6.5 to 54.6 feet radially and is divided into four compartments. The M zone extends from 48 365 to 71 996 feet and is divided into 16 compartments. For each compartment the average elevation has to be estimated in some manner and the difference from the station elevation calculated. This elevation difference in conjunction with a terrain correction table allows the terrain correction to be determined for each compartment. The corrections are summed over all compartments for all zones. The accuracy of the terrain corrections depends mainly on the density used, estimation of the average elevation for each compartment, accuracy of the topographic maps used (because of the range in the radial extent of the zones, maps with various scales are frequently used) and precision of the human element because the process is laborious and time consuming.

The uncertainty associated with the terrain corrections can only be some best estimate because of the numerous factors involved. Some general comments can be made however. Terrain corrections in gentle topography can be expected to be more accurate than in rugged terrain. The best that probably can be approached due to inherent procedures in the corrections is 0.02 mGal (Hammer 1939, p.199). Chapman (1983, p.175) has reported that in very rough topography the error could be as high as 0.10 mGal.

The sedimentary facies in Heaton Canyon have a measured density of approximately 2.35 gm/cm^3 with the exception of coal seams which have a density of 1.3 gm/cm^3 . The alluvial cover has a density of about 1.8 gm/cm^3. Thus 2.0 gm/cm^3 was a reasonable density to use for terrain corrections. While calculating terrain corrections it was noticed that the terrain corrections for zone K were small, contributing only 3.2 percent to the total correction. Some rough calculations for selected compartments in zone L showed a further decrease. Also the effect of distant terrain on a gravity survey of small lateral dimensions should be approximately the same for all stations. From this information it was decided to only calculate terrain corrections through zone K to a distance of 32 490 feet.

Terrain corrections were obtained for every fifth station along Heaton Canyon gravity line HC-G-4. The position of the line is shown in Map 1 and a graph of the terrain corrections is shown in Figure 2. If the terrain correction values for any two stations spaced 100 feet apart on line HC-G-4 are connected by a straight line, then the maximum vertical difference between this line and the bracketed terrain correction occurs for station 8+200 (Figure 2). The difference between the straight line connecting stations 8+150 and 8+250 and station 8+200 is approximately 0.04 mGal. Because the difference is small, terrain corrections were calculated only at 100 foot intervals, and the corrections for the remaining stations were obtained by interpolating between these stations. From this technique, a probable standard deviation of 0.04 mGal (Johnpeer et al., 1985, in press) was chosen as the uncertainty for the terrain corrections.

OTHER CORRECTIONS

Regional trends with wavelengths greater than those of interest can be removed or filtered by various techniques to find the residual anomaly. The error associated with this process can vary depending on the

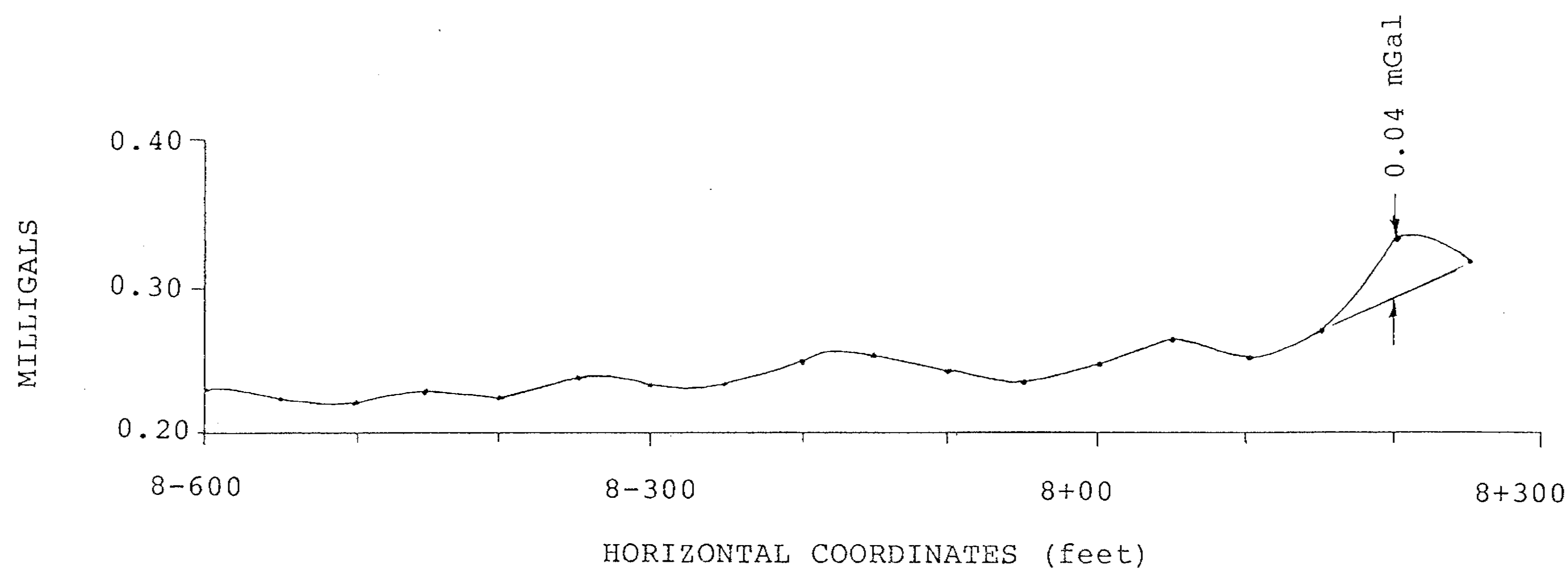

FIGURE 2. Technique used to determine maximum error associated with terrain corrections. The maximum difference between the straight line connecting any two stations separated by 100 feet and the encompassed station is 0.04 mGal. Thus 0.04 mGal was chosen as the error associated with terrain corrections.

wavelength, magnitude and technique employed. Here again a linear approach versus a polynomial curve can introduce uncertainties. For removing long wavelength anomalies with a linear technique one must make certain that the survey is extended into the non-anomalous regions. This insures that the regional anomaly can be deleted by extrapolating it back into the zone of interest.

TOTAL ERROR

Some sort of estimate of the total error is necessary if the limits of resolution of the gravity survey are to be defined. Because shallow mine tunnels should display small magnitude anomalies of relatively short wavelength, distinguishing them from possible errors in the data is of importance. As discussed above many factors affect the error, and obtaining a good estimate on the total error may not be easy. An analytical approach in determining the errors for certain corrections was demonstrated with data from the Heaton Canyon study. For other corrections such as the terrain correction only a best estimate could be attempted. Another possible approach is to consider the scatter in the reduced data. Here it is implied that the numerous small amplitude short wavelength anomalies are due to errors in the data. Many such anomalies have been deleted with a smooth continuous curve drawn through the data points. The analytical estimates of error can be compared to the small scale scatter in the profile. This comparison would lend confidence to the error estimate.

Another method for obtaining a better estimate on the errors, assuming a Gaussian distribution, is to increase the number of data points per unit length of the profile. This would imply that a greater number of data points would lie closer to the mean. Also because the probability of a data point lying above the mean should be equal to it lying below the mean, the wavelength of the errors should decrease. Thus it should be easier to delete the errors, but time and funding usually limit this approach.

Some reported standard deviations associated with reduced data are 0.1 mGal (Telford et al., 1976, p.48), 0.13 mGal (Johnpeer et al., 1985, in press) and 0.08 mGal (Richard et al., 1984, p.1786). However, it should be pointed out that the error associated with the reduced data is dependent very much on the individual nuances of each survey and thus should be determined separately for each survey.

DETECTION OF ANOMALIES

MAGNITUDE

The more the magnitude of the anomaly exceeds the estimate of the total error the easier it is to delineate the anomaly. If the anomaly magnitude is less than twice an error of one standard deviation, then a straight line will satisfy the data points because it intersects all the error bars (Figure 3). Thus to insure detection the anomaly should be at least twice the estimate of the total error. Hammer (1945, p.36) states that for an instrumental error of about 0.02 mGal the smallest reliably observable gravity anomaly should have a magnitude of 0.05 mGal.

SPACING

The station density must be high enough to allow some stations to be located near the maximum of the anomaly. By setting the station spacing equal to one-half the half-width of the anomaly, the number of stations falling within an interval where the anomaly magnitude is greater than one-half the maximum is two. With this station spacing, only 3/4 maximum amplitude of an anomaly can be guaranteed to be registered by any one data point. By increasing the density of the station spacing, some stations will be situated still closer to the anomaly maximum, but here again time and funding will limit this approach.

The spacing of the underground voids is also important. As their separation distance decreases their respective anomalies tend to merge into one anomaly (Figure 4). Depth is also a consideration here as shown below.

DEPTH

The anomalies generated by a tunnel gradually decrease in magnitude and broaden out with an increase in depth, all other factors being equal (Figure 5). Thus given an expected size for the tunnel, the maximum depth

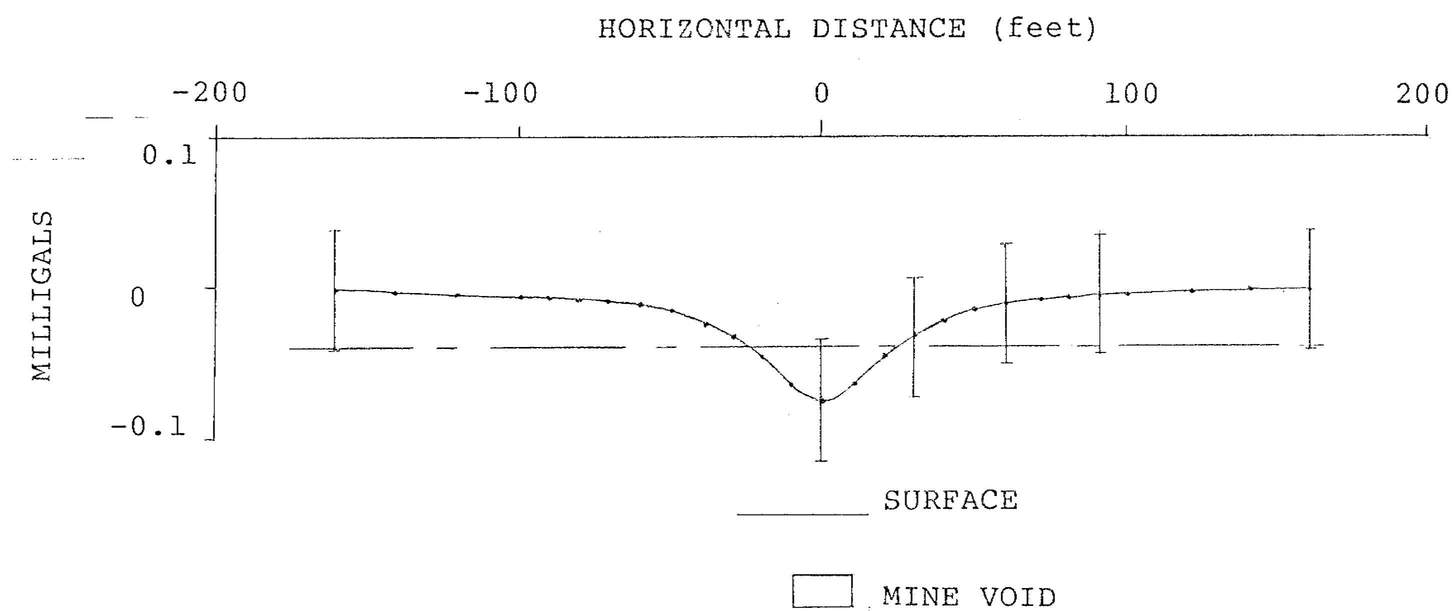

FIGURE 3. Non-detectability of an anomaly illustrated by a straight line intersecting all the error bars. Above anomaly was generated by a 10x20 foot tunnel of infinite strike length, at a depth of 20 feet and having a density contrast of -2.35 gm/cm^3. The maximum amplitude is -0.074 mGal and the uncertainty is +0.04 mGal. Thus the anomaly magnitude should be at least twice the error to insure detection of anomalies.

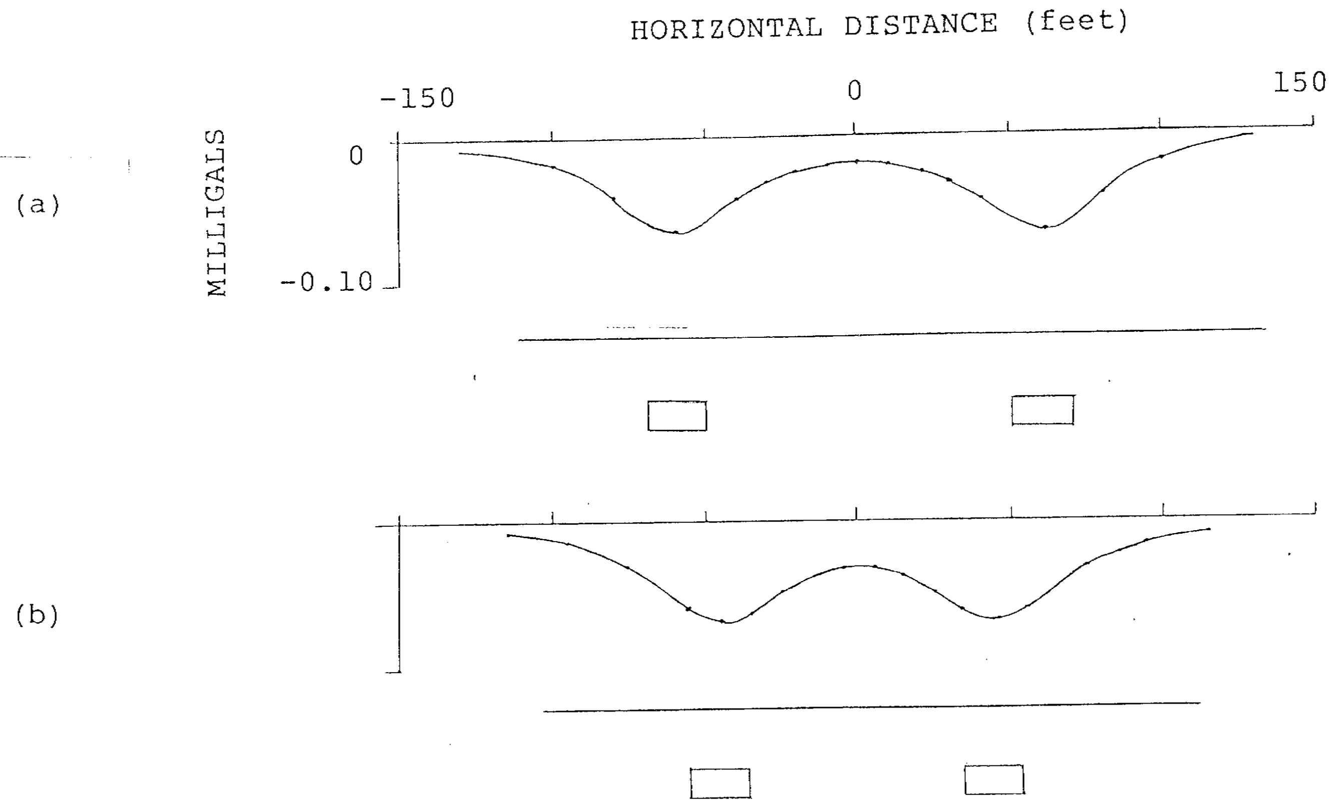

FIGURE 4. Variation in shape of gravity anomaly with change in distance between two mine voids. Voids are 10x20 feet in cross-section and infinitly long. Density contrast is -2.0 gm/cm^3. Tops of the voids are 20 feet beneath the surface. Separation is (a) 100 feet (b) 70 feet (c) 50 feet (d) 30 feet (e) 20 feet (f) 10 feet.

