Geophysical Open-File Report 38
Geoscience Department and
Geophysical Research Center
New Mexico Tech
Socorro, NM 87801

SURFACE WAVE ANALYSIS USING TWO-DIMENSIONAL FINITE ELEMENT TECHNIQUES

by

Philip J. Carpenter

Submitted in partial fulfillment

of the requirement for

Geophysics 590

and the

Master's Degree Program

at

New Mexico Institute of

Mining and Technology

May, 1981

Abstract

Two-dimensional finite element techniques are employed to model Love and Rayleigh waves propagating across structures with varying topography. Love and Rayleigh wave propagation through a model of the Magdalena Mountains is examined for periods from 1.0-6.0 seconds. For an incident fundamental mode Love wave, more than 80% of the energy is transmitted in the fundamental mode. For an incident Rayleigh wave, the energy in the transmitted fundamental mode increases from 85% to nearly 100% over this range of periods.

Several Rio Grande rift models are examined for incident fundamental Love and Rayleigh wave motion at periods of 1.0 to 8.0 seconds. For Rayleigh waves, lower periods (high frequencies) generally lose much of their energy to higher surface wave modes (body waves). For Love waves, the same is true, except for an anomalous (65%) loss of energy near 3.7 seconds. In all cases tested, nearly all energies are transmitted with virtually no reflection.

The conversion of energy into other modes suggests possible mechanisms for the existence or absence of features seen on regional earthquake seismograms (e.g. low-frequency codas or total lack of surface waves).

Table of Contents

LIST OF FIGURES

LIST OF FIGURES (CONTINUED)

LIST OF TABLES

I. Introduction

Aki (1969) suggested that the presence or absence of surface waves in the codas of local microearthquakes is a function of scattering by topographical features and irregular geology. If the mechanism of this scattering can be understood, and models developed, it may be possible to invert local microearthquake data and obtain information on the extent and nature of the scattering bodies. This study constitutes a step in such an analysis, and was undertaken to determine the degree of scattering from two prominent local features: the Magdalena Mountains and the Rio Grande rift. Also, in order to interpret results more accurately, it was necessary to identify problems in finite element modeling of realistic structures at periods shorter that those examined in previous studies.

The propagation of Love and Rayleigh waves across irregular geolgical structures with varying topograpy presents a formidable mathematical problem. Analytical solutions have been proposed for only the simplest of cases, such as a step over a half-space or a sudden change in horizontal layering over a half-space (Mal and Knopoff, 1965; McGarr and Alsop, 1967; McGarr, 1969). More realistic models must be examined numerically through the use of techniques like the finite element method (Zienkiewicz and Cheung, 1967) or finite difference analysis (Munasinghe and Farnell, 1973).

The study of surface waves using two-dimensional finite element techniques is well known (Lysmer and Drake, 1972). In brief, a cross-section of the earth is divided into a mesh composed of quadrilateral elements. Each element is assigned a characteristic P and S wave velocity and density; damping (or attenuation) may be included as well. These elements are interconnected at a discete number of nodal points. Three basic assumptions apply to the nodes:

1) All forces within the structure act through the nodes.

2) The displacements of the nodal points define the displacement within each element.

3) The displacements between nodal points are required to be linear.

For purposes of computation, the mesh is divided into three regions: two layered zones, one at each end, and an irregular zone encompassing all of the complex geology and topographical irregularities (Figure 1). The irregular structure is excited by a plane wave of period T incident upon its left-hand side. Mode shapes (displacement vs. depth curves) for both incident and reflected waves are found from a steady-state analysis of the left layered zone.

Likewise, mode-shapes for transmitted waves are determined from a steady-state analysis of the right layered zone.

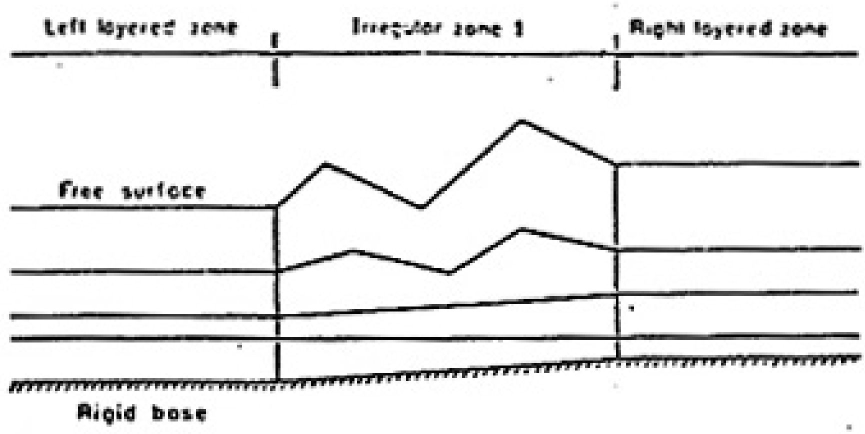

Figure 1. Division of the finite element mesh into 3 distinct regions (from Lysmer and Drake, 1972)

Lysmer and Drake (1972) have suggested several criteria to be met to insure that displacements are accurately modeled. First, they suggest that element lengths be less than 1/10 of a wavelength in the direction of propagation. Second, they note that the accuracy of the mode-shape with depth depends strongly on the number and positioning of layers. More vertical nodes, and nodes placed closer together, will produce a better approximation to the actual mode-shape. Finally, since the model is finite, a rigid boundary must be placed at depth. Lysmer and Drake (1972) suggest placing the rigid base at 2-3 times the longest wavelength of interest to insure that accurate mode-shapes are obtained.

II. Previous studies

The formulation used in this study was derived from several sources. Zienkiewicz and Cheung (1967) set down a general finite element formulation suitable for use in engineering applications. This was expanded upon by Lysmer (1970), who modeled Rayleigh modes in a layered structure with the lumped mass method. Lysmer and Drake (1972) provided an even more expanded formulation which analyzed two-dimensional Love and Rayleigh waves in an alluvial valley, as well as along a section through central California. Drake (1972) considered Rayleigh wave propagation across a step change in elevation and across an inclined interface. Lysmer and Drake (1971) studied Love wave transmission through a sinusoidal depression, continental boundary, and a subduction zone. More recently, Drake and Bolt (1980) analyzed an ocean-continent interface with the finite element technique, and derived energy transmission curves as a function of period. The results obtained in the above cases showed that the basic formulation was correct and that realistic models could be analyzed.

Laboratory studies have produced results which have been compared with the results from numerical modeling. Kuo and Thompson (1963) conducted an experiment in Rayleigh wave propagation with a gently sloping plexiglass-panelyte interface, and found that Rayleigh wave phase velocities were independent of the direction of propagation. Abe and Suzuki (1970), however, found phase velocities to be a function of the angle of incidence for Rayleigh waves propagating across an inclined surface layer over a half-space of aluminum and brass. Drake (1972) attributed this disagreement in phase velocities to Kuo and Thompson's neglect of body wave energy. Since the finite element method accounts for all energy, conversions will appear as higher mode surface waves. In fact, this is found to occur with several models investigated in this study.

III. Programming

Complete programs for the analysis of layered and irregular zones were coded in FORTRAN for the DEC 2060 on the basis of work done by Lysmer (1970), Lysmer and Drake (1971), Lysmer and Drake (1972), and Drake (1972). Program listings are presented in the Appendix. Although few additions or modifications were made to the original formulation, these programs were tested against published results to insure confidence both in the programs and their results (see Section IV).

A major problem in using finite element code is storage limitations. For the DEC 2060, a 300x300 matrix is about the largest that can be placed in memory at one time with a modest source code. To circumvent this problem, a parallel set of programs (LVIRRX, RYIRRX) were developed which construct and solve large matrices two lines at a time and thus have no maximum limit for matrix size (see Appendix, pages A-22 and A-42). However, the amount of CPU time required for a small matrix solution (100x100) is estimated at 10 hours, making these programs unfeasable for use on the DEC 2060.

IV. Testing

1. Love waves

Extensive tests were conducted using a model of a sinusoidal valley (Figure 2) with incident Love waves at a period of 62 seconds. The structure was designed to match a sinusoidal valley investigated by Lysmer and Drake (1971). The period was chosen to match the Slavin and Wolf (1970) model, in which the width of the depression was 1.2 times the wavelength of the incident surface wave. Lysmer and Drake added realistic elastic parameters and used a period of 62 seconds to maintain this same width/wavelength ratio. Element parameters are generally the same as those used by Lysmer and Drake, although they did not specify the depth of the top of the half-space. Results from this model may be compared to those obtained by Lysmer and Drake (1971) in Table 1.

TABLE 1
NORMALIZED ENERGY PERCENTAGES AND PHASE VELOCITIES FOR THE SINUSOIDAL VALLEY MODEL

Results	Fundamental Trans	Fundamental Refl	1st Higher Trans	1st Higher Refl	Higher modes Trans	Higher modes Ref	Fundamental Phase vel.
Lysmer and Drake (1971)	86.00%	0.20	0.70	0.29	9.91	2.90	———
LVIRR	83.12	5.63	1.31	4.18	2.72	3.04	4.367 km/s

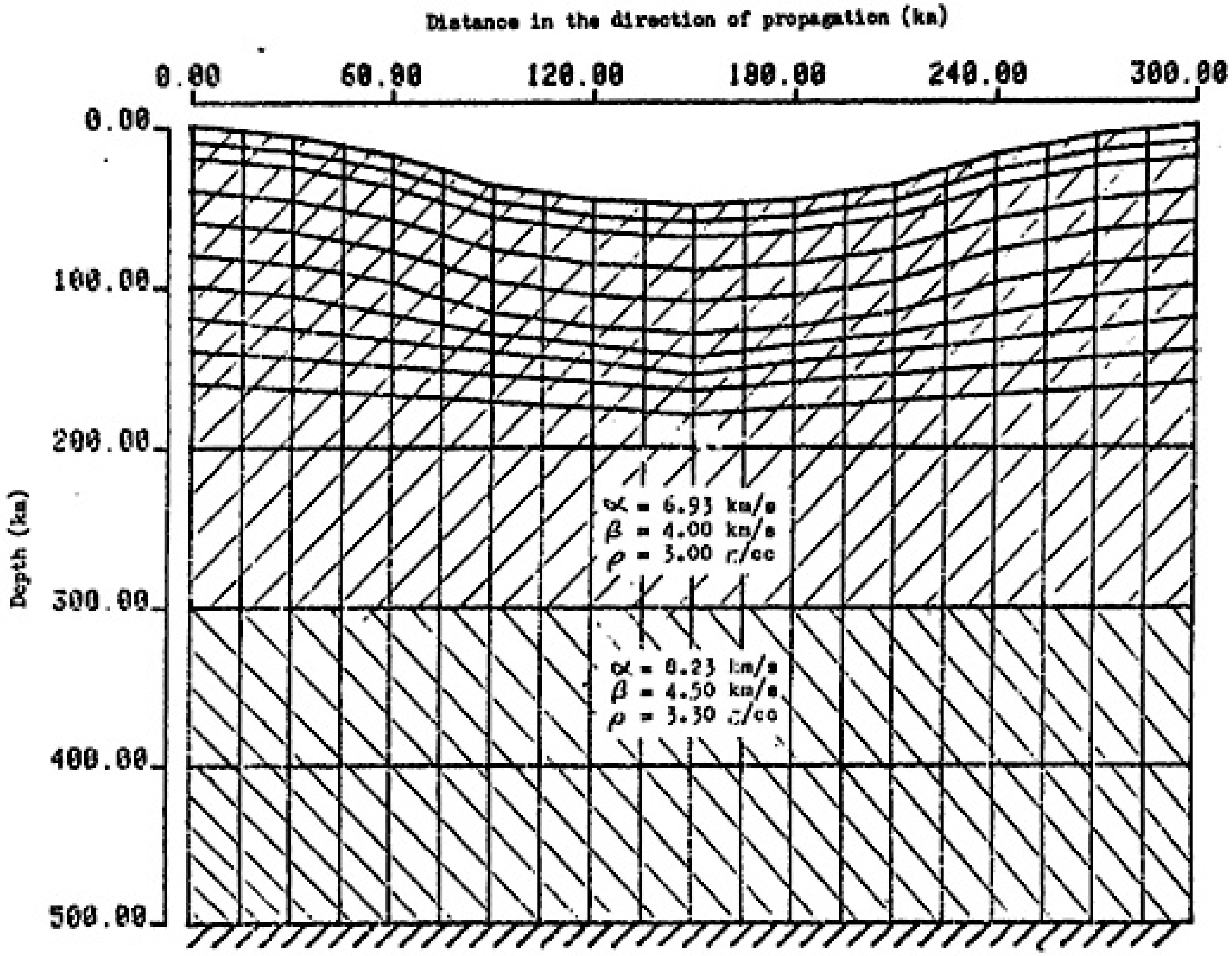

Figure 2. Sinusoidal valley over a half-space designed to match Lyemer and Drake (1971). 260 elements are arranged in 13 layers and 20 columns with 273 free nodes. All elements have identical elastic parameters.

The percentage of energy in the transmitted fundamental mode of this structure compares favorably with that of Lysmer and Drake (1971). If their energies are accepted as the standard, a 3.4% error exists in the energy transmitted in the fundamental mode. However, much larger discrepencies exist in the relative energies of higher modes. The relatively large amount of energy in reflected modes is probaby the result of small differences in model parameters and positioning of the nodes. In particular, my test case used but 60% of the number of free nodes of Lysmer and Drake (273 nodes with 260 elements in 13 layers versus 420 nodes with 400 elements in 20 layers). Thus, mode-shape approximations are probably not as accurate as in the Lysmer and Drake model. This may cause additional energy scattering, primarily into the reflected fundamental and 1st higher modes. Despite the differences in calculated energies, phase velocities for the two structures agree to within 3%.

2. Rayleigh wave tests

To test the accuracy of the Rayleigh wave modeling programs, Drake's (1972) model of a step of height H over a half-space was analyzed (Figures 3 and 4). Element parameters and geometry were identical to Drake's structure in the upper 10 km, and incident waves had a period of 13.31

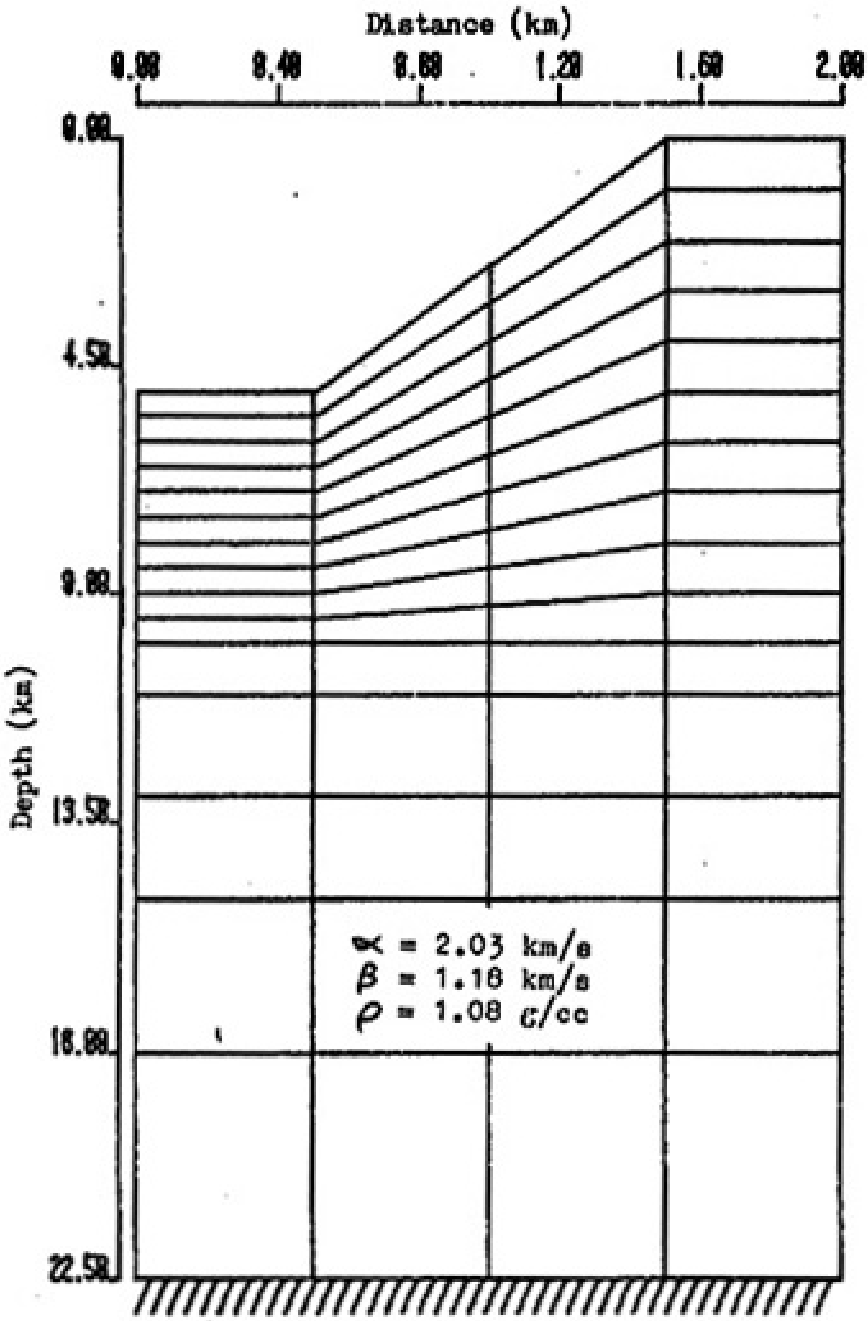

Figure 3. Model designed to match Drake's (1972) 'step' model. Note horizontal exaggeration. 273 free nodes exist with 260 elements in 13 layers.

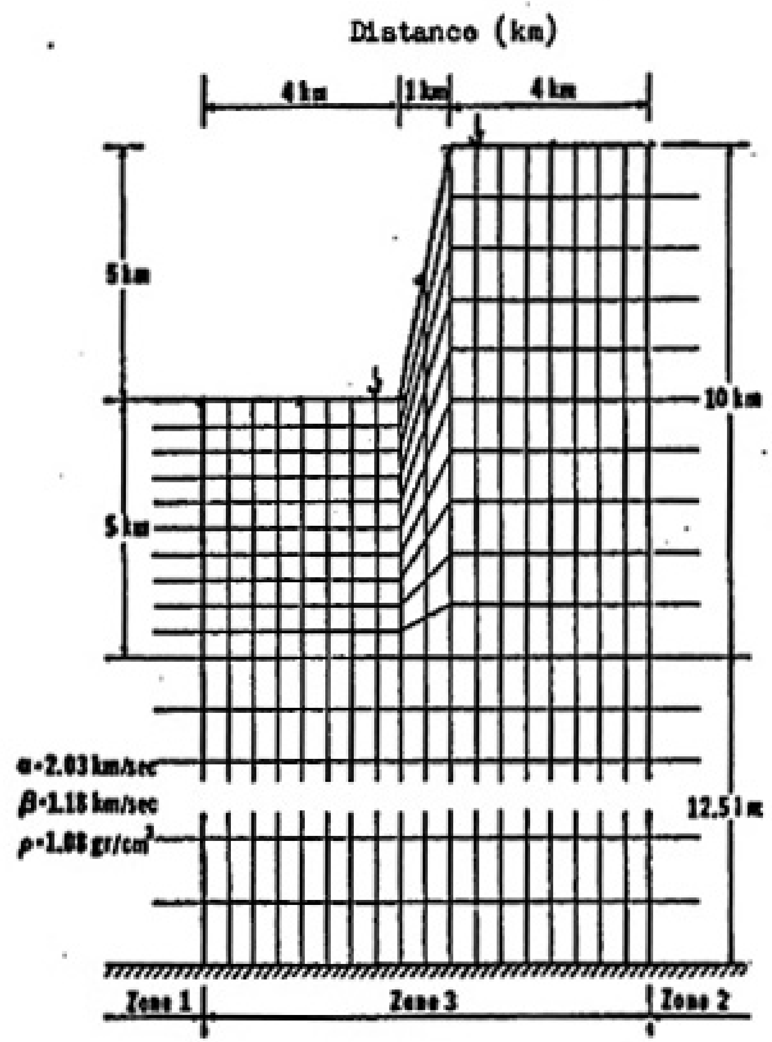

Figure 4. Drake's (1972) 'step' model. This model consists of 400 elements in 20 layers with 420 free nodes.

seconds. This period was chosen so that $\omega H/\beta = 2$.