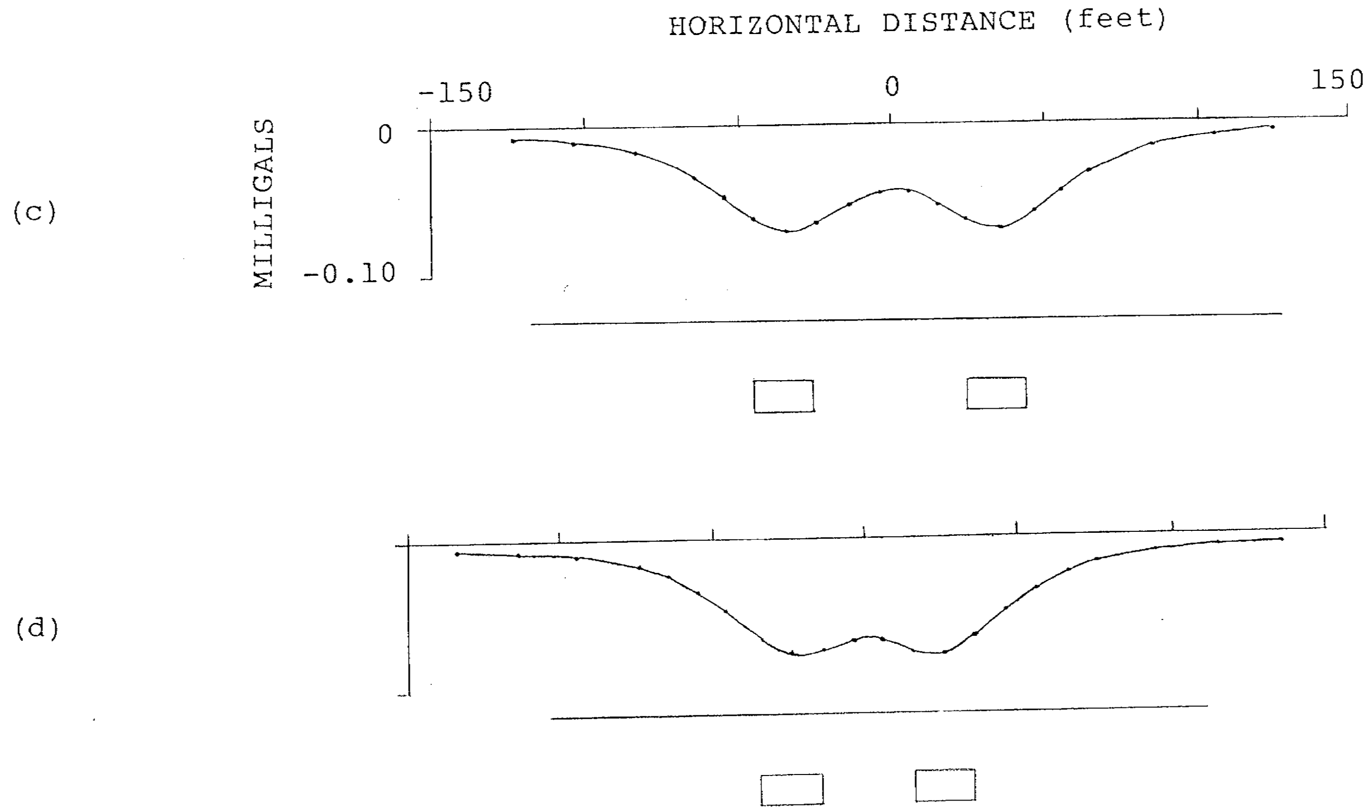

Figure 4. Continued

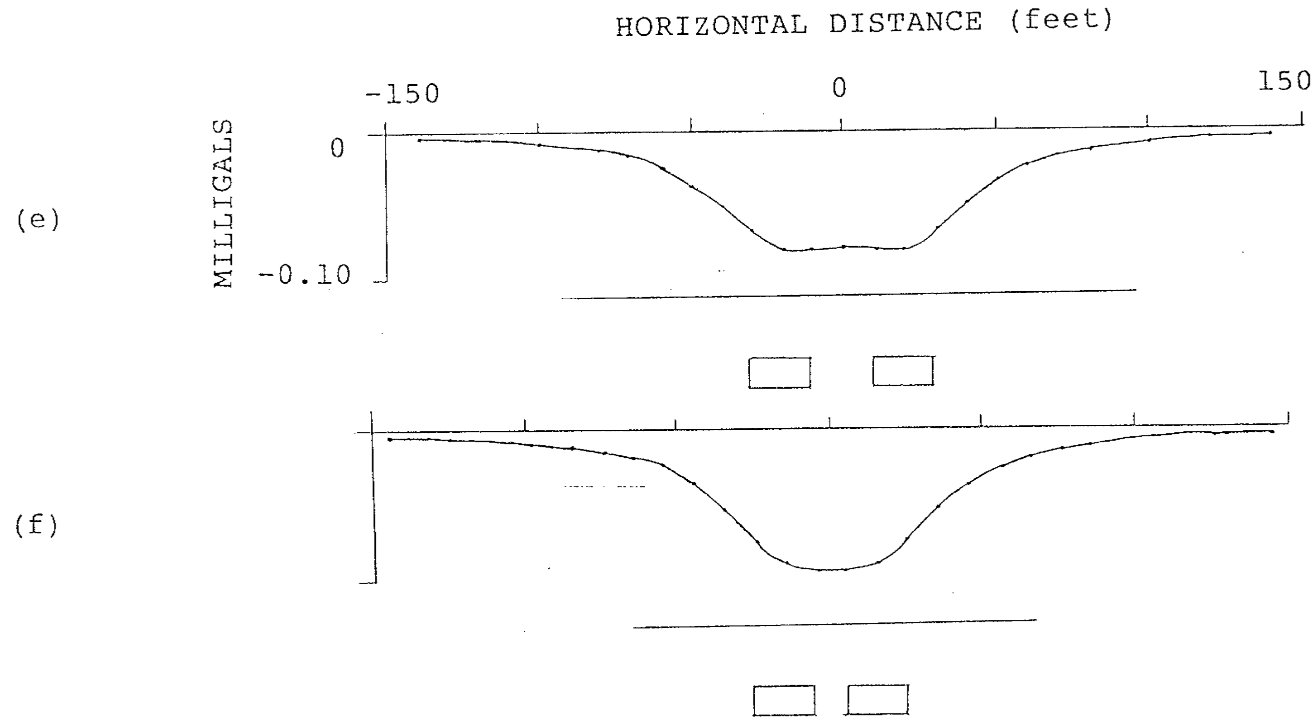

Figure 4. Continued

(a)

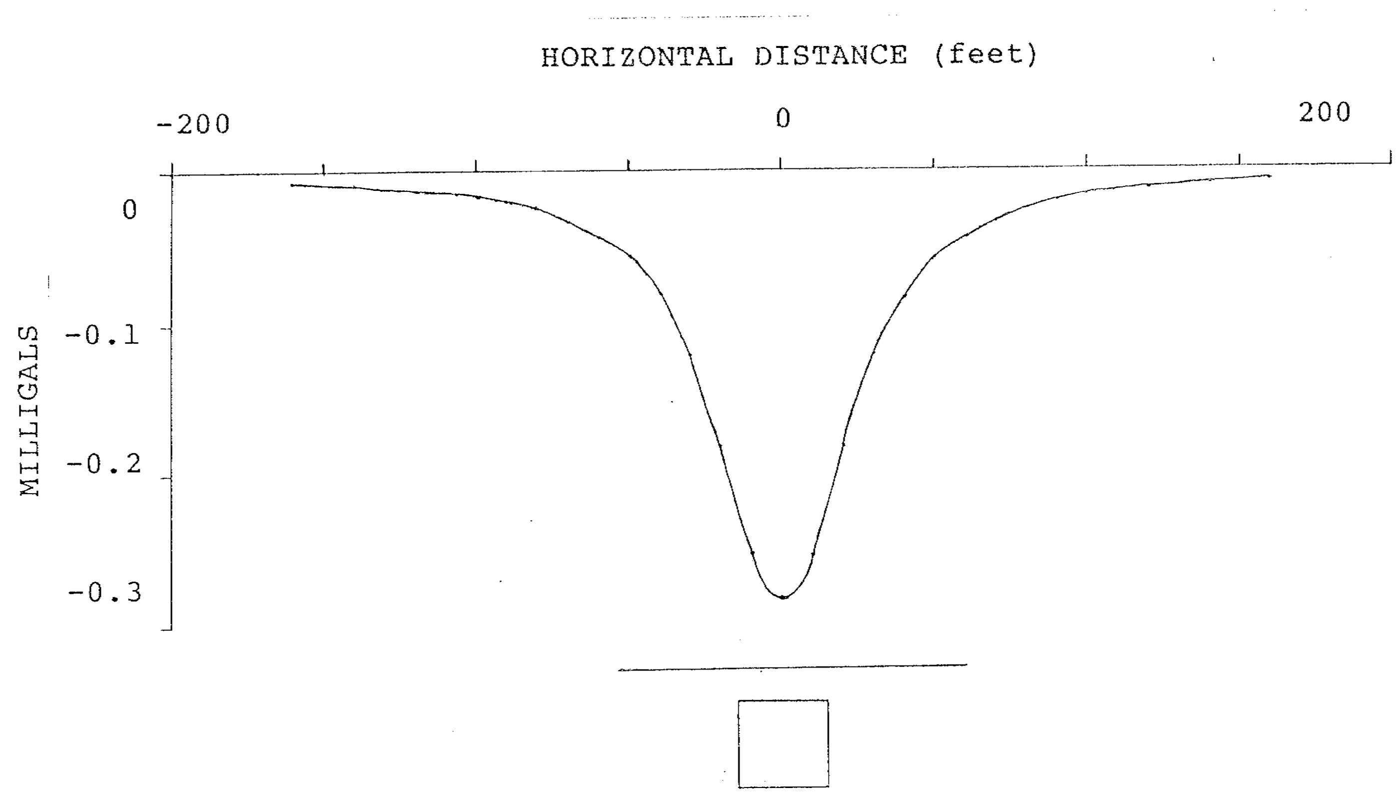

FIGURE 5. Variation in magnitude of gravity anomaly with depth of mine void. Voids are 30x30 feet in cross-section and infinitly long. Density contrast is - 2.0 gm/cm^3. Depth to top of mine void is (a) 10 feet (b) 50 feet (c) 130 feet.

(b)

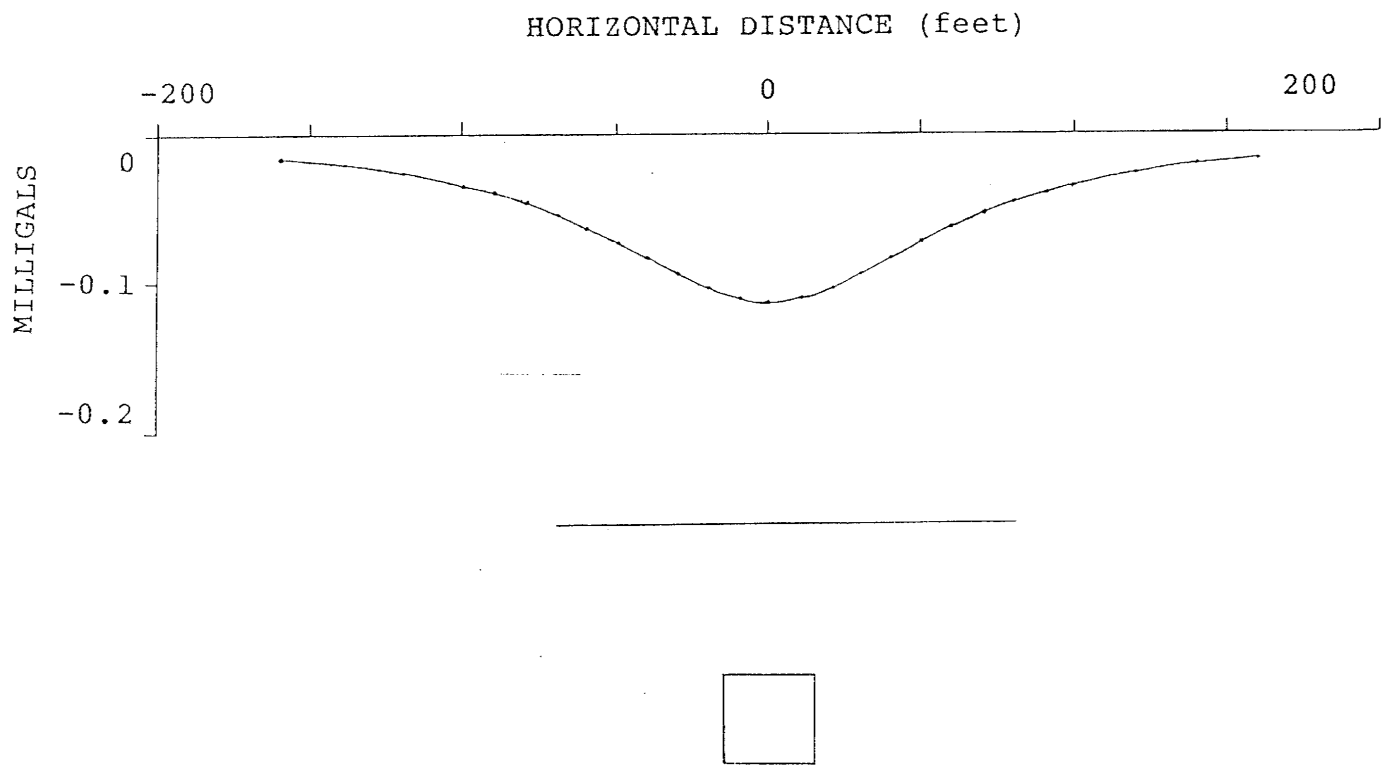

FIGURE 5. Continued

(c)