Transmitted and reflected energies computed by RYIRR agree closely with those found by Drake (see Table 2). Slight differences may be attributed to differences in the number of nodes between the two structures. Computer storage limitations require that fewer nodes be used here than in the Love wave case, since the global matrix is of order 2Nx2N (see program RYIRR in Appendix, page A-38). Thus only 60 elements were used in this analysis compared to Drake's 400. These energies agree more closely with published results than those derived from the Love wave programs. One reason for this is that the structure modeled here is closer to Drake's structure due to his more detailed description.

TABLE 2
NORMALIZED ENERGY PERCENTAGES AND PHASE VELOCITIES
FOR THE STEP MODEL

Results	Fundamental Trans	Fundamental Refl	Higher modes Trans	Higher modes Refl	Fundamental Phase vel.
Left to right:					
Drake (1972)	29.84	1.70	67.11	1.35	1.0248
RYIRR	27.34	1.79	69.57	1.29	1.0230
Right to left:					
Drake (1972)	29.84	11.97	16.53	41.66	1.0248
RYIRR	27.34	9.68	18.24	44.75	1.0226

Another test of the Rayleigh wave routines was to compare results with those obtained by Fuyuki and Matsumoto (1980) for Rayleigh waves propagating across a trench. Fuyuki and Matsumoto used finite difference techniques to analyze an idealized trench model (square well of depth h) in acrylite. Their model consisted of 322x801 nodal points. They obtained curves that related transmitted and reflected amplitudes to h/λ , where λ is the incident wavelength. Curves were generated for the structure shown in Figure 5, and compared to the Fuyuki and Matsumoto curves (Figures 6 and 7). Considering the disparity in nodal points between the two structures, and the slight inclination of the trench walls in this model, the curves show many similarities. The amplitudes found by the finite element method are slightly higher than those found by Fuyuki and Matsumoto. It also appears that the finite element curves are slightly shifted to higher values of h/λ when compared to the Fuyuki and Matsumoto curves. It should be noted that the Fuyuki and Matsumoto curves presented in Figures 6 and 7 represent the average of several runs.

As a result of these tests, it was concluded that the routines used in this study are accurate and may be used to investigate new structures. However, some of the problems encountered in examining new structures must be considered before a presentation of the final results.

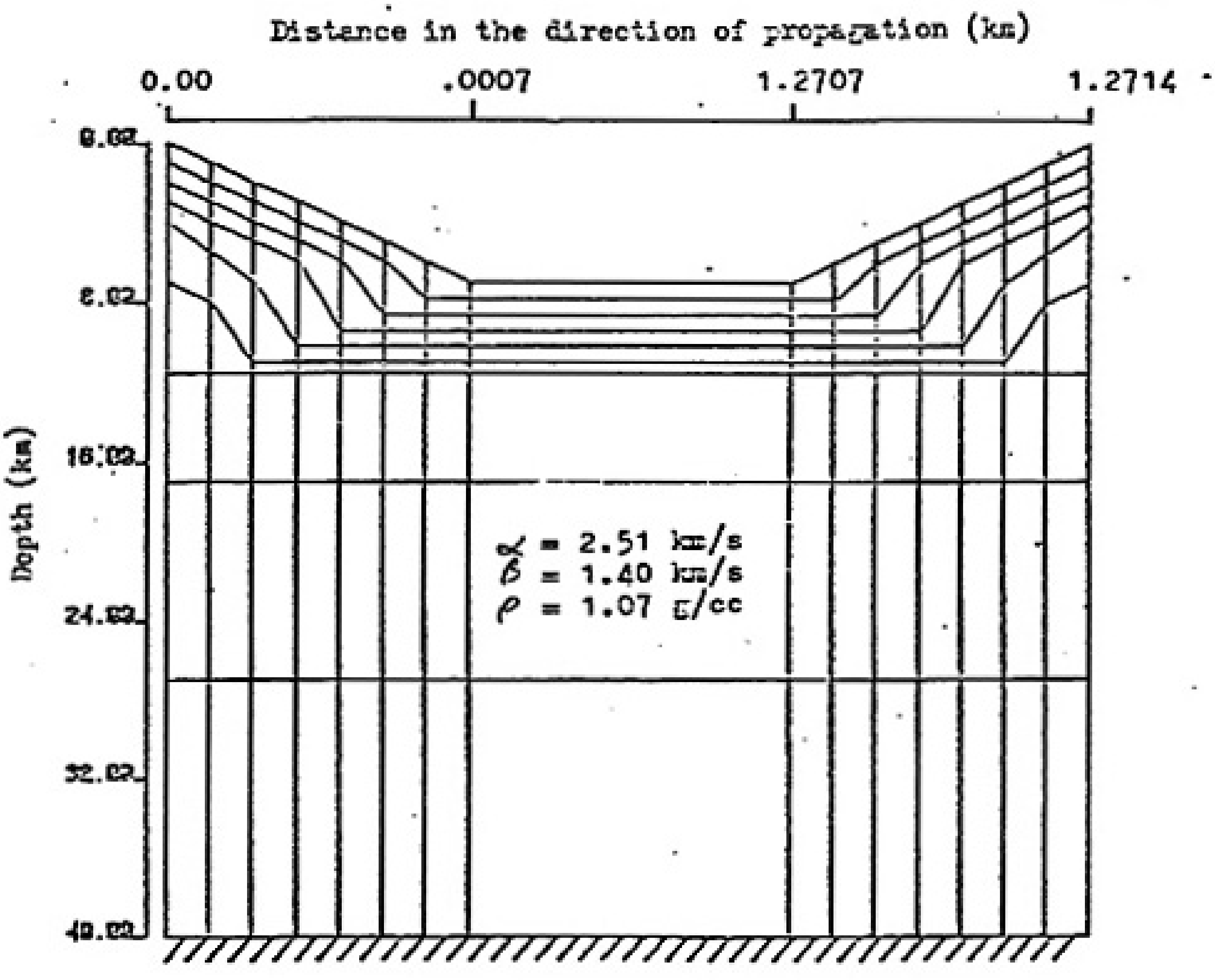

Figure 5. Structure used to model the 'trench' analyzed by Fuyuki and Matsumoto (1980). Element widths near edges are exaggerated to show geometry. All elements have identical elastic parameters.

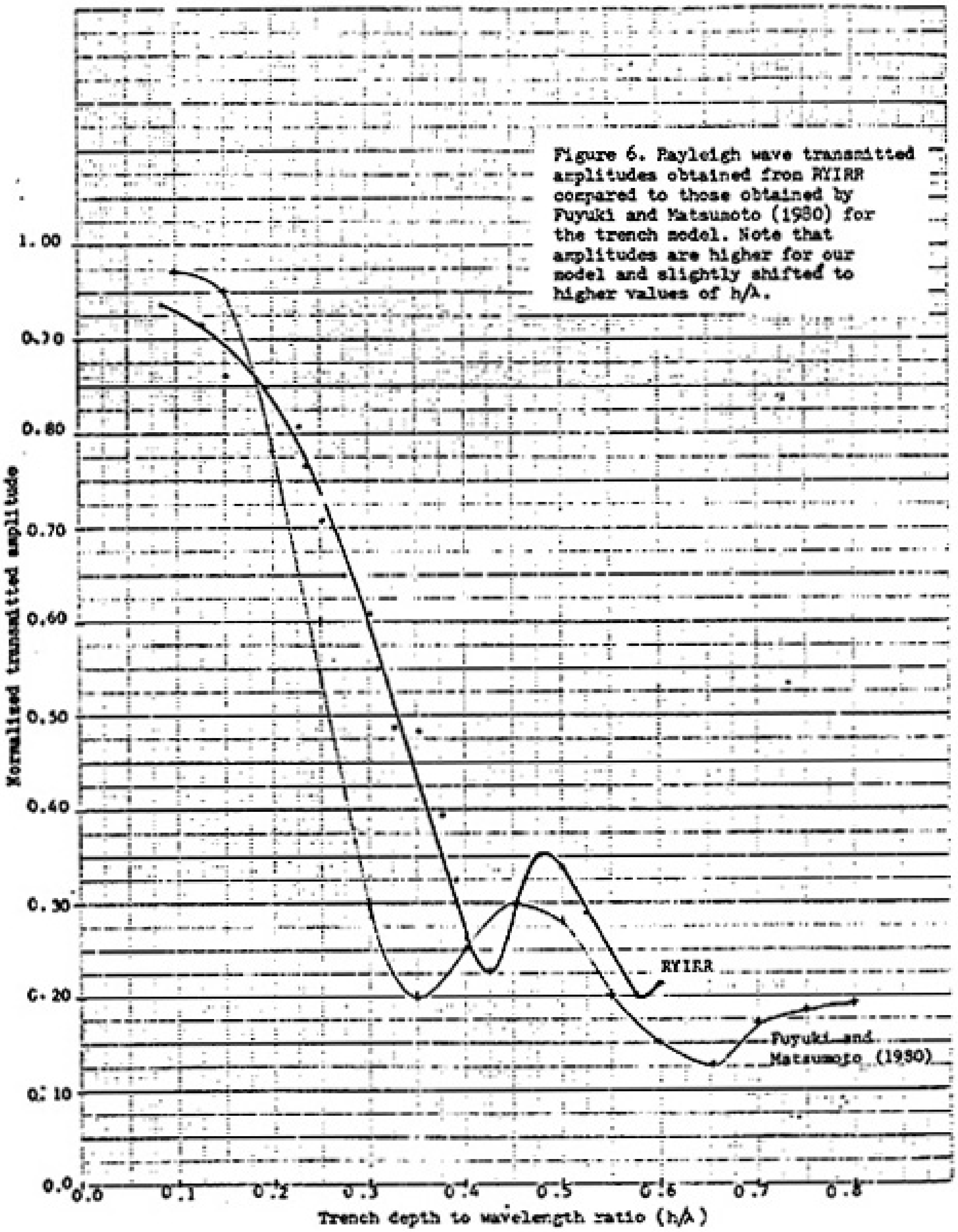

Figure 6. Rayleigh wave transmitted amplitudes obtained from RYIRR compared to those obtained by Fuyuki and Matsumoto (1980) for the trench model. Note that amplitudes are higher for our model and slightly shifted to higher values of h/λ.

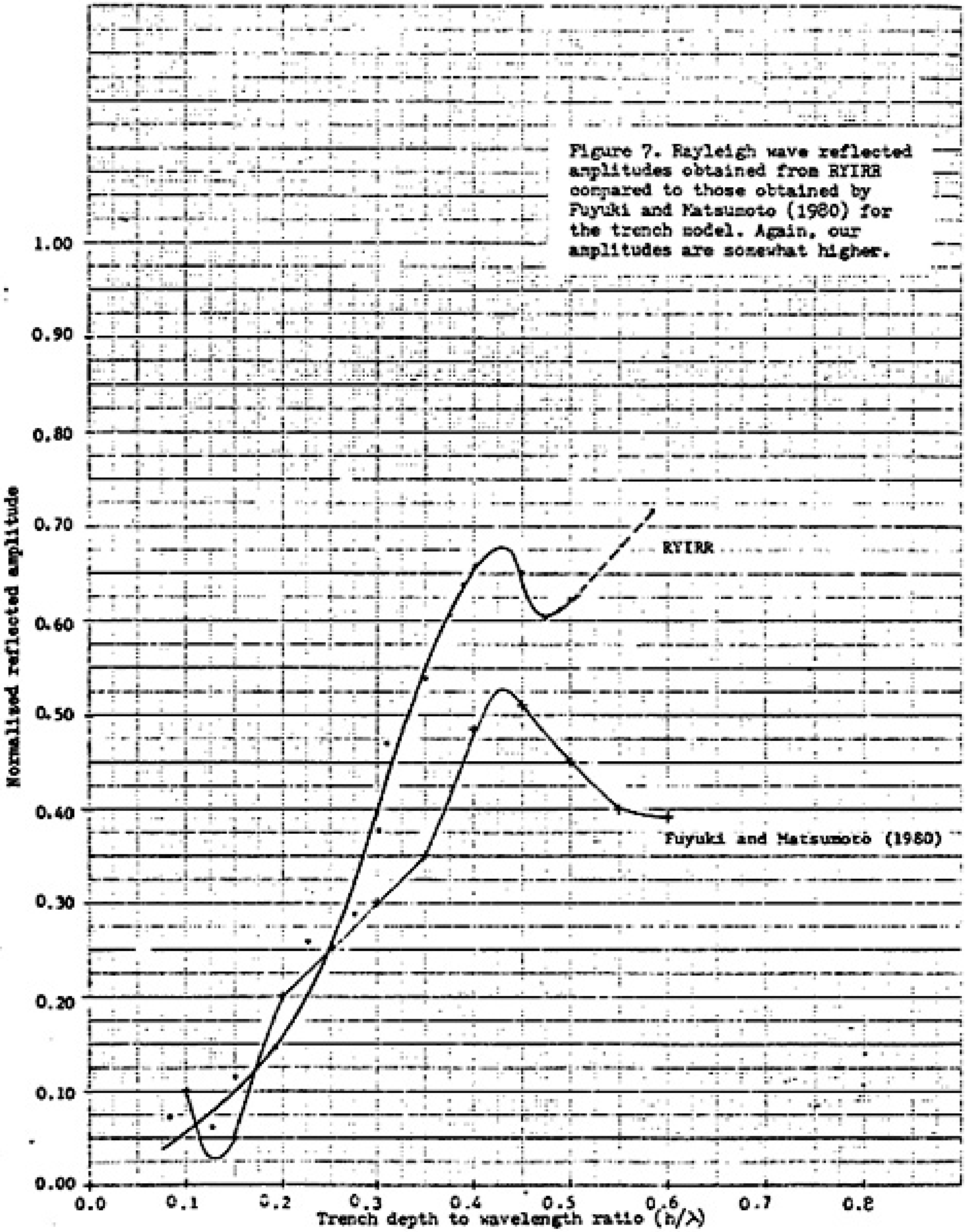

Figure 7. Rayleigh wave reflected amplitudes obtained from RYIRR compared to those obtained by Fuyuki and Matsumoto (1980) for the trench model. Again, our amplitudes are somewhat higher.

V. Problems and Interpretation Pitfalls

In order to interpret the results accurately, it is necessary to discuss sources of error. Of primary interest are the effects of including elements with lengths in the direction of propagation larger than the 1/10 wavelength criterion (Lysmer and Drake, 1972). In general, very little energy is transmitted in the fundamental mode where element lengths >> 1/10 wavelength are used. Tests show that substantial fundamental mode energy is reflected as well as converted into body waves at these periods.

While violation of the element length condition provides a simple explanation for the small amounts of energy transmitted in the fundamental mode at short periods, it must be remembered that short period waves are also more sensitive to irregularities in the structure. Thus, it is difficult to separate out the effects of poorly modeled displacements from actual scattering at these periods.

Figures 8, 9, and 10 show irregular structures Ridge A and Ridge B, both consisting of a 0.25-km high symmetrical ridge in a 2.25 km layer overlaying a half-space. Ridge A and Ridge B produced the energy curves given in Figures 11 and 12 and Tables 3 and 4. The energy in the transmitted fundamental mode drops drastically in the case of Ridge A, forming an energy 'hole' at $\lambda/h \approx 13.50$, where h is the height of the ridge. However, Ridge B transmits almost all of the

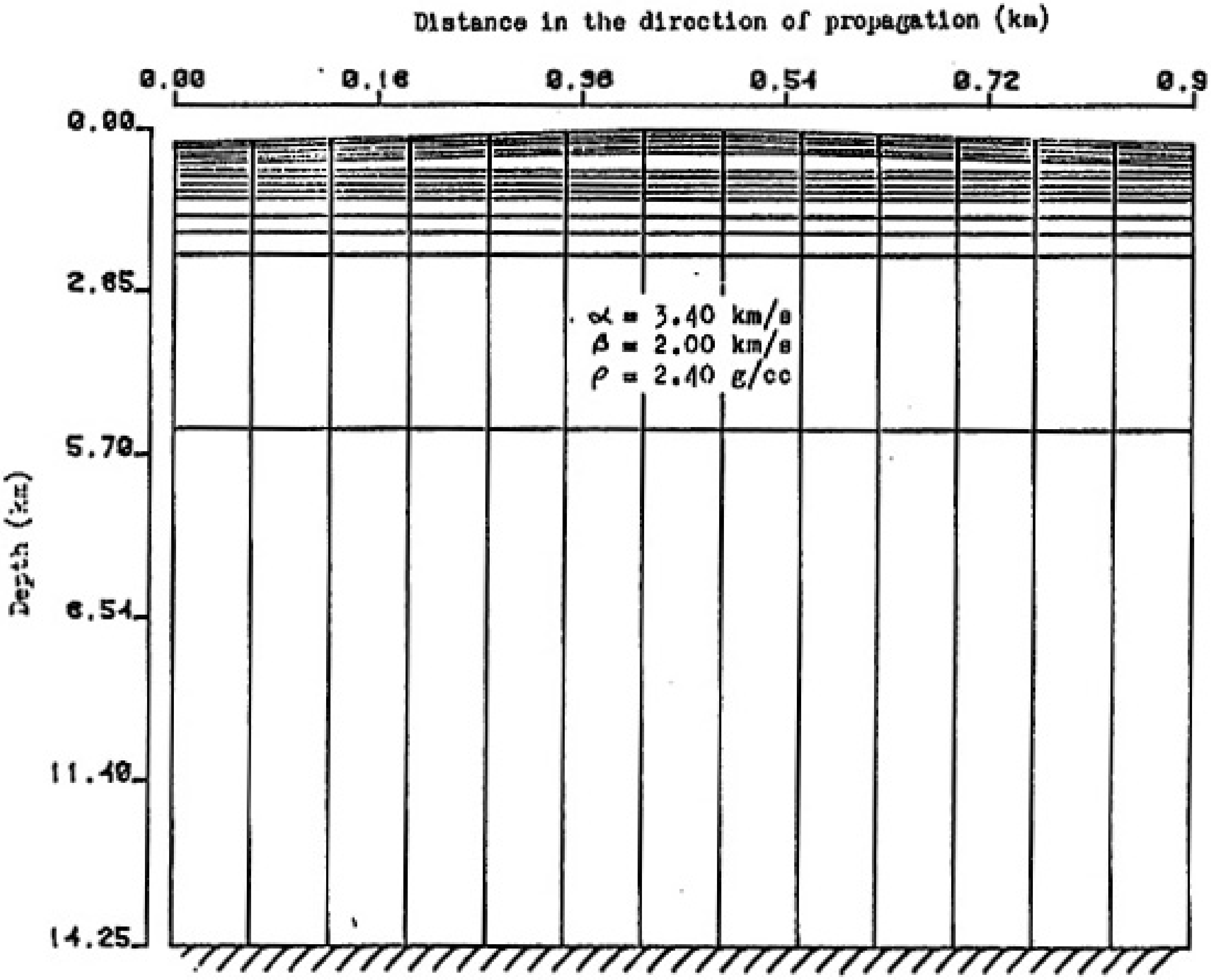

Figure 8. Ridge A model consisting of 182 elements arranged in 14 layers and 13 columns giving a total of 196 free nodes. All elements have identical elastic parameters. See Figure 9 for an enlargement of the upper 11 layers.