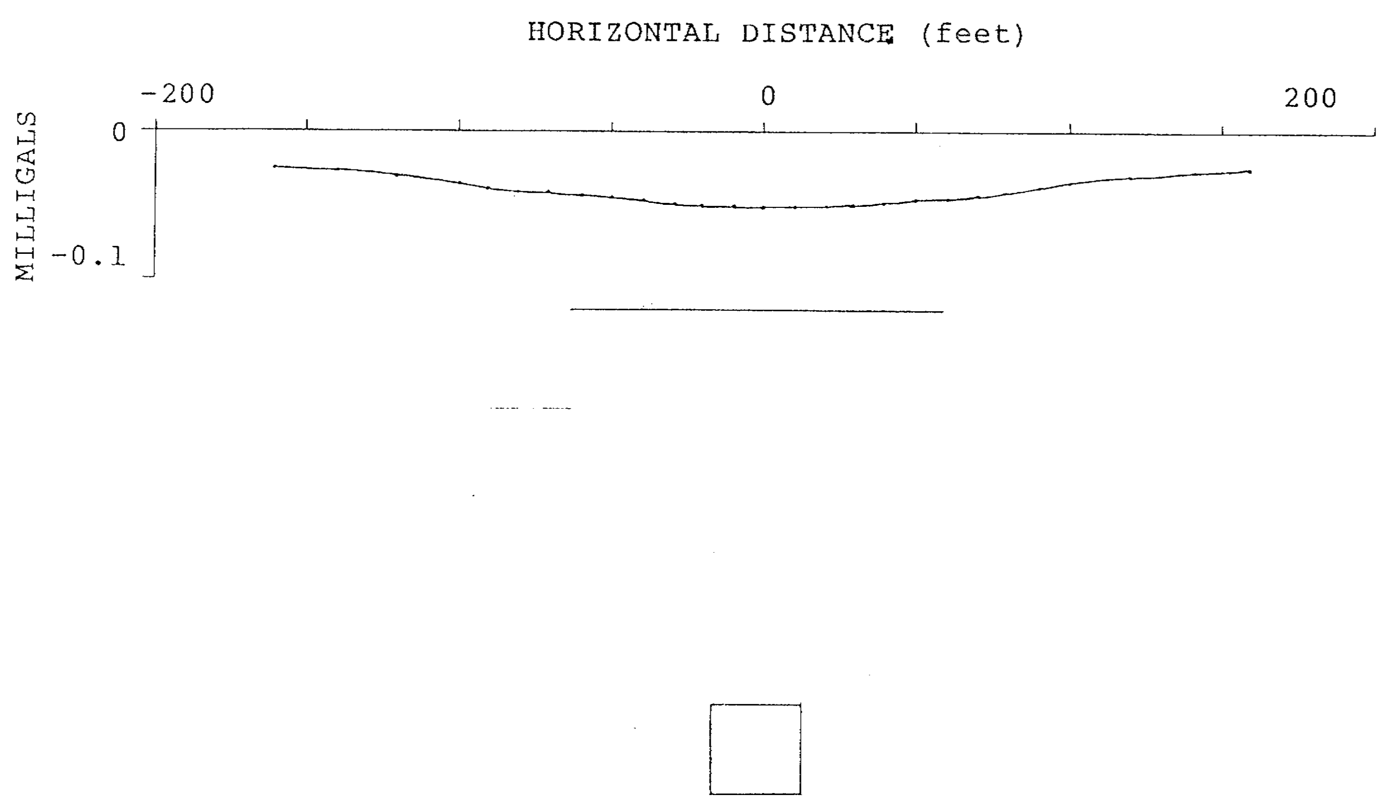

FIGURE 5. Continued

at which it can be detected is limited by the error associated with the reduced data. Also, as the depth for two or more adjacent tunnels increases their anomalies broaden out and merge into one indistinguishable anomaly (Figure 6).

APPROXIMATIONS

Theoretical gravity profiles across mine voids can be easily calculated, if tunnels can be approximated by horizontal cylinders. Various cross-sectional shapes can be approximated by horizontal cylinders as shown in Figure 7. The formula giving the anomaly produced by a horizontal cylinder of infinite strike length is given by (Telford et al., 1976, p.61):

$$g = (2\pi G \sigma R^2)/[Z(1+X^2/Z^2)], \tag{5}$$

where

g = Gravity anomaly,

G = Gravitational constant,

σ = Density contrast,

R = Radius of cylinder,

Z = Depth to center of cylinder,

X = Horizontal coordinate system centered above the cylinder.

This formula allows for easy and fast calculations of anomalies. Also the half-width distance is equal to Z which can be used to set the spacing for the data points. However, one should be careful when using this formula for an

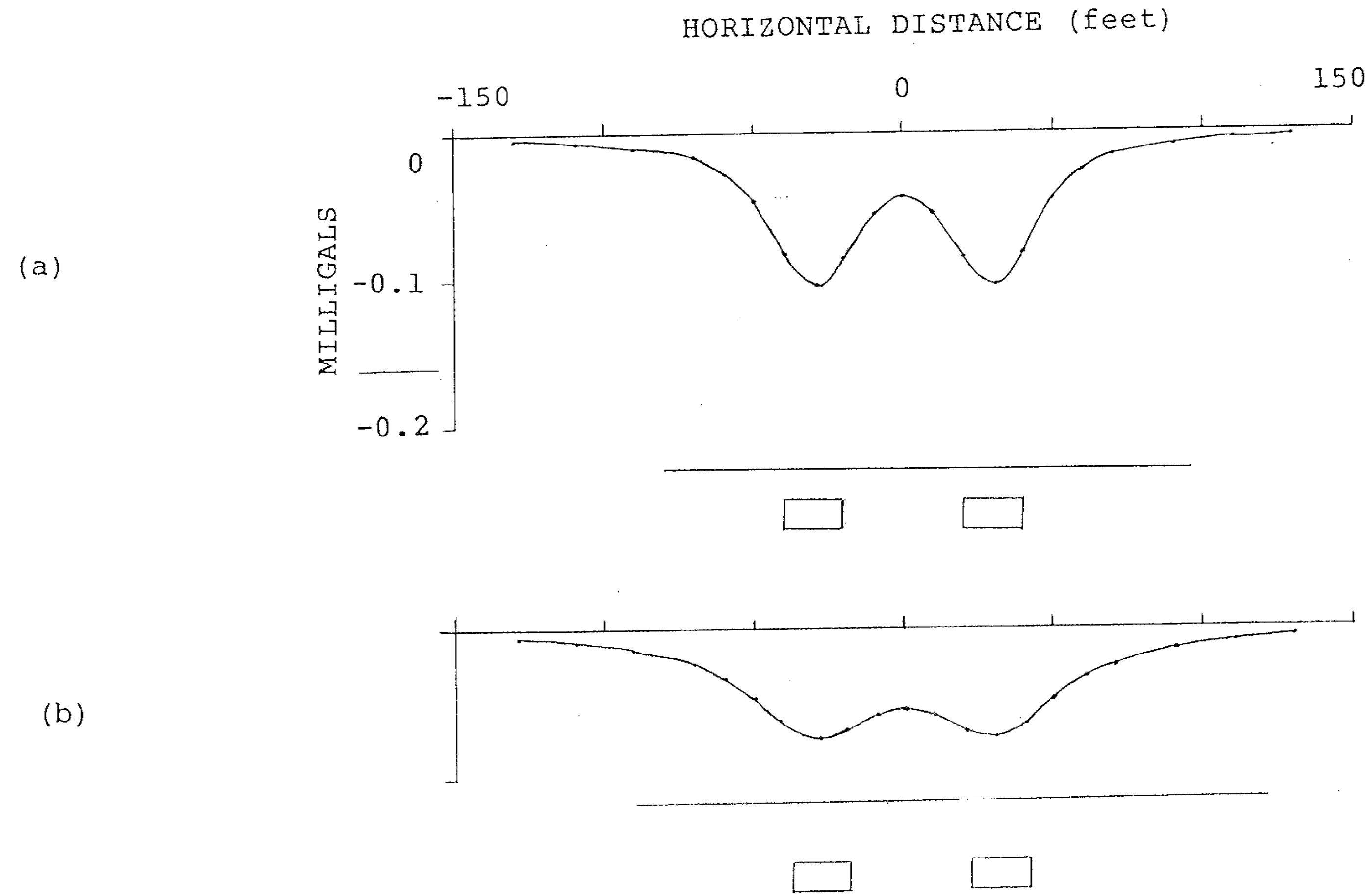

FIGURE 6. Variation of anomaly with depth of two mine voids. Voids are 10x20 feet and separated by 40 feet. Density contrast is -2.0 gm/cm^3. Depth to top of voids is (a) 10 feet (b) 20 feet (c) 30 feet (d) 70 feet.

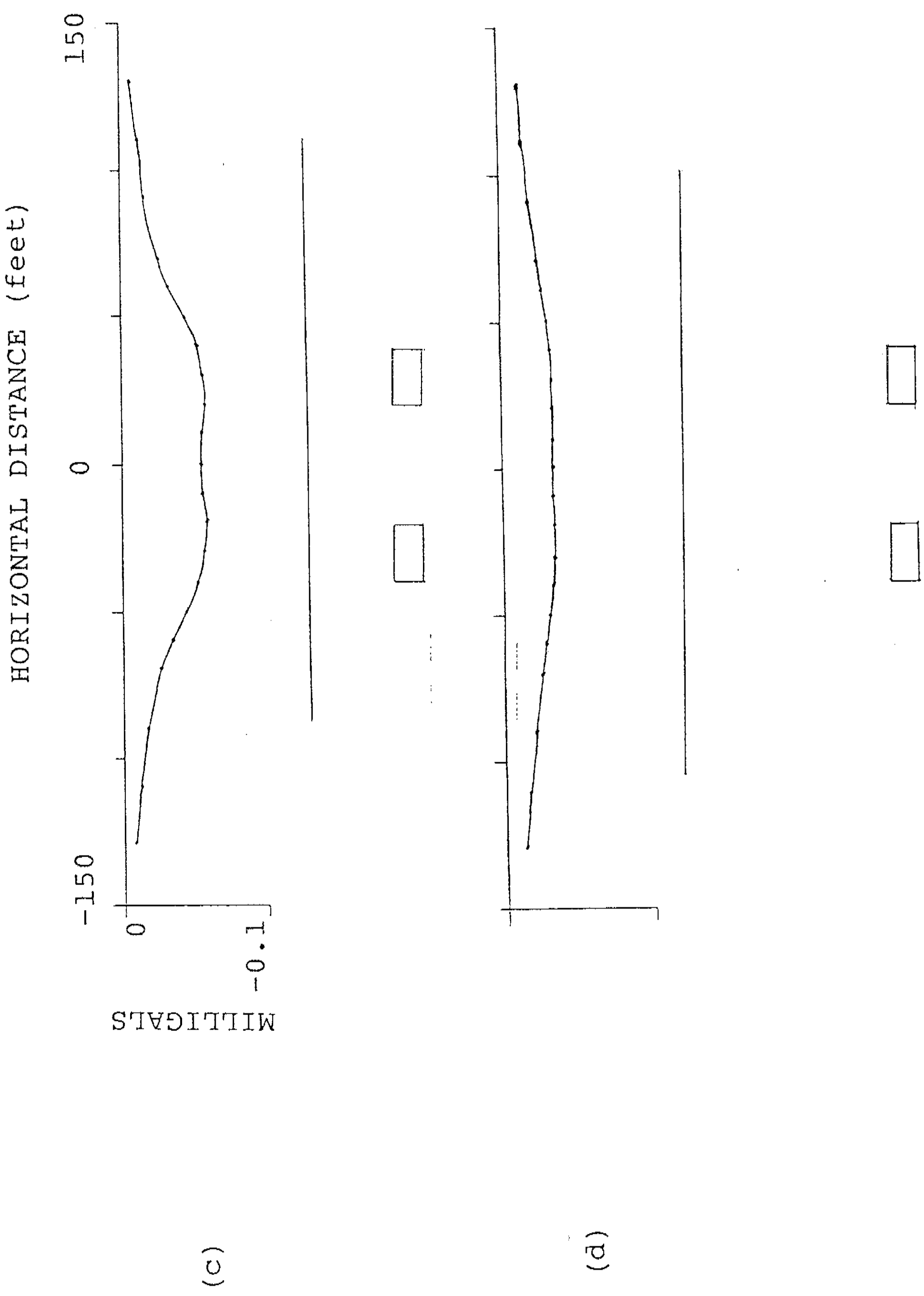

FIGURE 6. Continued

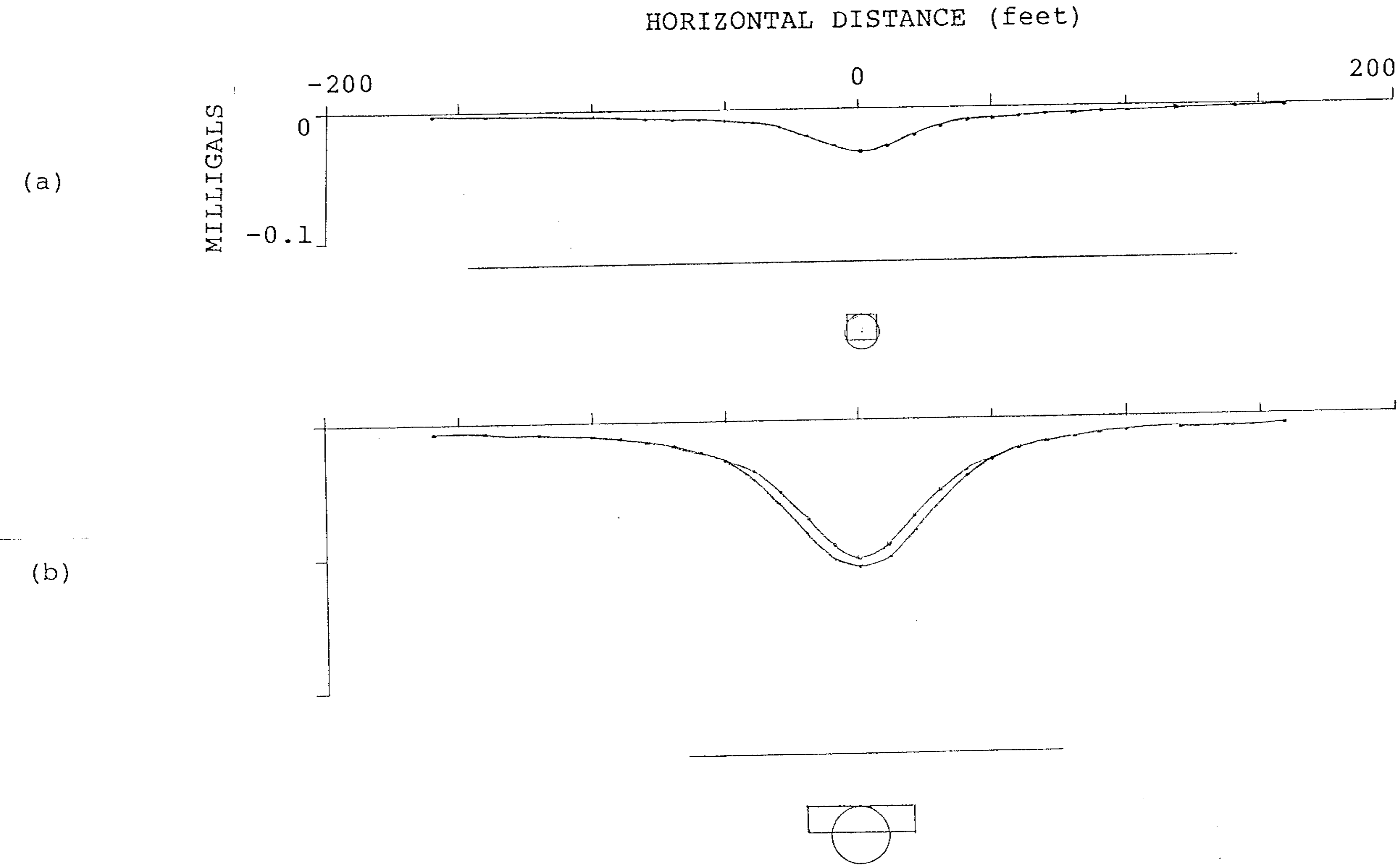

FIGURE 7. Comparison of anomalies produced by voids of rectangular cross-section with voids of circular cross-section. Voids are at a depth of 20 feet, infinitly long and have the same cross-sectional area. Density contrast is -2.0 gm/cm^3.
(a) 10x10 foot rectangle versus a 5.64 foot radius circle.
(b) 10x40 foot rectangle versus a 11.28 foot radius circle.
(c) 30x10 foot rectangle versus a 9.77 foot radius circle.
(d) 40x10 foot rectangle versus a 11.28 foot radius circle.

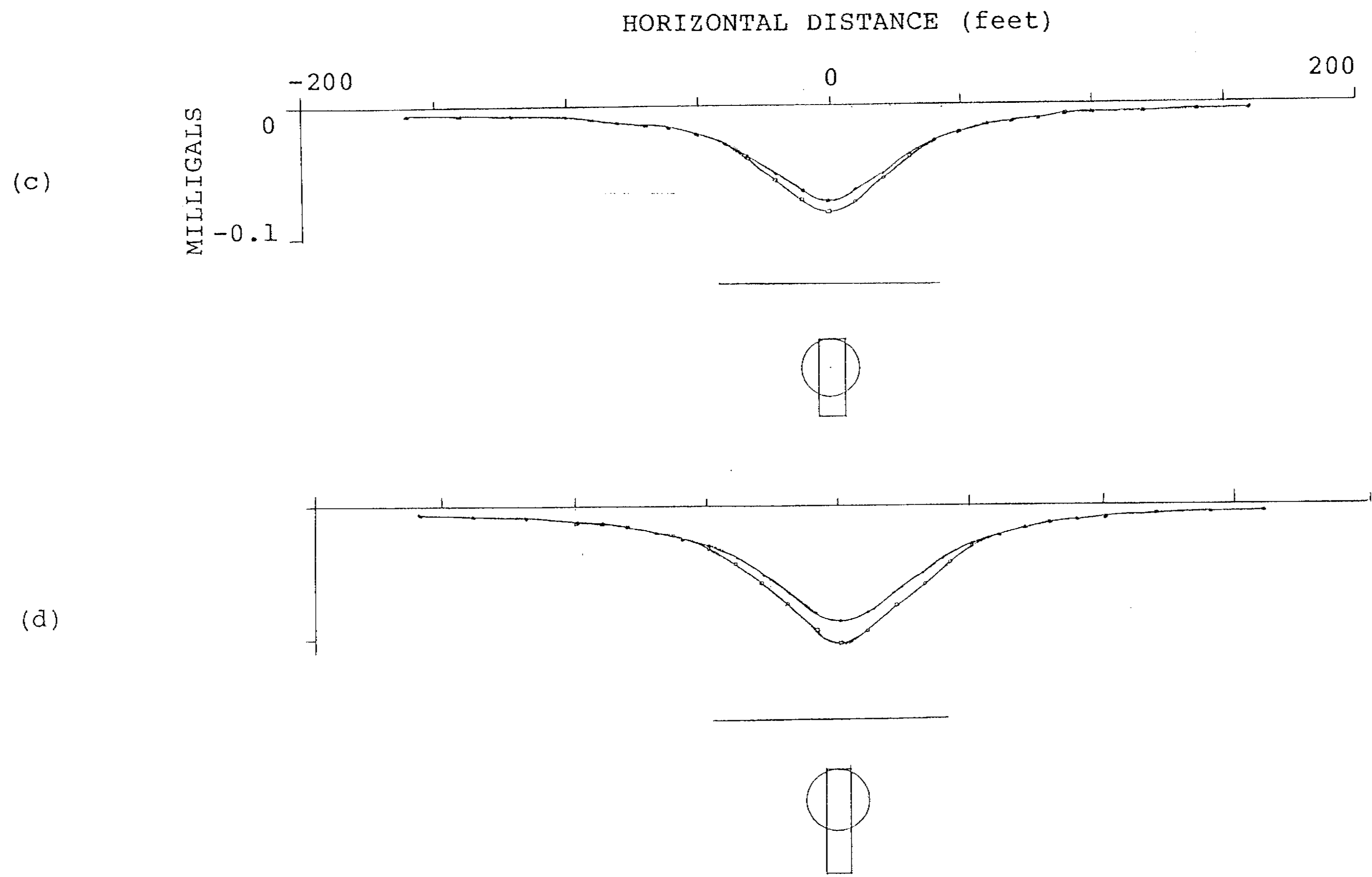

FIGURE 7. Continued

approximation because it deteriorates as the cross-sectional shapes elongate in one dimension (Figures 8 and 9).

DISCUSSION

OTHER DATA

Auxiliary data sources can constrain parameters and thus be of assistance in modeling the structure and/or in generating anomalies from models. Drill holes are useful for obtaining lithologic depths, and thicknesses and for running density logs. Seismic refraction surveys can constrain depths to various layers. Magnetic surveys could be of use in correlating old mine maps with the surface topography by locating buried magnetic objects (water pipes) shown on the maps. Regional gravity trends may sometimes be obtained from larger scale maps. Old reports and mine maps can constrain the depth, height, width and location on mine voids.

DETECTION FEASIBILITY

An estimate on the error associated with the reduced data along with an estimate on the maximum anomaly magnitude needs to be made to be able to predict the detectability of a mine void. If the expected anomaly magnitude is greater than two times the expected error, then the mine void anomaly probably can be identified. If constraints on the mine parameters can be found from various sources, then an expected anomaly can be modeled. The confidence in this anomaly would be proportional to the

(a)

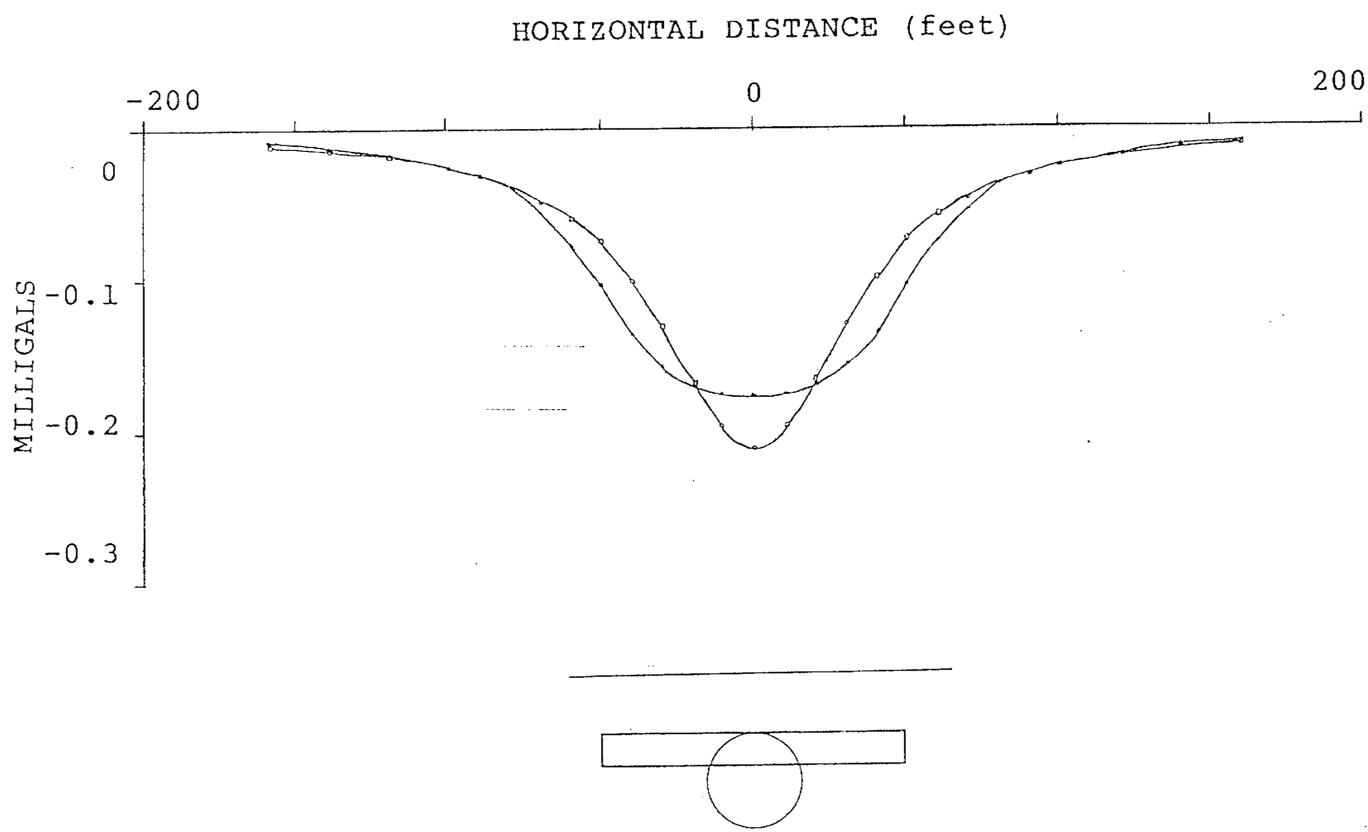

FIGURE 8 Comparison of anomalies due to rectanglar with circular cross-sectional shapes of equal area. Voids are at a depth of 20 feet and infinitly long. Density contrast is -2 0 gm/cm^3. Note poor approximations.
(a) 10x100 foot rectangle versus a 17 84 foot radius circle.
(b) 100x10 foot rectangle versus a 17 84 foot radius circle.

(b)

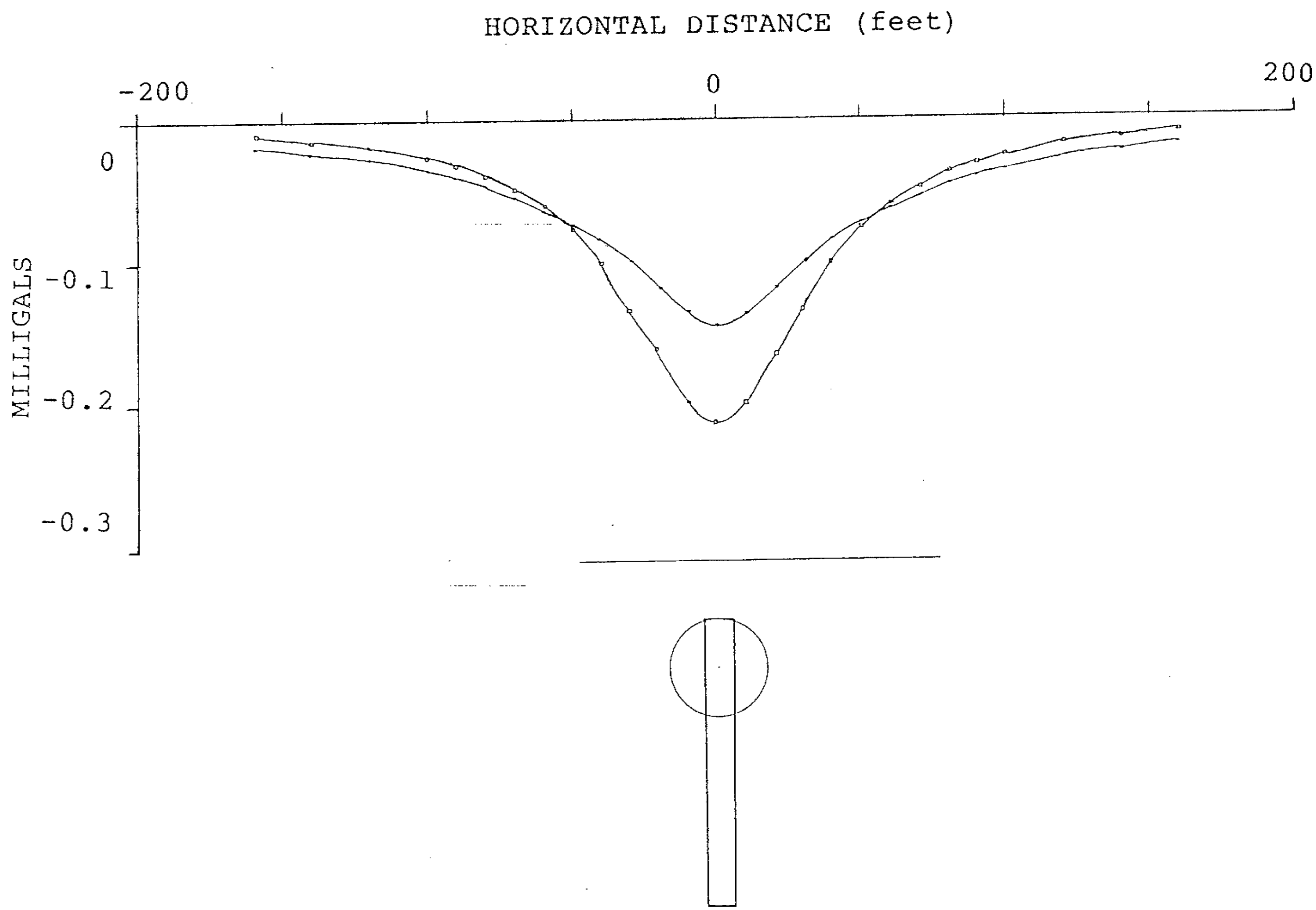

FIGURE 8. Continued

(c)