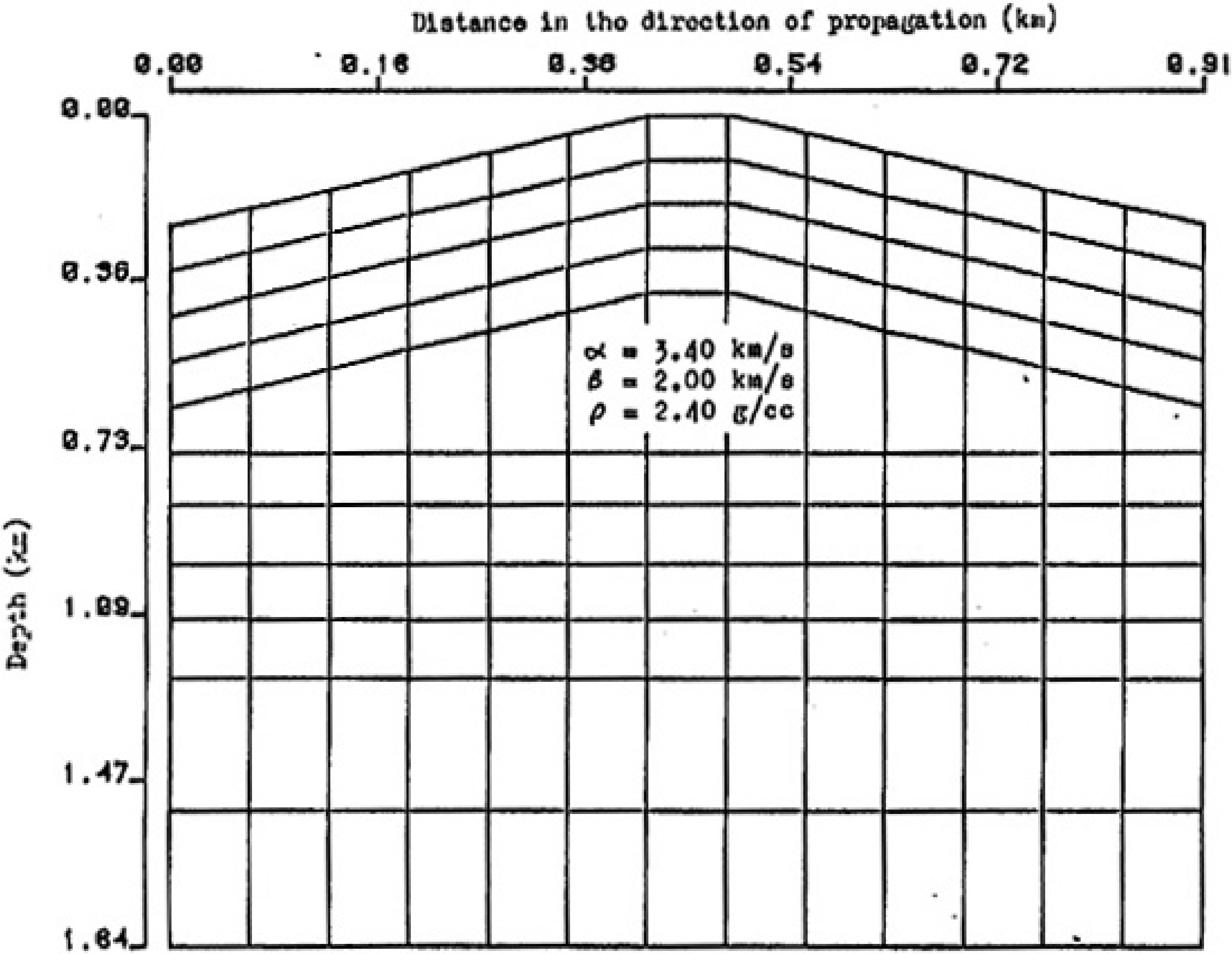

Figure 9. Upper 11 layers of Ridge A. The increase in thickness of Layer 5 (from the top) is probably responsible for the 'hole' in fundamental mode energy transmission at $\lambda/h \approx 13.50$ (Figure 11).

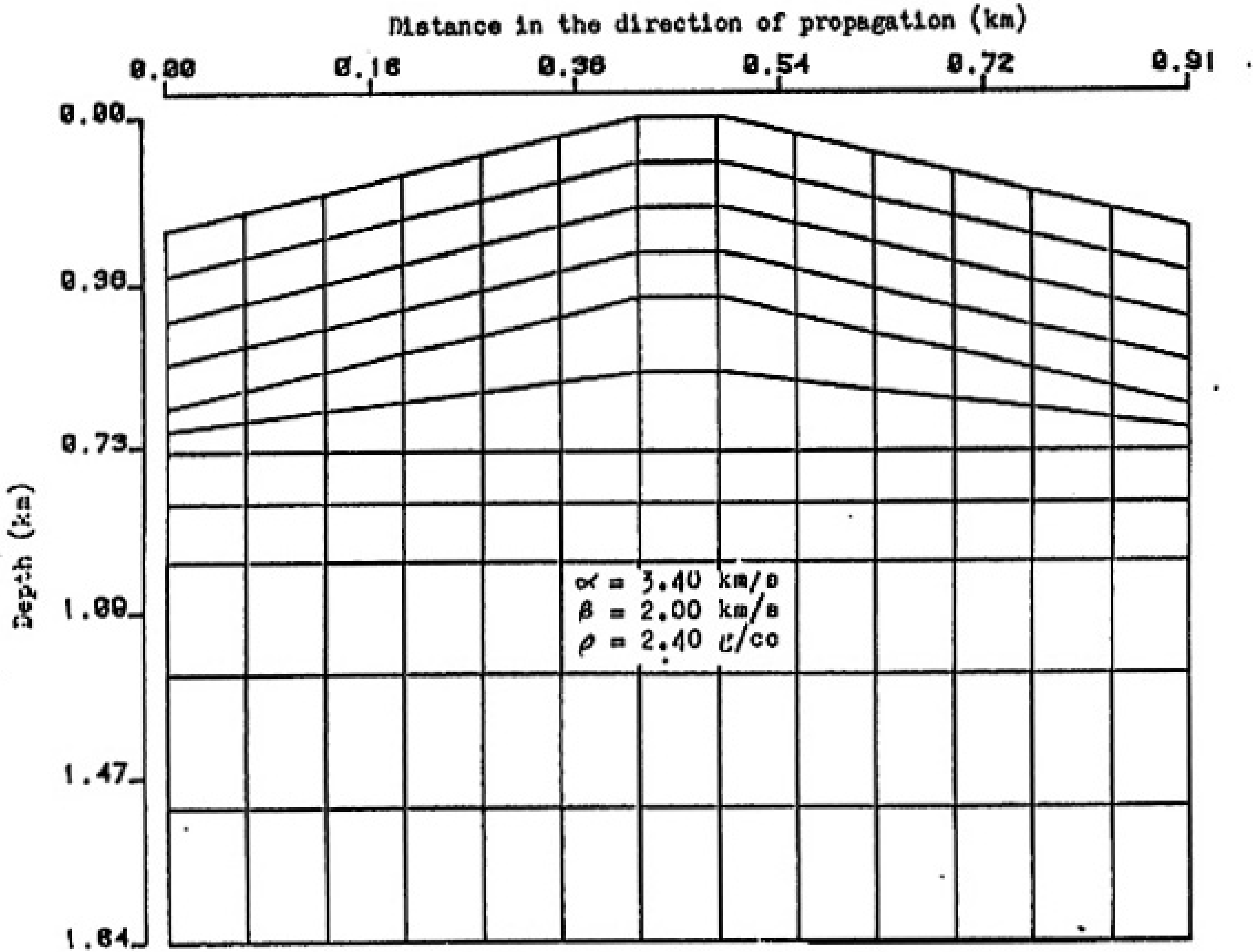

Figure 10. Upper 11 layers of Ridge B. Lower layers are identical to Ridge A. Note that the sudden increase in layer thickness between horizontal and non-horizontal parts of the model is averaged over two layers. Higher energy transmission in the fundamental mode results, compared to Ridge A (Figures 11 and 12).

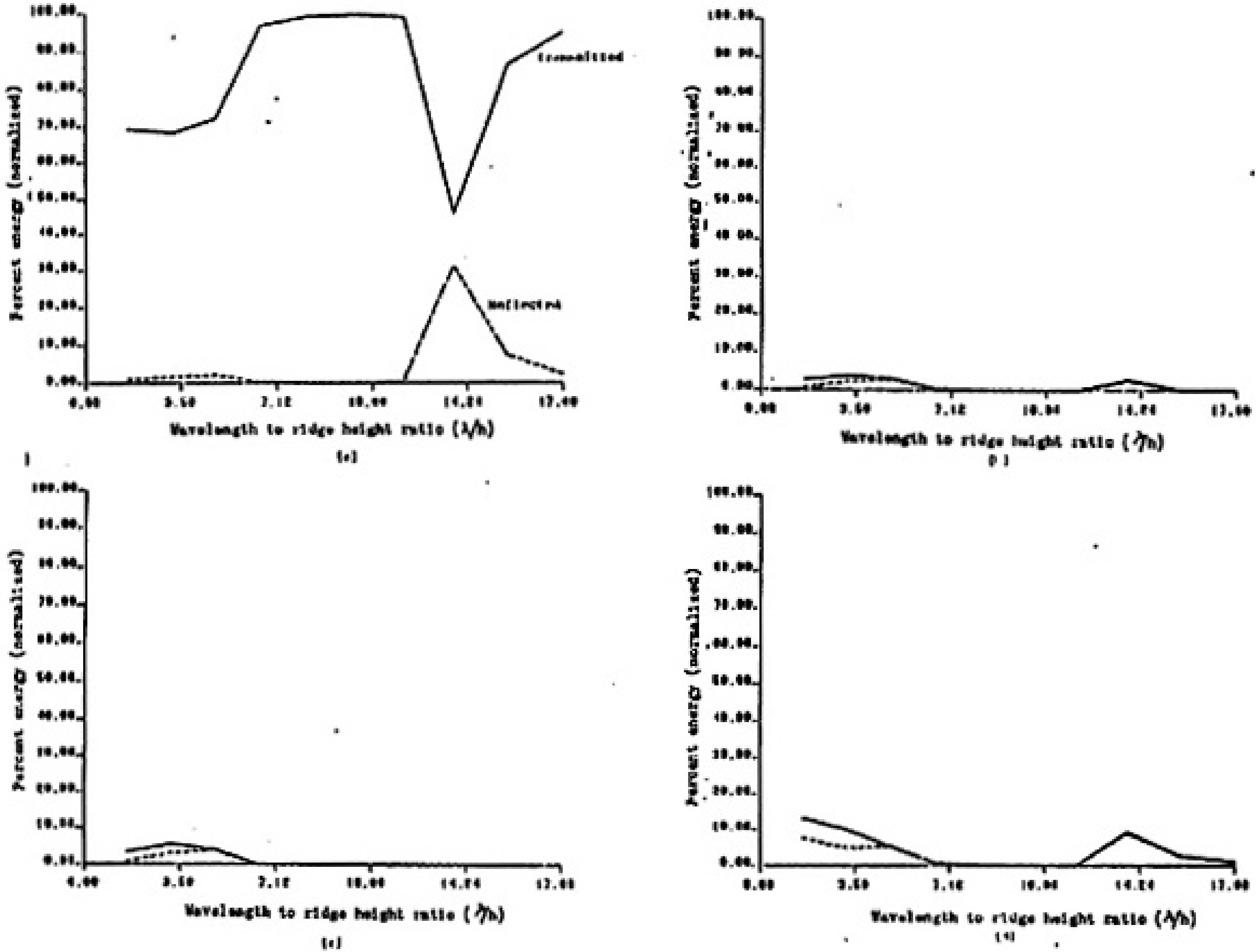

Figure 11. Love wave energy transmission and reflection for Ridge A (Figures 8 and 9) as a function of λ/h. (a) shows energy transmitted and reflected in the fundamental mode. The sharp dip in transmitted energy at $\lambda/h \approx 13.50$ probably results from inaccurately modeled displacements. (b) and (c) show transmitted and reflected energies in the 1st and 2nd higher modes. In both cases very little energy is transmitted or reflected. (d) shows energy propagating in modes above the second higher mode. Note the increase at $\lambda/h \approx 13.50$ and at short periods. See Table 3 for energy values.

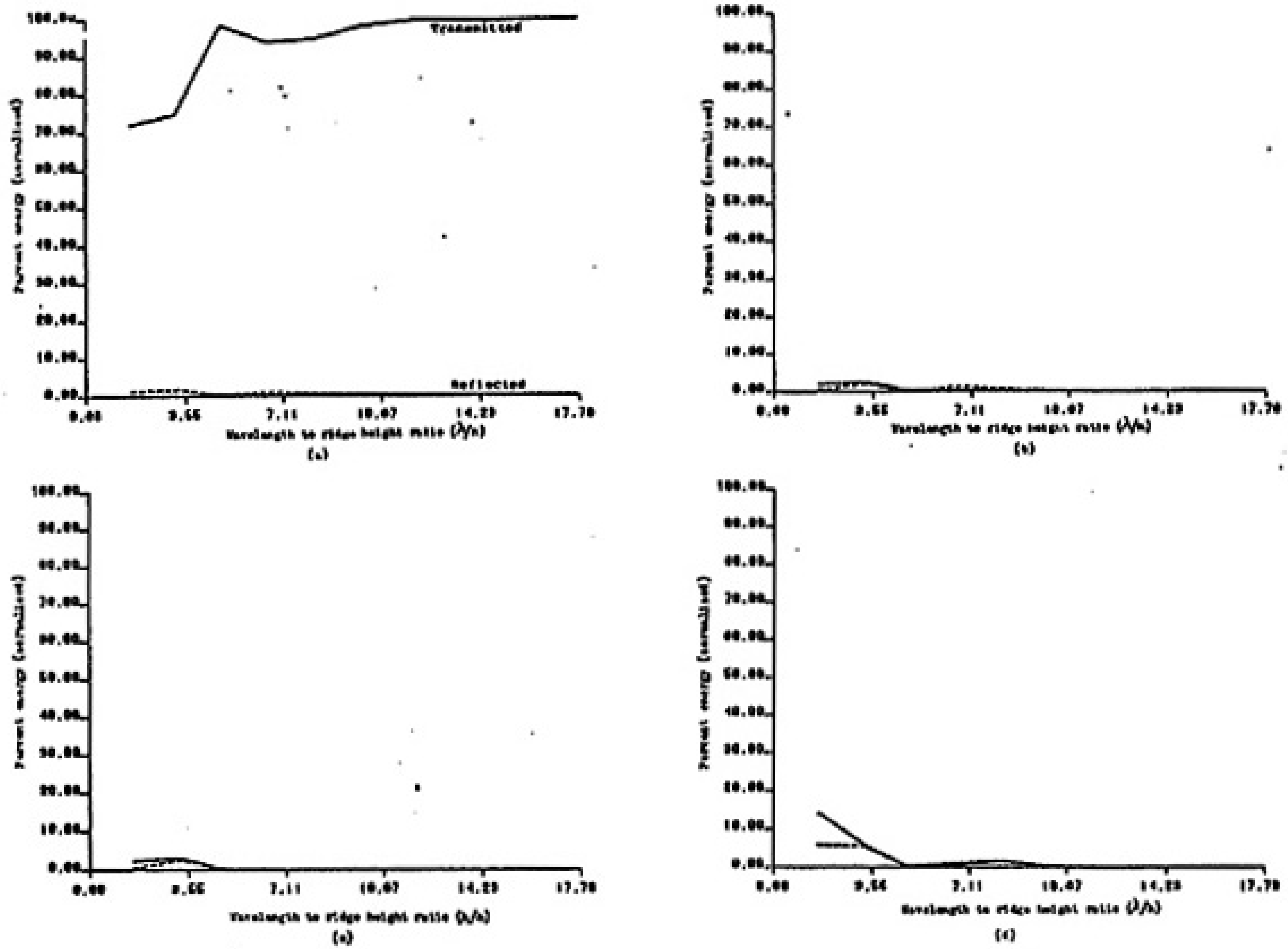

Figure 12. Energy transmission and reflection for Ridge B (Figure 10) as a function of λ/h. (a) shows the energy transmitted and reflected in the fundamental mode. Note no sharp energy dips occur near λ/h ≈ 13.50. This is probably due to a closer spacing of nodes in critical parts of the model. (b) and (c) show very little energy transmitted or reflected in the 1st and 2nd higher modes. (d) shows the energy in modes above the 2nd higher mode. Note no peaks occur near 3.7 seconds, although some energy is transmitted and reflected for short periods. See Table 4 for energy values.

TABLE 3
NORMALIZED ENERGY PERCENTAGES FROM RIDGE A

λ/h	Fundamental Trans	Fundamental Refl	1st Higher Trans	1st Higher Refl	2nd Higher Trans	2nd Higher Refl	Other Higher Trans	Other Higher Refl
1.60	69.34	1.31	3.04	1.00	3.43	1.05	13.12	7.71
3.22	68.16	1.94	3.98	2.42	5.55	3.11	9.94	4.90
4.85	72.22	2.40	3.10	3.09	4.18	4.15	5.45	5.41
6.52	96.93	0.43	0.10	0.62	0.01	0.04	0.65	1.22
8.24	99.43	0.17	0.00	0.00	----	----	0.02	0.38
10.00	99.89	0.01	0.03	0.02	----	----	0.00	0.04
11.83	99.06	0.42	0.17	0.07	----	----	0.19	0.00
13.74	45.64	30.88	2.79	2.61	----	----	9.30	8.78
15.72	87.06	7.49	----	----	----	----	2.53	2.92
17.80	95.15	2.61	----	----	----	----	1.01	1.23

TABLE 4
NORMALIZED ENERGY PERCENTAGES FROM RIDGE B

λ/h	Fundamental Trans	Fundamental Refl	1st Higher Trans	1st Higher Refl	2nd Higher Trans	2nd Higher Refl	Other Higher Trans	Other Higher Refl
1.60	72.02	1.42	2.30	0.82	2.47	0.85	14.06	6.06
3.22	74.91	1.93	2.92	2.26	3.39	2.66	6.26	5.67
4.85	98.19	0.34	0.38	0.39	0.23	0.26	0.09	0.29
6.52	93.98	1.27	0.63	1.62	0.07	0.14	0.87	1.42
8.24	94.76	0.74	0.20	0.91	----	----	1.72	1.67
10.00	98.14	0.53	0.12	0.43	----	----	0.26	0.54
11.83	99.76	0.16	0.00	0.04	----	----	0.00	0.04
13.74	99.59	0.26	0.01	0.03	----	----	0.03	0.09
15.72	99.71	0.18	----	----	----	----	0.02	0.03
17.79	99.80	0.12	----	----	----	----	0.02	0.05

incident energy in the fundamental mode at $\lambda/h \approx 13.50$. Also, the energy transmitted in the fundamental mode drops sharply for $\lambda/h \approx 7.0$, while that of Ridge B begins to drop off at $\lambda/h \approx 4.5$.

According to the 1/10 wavelength criterion, poor displacements should occur only below a λ/h of 6.65. Thus, the energy hole at $\lambda/h \approx 13.50$ is probably not the result of elements being too long. A more likely cause is a poor mode shape produced by the sudden change in thickness in the fifth layer of Ridge A (between the horizontal and non-horizontal parts of the model). In Ridge B, this thickness change is distributed over two layers. It is likely that the high percentage of incident energy transmitted in the fundamental mode for Ridge B is the result of a more accurate mode-shape approximation by the model. Ridge B has a closer spacing of nodes in the critical regions of rapidly changing structure and thus can provide a more detailed model. Interestingly, most of the energy lost by the transmitted fundamental mode in Ridge A at $\lambda/h \approx 13.50$ goes into the reflected fundamental mode and higher modes above the second higher mode. This distribution is similar to the sinusoidal valley model (Figure 2 and Table 1) considered earlier, although the energy percentages differ greatly. This observation will prove useful in the analysis of Love wave propagation through the Rio Grande rift, which shows a similar dip in fundamental mode energy transmission.

Two other characteristics of energy holes caused by modeling problems must be considered before a discussion of the final results. First, the periods at which energy minima occur are found to change with a slight rearrangement of vertical nodes. This lends support to the argument that the holes are created by poor mode-shape approximations at certain periods.

Second, large differences in the elastic element parameters of the elements were observed to lower the amount of energy transmitted in the fundamental mode. In some structures, significant changes in elastic parameters from element to element produced a deep energy hole (20-40%) over a narrow band of periods (0.2-0.5 seconds). To determine whether this energy drop is model-dependent, a slightly modified model with discontinuities 'cushioned' by thin transition elements should be examined. If a hole is the function of abrupt changes, it will disappear when discontinuities are spread out over a distance. At the same time, more accurate displacements are produced by a closer nodal spacing and this may also lead to the vanishing of the energy hole.

VI. Results of models

It is of interest to know the effects of complex geological structure and rugged topography on surface wave propagation in the Socorro region. Models of the Rio Grande rift and Magdalena Mountains were constructed to study the effects of their geology and topography on surface waves from a zone of microearthquakes near station SC, as well as from the Nevada Test Site (Figure 13).

1. Magdalena Mountains

This structure was chosen to evaluate the effect of a mountain range on surface waves with periods ranging from 1.0 to 6.0 seconds. The finite element structure modeled a 16 km line from station SC to the mouth of Hop Canyon (Figure 13). The path crosses irregular topography which has a total vertical relief of about 0.76 km. Although complex geology abounds, detailed structure could not be accurately modeled with the limited number of elements available. To simulate complex geology, an average of rock types within any particular element was formed from information in Chapin and others (1975), Sanford and others (1977), Sanford (1978), Rinehart (1979), Brown and others (1979), and Ward (1980). The resulting uncertainties in velocities are thought to be about 15%.

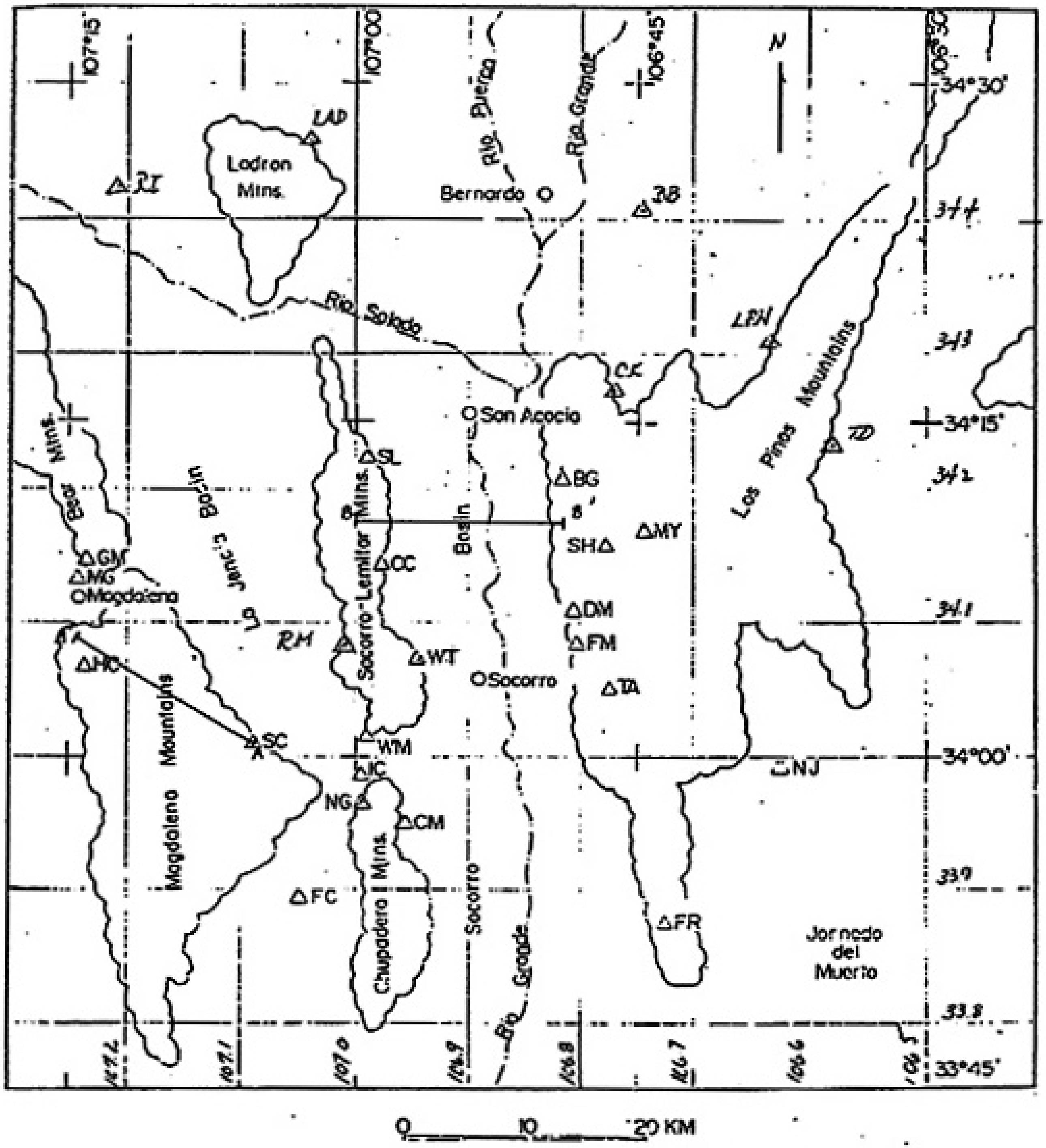

Figure 13. Map of Socorro area showing station locations and physiography. Magdalena Mountain model is along AA′ and Rio Grande rift model along BB′.

A second reason for choosing this line was that it closely parallels the raypath of surface waves propagating from events clustered in an active zone near SC (the intersection of the Morenci and Capitan lineaments) to stations HC and GM (Figure 13). Station GM shows unusual high-frequency surface waves on short period seismograms.

The smallest period used in this study was taken at the point where rapidly falling fundamental mode energies resembled the energy curves obtained from inaccurate models described in Section V. The period was then increased until most of the energy versus period curves showed complete (~100%) transmission of incident energy in the fundamental mode. The range 1.0 to 6.0 seconds was chosen on this basis, even though 1.0 second produced wavelengths significantly less than the suggested element-wavelength ratio of 0.1 (Lysmer and Drake, 1972). Results should be regarded carefully for this reason. In this model, maximum element lengths were about 30% of the wavelength at 1.00 seconds. While this might seem large, Lysmer and Drake (1971) demonstrated the tolerance of the element length condition for Love waves by using lengths 20% of the wavelength and finding only a 0.8% error in transmitted energies. Since the interesting energy transmission bands occur below 6 seconds for both Love and Rayleigh waves, it was decided to push the finite element method to its limit in investigating short periods, and then note any symptoms of inaccurate modeling.

A. Love wave analysis:

The structure for the Love wave analysis has 15 layers and 20 columns for a total of 300 elements (Figures 14 and 15). Element parameters for the upper 7 layers are presented in Table 5. Element lengths in the irregular zone are 0.8 km, and thus the 1/10 wavelength criterion of Lysmer and Drake is satisfied above a period of 2.5 seconds (assuming an average Love wave velocity of 3.17 km/s).

Results are shown in Figure 16 and Table 6. The fraction of incident energy transmitted in the fundamental mode is large for all periods, and increases slightly from about 90% at 1.0 second to nearly 100% at 6.0 seconds. Dips in the percentage of energy transmitted in the fundamental mode occur at 2.5 and 4.0 seconds and are probably the result of poor mode-shape approximations. This is suggested by their resemblence to holes in the energy curves of Ridge A, which were thought to be caused by poor mode-shape approximations. In this case, most of the converted incident energy goes into higher modes (above the 2nd), with little energy in the reflected fundamental mode. Thus, in general, it would be expected that nearly all incident Love

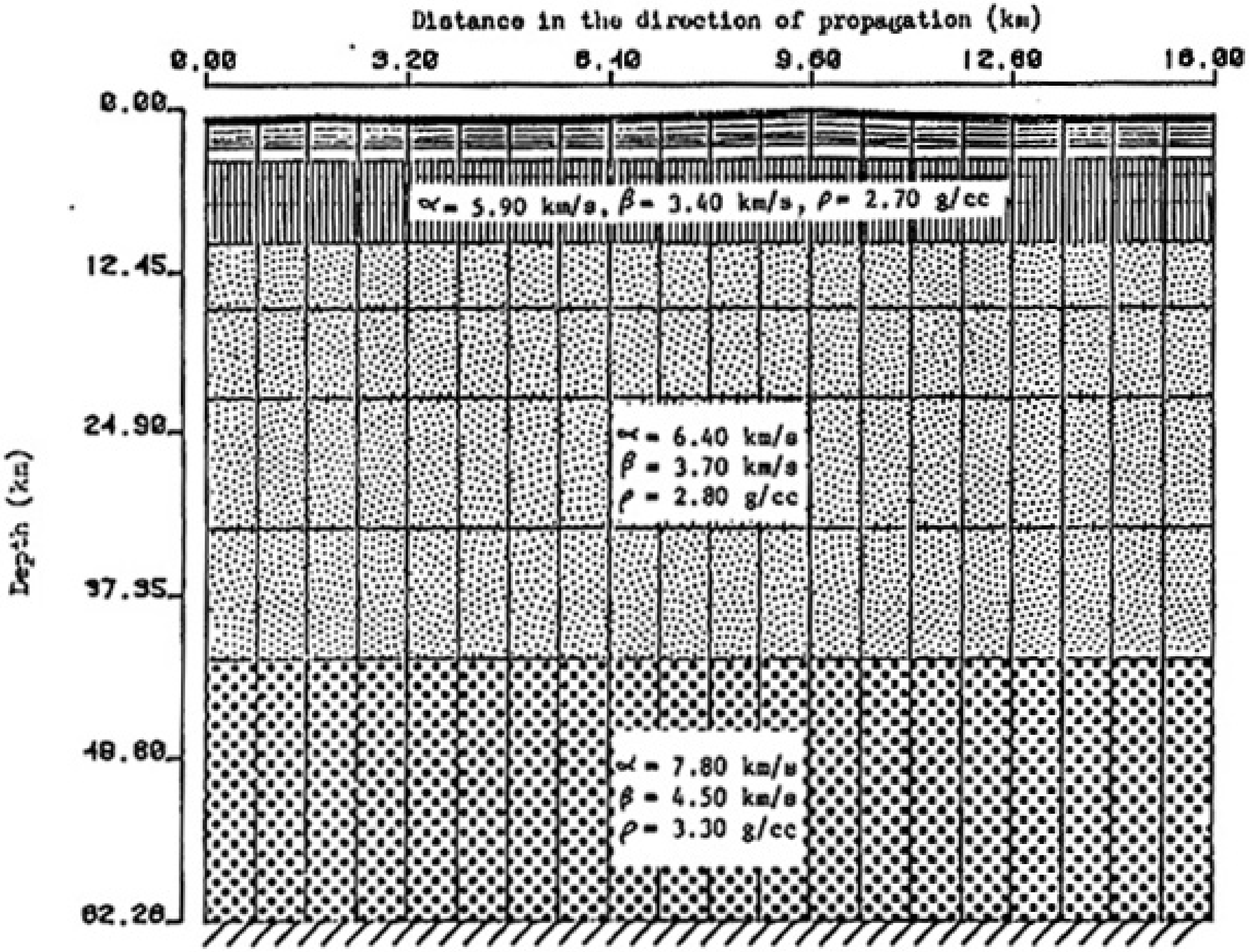

Figure 14. Love wave Magdalena Mountain model consisting of 300 elements in 15 layers and 20 columns giving a total of 315 free nodes. An enlarged view of the upper layers is presented in Figure 15.

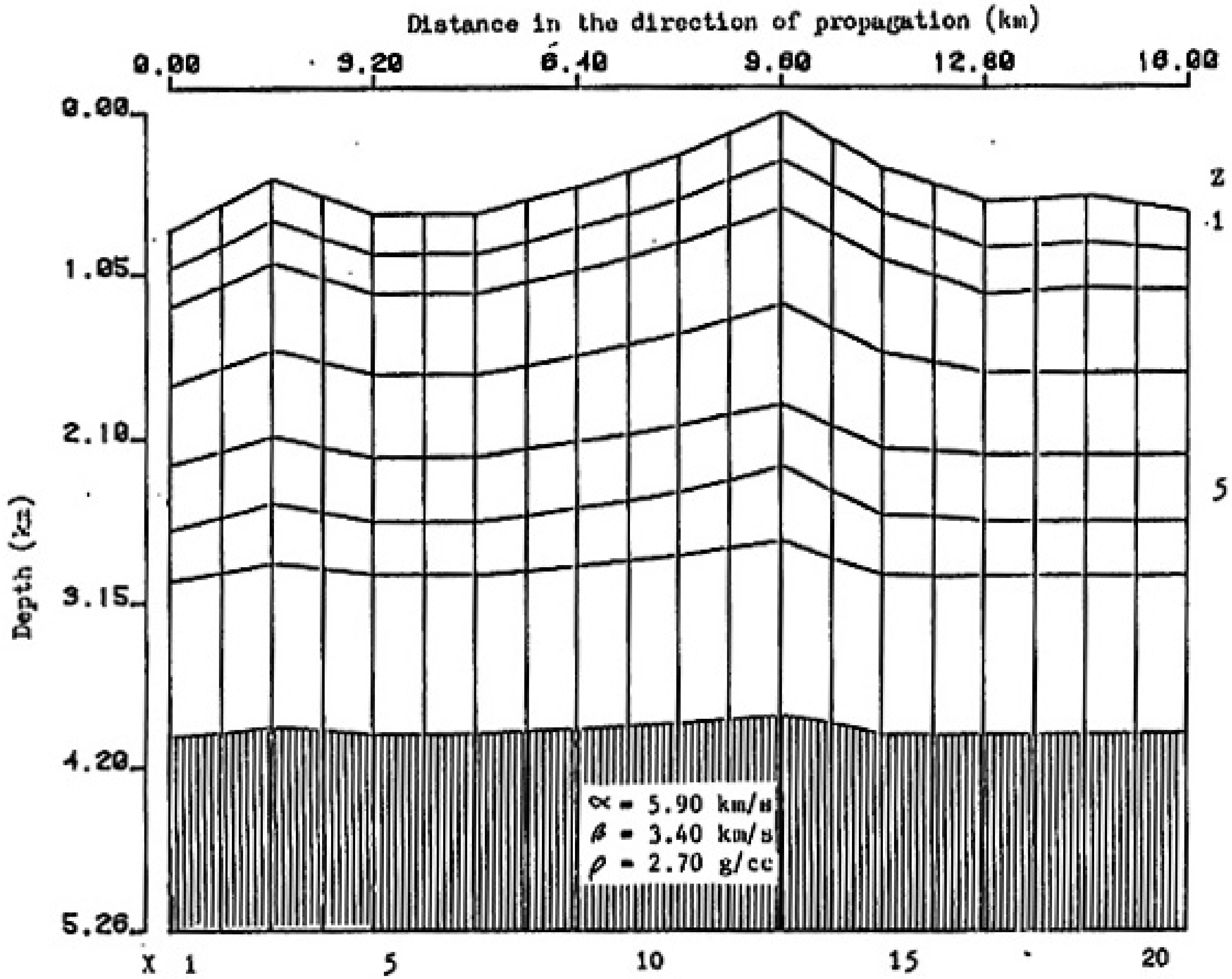

Figure 15. Upper 7 layers of Figure 14, showing detail. See Table 5 for parameters of each element in the upper 7 layers. The X axis designates the horizontal coordinate and the Z axis the vertical coordinate of the element as listed in Table 5.

TABLE 5
ELASTIC PARAMETERS FOR THE MAGDALENA MOUNTAIN MODEL (LOVE WAVES)

ELEMENT (X,Z)	ALPHA (KM/S)	EETA (KM/S)	RHO (G/CC)
(1, 1)	4.03	2.33	2.40
(1, 2)	4.23	2.33	2.40
(1, 3)	4.58	2.63	2.60
(1, 4)	5.20	3.00	2.65
(1, 5)	5.46	3.15	2.65
(1, 6)	5.46	3.15	2.65
(1, 7)	5.50	3.15	2.65
(2, 1)	4.31	2.49	2.45
(2, 2)	4.31	2.49	2.45
(2, 3)	4.50	2.60	2.50
(2, 4)	5.02	2.90	2.63
(2, 5)	5.46	3.15	2.65
(2, 6)	5.46	3.15	2.65
(2, 7)	5.90	3.15	2.65
(3, 1)	4.58	2.64	2.50
(3, 2)	4.58	2.64	2.50
(3, 3)	4.10	2.37	2.40
(3, 4)	4.84	2.80	2.60
(3, 5)	5.46	3.15	2.65
(3, 6)	5.46	3.15	2.65
(3, 7)	5.50	3.15	2.65
(4, 1)	4.00	2.31	2.40
(4, 2)	4.00	2.31	2.40
(4, 3)	4.46	2.58	2.50
(4, 4)	5.07	2.93	2.65
(4, 5)	5.46	3.15	2.65
(4, 6)	5.46	3.15	2.65
(4, 7)	5.50	3.15	2.65
(5, 1)	3.43	1.98	2.30
(5, 2)	3.43	1.90	2.30
(5, 3)	4.83	2.79	2.60
(5, 4)	5.30	3.06	2.70
(5, 5)	5.46	3.15	2.65
(5, 6)	5.46	3.15	2.65
(5, 7)	5.90	3.15	2.65
(6, 1)	4.13	2.38	2.45
(6, 2)	4.13	2.30	2.45
(6, 3)	4.94	2.85	2.60
(6, 4)	5.30	3.06	2.70
(6, 5)	5.46	3.15	2.65
(6, 6)	5.46	3.15	2.65
(6, 7)	5.90	3.15	2.65
(7, 1)	4.83	2.75	2.60
(7, 2)	4.83	2.75	2.60
(7, 3)	5.05	2.92	2.60
(7, 4)	5.32	3.06	2.70
(7, 5)	5.45	3.15	2.65
(7, 6)	5.46	3.15	2.65
(7, 7)	5.50	3.15	2.65
(8, 1)	4.83	2.79	2.60
(8, 2)	4.83	2.79	2.60
(8, 3)	5.05	2.92	2.60
(8, 4)	5.30	3.06	2.70
(8, 5)	5.45	3.15	2.65
(8, 6)	5.46	3.15	2.65
(8, 7)	5.90	3.15	2.65
(9, 1)	4.83	2.79	2.60
(9, 2)	4.83	2.79	2.60
(9, 3)	5.06	2.92	2.60
(5, 4)	5.30	3.06	2.70
(5, 5)	5.46	3.15	2.65
(5, 6)	5.46	3.15	2.65

TABLE 5 (CONTINUED)

ELASTIC PARAMETERS FOR THE MAGDALENA MOUNTAIN MODEL (LOVE WAVES)

ELEMENT (X,Z)	ALPHA (KM/S)	BETA (KM/S)	RHO (G/CC)
(9, 7)	5.90	3.15	2.65
(10, 1)	4.83	2.79	2.60
(10, 2)	4.83	2.79	2.60
(10, 3)	5.06	2.92	2.60
(10, 4)	5.30	3.06	2.70
(10, 5)	5.46	3.15	2.65
(10, 6)	5.46	3.15	2.65
(10, 7)	5.90	3.15	2.65
(11, 1)	4.83	2.79	2.60
(11, 2)	4.83	2.79	2.60
(11, 3)	5.06	2.92	2.60
(11, 4)	5.30	3.06	2.70
(11, 5)	5.46	3.15	2.65
(11, 6)	5.46	3.15	2.65
(11, 7)	5.90	3.15	2.65
(12, 1)	4.83	2.79	2.60
(12, 2)	4.83	2.79	2.60
(12, 3)	5.06	2.92	2.60
(12, 4)	5.30	3.06	2.70
(12, 5)	5.46	3.15	2.65
(12, 6)	5.46	3.15	2.65
(12, 7)	5.90	3.15	2.65
(13, 1)	4.83	2.79	2.60
(13, 2)	4.83	2.79	2.60
(13, 3)	5.06	2.92	2.60
(13, 4)	5.30	3.06	2.70
(13, 5)	5.46	3.15	2.65
(13, 6)	5.46	3.15	2.65
(13, 7)	5.90	3.15	2.65
(14, 1)	4.83	2.79	2.60
(14, 2)	4.83	2.79	2.60
(14, 3)	5.06	2.92	2.60
(14, 4)	5.30	3.06	2.70
(14, 5)	5.46	3.15	2.65
(14, 6)	5.46	3.15	2.65
(14, 7)	5.90	3.15	2.65
(15, 1)	4.83	2.79	2.60
(15, 2)	4.83	2.79	2.60
(15, 3)	5.06	2.92	2.60
(15, 4)	5.30	3.06	2.70
(15, 5)	5.46	3.15	2.65
(15, 6)	5.46	3.15	2.65
(15, 7)	5.90	3.15	2.65
(16, 1)	4.26	2.47	2.45
(16, 2)	4.26	2.47	2.45
(16, 3)	4.50	2.65	2.50
(16, 4)	5.07	2.93	2.65
(16, 5)	5.43	3.13	2.65
(16, 6)	5.43	3.13	2.65
(16, 7)	5.90	3.15	2.65
(17, 1)	3.70	2.14	2.30
(17, 2)	3.70	2.14	2.30
(17, 3)	4.10	2.37	2.40
(17, 4)	4.84	2.80	2.60
(17, 5)	5.39	3.11	2.65
(17, 6)	5.39	3.11	2.65
(17, 7)	5.90	3.15	2.65
(18, 1)	4.14	2.39	2.40
(18, 2)	4.14	2.39	2.40
(18, 3)	4.50	2.60	2.50
(18, 4)	5.07	2.90	2.65
(18, 5)	5.43	3.13	2.65
(18, 6)	5.43	3.13	2.65
(18, 7)	5.90	3.15	2.65
(19, 1)	4.50	2.64	2.50
(19, 2)	4.50	2.64	2.50
(19, 3)	4.90	2.83	2.60
(19, 4)	5.30	3.00	2.70
(19, 5)	5.46	3.15	2.65
(19, 6)	5.46	3.15	2.65
(19, 7)	5.90	3.15	2.65
(20, 1)	4.50	2.64	2.50
(20, 2)	4.50	2.64	2.50
(20, 3)	4.90	2.63	2.60
(20, 4)	5.30	3.00	2.70
(20, 5)	5.46	3.15	2.65
(20, 6)	5.46	3.15	2.65
(20, 7)	5.90	3.15	2.65