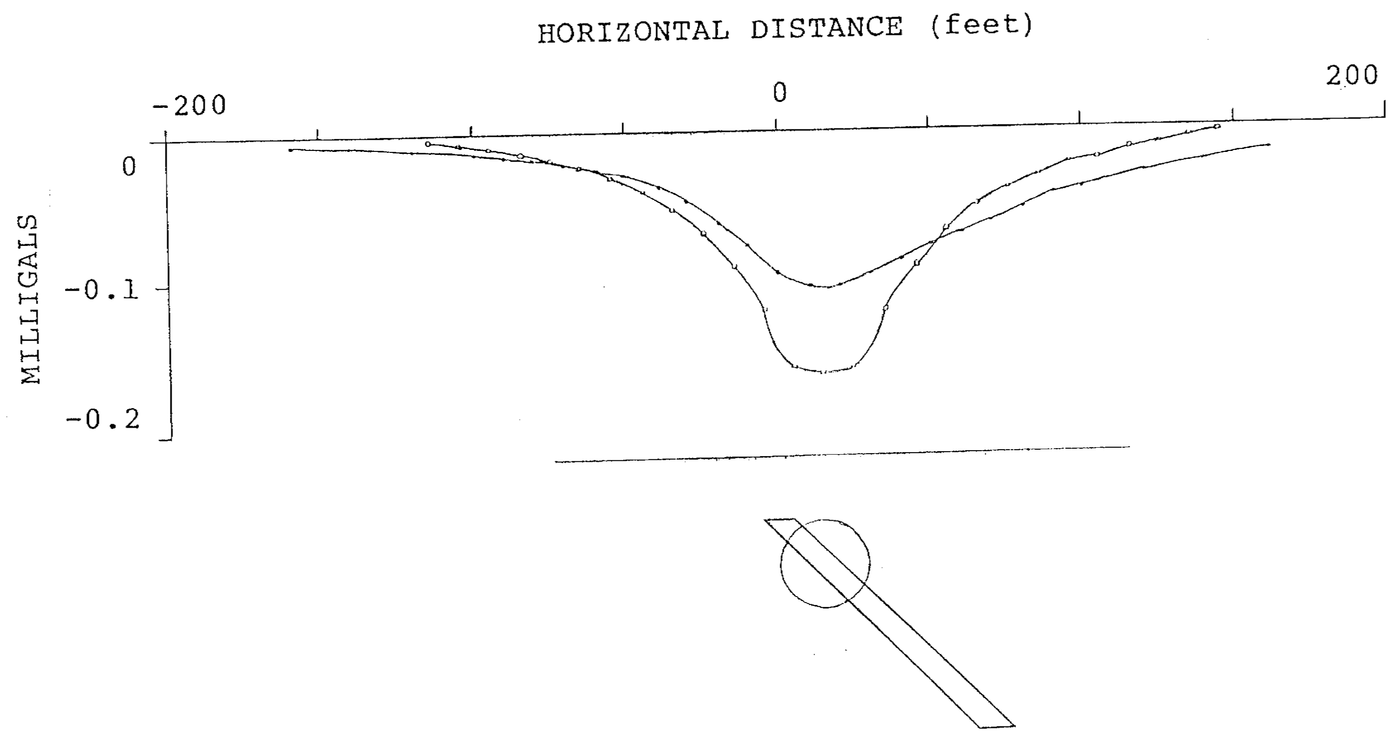

FIGURE 9. Comparison of anomaly due to a parallelogram with one of circular cross-sectional area. Voids are at a depth of 20 feet and infinitly long Density contrast is -2 0 gm/cm^3. Parallelogram is 100x7 feet (high) and dips at 45 degrees. The circle has a radius of 14 92 feet. Both figures have equal cross-sectional areas.

confidence in the parameters. The estimate for the expected error is dependent on many factors as shown in the above discussion and could even be indeterminable. Reference to errors obtained in other reports could be of help. Another factor to consider is the spacing between two or more adjacent tunnels. Here again modeling may foretell if they are separable.

Cross-sectional shapes of approximately equal dimensions may be modeled as horizontal cylinders of equal area for first order approximations. Given a maximum amplitude for an anomaly produced by a horizontal cylinder, the depths versus the radii which would generate an anomaly with this specific amplitude can be plotted. Figure 10 (Colley 1963, p.3) is such a plot for various anomaly magnitudes for a density contrast of 2.0 gm/cm^3 . For example, if 0.06 mGal was determined to be an expected error, then only an anomaly greater then 0.12 mGal could be detected. Further, if the mine tunnel dimensions were constrained to 10 x 20 feet, then a horizontal cylinder with a 7.97 foot radius would produce the same cross-sectional area. Then using the 0.12 mGal curve on Figure 25 and a radius of eight feet, the depth to the top of this tunnel should be less than 5.5 feet if this tunnel was to be located.

Horizontal cylinders employed as simple mine models may also be used to give an indication of when

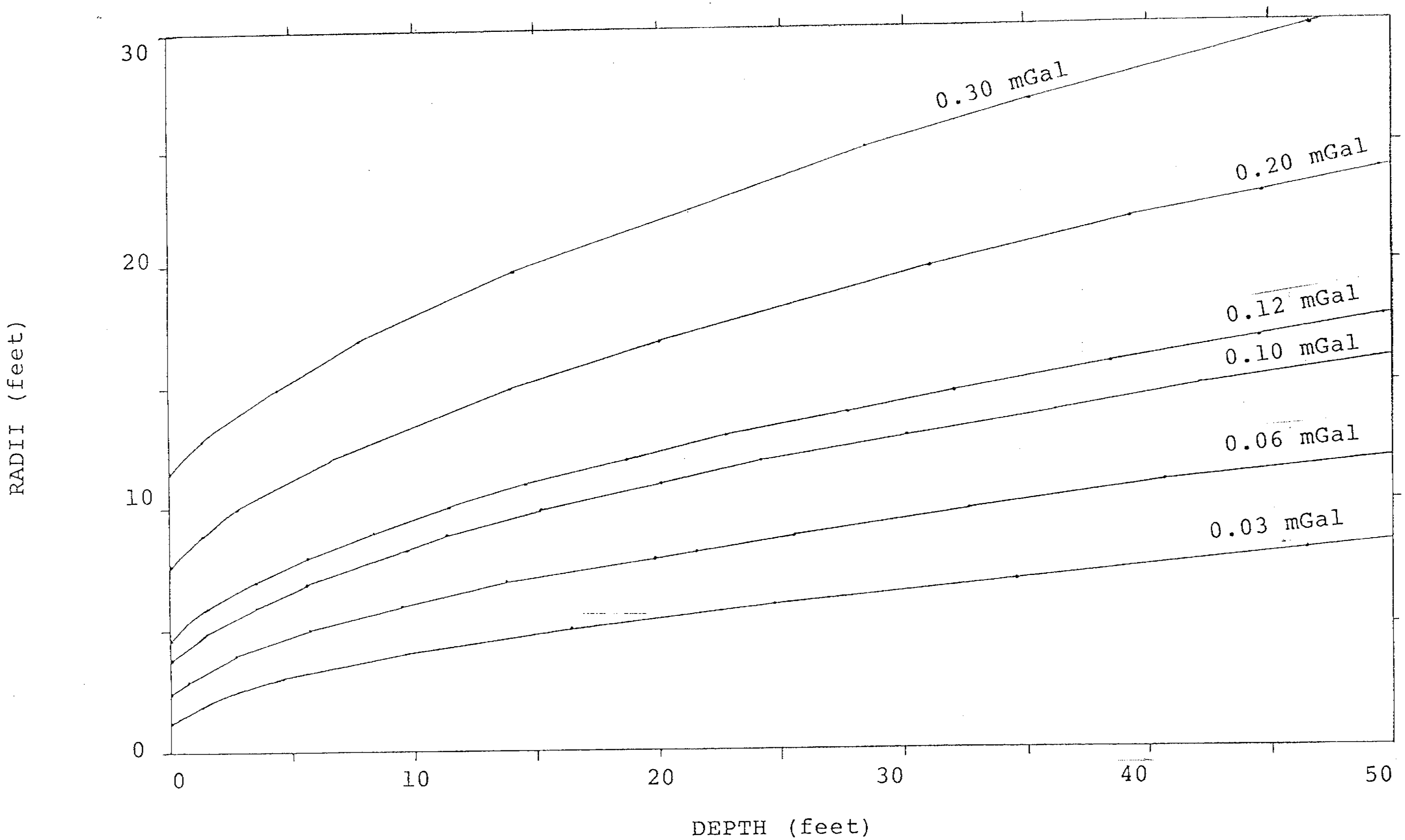

FIGURE 10. Each curve plots depths versus radii which generate a given maximum anomaly. A horizontal cylinder infinitly long having a density contrast of -2.0 gm/cm^3 was used to calculate the curves. The depth is measured to the top of the cylinder. Thus given an estimated error and the dimensions of a mine void, the maximum depth at which it can be detected can be read off the graph.

two adjacent tunnels are too close to be separated by a gravity survey. Consider two horizontal cylinders of equal cross-sectional areas emplaced at a depth of 30 feet. If these two cylinders were separated by 30 feet center to center, then the surface anomaly at the midpoint between their centers would be greater than the maximum magnitude for either one. Also the anomaly above the center of either cylinder would be 1.5 times the maximum anomaly for one cylinder. Thus it should be suspected that the anomalies from these two tunnels may not be separable and further modeling with specific values given to the parameters should be done to clear up this ambiguity.

HEATON CANYON EXAMPLES

MINE MODELS. - The Bouguer anomalies for Heaton Canyon profile HC-G-5 are portrayed in Figure 11a. This profile traverses Heaton Canyon which is buried by an alluvial cover, as evidenced by exposures in the canyon and by drill hole data. The profile is not corrected for the alluvial fill because the survey was not extended a sufficient distance away from the valley in either direction. Also if shallow mines were detected, it was assumed that their shorter wavelength anomaly could be differentiated from a longer wavelength anomaly caused by the alluvial filled valley. This assumption proved to be incorrect due to the close spacing of the mines.

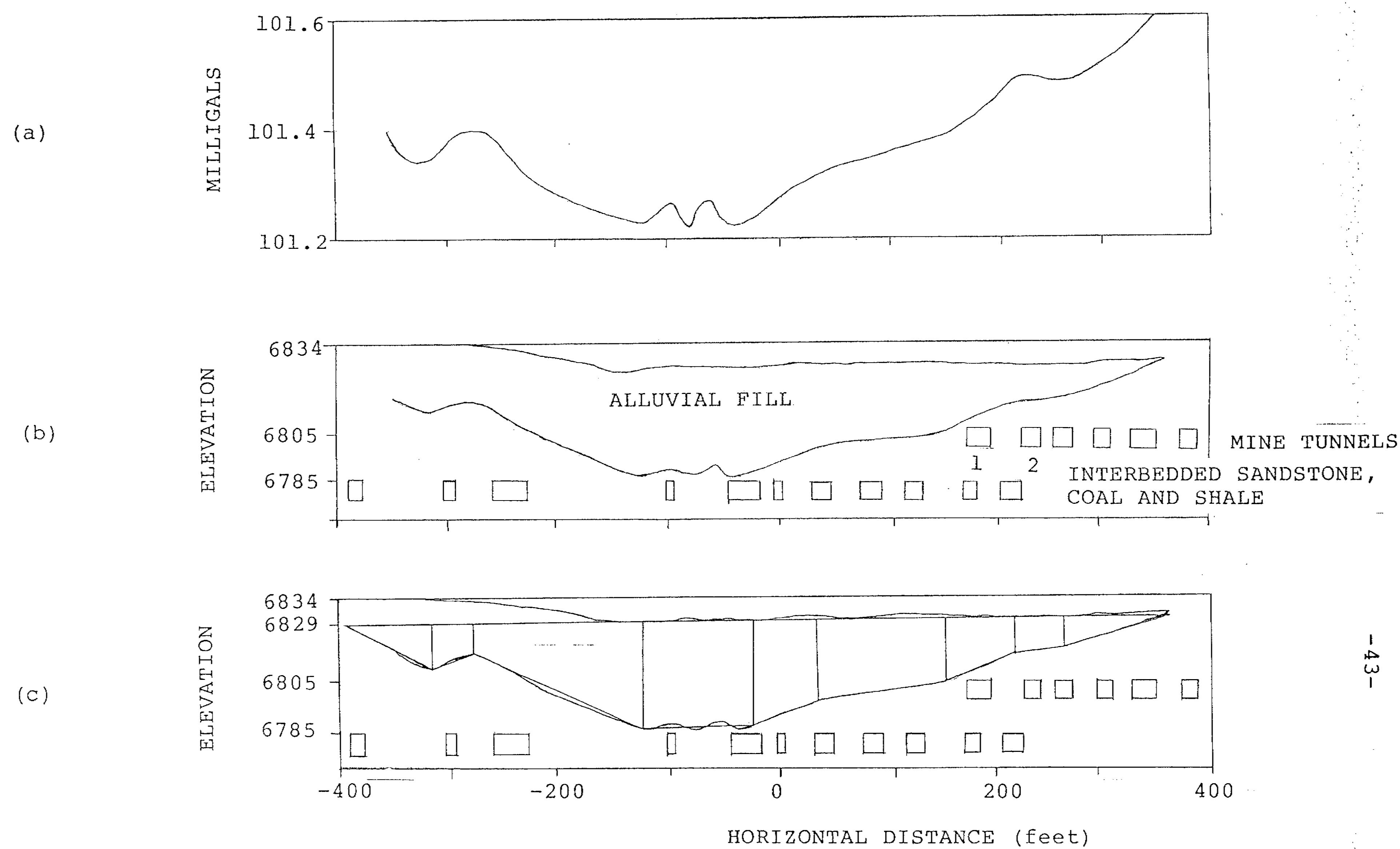

FIGURE 11. Heaton Canyon profile HC-G-5.
(a) Bouguer anomaly
(b) Subsurface interpretation
(c) Model used for calculating Bouguer anomalies. Alluvial fill density contrast is -0.2 gm/cm^3 and the mine void density contrast is -2.0 gm/cm^3.