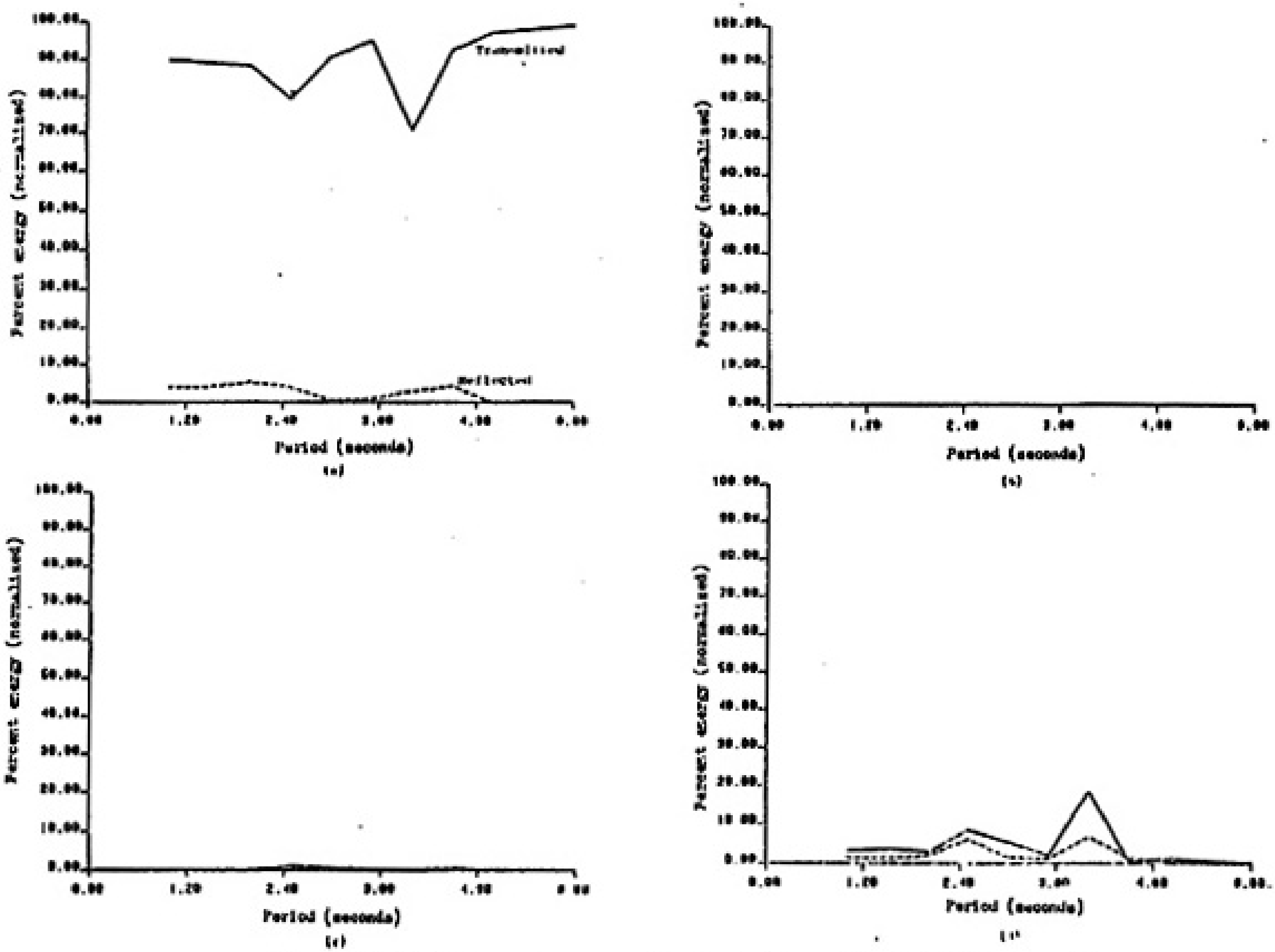

Figure 16. Love wave energy transmission and reflection for the Magdalena Mountain model (Figures 14 and 15). (a) shows energy in the fundamental mode. The energy dips for the transmitted fundamental mode at 2.5 and 4.5 seconds are believed to be caused by the model. Essentially no Love wave energy is transmitted or reflected in the 1st and 2nd higher modes as is shown in (b) and (c). (d) shows the transmitted and reflected energy in modes above the 2nd higher mode. Increases in both transmitted and reflected energy at 2.5 and 4.5 seconds coincide with minima for fundamental mode energy transmission. See Table 6 for energy values and phase velocities.

TABLE 6
NORMALIZED ENERGY PERCENTAGES AND
PHASE VELOCITIES FOR THE MAGDALENA MOUNTAIN MODEL (LOVE WAVES)

Period	Fundamental Trans	Fundamental Refl	1st Higher Trans	1st Higher Refl	2nd Higher Trans	2nd Higher Refl	Other Higher Trans	Other Higher Refl
1.00	90.17%	4.16	0.10	0.12	0.14	0.33	3.34	1.64
1.50	87.18	4.27	0.45	0.06	0.05	0.00	3.76	1.50
2.00	88.65	5.54	0.53	0.27	0.04	0.02	2.90	2.05
2.50	79.63	4.08	0.39	0.09	1.09	0.09	8.46	6.17
3.00	91.02	0.61	0.27	0.09	0.58	0.19	5.47	1.77
3.50	95.18	1.12	0.04	0.05	0.14	0.14	2.11	1.22
4.00	70.78	3.20	0.68	0.06	0.06	0.12	18.34	6.76
4.50	92.79	4.42	0.11	0.27	0.04	0.66	0.31	1.40
5.00	97.43	0.02	0.36	0.02	0.14	0.04	1.39	0.60
5.50	98.27	0.55	0.05	0.04	0.11	0.06	0.65	0.27
6.00	99.29	0.39	0.05	0.04	0.03	0.05	0.01	0.06

Period	Ph. vel.
1.00 sec	3.17 km/s
1.50	3.14
2.00	3.14
2.50	2.17
3.00	3.22
3.50	3.26
4.00	3.35
4.50	3.31
5.00	3.33
5.50	3.38
6.00	3.41

wave energy passes through the Magdalena Mountains in the fundamental mode for these periods (i.e., that the dips are artifacts of the modeling process).

Phase velocities for Love waves cluster around 3.20 km/s with the exception of a value of 2.17 km/s at a period of 2.50 seconds. As mentioned above, displacements at this period are probably inaccurate and thus velocities are probably also inaccurate.

B. Rayleigh wave analysis:

The structure used to model Rayleigh waves is shown in Figures 17 and 18; listings of parameters are presented in Table 7. The irregular zone consists of 13 layers and 10 horizontal elements. Since element lengths are 1.6 km, the 1/10 wavelength condition of Lysmer and Drake (1972) is met above a period of 5.6 seconds (assuming an average Rayleigh wave velocity of 2.9 km/s). Results are presented in Figure 19 and Table 8. The energy transmitted in the fundamental mode rises from near 90% to almost 100% over the period band examined (1.5-6.0 seconds). Although the direction of propagation was east-to-west, energies transmitted in the fundamental mode should be the same for propagation in a west-to-east direction (Lysmer and Drake, 1971).

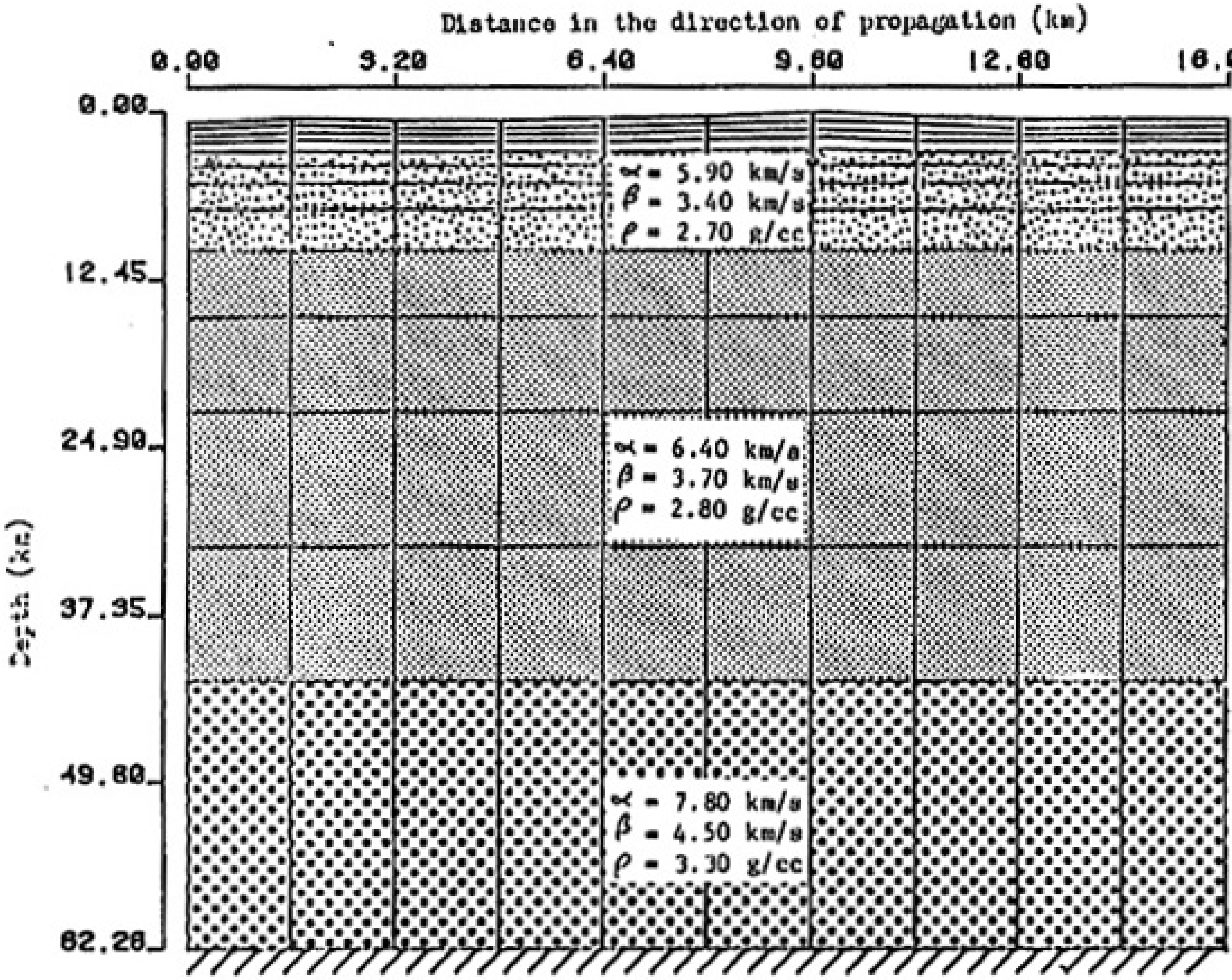

Figure 17. Magdalena Mountain model for Rayleigh waves consisting of 130 elements arranged in 13 layers and 10 columns giving a total of 143 free nodes. See Figure 18 for an enlargement of the upper 6 layers.

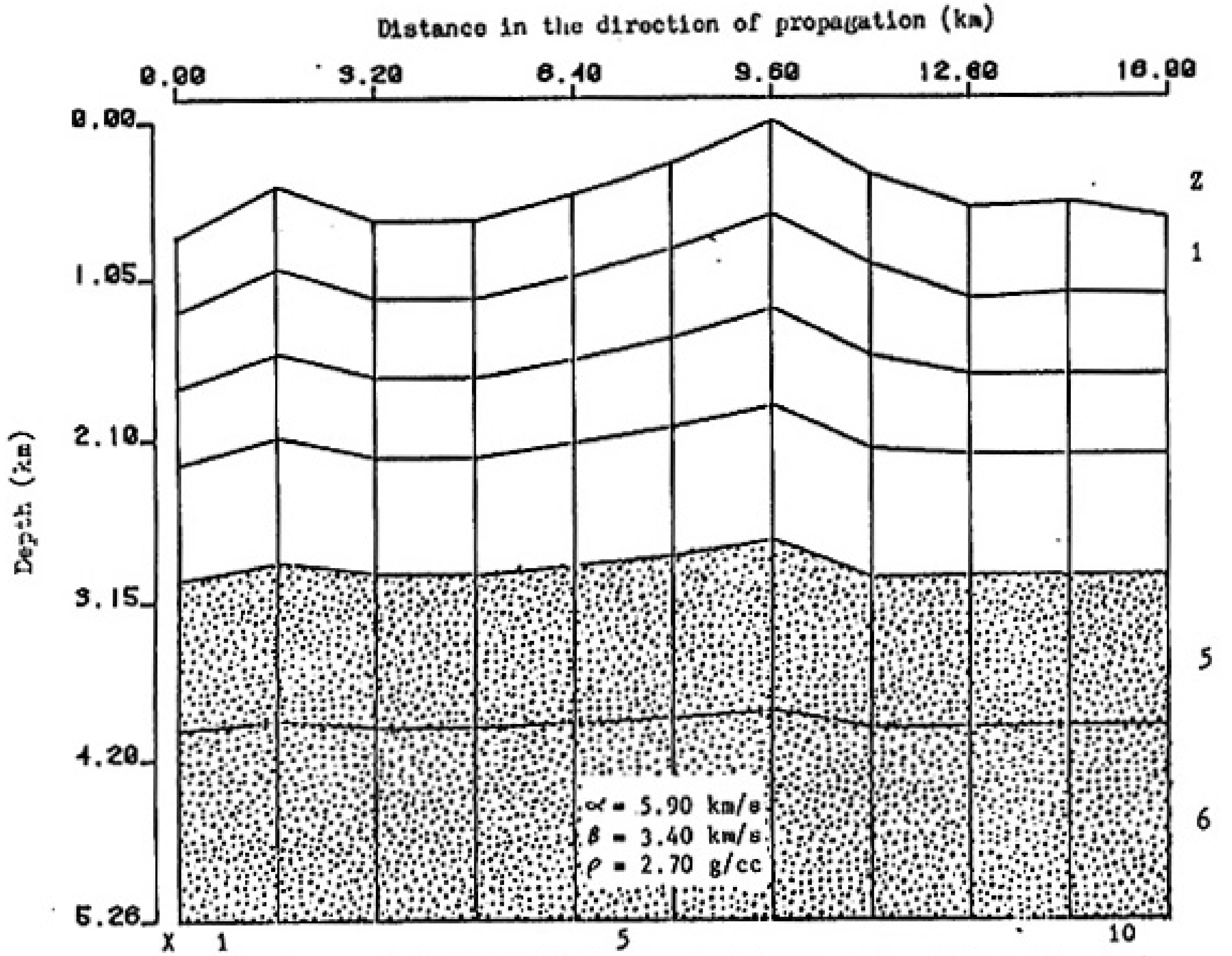

Figure 18. Upper 6 layers of the Rayleigh wave Magdalena Mountain model. See Table 7 for a listing of elastic parameters in the upper 4 layers. The X axis designates the horizontal coordinate and the Z axis the vertical coordinate of each element as listed in Table 7.

TABLE 7
ELASTIC PARAMETERS FOR THE MAGDALENA MOUNTAIN MODEL (RAYLEIGH WAVES)

ELEMENT (X,Z)	ALPHA (KM/S)	BETA (KM/S)	RHO (G/CC)
(1, 1)	4.03	2.33	2.40
(1, 2)	4.58	2.83	2.60
(1, 3)	5.20	3.00	2.63
(1, 4)	5.46	3.15	2.65
(2, 1)	4.38	2.64	2.50
(2, 2)	4.10	2.37	2.40
(2, 3)	4.84	2.80	2.60
(2, 4)	5.46	3.15	2.65
(3, 1)	3.43	1.98	2.30
(3, 2)	4.83	2.79	2.60
(3, 3)	5.30	3.06	2.70
(3, 4)	5.46	3.15	2.63
(4, 1)	4.83	2.79	2.60
(4, 2)	5.06	2.92	2.60
(4, 3)	5.30	3.06	2.70
(4, 4)	5.46	3.15	2.65
(5, 1)	4.83	2.79	2.60
(5, 2)	5.06	2.92	2.60
(5, 3)	5.30	3.06	2.70
(5, 4)	5.46	3.15	2.65
(6, 1)	4.03	2.79	2.60
(6, 2)	5.06	2.92	2.60
(6, 3)	5.30	3.06	2.70
(6, 4)	5.46	3.15	2.63
(7, 1)	4.83	2.79	2.60
(7, 2)	5.06	2.92	2.60
(7, 3)	5.30	3.06	2.70
(7, 4)	5.46	3.15	2.63
(8, 1)	4.83	2.79	2.60
(8, 2)	5.06	2.92	2.60
(8, 3)	5.30	3.06	2.70
(8, 4)	5.46	3.15	2.65
(9, 1)	3.70	2.14	2.30
(9, 2)	4.10	2.37	2.40
(9, 3)	4.84	2.80	2.60
(9, 4)	5.39	3.11	2.63
(10, 1)	4.58	2.64	2.50
(10, 2)	4.90	2.83	2.60
(10, 3)	5.30	3.00	2.70
(10, 4)	5.46	3.15	2.65

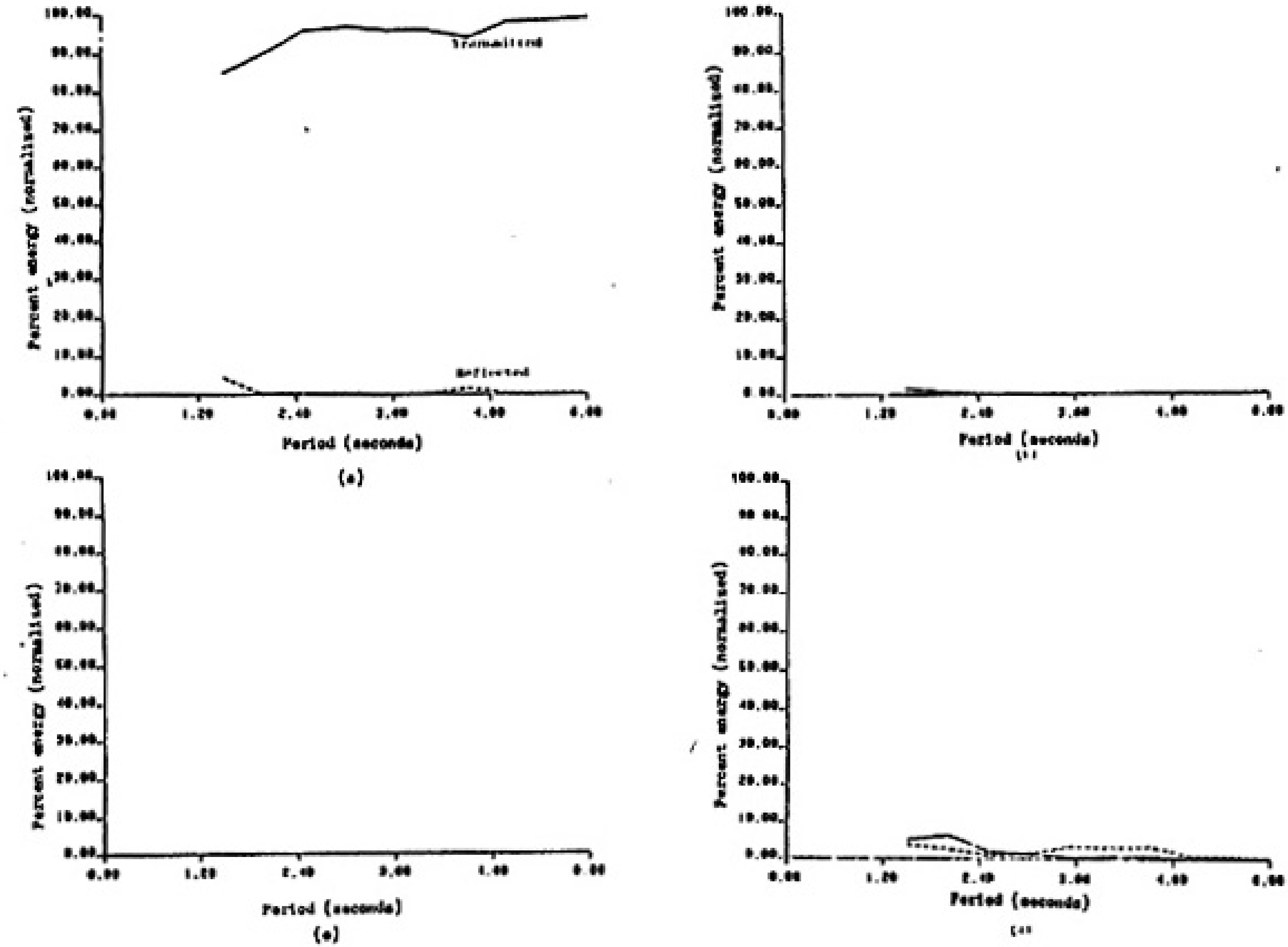

Figure 19. Energy transmission as a function of period for Rayleigh waves passing through the Magdalena Mountain model (Figures 17 and 18). (a) shows energy in the fundamental mode. Note that the amount of energy transmitted is greater than 80% of the incident energy for all periods examined. (b), (c), and (d) show that little energy is transmitted or reflected for the 1st and 2nd higher modes, and higher modes above the 2nd higher mode. See Table 8 for energy values and phase velocities.

TABLE 8

NORMALIZED ENERGY PERCENTAGES AND PHASE VELOCITIES FOR THE MAGDALENA MOUNTAIN MODEL (RAYLEIGH WAVES)

Period	Fundamental Trans	Fundamental Refl	1st Higher Trans	1st Higher Refl	2nd Higher Trans	2nd Higher Refl	Other Higher Trans	Other Higher Refl
1.50	85.01	4.21	1.57	0.15	0.03	0.01	5.19	3.83
2.00	90.15	0.14	0.71	0.06	0.02	0.00	6.27	2.65
2.50	95.91	0.65	0.12	0.00	0.04	0.00	2.14	1.14
3.00	96.61	0.46	0.03	0.00	0.02	0.00	1.28	1.62
3.50	95.78	0.06	0.01	0.00	0.02	0.00	0.66	3.48
4.00	95.65	0.39	0.01	0.00	0.02	0.00	0.85	3.10
4.50	93.87	1.71	----	----	----	----	0.86	3.56
5.00	98.01	0.47	0.01	0.06	0.01	0.00	0.35	1.09
5.50	98.22	0.34	0.01	0.01	0.07	0.00	0.44	0.91
6.00	98.96	0.25	0.01	0.01	0.09	0.01	0.16	0.51

Period	Ph. vel.
1.50 sec	3.20 km/s
2.00	3.17
2.50	2.16
3.00	2.99
3.50	3.00
4.00	3.32
4.50	----
5.00	2.33

Phase velocities for the Magdalena Mountain structure are presented in Table 8. Although a 1.16 km/s variation exists over the range of periods, no clear pattern emerges as to why this should be so. Phase velocities are highly model dependent and, in fact, these are less well determined than those of the Love wave case because fewer elements are involved.

2. Rio Grande rift

A series of models were constructed to examine surface wave propagation across the Rio Grande rift. The actual cross-section was chosen on the basis of a gravity survey by Sanford (1968); the line in this study closely parallels Sanford's and is 18 km long (from 106°47′W to 107°00′W at latitude 34°10′ as shown in Figure 13). Elastic parameters were chosen on the basis of Sanford's densities as well as work done by Sanford and others (1977), Sanford (1978), Brown and others (1979), and Rinehart (1979). While topographic variations are slight, geology is complex and valley alluvial fill has a sharp acoustic impedance against upfaulted granitic blocks bordering the rift. Here again, only major structural features can be modeled.

A. Love wave analysis:

The first Love wave model, Rift A, is shown in Figures 20 and 21 with parameters listed in Table 9. Element lengths are within the 1/10 wavelength criterion for periods above 2.44 seconds (assuming an average Love wave velocity of 3.28 km/s).

The results are presented in Figure 22 and tabulated in Table 10. A deep and rather broad dip in fundamental mode energy transmission occurs near 3.7 seconds. This is similar to the energy holes observed in the ridge models of Section V, although this is much broader.

Since model-dependent energy holes usually occur at different periods with slight changes in vertical structure, a second rift model, Rift B, was constructed (Figure 23) with the same elastic parameters for each element but with a slightly different nodal configuration.

Results from Rift B are presented in Figure 24 and Table 11. Note that the hole remains at the same period. A new, unexplained peak appears at 4.5 seconds. In addition, note that in both models the energy transmitted in the 1st higher mode grows at the expense of energy transmitted in the fundamental mode near 3.7 seconds. This energy distribtion is important since it distinguishes this energy dip from model-dependent holes where most of the energy ends

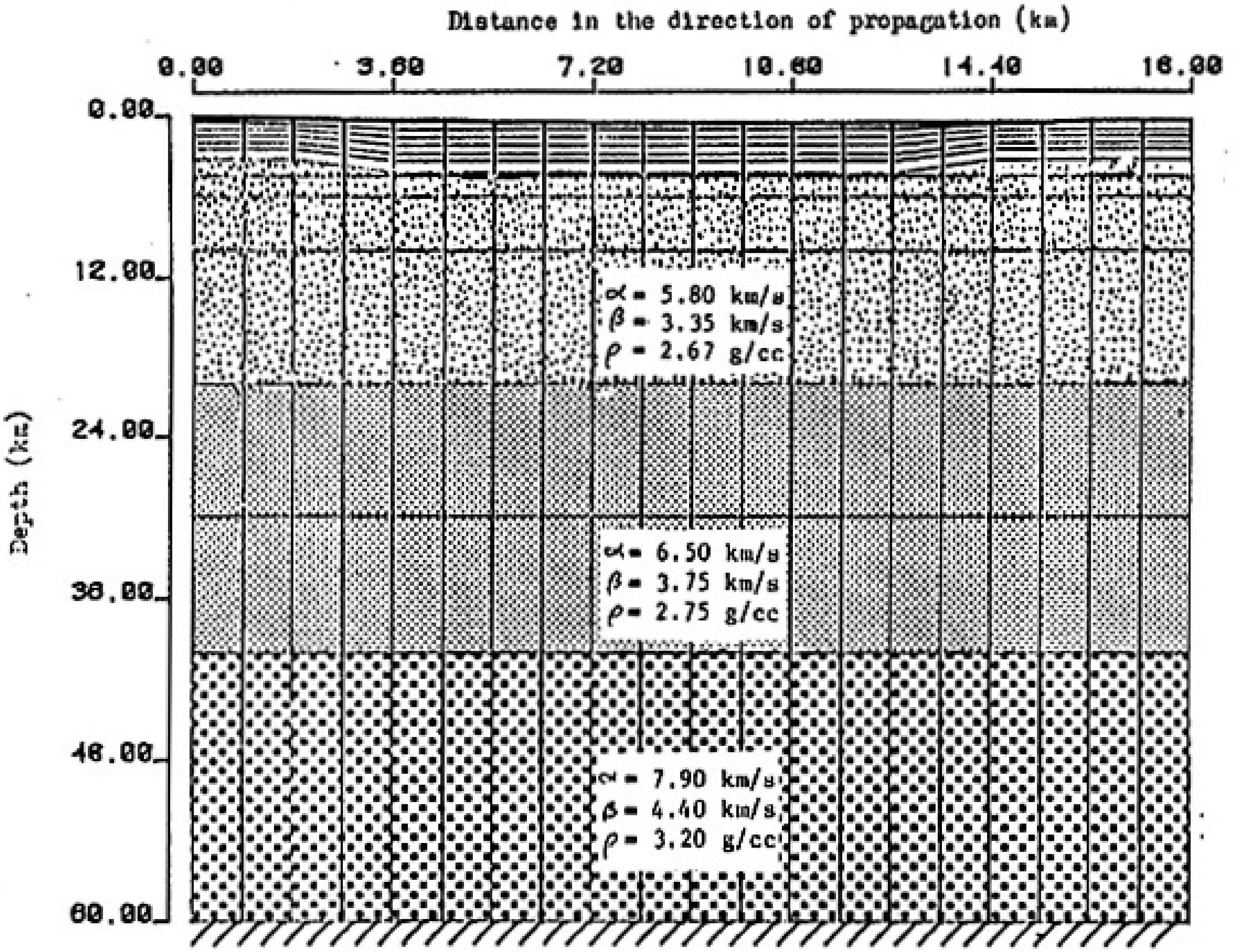

Figure 20. Rift A model for Love waves consisting of 280 elements in 14 layers and 20 columns. The total number of free nodes is, 294. See Figure 21 for an enlargemant of the upper 9 layers.

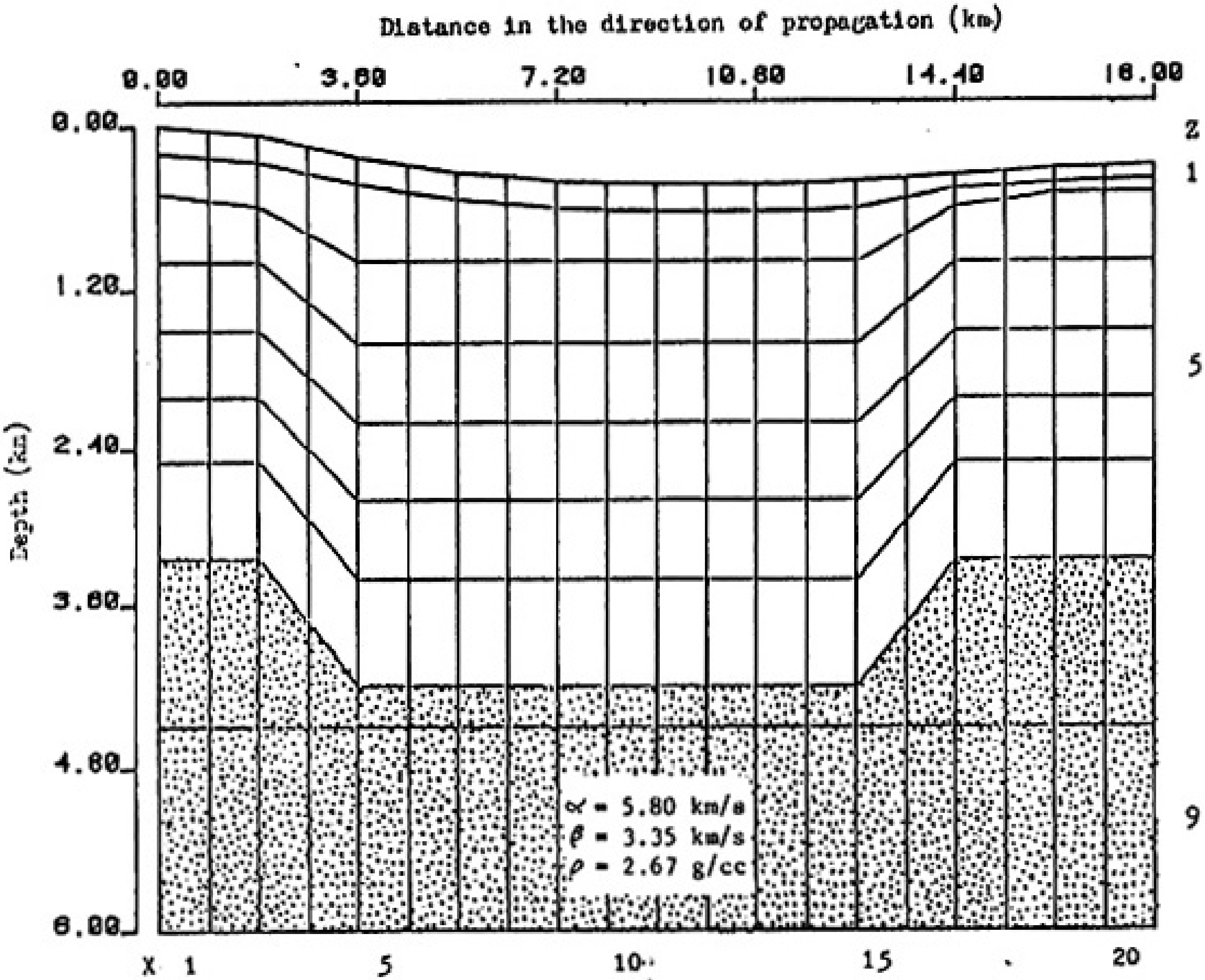

Figure 21. Upper 9 layers of the Rift A model. Note that in this case, as opposed to the Magdalena Mountain models, nodes follow geology. Element parameters for the upper 7 layers are presented in Table 9. The X axis designates the horizontal coordinate and the Z axis the vertical coordinate of each element as listed in Table 9.

TABLE 9
ELASTIC PARAMETERS FOR RIFT A

ELEMENT (X,Z)	ALPHA (KM/S)	BETA (KM/S)	RHO (G/CC)
(1, 1)	5.80	3.35	2.67
(1, 2)	5.80	3.35	2.67
(1, 3)	5.80	3.35	2.67
(1, 4)	5.80	3.35	2.67
(1, 5)	5.80	3.35	2.67
(1, 6)	5.80	3.35	2.67
(1, 7)	5.80	3.35	2.67
(2, 1)	4.30	2.49	2.44
(2, 2)	4.30	2.49	2.44
(2, 3)	4.45	2.57	2.49
(2, 4)	4.45	2.57	2.49
(2, 5)	5.23	3.01	2.59
(2, 6)	5.40	3.12	2.65
(2, 7)	5.80	3.35	2.67
(3, 1)	2.80	1.62	2.20
(3, 2)	2.80	1.62	2.20
(3, 3)	3.10	1.79	2.30
(3, 4)	3.10	1.79	2.30
(3, 5)	4.65	2.68	2.50
(3, 6)	5.00	2.89	2.63
(3, 7)	5.80	3.35	2.67
(4, 1)	2.80	1.62	2.20
(4, 2)	2.80	1.62	2.20
(4, 3)	3.10	1.79	2.30
(4, 4)	3.10	1.79	2.30
(4, 5)	4.43	2.55	2.45
(4, 6)	4.82	2.78	2.56
(4, 7)	5.30	3.06	2.62
(5, 1)	2.80	1.62	2.20
(5, 2)	2.80	1.62	2.20
(5, 3)	3.10	1.79	2.30
(5, 4)	3.10	1.79	2.30
(5, 5)	4.20	2.42	2.40
(5, 6)	4.65	2.68	2.50
(5, 7)	4.80	2.77	2.57
(6, 1)	2.80	1.62	2.20
(6, 2)	2.80	1.62	2.20
(6, 3)	3.10	1.79	2.30
(6, 4)	3.10	1.79	2.30
(6, 5)	4.20	2.42	2.40
(6, 6)	4.65	2.68	2.50
(6, 7)	4.80	2.77	2.57
(7, 1)	2.80	1.62	2.20
(7, 2)	2.80	1.62	2.20
(7, 3)	3.10	1.79	2.30
(7, 4)	3.10	1.79	2.30
(7, 5)	4.20	2.42	2.40
(7, 6)	4.65	2.68	2.50
(7, 7)	4.80	2.77	2.57
(8, 1)	2.80	1.62	2.20
(8, 2)	2.80	1.62	2.20
(8, 3)	3.10	1.79	2.30
(8, 4)	3.10	1.79	2.30
(8, 5)	4.20	2.42	2.40
(8, 6)	4.65	2.68	2.50
(8, 7)	4.80	2.77	2.57
(9, 1)	2.80	1.62	2.20
(9, 2)	2.80	1.62	2.20
(9, 3)	3.10	1.79	2.30
(9, 4)	3.10	1.79	2.30
(9, 5)	4.20	2.42	2.40
(9, 6)	4.65	2.68	2.50

TABLE 9 (CONTINUED)
ELASTIC PARAMETERS FOR RIFT A

ELEMENT (X,Z)	ALPHA (KM/S)	BETA (KM/S)	RHO (G/CC)
(9, 7)	4.80	2.77	2.57
(10, 1)	2.80	1.62	2.20
(10, 2)	2.80	1.62	2.20
(10, 3)	3.10	1.79	2.30
(10, 4)	3.10	1.79	2.30
(10, 5)	4.20	2.42	2.40
(10, 6)	4.65	2.68	2.50
(10, 7)	4.80	2.77	2.57
(11, 1)	2.80	1.62	2.20
(11, 2)	2.80	1.62	2.20
(11, 3)	3.10	1.79	2.30
(11, 4)	3.10	1.79	2.30
(11, 5)	4.20	2.42	2.40
(11, 6)	4.65	2.68	2.50
(11, 7)	4.80	2.77	2.57
(12, 1)	2.80	1.62	2.20
(12, 2)	2.80	1.62	2.20
(12, 3)	3.10	1.79	2.30
(12, 4)	3.10	1.79	2.30
(12, 5)	4.20	2.42	2.40
(12, 6)	4.65	2.68	2.50
(12, 7)	4.80	2.77	2.57
(13, 1)	2.80	1.62	2.20
(13, 2)	2.80	1.62	2.20
(13, 3)	3.10	1.79	2.30
(13, 4)	3.10	1.79	2.30
(13, 5)	4.20	2.42	2.40
(13, 6)	4.65	2.68	2.50
(13, 7)	4.80	2.77	2.57
(14, 1)	2.80	1.62	2.20
(14, 2)	2.80	1.62	2.20
(14, 3)	3.10	1.79	2.30
(14, 4)	3.10	1.79	2.30
(14, 5)	4.43	2.55	2.45
(14, 6)	4.82	2.78	2.56
(14, 7)	5.30	3.06	2.62
(15, 1)	2.80	1.62	2.20
(15, 2)	2.80	1.62	2.20
(15, 3)	3.10	1.79	2.30
(15, 4)	3.10	1.79	2.30
(15, 5)	4.65	2.68	2.50
(15, 6)	5.00	2.89	2.63
(15, 7)	5.80	3.35	2.67
(16, 1)	3.75	2.16	2.37
(16, 2)	3.75	2.16	2.37
(16, 3)	3.90	2.25	2.41
(16, 4)	3.90	2.25	2.41
(16, 5)	5.23	3.01	2.59
(16, 6)	5.40	3.12	2.65
(16, 7)	5.80	3.35	2.67
(17, 1)	4.70	2.71	2.53
(17, 2)	4.70	2.71	2.53
(17, 3)	4.70	2.71	2.53
(17, 4)	4.70	2.71	2.53
(17, 5)	5.80	3.35	2.67
(17, 6)	5.80	3.35	2.67
(17, 7)	5.80	3.35	2.67
(18, 1)	4.75	2.71	2.53
(18, 2)	4.75	2.71	2.53
(18, 3)	4.70	2.71	2.53
(18, 4)	4.70	2.71	2.53
(18, 5)	5.80	3.35	2.67
(18, 6)	5.80	3.35	2.67
(18, 7)	5.80	3.35	2.67
(19, 1)	4.79	2.71	2.53
(19, 2)	4.79	2.71	2.53
(19, 3)	4.70	2.71	2.53
(19, 4)	4.70	2.71	2.53
(19, 5)	5.80	3.35	2.67
(19, 6)	5.80	3.35	2.67
(19, 7)	5.80	3.35	2.67
(20, 1)	4.79	2.71	2.53
(20, 2)	4.79	2.71	2.53
(20, 3)	4.70	2.71	2.53
(20, 4)	4.70	2.71	2.53
(20, 5)	5.80	3.35	2.67
(20, 6)	5.80	3.35	2.67
(20, 7)	5.80	3.35	2.67