Figure 11b is a subsurface interpretation of this profile. Drill hole data, seismic data and the Bouguer anomalies were used in compiling the alluvial thicknesses across the valley. Information regarding the placement of the subsurface mines was extracted from drill hole data and old mine maps. A drilling success rate of 70 percent in intersecting mines by using this map lends confidence to the subsurface mine interpretation.

Figure 11c represents the model used to generate gravity anomalies. The alluvium is modeled by nine blocks of infinite strike length with a $-0.2\ gm/cm^3$ density contrast. The upper level mine tunnels are modeled as having infinite strike length, as being at a depth of 24 feet to their tops, as having heights of 10 feet, as having widths scaled from the mine map and as having a density contrast of $-2.0\ gm/cm^3$. The lower level tunnels have similar parameters except for their depth which is 42 feet. The 6 829 foot elevation was used as the base line for which the gravity anomalies were calculated.

The anomaly calculated by the S.K.W. computer program for all the upper level mines (Figure 12a) does not allow individual mines to be differentiated. The group as a whole is below the 0.26 mGal threshold (twice the estimated error). Thus the upper level mines as a group probably could not be detected. The same also applies to the lower level mines (Figure 12b). The

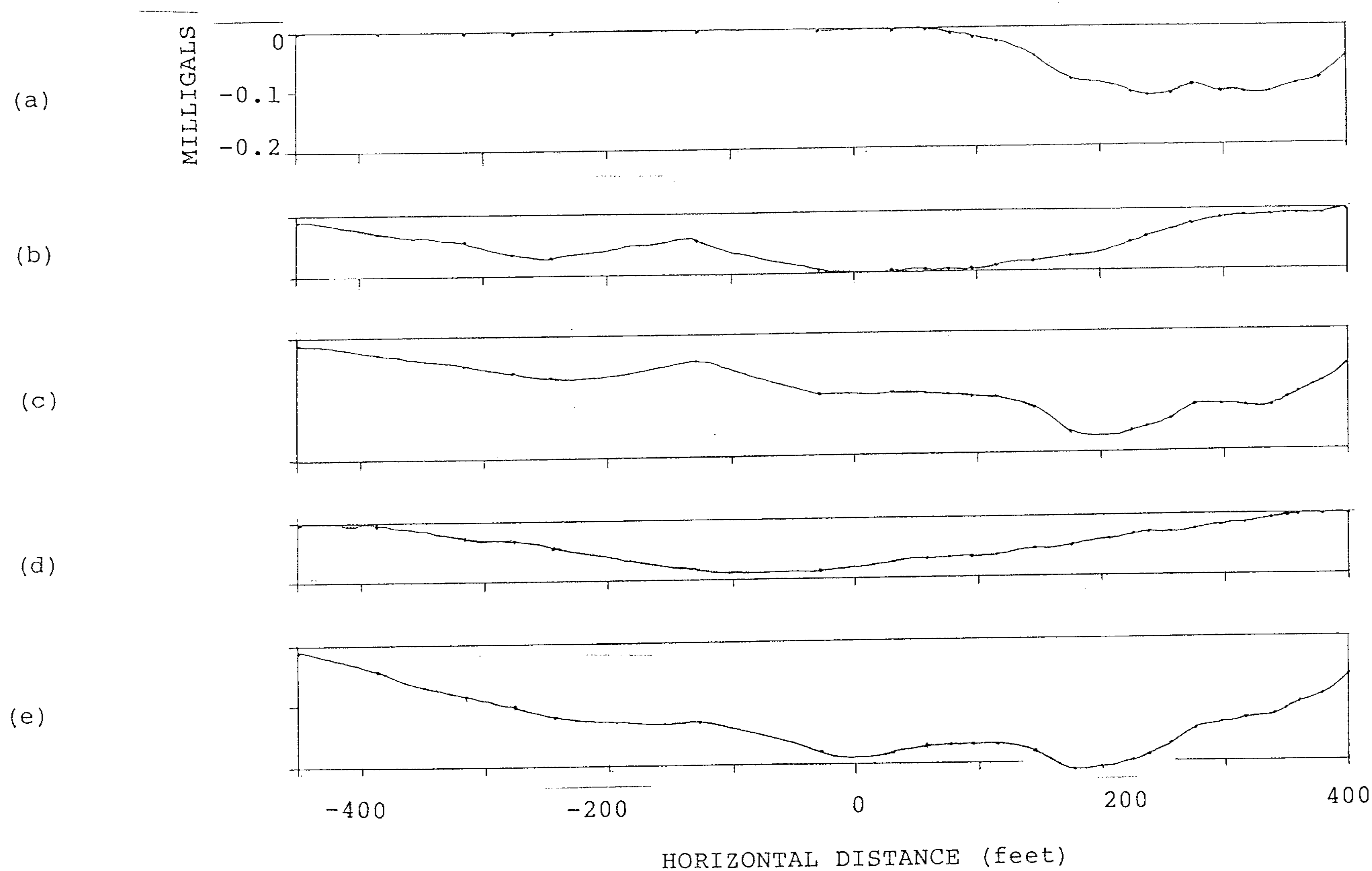

FIGURE 12. Bouguer anomalies for various models in Figure 10c. Mines are infinitly long and 10 feet high.
(a) Anomaly for upper level mines.
(b) Anomaly for lower level mines.
(c) Anomaly for upper and lower level mines.
(d) Anomaly for alluvial filled valley.
(e) Anomaly for alluvial fill and mine voids.

anomaly produced by all the mines (Figure 12c) gives no indication that the anomaly is caused by a series of parallel mine tunnels.

Considering only mine tunnels one and two (Figure 12b), first order approximations give a 24 foot depth, dimensions of 10 x 20 feet, an infinite strike length and a separation distance of 25 feet. If 0.13 mGal is the estimated error, then Figures 4d and 4e can be used to predict that the combined anomaly is less than 0.26 mGal and thus would not be detected. Further, the estimated error would have to be decreased by two orders of magnitude to differentiate these two tunnels.

Figure 12d depicts the anomaly for the alluvial-filled valley which, when added to the total mine anomaly (Figure 12c), produces the total anomaly (Figure 12e). From these three figures it can be surmised that one way to distinguish the alluvial anomaly from the total mine anomaly is with auxiliary information such as alluvial depths and density contrast, which may not be easily obtainable in some situations. Thus for this example the anomalies from all the mines merge together to form one smooth anomaly with a wavelength comparable to that of the alluvial anomaly rendering it difficult to differentiate the two anomalies. Individual mine anomalies under these conditions are completely indistinguishable.

Likewise the Heaton Canyon Bouguer anomaly for profile HC-G-4 (Figure 13a) is a smooth curve displaying no short wavelength anomalies. The subsurface interpretation (Figure 13b) was determined as for profile HC-G-5. Only the five upper level mine tunnels between 0 and 200 feet were modeled. Every allowable variance in the parameters was shifted in favor of mine detection. A depth of 25 feet, a height of 12 feet, a density contrast of -2.35 gm/cm^3, infinite strike lengths and widths scaled from Figure 13b were used to generate the gravity anomaly. These five tunnels generate an anomaly (Figure 14a) less than 0.26 mGal and are spaced so closely that individual tunnels can not be differentiated. By deleting every other tunnel (Figure 14b) the anomaly is beginning to differentiate into separate anomalies caused by single tunnels. Finally using only the two outer tunnels, the profile (Figure 14c) clearly shows two separate anomalies. The relative anomaly magnitude is 0.06 mGal, and the total anomaly has slipped further below the threshold level of 0.26 mGal. Thus even for the best of circumstances, the mines in Heaton Canyon could not be differentiated by a gravity survey.

MODEL FOR A SUBSIDENCE FEATURE. - Modeling a subsidence feature not yet expressed in the surface morphology and calculating the gravity anomaly produced by this structure will allow for a better assessment of the detectability of such features. The model (Figure 15a)

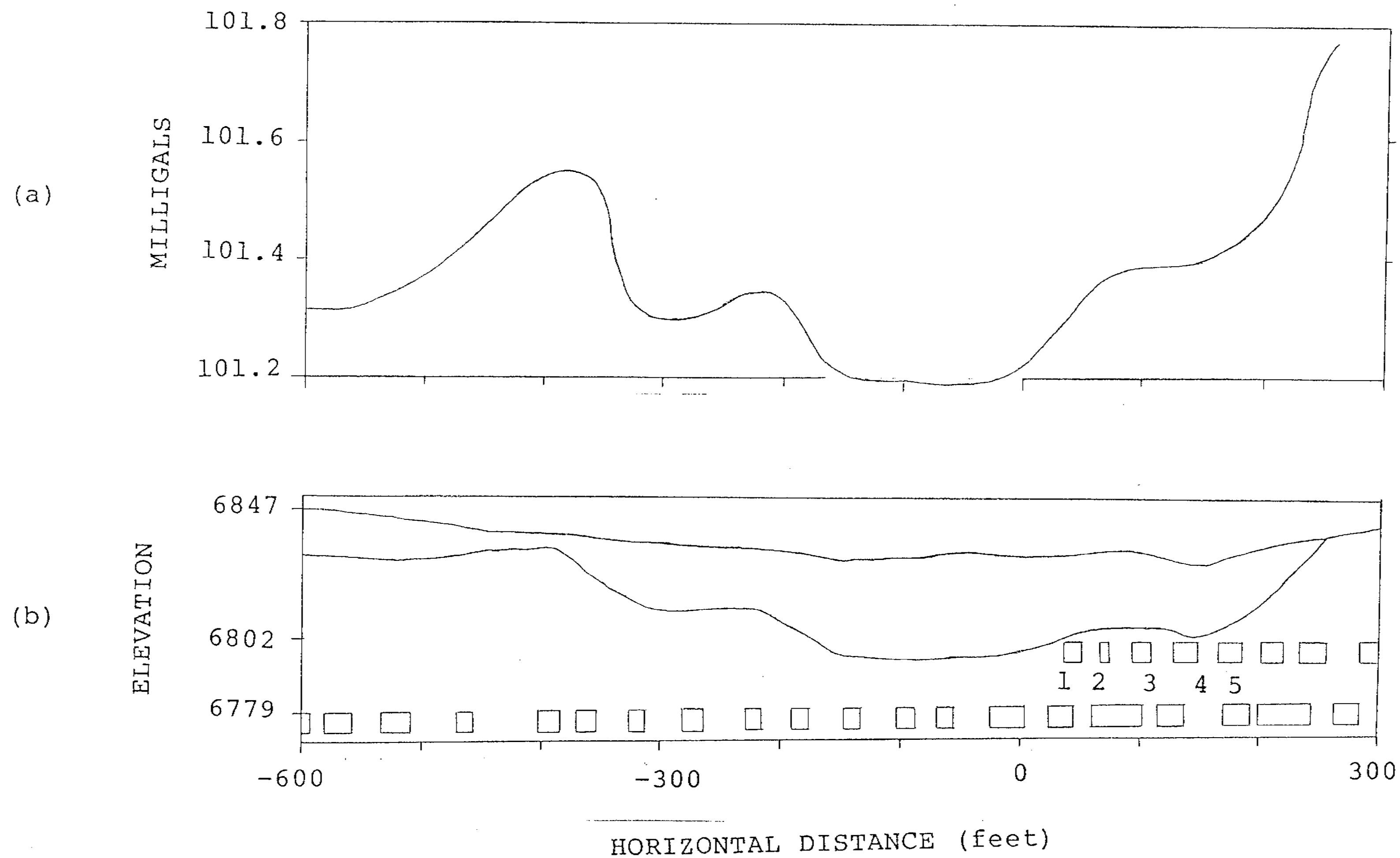

FIGURE 13 Heaton Canyon profile HC-G-4
(a) Bouguer anomaly
(b) Subsurface interpretation