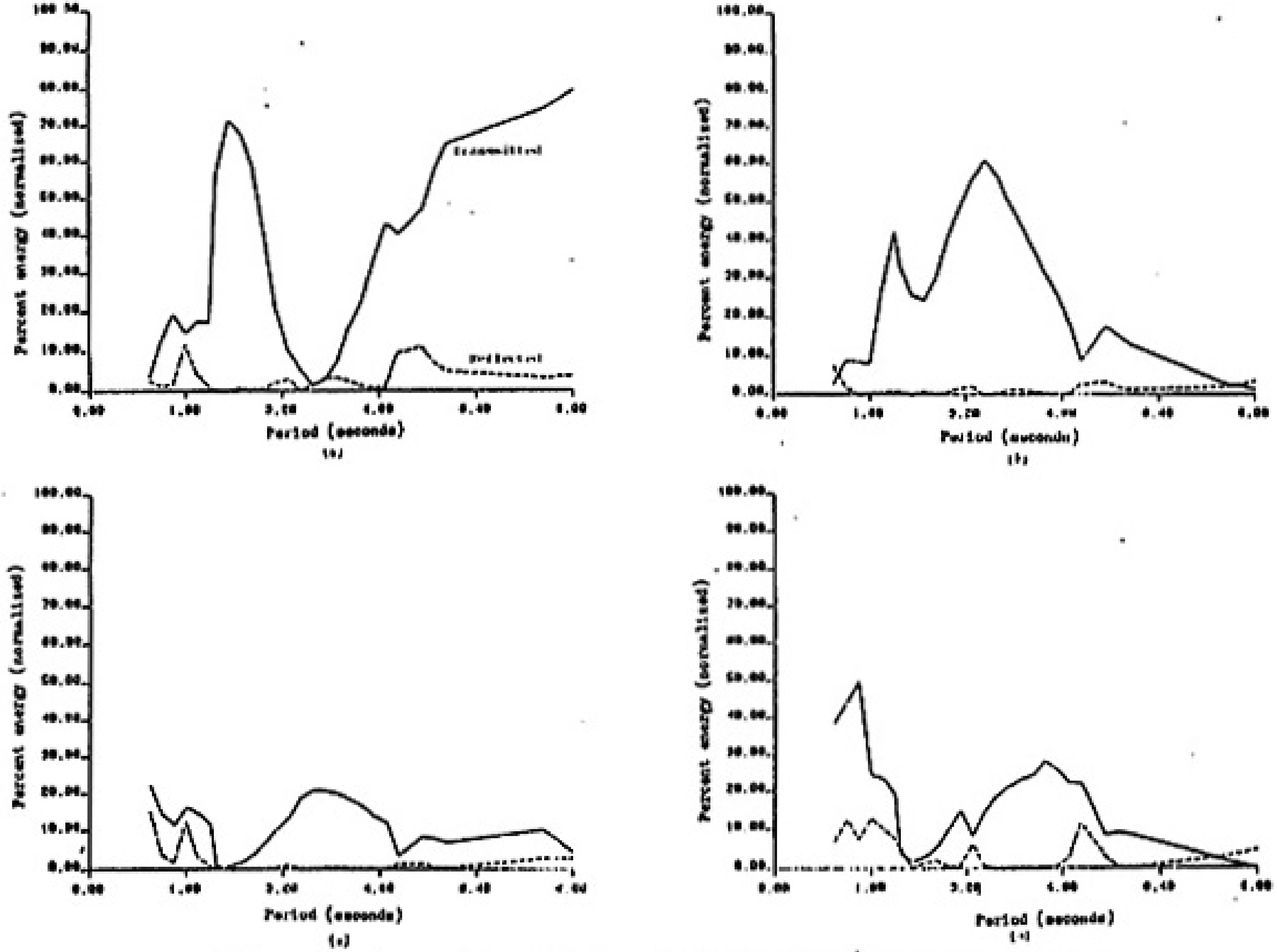

Figure 22. Energy transmitted and reflected as a function of period for Love waves propagating through the Rift A model (Figures 20 and 21). (a) shows energy in the fundamental mode. The minimum transmitted energy occurs near 3.7 seconds. (b) shows energy in the 1st higher mode. Note that most of the energy lost by the transmitted fundamental mode near 3.7 seconds is converted into the transmitted 1st higher mode. (c) shows energy in the 2nd higher mode. (d) includes energy transmitted and reflected in modes above the 2nd higher mode. Note the rapid rise in transmitted and reflected energy near 4.5 seconds, which is at a slightly longer period than the transmitted fundamental mode energy minumum. See Table 10 for energy values and phase velocities.

TABLE 10

NORMALIZED ENERGY PERCENTAGES AND PHASE VELOCITIES FROM RIFT A

Period	Fundamental Trans	Fundamental Refl	1st Higher Trans	1st Higher Refl	2nd Higher Trans	2nd Higher Refl	Other Higher Trans	Other Higher Refl
1.00	3.87%	2.47	2.94	7.57	22.23	15.05	30.95	6.92
1.20	13.07	1.41	8.84	1.75	14.17	3.67	44.52	12.37
1.40	19.32	1.90	8.65	0.10	11.42	1.69	49.61	7.39
1.60	14.56	11.80	7.95	0.40	15.96	12.31	24.35	12.67
1.80	17.40	4.34	24.96	0.47	14.32	3.06	23.26	10.17
2.00	17.34	0.95	42.33	1.09	11.35	0.74	18.79	7.41
2.10	57.04	0.13	32.82	0.75	1.02	0.55	3.27	4.42
2.30	71.50	0.04	25.62	0.04	0.83	0.02	1.80	0.15
2.50	67.67	0.80	24.46	0.72	1.53	0.19	3.04	1.59
2.70	50.51	0.37	30.11	0.18	3.40	0.10	5.20	2.05
2.90	41.24	0.68	40.77	0.58	6.69	0.16	9.44	0.25
3.10	21.01	2.28	40.88	1.73	10.15	0.56	14.39	0.00
3.30	10.10	3.01	56.10	2.05	13.40	1.07	8.20	6.07
3.50	5.23	0.15	60.83	0.05	18.81	0.01	14.49	0.43
3.70	1.46	1.63	56.86	0.54	20.72	0.13	18.63	0.03
3.90	2.80	3.29	50.22	1.24	20.67	0.46	21.15	0.17
4.10	7.53	3.52	44.10	1.21	19.91	0.46	23.10	0.17
4.30	16.25	2.56	37.67	0.72	18.20	0.22	24.29	0.09
4.50	21.87	1.48	31.31	0.49	16.54	0.31	27.99	0.11
4.70	32.82	0.77	25.83	0.21	13.71	0.12	25.89	0.65
4.90	43.01	1.09	18.49	0.31	11.91	0.26	22.26	2.67
5.10	40.39	9.67	8.98	2.70	3.63	1.37	21.97	11.29
5.50	46.35	11.26	17.51	3.28	8.20	1.45	8.35	3.02
5.70	57.64	7.15	15.17	1.98	7.71	0.86	9.07	0.42
5.90	64.73	5.03	13.04	1.28	6.81	0.51	8.42	0.18
7.50	74.22	3.27	2.72	2.10	10.18	2.81	1.66	3.04
8.00	79.42	3.89	1.28	3.55	4.49	2.91	0.00	4.47

Period	Ph. vel.
2.10 sec	3.28 km/s
2.30	3.36
2.50	3.47
2.70	3.56
2.90	3.69
3.10	3.82
3.30	4.10
3.50	4.39
3.70	2.53
3.90	2.76
4.10	2.85
4.30	2.90
4.50	2.96
4.70	2.98
4.90	3.03
5.10	3.15
5.30	2.97
5.50	3.45
5.70	3.17
5.90	3.20

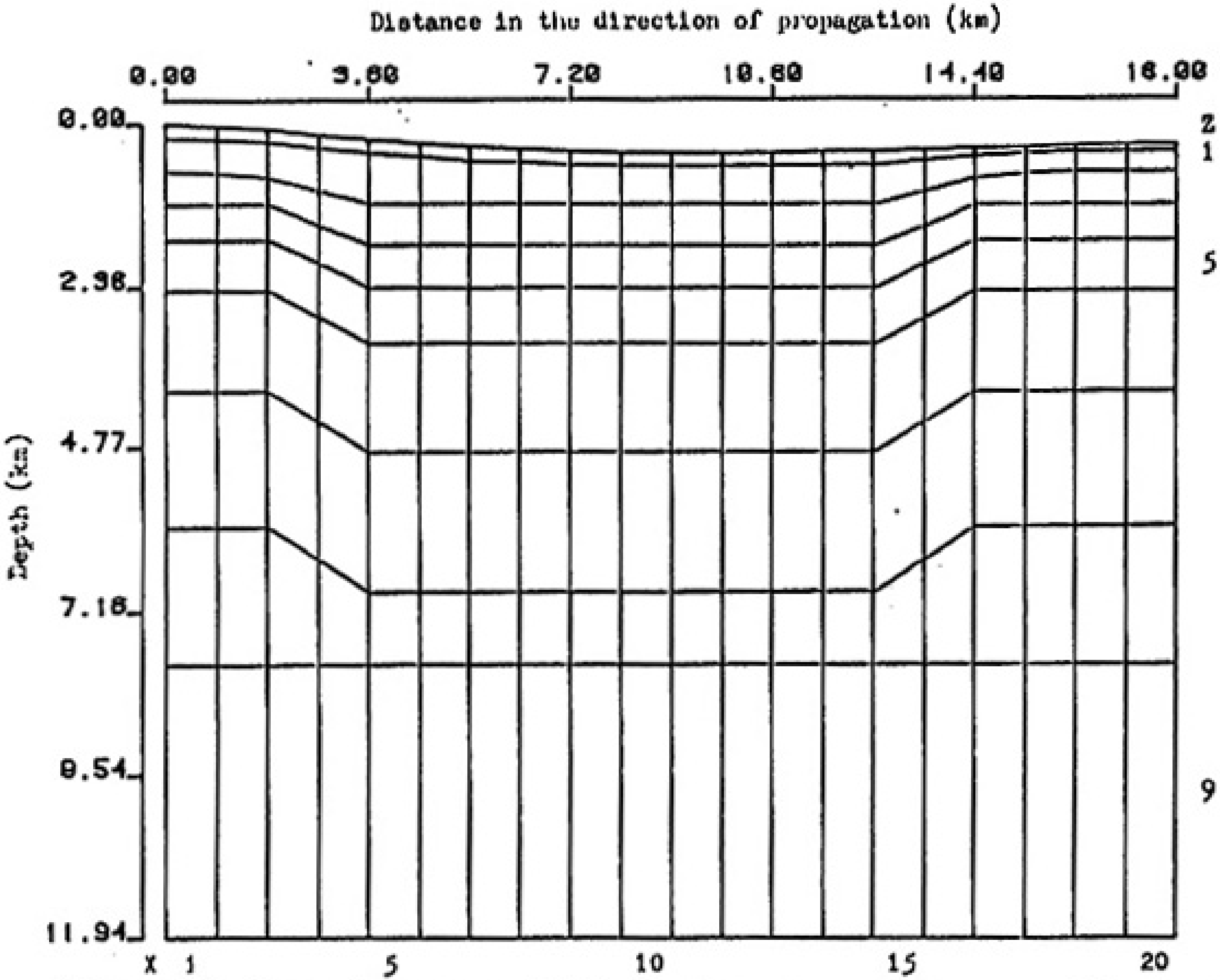

Figure 23. Upper 9 layers of Rift B. Element parameters are identical to those used in Rift A (Table 9) as is the structure below 11.94 km. Positions of nodes, however, have been changed. The X axis designates the horizontal coordinate and the Z axis the vertical coordinate of each element as listed in Table 9. It was found from Rift B that the period for minimum energy transmission in the fundamental mode did not change with a new nodal point configuration. This strengthens the argument that the energy minimum is real (Figure 24).

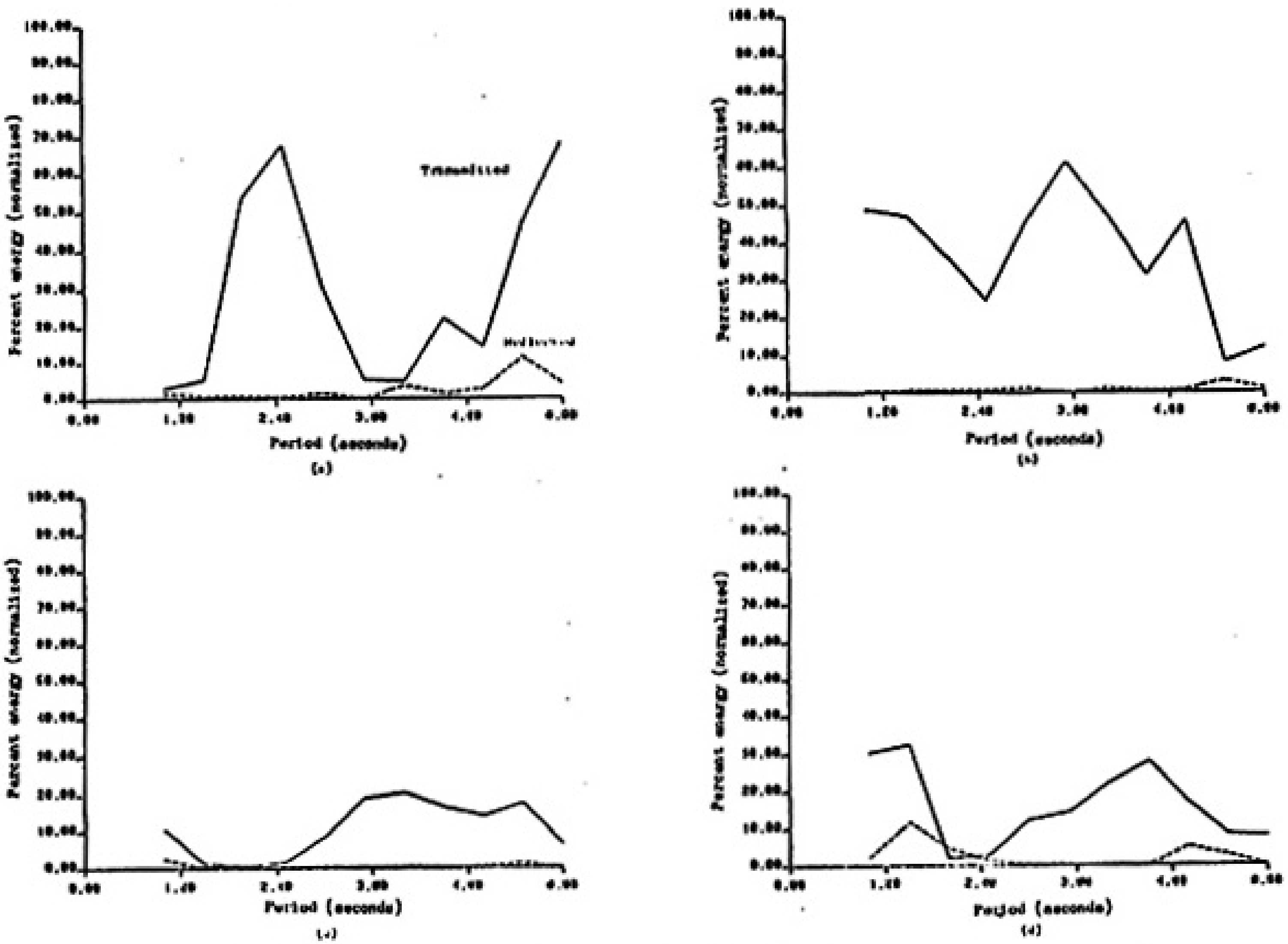

Figure 24. Energy transmission and reflection for Love waves propagating across the Rift B model (Figure 23). In (a), the fundamental mode energy minimum occurs near 3.7 seconds which agrees with results obtained with Rift A. (b), (c), and (d) show similarities to Rift A for the 1st and 2nd higher modes, and higher modes above the 2nd. Note that in (d), the energy curve at short periods has a different maximum than for Rift A. This implies model-dependency at these periods (Table 11).

TABLE 11
NORMALIZED ENERGY PERCENTAGES FROM RIFT B

Period	Fundamental		1st Higher		2nd Higher		Other Higher	
	Trans	Refl	Trans	Refl	Trans	Refl	Trans	Refl
1.00	3.07%	1.51	48.81	0.59	10.61	2.48	30.43	3.03
1.50	5.40	0.96	31.57	0.76	1.11	0.10	48.14	11.96
2.00	53.61	0.75	36.33	0.79	0.66	0.45	2.24	4.97
2.50	67.68	0.80	24.46	0.73	0.44	0.19	4.12	1.59
3.00	29.93	1.59	45.32	0.19	4.21	0.42	16.66	1.48
3.50	5.23	0.00	60.83	0.05	18.81	0.10	14.49	0.49
4.00	4.75	3.59	47.08	1.31	14.22	0.51	28.34	0.20
4.50	21.87	1.48	31.31	0.49	16.54	0.21	27.99	0.11
5.00	14.06	2.72	45.62	0.76	14.06	0.50	17.04	5.24
5.50	46.85	11.26	17.51	3.28	0.13	1.45	16.49	3.03
6.00	67.82	4.14	12.12	0.38	8.03	1.00	6.42	0.09

up in higher modes or the reflected fundamental mode.

These two factors imply that the dip near 3.7 seconds is real; this has important consequences for surface waves passing through the rift. The arrival of the 1st higher mode may be lost amid late oscillations of the S-phase in regional events where a substantial distance seperates the rift and the station. The 2nd higher, and other higher modes, split the remaining energy about evenly. In all cases, more than 90% of the incident energy is transmitted, and less than 10% reflected.

Phase velocities are listed in Table 10. These velocities vary from about 2.53 km/s at 3.70 seconds to a maximum of of 4.39 km/s at 3.5 seconds. Interestingly, the highest phase velocities are found at periods where the energy transmission in the fundamental mode is lowest. One reason for this could be phase shifts caused by mode conversions at element boundaries.

B. Rayleigh wave analysis

The Rayleigh wave analysis of the rift provided a far different energy distribution for waves with periods of 1.5-6.0 seconds. The structure is shown in Figures 25 and 26 and parameters for the upper 7 layers listed in Table 12. The 1/10 element length/wavelength condition is met for periods above 5.6 seconds (assuming an average Rayleigh wave

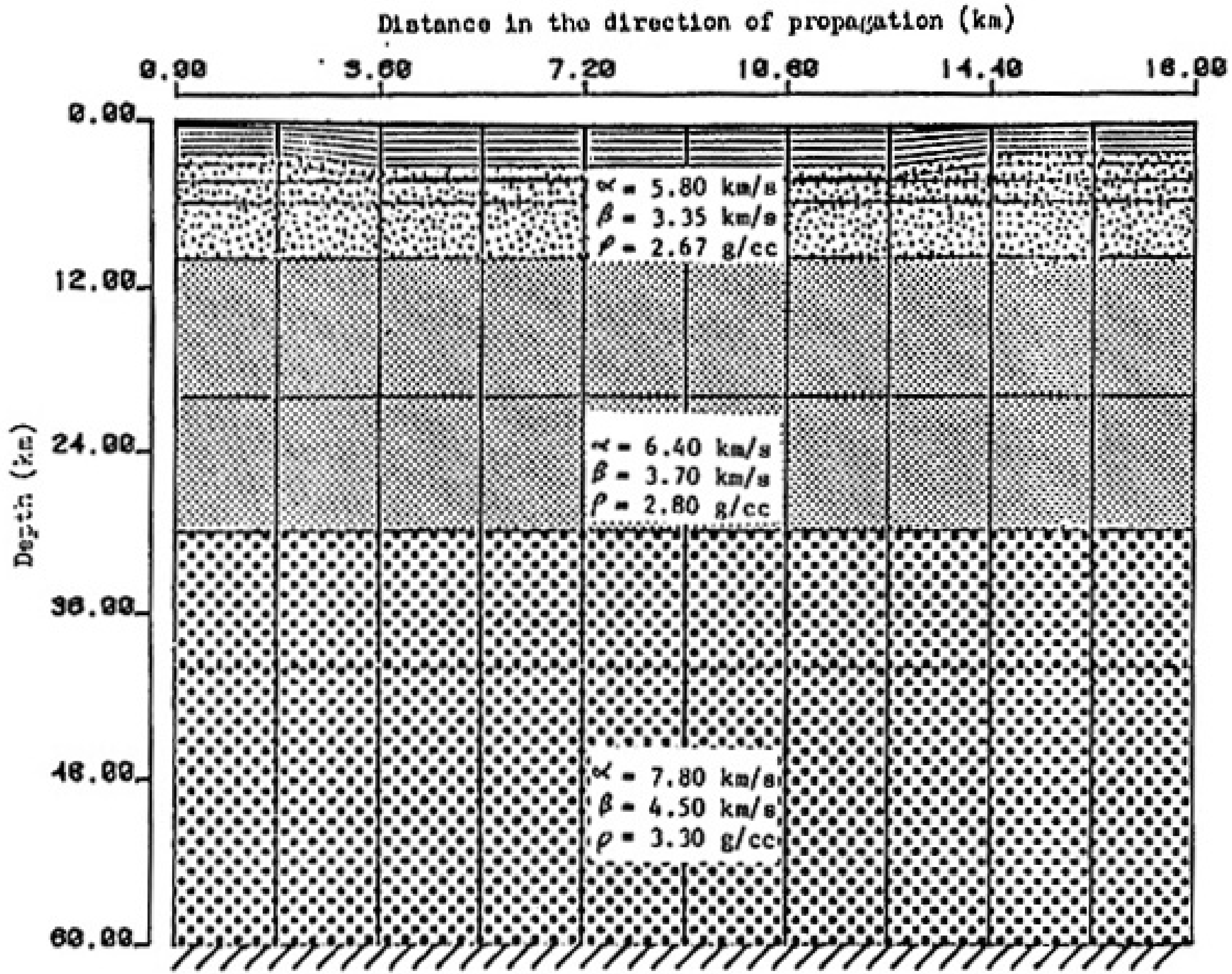

Figure 25. Rayleigh wave rift model consisting of 150 elements in 15 layers and 10 columns giving a total of 165 free nodes. See Figure 26 for an enlargement of the upper 9 layers.