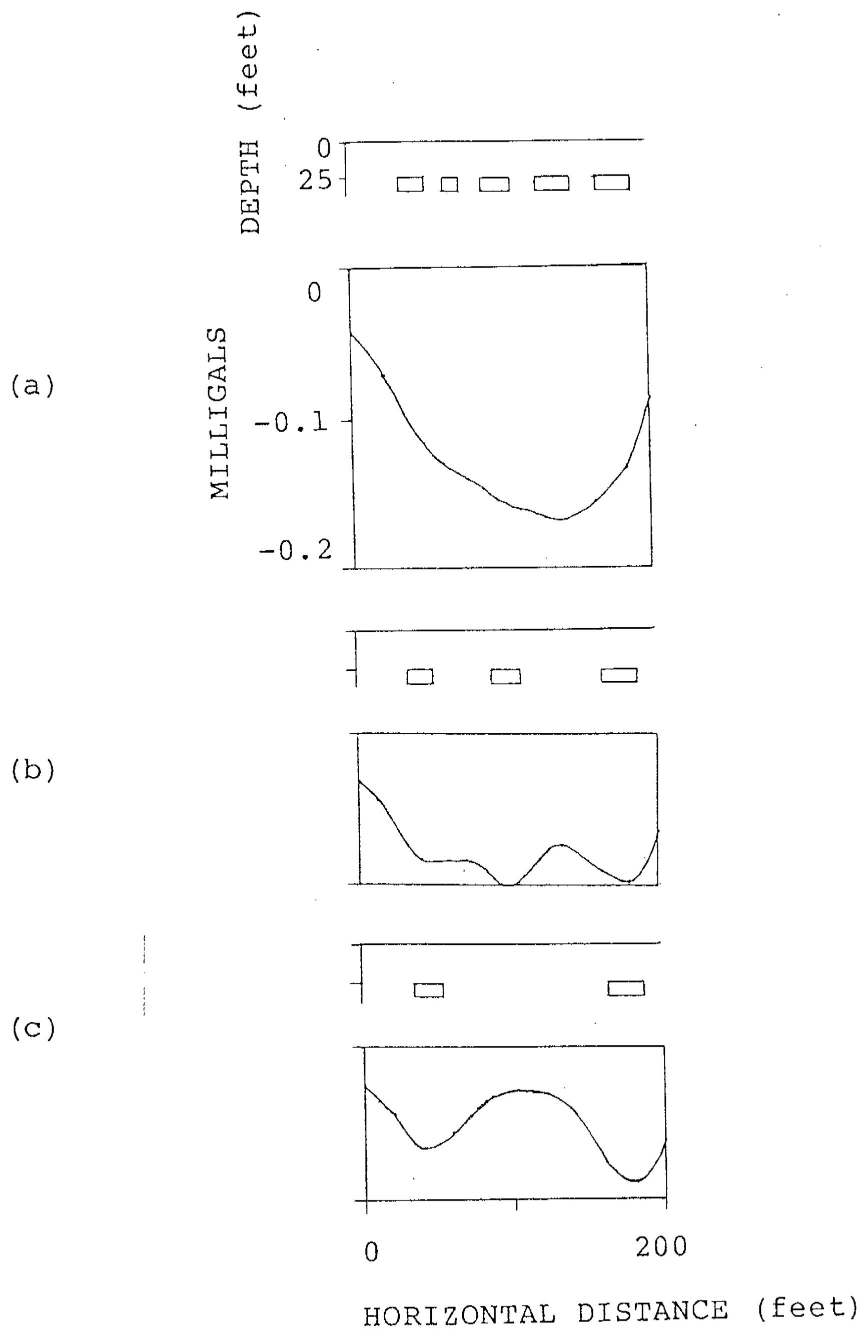

FIGURE 14a,b, and c. Various combinations of the upper level mines and the associated Bouguer anomaly. Depth to top of the mines is 25 feet. Mine height is 12 feet and density contrast is -2.35 gm/cm^3. Upper figure depicts modeled mines and lower figure displays the Bouguer anomaly.

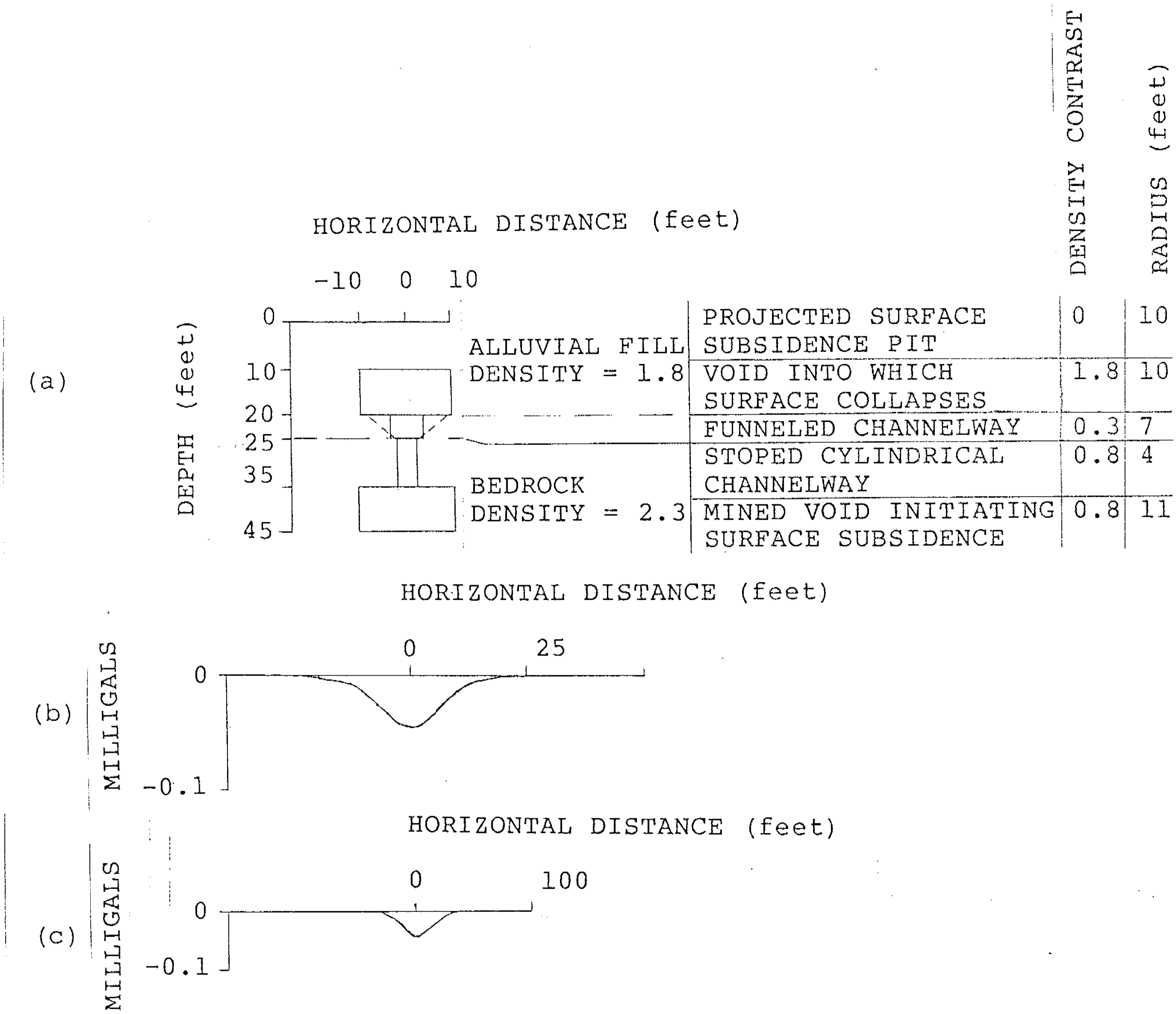

FIGURE 15. (a) Model for a subsidence feature in the pre-surface subsidence stage.
(b) and (c) Bouguer anomaly for the subsidence feature.

consists of a series of vertical cylinders centered on the same axial line. The lower cylinder which models part of the mine void consists of a 10 foot long cylinder with a 11 foot radius. T' is cylinder will be assumed to be filled with rubble caved in from the roof. A density of 1.5 gm/cm^3 is assumed for this rubble. Conservation of mass will be taken up by debris being distributed outwardly from this cylinder along the mine tunnel. Otherwise this part of the mine tunnel will not be involved in any of the calculations. The top of the mine tunnel is at a depth of 35 feet. With 2.3 gm/cm^3 for the density of the surrounding bedrock, the density contrast is -0.8 gm/cm^3 . Next a 10 foot long cylinder with a 4 foot radius also filled with caved material of density 1.5 gm/cm^3 connects the mine tunnel to the alluvium above. Above this, a funnel shaped structure modeled as a 5 foot long cylinder with a radius of 7 feet and also filled with debris with a 1.5 gm/cm^3 density connects to a void above. The alluvial density of 1.8 gm/cm^3 gives a -0.3 gm/cm^3 density contrast for this cylinder. The upper cylinder modeling the void is 10 feet long, has a 10 foot radius, has a -1.8 gm/cm^3 density contrast and is 10 feet under the surface.

The surface material is expected to collapse into this void to produce a surface subsidence feature. The known surface subsidence features are approximately 10 feet deep, have steep sides and have a surface area greater than this model. This model, although somewhat extreme, will answer some questions about the detectability of subsurface subsidence stoping its way toward the surface. The anomaly produced by this model (Figure 15b) is drawn to the same scale in Figure 15c as the Heaton Canyon profiles. The magnitude of this anomaly, 0.045 mGal, is below the magnitude for twice the estimated error for the reduced Heaton Canyon data. Thus this model would support the statement that even subsidence features in the latter stages of formation, but not yet expressed on the surface, could not be detected with a gravity survey.

CONCLUSIONS

Can the feasibility of detecting mine voids be predicted? In general because of the numerous factors involved there exists no unequivocal answer to this question. Each case must be looked into individually. However, it is hoped that most of the pertinent factors involved with this question have been mentioned.

First a calculation or estimate of the expected error associated with the reduced gravity data is needed. A major step here is to find the error incurred in calculating the Bouguer anomaly. Some factors contributing to the error in the reduced data are density, elevation determinations, location of data points, repeatability of gravity observations, ruggedness of topography, topographic maps employed and the precision of the terrain corrections.

Uncertainties in the above factors produce an error which accumulates into the final error associated with the reduced data. The errors for some factors may "degenerate into guesses" and thus shroud the error estimate in some uncertainty. In particular the error associated with the terrain corrections is the most uncertain and is also an extremum and thus a limiting factor for mine detection. Deleting other anomalies of non-interest can also introduce errors depending on the type of technique used.

Next an estimate of the anomaly magnitude generated by the structure of interest is needed. Here auxiliary sources of data such as existing mine maps and/or drill hole data can limit the mine parameters and thus allow for an anomaly calculation. Computer programs are expedient in calculating an expected anomaly. For approximately squarish shapes the expected anomaly can be calculated approximately by use of a horizontal cylinder. The expected anomaly should be at least twice the magnitude of the expected error to stand a chance at being detected. Even then, auxiliary factors could affect the above condition. Spacing of the data points must be such as to insure that at least one data point is located near the maximum magnitude of the anomaly of interest. Another factor is the spacing between a series of mine tunnels. This spacing must be such that the anomalies due to individual mines are differentiable.

Other anomalies of non-interest must be removed or shown not to interfere with detecting the location of mines.

The theoretical Heaton Canyon profiles for mines at the shallowest levels (25 foot depths) did not meet the basic requirement that the mine anomalies be at least two times the estimated error. Thus it should be expected that these mines would not be detected with a gravity survey as was confirmed in practice. By computer modeling it was shown that another factor was also detracting from the quality of the Heaton Canyon anomaly profiles. The close spacing of the mines merged the individual anomalies into one broad anomaly with a wavelength approaching that of the alluvial fill in Heaton Canyon. In theory, the gravity effects of the mine voids was similar to the gravity effect of an increased thickness in the alluvial fill.

From the standpoint of mine detection with a gravity survey, Heaton Canyon was a relatively ideal location. Factors favoring mine detection were tunnels of large cross-sectional area. The tunnels being slightly inclined to the horizontal were projected from mine maps to lie about 25 feet under the surface. The topography also was smooth. Because in theory as well as in practice these tunnels were not detected, the location of underground voids by gravimetric methods probably would be successful only under very ideal conditions.

ACKNOWLEDGMENTS

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APPENDIX

The following 13 tables display and tabulate information necessary in plotting Figures 1 through 15.

Table 1

Table 1 tabulates the data used to plot Figure 1.

Worden gravimeter - serial no. 771590 - scale constant 0.0839 mGal/division.

Observer	Time min:sec	Reading mGal
1	0:0	151.50
	0:35	151.53
	1:08	151.53
	1:55	151.50
	2:25	151.52
	3:13	151.47
	4:10	151.52
	4:49	151.52
	5:25	151.50
	6:04	151.47
2	9:04	151.48
	10:01	151.51
	10:54	151.48
	11:44	151.49
	12:44	151.52
	13:54	151.48
	14:39	151.52
	15:24	151.49
	15:59	151.49
	16:39	151.50

Table 2

Table 2 which is used to graph Figure 2, tabulates data from the Heaton Canyon study (Johnpeer 1985, in press).

Station (feet)	Terrain correction (mGal)
8 - 600	.232
8 - 550	.224
8 - 500	.222
8 - 450	.230
8 - 400	.224
8 - 350	.240
8 - 300	.234
8 - 250	.235
8 - 200	.252
8 - 150	.256
8 - 100	.243
8 - 50	.235
8 + 00	.249
8 + 50	.267
8 + 100	.251
8 + 150	.271
8 + 200	.334
8 + 250	.318

TABLE 3

Table 3 tabulates the data used to plot Figure 3.

The S.K.W. Enterprises program was used to calculate the gravity anomaly for this model.

Mine void - 10 x 20 (feet), infinite strike length

D (depth to top) - 20 (feet)

Density contrast - -2.35 gm/cm^3

X (feet) - Coordinate system centered on the symmetry plane

g (mGal) - Gravity anomaly

X	g
0	-.074
10	-.065
20	-.047
30	-.032
40	-.022
50	-.016
60	-.011
70	-.009
80	-.007
90	-.006
100	-.005
120	-.003
140	-.002
160	-.002

TABLE 4

Table 4 tabulates data for Figure 4.
The S.K.W. Enterprises program was used to calculate the gravity anomalies.

Mine voids	- Two 10 x 20 tunnels of infinite strike length and separated by S feet between edges of inside boundries.
D (Depth to top)	- 20
Density contrast	- -2.0 gm/cm^3
X (feet)	- Coordinate system centered on the symmetry plane
g (mGal)	- Gravity anomaly
S (feet)	- Separation distance between the inside edges of the mine tunnels

	S					
	100	70	50	30	20	10
X	g					
0	-.020				-.080	
5		-.032	-.046	-.054		-.095
10	-.021				-.082	
15		-.037	-.053	-.055		-.090
20	-.025				-.081	
25		-.048	-.065	-.058		-.074
30	-.032				-.068	
35		-.061	-.070	-.057		-.053
40	-.044				-.050	
45		-.067	-.061	-.052		-.037
50					-.035	
55		-.059	-.045	-.043		-.026
60	-.065				-.025	
65			-.031	-.033		-.019
70					-.018	
75		-.030		-.025		-.014
80	-.042				-.014	
85			-.016			-.011
95		-.015		-.016		
100	-.020				-.009	
105			-.009			-.007
115		-.009		-.010		
120					-.006	
125			-.006			-.005
135				-.007		
140					-.004	
145						-.004

TABLE 5

Table 5 tabulates data for Figure 5.

The S.K.W. Enterprises program was used to calculate the gravity anomalies.

Mine void - 30 x 30 tunnel of infinite strike length

Depth to top (D) - variable (feet)

Density contrast - -2.0 gm/cm^3

	D		
X	10	50	130
0	-.284	-.113	-.050
10	-.254	-.110	-.050
20	-.182	-.103	-.050
30	-.120	-.093	-.048
40	-.082	-.082	-.047
50	-.058	-.071	-.045
60	-.043	.061	-.043
70	-.033	-.052	-.041
80	-.026	-.045	-.039
90	-.021	-.039	-.036
100	-.017	-.033	-.034
120	-.012	-.026	-.030
140	-.009	-.026	-.026
160	-.007	-.016	-.023

TABLE 6

Table 6 tabulates information for plotting Figure 6. The S.K.W. Enterprises program calculated the gravity anomalies.

Mine voids	- Two 10 x 20 tunnels of infinite strike length, spaced 40 feet apart
Depth to top (D)	- variable (feet)
Density contrast	- $-2.0\ gm/cm^3$
X	- Coordinate system centered on the symmetry plane.
g (mGal)	- Gravity anomaly

	D			
	10	20	30	70
X	g			
0	-.046	-.055	-.054	-.037
10	-.055	-.059	-.055	-.037
20	-.085	-.068	-.058	-.036
30	-.104	-.072	-.057	-.035
40	-.081	-.063	-.052	-.033
50	-.045	-.046	-.043	-.030
60	-.026	-.032	-.033	-.028
70	-.016	-.023	-.025	-.025
90	-.008	-.012	-.016	-.019
110	-.005	-.008	-.010	-.015
130	-.003	-.005	-.007	-.012

TABLE 7

Table 7 lists the information employed in constructing Figures 7a, 7b and 8a. The S.K.W. Enterprises program calculated the gravity anomalies. Mine tunnels are compared to horizontal cylinders in these Figures.

Mine voids - (1) 10 x W (width), infinite strike length

(2) Horizontal cylinder, R (radius) gives this cylinder a cross-sectional area equal to the rectangles, infinite strike length

Depth to top - 20 feet

Density contrast - -2.0 gm/cm^3

X - Coordinate system centered on the symmetry plane

g (mGal) - Gravity anomaly

	W	R	W	R	W	R	W	R
	10	5.64	40	11.28	100	17.84	150	21.85
X	g							
0	-.033	-.032	-.110	-.104	-.180	-.215	-.203	-.291
10	-.028	-.028	-.103	-.094	-.178	-.201	-.203	-.276
20	-.020	-.020	-.082	-.074	-.171	-.168	-.200	-.237
30	-.013	-.013	-.059	-.054	-.158	-.132	-.195	-.193
40	-.009	-.009	-.041	-.039	-.137	-.101	-.188	-.152
50	-.007	-.007	-.028	-.029	-.108	-.078	-.176	-.120
60	-.005	.-005	-.021	-.022	-.078	-.061	-.157	-.095
70	-.004	-.004	-.016	-.017	-.056	-.049	-.130	-.077
80	-.003	-.003	-.012	-.014	-.041	-.039	-.099	-.063
90	-.002	-.002	-.010	-.011	-.031	-.032	-.071	-.052
100	-.002	-.002	-.008	-.009	-.024	-.027	-.052	-.043
120	-.001	-.001	-.006	-.007	-.016	-.019	-.031	-.032
140	-.001	-.001	-.004	.005	-.011	-.015	-.020	-.024
160	-.001	-.001	-.003	-.004	-.009	-.011	-.015	-.019

TABLE 8

Table 8 tabulates information used to plot Figures 7c, 7d and 8b. Vertical mine stopes are compared to equal area cross-sectional horizontal cylinders. The S.K.W. Enterprises program calculated the gravity anomalies.

Mine voids - (1) 10 foot wide rectangular stope of variable height (H) and of infinite strike length

(2) Horizontal cylinder with radius R, infinite strike length

Density contrast - -2.0 gm/cm^3

Depth to top - 20 feet

X - Coordinate system centered on symmetry plane

g (mGal) - Gravity anomaly

	H	R	H	R	H	R
	30	9.77	40	11.28	100	17.84
X	g					
0	-.074	-.082	-.089	-.104	-.145	-.215
10	-.067	-.074	-.081	-.094	-.137	-.201
20	-.052	-.056	-.066	-.074	-.119	-.168
30	-.039	-.041	-.051	-.054	-.100	-.132
40	-.029	-.029	.039	-.039	-.085	-.101
50	-.022	-.021	-.030	-.029	-.072	-.078
60	-.017	-.016	-.024	-.022	-.061	-.061
70	-.014	-.013	-.019	-.017	-.053	-.049
80	-.011	-.010	-.016	-.014	-.046	-.039
90	-.009	-.008	-.013	-.011	-.040	-.032
100	-.007	-.007	-.011	-.009	-.035	-.027
120	-.005	-.005	-.008	-.007	-.027	-.019
140	-.004	-.004	-.006	-.005	-.022	-.015
160	-.003	-.003	-.005	-.004	-.018	-.011

TABLE 9

Table 9 displays the information used in plotting Figure 9. The S.K.W. Enterprises program was used to calculate the gravity anomaly.

Mine void	- (1) 7 x 100 (foot) parallelogram inclined 45 degrees, infinite strike length (2) Horizontal cylinder, R =14.92 feet, infinite strike length
Density contrast	- $-2.0 gm/cm^3$
Depth to top	- 20 feet
X	- Coordinate system
g (mGal)	- Gravity anomaly

	Parallelogram	Cylinder
X	g	
-160	-.007	
-140	-.009	
-120	-.011	-.007
-110		-.009
-100	.014	-.012
-90	-.016	-.018
-80	-.019	-.021
-70	-.022	-.026
-60	-.026	-.032
-50	-.031	-.041
-40	-.038	-.053
-30	-.048	-.070
-20	-.061	-.093
-10	-.079	-.122
0	-.097	-.160
10	-.106	-.162
20	-.105	-.160
30	-.098	-.122
40	-.089	-.093
50	-.079	-.070
60	-.070	-.053
70	-.062	-.041
80	-.054	-.032
90	-.046	-.026
100	-.040	-.021
110		-.018
120	-.030	-.012
130		-.009
140	-.023	-.007
160	-.017	

TABLE 10

Table 10 displays the formulas and values used to graph Figure 10. Equation 5 (see text) describes the anomaly produced by a horizontal cylinder,

$$g = 2\pi G \sigma R^2 / [\, Z\ (1 + X^2/Z^2)] . \qquad (5)$$

Differeniating equation 5 with espect to X, setting Dg/dX = 0 and then solving for X gives the coordinate at which g is an extrema.

$$Dg/dX = -2\pi G \sigma R^2 Z\ 2\ X\ /\ (Z^2 + X^2)^2 . \qquad (10\text{-}1)$$

Setting Dg/dX = 0 it is found that X =0 gives the extrema. Also

$$Z = D + R, \qquad (10\text{-}2)$$

where

D = depth to the top of the horizontal cylinder.

Because of interest only in the extrema, equation 10-2 and X = 0 are substituted into equation 5 giving:

$$D = R[((2\pi \sigma G)/g)R-1] . \qquad (10\text{-}3)$$

Equation 10-3 is used to plot Figure 10 for various values of g, which are tabulated below. Density contrast is -2.0 gm/cm^3 .

TABLE 10

g	0.03		0.06		0.10		0.12		0.20		0.30	
	D	R	D	R	D	R	D	R	D	R	D	R
	0	1.17	0	2.5	0	3.9	0	4.6	0	7.8	0	11.6
	1.4	2	0.7	3	0.086	4	0.32	5	1.34	9	1.2	13
	4.6	3	2.6	4	1.38	5	1.6	6	2.77	10	4.1	15
	9.6	4	5.6	5	3.2	6	3.4	7	6.38	12	7.5	17
	16.2	5	9.4	6	5.5	7	5.6	8	13.7	15	14.0	20
	24.6	6	13.6	7	8.3	8	8.2	9	19.9	17	28.3	25
	34.7	7	19.7	8	11.1	9	11.2	10	31.0	20	35.0	27
	46.4	8	25.4	9	15.0	10	14.7	11	39.0	22	46.5	30
	75.0	10	32.5	10	19.9	11	18.6	12	44.5	23		
			40.5	11	24.01	12	22.9	13	49.5	24		
			49.4	12	30.0	13	27.7	14	54.8	25		
					36.0	14	32.0	15				
					42.0	15	38.4	16				
					49.0	16	44.5	17				
							50.9	18				

TABLE 11

The first set of tables display the coordinates, of the corners of the alluvial blocks and mine tunnels, used in calculating the gravity anomalies for various models in Figure 11c. The S.K.W. Enterprises program was used to calculate the second table which tabulates the gravity data for Figure 12. The alluvial blocks have a density contrast of -0.2 gm/cm^3 . The mine tunnels are given a density contrast of -2.0 gm/cm^3 . All blocks are of infinite strike length.

Alluvium

block	coordinates
A1	(-400,0) (-315,0) (-315,15)
A2	(-315,0) (-278,0) (-315,15) (-278,11)
A3	(-278,0) (-128,0) (-278,11) (-128,39)
A4	(-128,0) (-25,0) (-128,39) (-25,39)
A5	(-25,0) (30,0) (-25,39) (30,29)
A6	(30,0) (145,0) (30,29) (145,23)
A7	(145,0) (210,0) (145,23) (210,13)
A8	(210,0) (260,0) (210,13) (260,11)
A9	(260,0) (350,0) (260,11)

lower level mines

block	coordinates
T10	(-105,42) (-95,42) (-105,52) (-95,52)
T12	(-8,42) (2,42) (-8,52) (2,52)
T14	(68,42) (88,42) (68,52) (88,52)
T16	(160,42) (174,42) (160,52) (174,52)

upper level mines

block	coordinates
T1	(163,-24) (186,-24) (163,-34) (186,-34)
T2	(214,-24) (232,-24) (214,-34) (232,-34)
T3	(241,-24) (261,-24) (241,-34) (261,-34)
T4	(288,-24) (305,-24) (288,-34) (305,-34)
T5	(323,-24) (348,-24) (323,-34) (348,-34)
T6	(368,-24) (387,-24) (368,-34) (387,-34)

lower level mines

block	coordinates
T7	(-390,42) (-378,42) (-390,52) (-378,52)
T8	(-306,42) (-294,42) (-306,52) (-294,52)
T9	(-262,42) (-228,42) (-262,52) (-228,52)
T11	(-50,42) (-18,42) (-50,52) (-18,52)
T13	(25,42) (45,42) (24,52) (45,52)
T15	(107,42) (124,42) (107,52) (124,52)
T17	(193,42) (214,42) (193,52) (214,52)

TABLE 11

The following table tabulates the gravity anomalies calculated for the various models in Figure 12.

X	Upper level mines	lower level mines	alluvium	upper & lower level m.	total anomaly
	g				
-450	-.001	-.014	-.002	-.015	-.017
-385	-.001	-.035	-.010	-.036	-.046
-315	-.001	-.049	-.035	-.050	-.085
-278	-.001	-.066	-.037	-.067	-.104
-245	-.001	-.075	-.048	-.076	-.124
-128	-.002	-.043	-.085	-.045	-.130
-30	-.004	-.098	-.087	-.102	-.189
30	-.006	-.099	-.075	-.105	-.180
56	-.008	-.098	-.070	-.106	-.176
78	-.011	-.097	-.067	-.108	-.175
96	-.015	-.094	-.064	-.109	-.173
115	-.022	-.088	-.061	-.110	-.171
145	-.047	-.079	-.056	-.126	-.182
175	-.089	-.077	-.048	-.166	-.214
223	-.109	-.055	-.035	-.164	-.199
237	-.115	-.044	-.033	-.159	-.192
251	-.113	-.034	-.030	-.147	-.177
274	-.099	-.023	-.026	-.122	-.148
296	-.105	-.017	-.019	-.122	-.141
314	-.107	-.014	-.014	-.121	-.135
335	-.113	-.011	-.007	-.124	-.131
358	-.098	-.009	-.002	-.107	-.109
378	-.086	-.008	-.002	-.094	-.096
400	-.053	-.006	-.001	.059	-.060

TABLE 12

The first table displays the coordinates of the tunnels in Figure 13. The gravity anomalies for the various models in Figure 14 are tabulated in the second table.

Density contrast - $-2.0\ gm/cm^3$
X - Coordinate system
g (mGal) - Gravity anomaly

Tunnel no.	Coordinates
1	(35,25) (52,25) (35,37) (52,37)
2	(65,25) (75,25) (65,37) (75,37)
3	(90,25) (110,25) (90,37) (110,37)
4	(125,25) (150,25) (125,37) (150,37)
5	(164,25) (188,25) (164,37) (188,37)

	Tunnels		
	all	1,3&5	1 & 5
X	g		
0	-.041	-.031	-.024
20	-.070	-.053	-.043
40	-.109	-.082	-.066
60	-.130	-.083	-.055
70	-.136	-.082	-.044
75	-.139	-.084	-.039
80	-.142	-.087	-.035
85	-.146	-.092	-.032
90	-.150	-.096	-.030
95	-.154	-.099	-.028
100	-.157	-.100	-.027
105	-.159	-.098	-.027
110	-.160	-.094	-.028
115	-.161	-.088	-.029
120	-.163	-.083	-.030
125	-.165	-.070	-.032
135	-.168	-.072	-.039
160	-.155	-.090	-.074
180	-.133	-.097	-.088
200	-.086	-.065	-.059

TABLE 13

Information and parameters concerning the subsidence model in Figure 15 are tabulated in these tables.

X - Coordinate system centered on the cylindrical axis

g (mGal) - Gravity anomaly

R - Radius (feet)

Density contrast- $-2.0\ gm/cm^3$

length - (feet)

Depth - (feet)

Cylinder no.	R	Density	Length	Depth
1	10	$-1.8 gm/cm^3$	10	10
2	7	-0.3	5	20
3	4	-0.8	10	25
4	11	-0.8	10	35

X	g Cylinder no. 1	g Cylinder no. 2	g Cylinder no. 3	g Cylinder no. 4	Total anomaly
0	-.039	-.001	-.001	-.004	-.045
+ 5	-.037	-.001	-.001	-.004	-.042
10	-.028	-.001	-.001	-.003	-.033
15	-.019	-.001	-.001	-.003	-.024
20	-.012	-.000	-.001	-.003	-.016
25	-.007		-.000	-.002	-.009
30	-.005			-.002	-.007
40	-.002			-.001	-.003
50	-.001			-.001	-.002
70	-.001			-.000	-.001
90	-.000				-.000