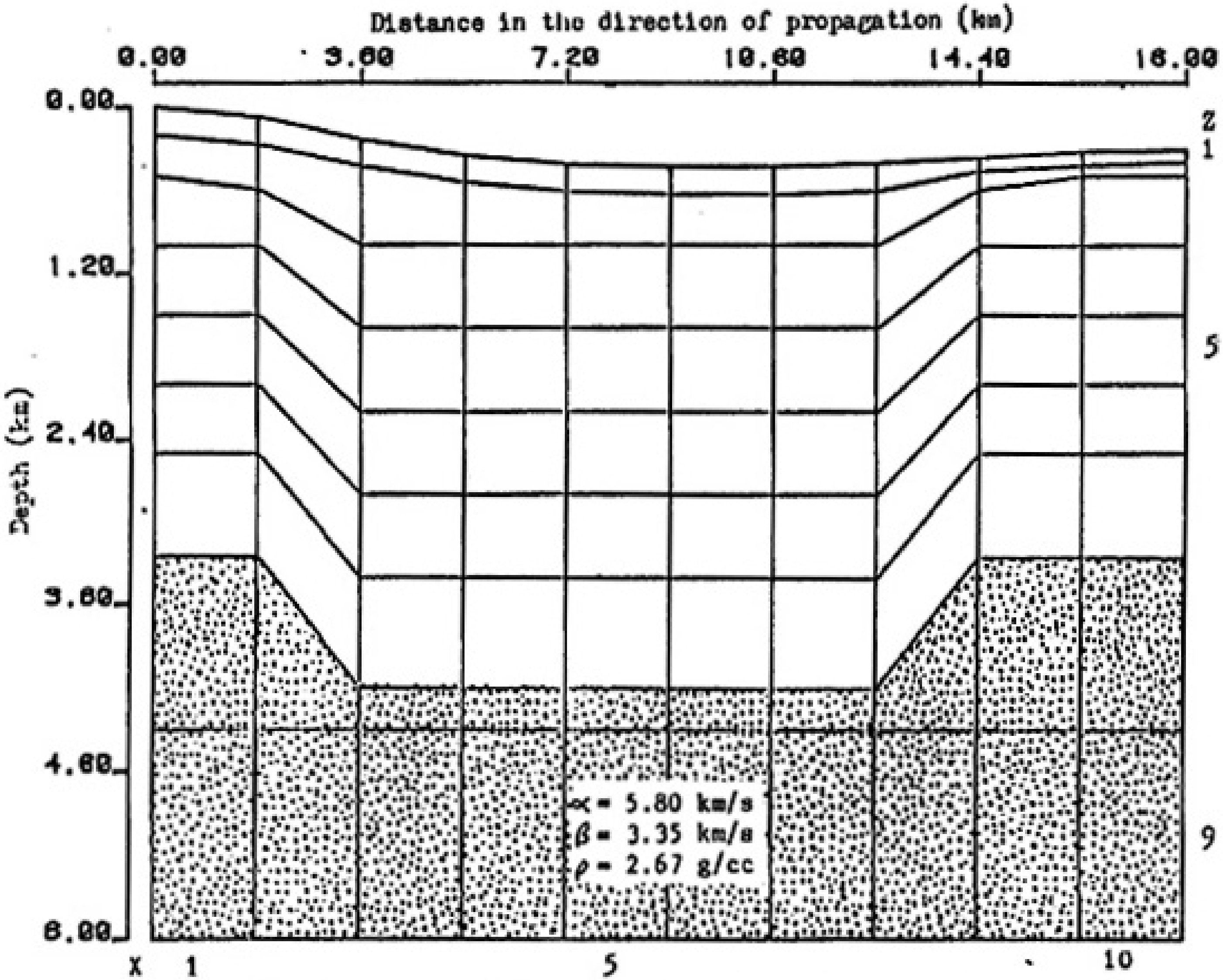

Figure 26. The upper 9 layers of the Rayleigh wave rift model. Element parameters for the upper 7 layers are presented in Table 12. The X axis designates the horizontal coordinate and the Z axis the vertical coordinate of each element as listed in Table 12.

TABLE 12
ELASTIC PARAMETERS FOR THE RIFT MODEL (RAYLEIGH WAVES)

ELEMENT (X,Z)	ALPHA (KM/S)	BETA (KM/S)	RHO (G/CC)
(1, 1)	5.80	3.35	2.67
(1, 2)	5.80	3.35	2.67
(1, 3)	5.80	3.35	2.67
(1, 4)	5.80	3.35	2.67
(1, 5)	5.80	3.35	2.67
(1, 6)	5.80	3.35	2.67
(1, 7)	5.80	3.35	2.67
(2, 1)	2.80	1.62	2.20
(2, 2)	3.10	1.79	2.30
(2, 3)	3.10	1.79	2.30
(2, 4)	4.65	2.60	2.50
(2, 5)	5.00	2.89	2.63
(2, 6)	5.80	3.35	2.67
(2, 7)	5.80	3.35	2.67
(3, 1)	2.80	1.62	2.20
(3, 2)	3.10	1.79	2.30
(3, 3)	3.10	1.79	2.30
(3, 4)	4.20	2.42	2.40
(3, 5)	4.65	2.60	2.50
(3, 6)	4.80	2.77	2.57
(3, 7)	5.80	3.35	2.67
(4, 1)	2.80	1.62	2.20
(4, 2)	3.10	1.79	2.30
(4, 3)	3.10	1.79	2.30
(4, 4)	4.20	2.42	2.40
(4, 5)	4.65	2.60	2.50
(4, 6)	4.80	2.77	2.57
(4, 7)	5.80	3.35	2.67
(5, 1)	2.80	1.62	2.20
(5, 2)	3.10	1.79	2.30
(5, 3)	3.10	1.79	2.30
(5, 4)	4.20	2.42	2.40
(5, 5)	4.65	2.60	2.50
(5, 6)	4.80	2.77	2.57
(5, 7)	5.80	3.35	2.67
(6, 1)	2.80	1.62	2.20
(6, 2)	3.10	1.79	2.30
(6, 3)	3.10	1.79	2.30
(6, 4)	4.20	2.42	2.40
(6, 5)	4.65	2.60	2.50
(6, 6)	4.80	2.77	2.57
(6, 7)	5.80	3.35	2.67
(7, 1)	2.80	1.62	2.20
(7, 2)	3.10	1.79	2.30
(7, 3)	3.10	1.79	2.30
(7, 4)	4.20	2.42	2.40
(7, 5)	4.65	2.60	2.50
(7, 6)	4.80	2.77	2.57
(7, 7)	5.80	3.35	2.67
(8, 1)	2.80	1.62	2.20
(8, 2)	3.10	1.79	2.30
(8, 3)	3.10	1.79	2.30
(8, 4)	4.65	2.60	2.50
(8, 5)	5.00	2.89	2.63
(8, 6)	5.80	3.35	2.67
(8, 7)	5.30	3.35	2.67
(9, 1)	4.70	2.71	2.53
(9, 2)	4.70	2.71	2.53
(9, 3)	4.70	2.71	2.53
(9, 4)	5.80	3.35	2.67
(9, 5)	5.80	3.35	2.67
(9, 6)	5.80	3.35	2.67
(9, 7)	5.80	3.35	2.67
(10, 1)	4.79	2.71	2.53
(10, 2)	4.70	2.71	2.53
(10, 3)	4.70	2.71	2.53
(10, 4)	5.80	3.35	2.67
(10, 5)	5.80	3.35	2.67
(10, 6)	5.80	3.35	2.67
(10, 7)	5.80	3.35	2.67

phase velocity of 3.2 km/s).

Figure 27 and Table 13 show the energy distribution for this model. From Figure 27 it can be seen that the energy transmitted in the fundamental mode increases nearly linearly between periods of 1.5 and 3.5 seconds. One possible explanation for this is that the approximations to the displacements improve as periods increase above 1.5 seconds. This hypothesis is strengthened by the observation that most of the scattered energy goes into higher modes. It is just such an increase in higher mode energy (above the 2nd) that accompanies the drop in energies transmitted by the fundamental mode for the ridge models (Figures 11 and 12). At short periods, this drop was attributed to poor model displacements.

These results, if correct, imply that at periods of less than about 2.5 seconds, strong Rayleigh phases would not be seen in vertical-component seismograms for events from opposite sides of the rift. An important practical consequence of this energy distribution is that it may well be impossible to resolve fine detail within the rift from surface waves due to the lack of transmitted short period waves.

Phase velocities for the Rayleigh wave rift model are presented in Table 13. Phase velocities range from 2.4 km/s at 5.5 seconds to 4.6 km/s at 3.00 seconds. Here again, the maximum phase velocities coincide with the minimum energy

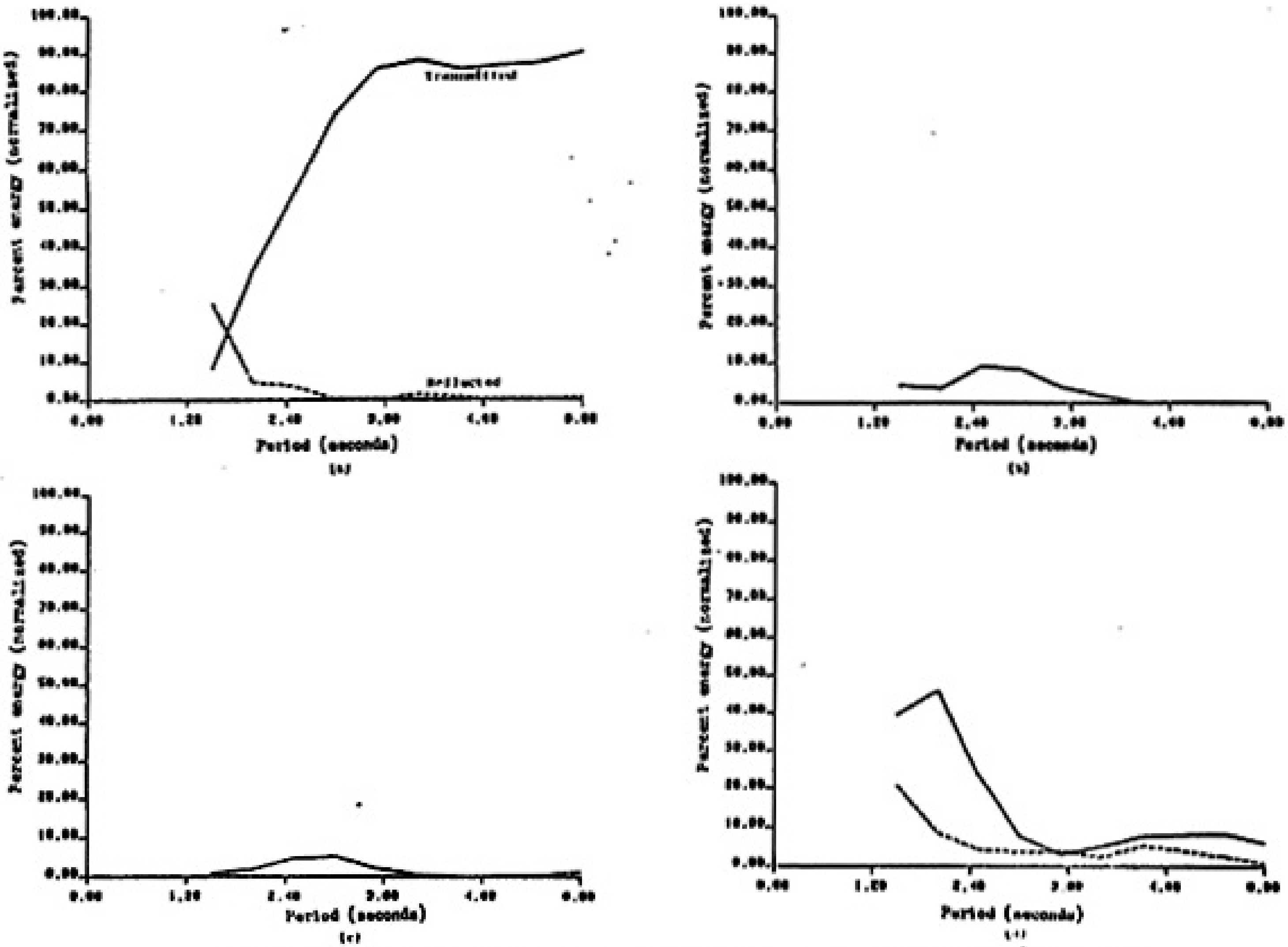

Figure 27. Rayleigh wave energy transmitted and reflected for the rift model (Figures 25 and 26). For the fundamental mode, (a), an almost constant increase in energy with period exists for transmitted waves between 1.5 and 3.5 seconds. This may be due to model displacements improving with period. (b) and (c) show little energy transmitted and reflected in the 1st and 2nd higher modes. (d) shows energy distribution in modes above the 2nd higher mode. As for the Love wave case, the rise in energy at short periods is probably model-dependent. See Table 13 for energy values and phase velocities.

TABLE 13

NORMALIZED ENERGY PERCENTAGES FROM THE RIFT MODEL (RAYLEIGH WAVES)

Period	Fundamental Trans	Fundamental Refl	1st Higher Trans	1st Higher Refl	2nd Higher Trans	2nd Higher Refl	Other Higher Trans	Other Higher Refl
1.50	8.38%	25.40	4.39	0.20	0.98	0.37	39.84	20.64
2.00	34.60	4.60	3.63	0.19	1.84	0.21	46.16	8.77
2.50	54.26	3.67	9.23	0.14	4.83	0.08	23.49	4.30
3.00	74.26	0.37	8.42	0.07	5.29	0.13	7.74	3.79
3.50	86.02	0.35	3.96	0.01	2.20	0.08	3.25	4.13
4.00	88.25	1.60	1.77	0.01	0.70	0.06	5.13	2.48
4.50	85.95	0.97	0.00	0.01	0.15	0.03	7.71	5.18
5.00	96.91	0.24	0.34	0.00	0.24	0.04	8.15	4.08
5.50	87.74	0.48	0.15	0.01	0.64	0.07	8.29	2.63
6.00	90.58	0.94	0.08	0.02	1.42	0.10	5.88	0.98

Period	Ph. vel.
1.50 sec	3.32 km/s
2.00	3.44
2.50	2.55
3.00	4.61
3.50	4.46
4.00	3.21
4.50	2.50
5.00	2.89
5.50	2.40
6.00	3.10

transmission in the fundamental mode (between 3.0 and 4.0 seconds).

VII. Suggestions for Further Work

In order to extend these results to shorter periods and to increase confidence in results presented, more elements should be added to the Magdalena Mountain and Rio Grande rift structures. Improvements afforded by additional elements would be more accurate mode-shape approximations and the ability to include transition zones for elastic parameters. One drawback of including more elements is the amount of computer time (money) involved. In general, the computation time increases with the square of the number of elements. Thus partitioning of large matrices, or finding routines that solve large complex matrices in symmetric storage mode, will be necessary to handle the sizable matrices generated by larger models. In addition, the use of a smaller period increment would produce more detailed curves, and may show additional peaks and dips. Finally, including damping may make the structures more realistic if accurate values for local attenuation become available.

VIII. Conclusions

Analysis of the results suggest that virtually all Love and Rayleigh wave energy incident upon the Magdalena Mountains in the period range 1.0-6.0 seconds (and probably above 6.0 seconds) is transmitted in the fundamental mode. For the Rio Grande rift, however, incident fundamental Love modes appear to be scattered into higher modes, particularly the first higher mode below 2.3 seconds and between 3.0-5.0 seconds. Incident fundamental Rayleigh modes are likewise scattered at short periods, although the energy transmitted in the fundamental mode increases nearly linearly from 8% at a period of 1.5 seconds to nearly 90% at periods of 4.0 seconds and larger. With the exception of very short periods, virtually no reflections occur. At these short periods it is likely that inaccurate displacements due to wide nodal spacing may be the cause of substantial reflected energy in higher modes (body waves).

Two major problems were encountered in the analysis of the models. First, inaccurate mode-shapes generated by some models are believed to have led to sharp drops in the energy transmitted in the fundamental mode over very narrow ranges of periods. A second problem was that elements in the direction of propagation often exceeded 1/10 wavelength and effects caused by inaccurate modeling became important at short periods. Thus, a lower limit was placed on the periods that could be used. Both of these problems could be solved with the introduction of additional elements. To include these additional elements, however, new computer codes must be obtained or written that will handle the larger matrices generated by larger models.

While only about 300 nodes were used in the Love wave models, and 150 in the Rayleigh wave models, this study provides a basis for explaining the existence or absence of high- or low-frequency surface waves on seismograms from regional events. While the original intention of this study was to explain surface waves in the codas of local microearthquake seismograms, two factors make any application of the results to local microearthquakes a large extrapolation. First, distances to microearthquake hypocenters are so small that the assumption of an incident plane wave is not likely to be valid. Second, the local microearthquake recording systems have extremely low magnifications at periods above 1.5 seconds and background noise can obscure the weak surface waves that might be generated from these events (Sanford, 1981, personal communication). Nevertheless, the short period surface waves in the seismograms of stations GM can be explained by just such a large extrapolation.

In addition, this study suggests that surface waves from regional events with periods as low as 6 seconds have not been scattered by topographic features along their paths of transmission, and thus might be used for studying local structures using local, two-station networks.

Further investigations are needed to verify the results presented above. The addition of more elements, including attenuation, the use of a smaller period increment, and extension of the results to structures at shorter periods will form an integral part of further studies. These theoretical studies, combined with complementary studies of recorded traces, should help in the unraveling of local and regional structures.

IX. Acknowledgements

I would like to thank Dr. John Schlue for providing valuable assistance in the execution of this project as well as in the editing of manuscripts. Also, Dr. Allan Sanford provided many useful suggestions, particularly on the relation of these results to microearthquake waves. In addition, I would like to express my appreciation to the National Science Foundation and Geophysical Research Center for funding this project. Finally, the New Mexico Tech Computing Center has provided generous amounts of computer funds.

X. Bibliography

Aki, K. (1969). Analysis of the seismic coda of local microearthquakes as scattered waves, J. Geophys. Res., 74, 615-631.

Birch, F. (1966). Compressibility; elastic constants, in Handbook of Physical Constants, S. P. Clark, Editor, 97-103, Memoir 97, Geol. Soc. Am.

Brown, L. D., P. A. Krumhansl, C. E. Chapin, A. R. Sanford, F. A. Cook, S. Kaufman, J. E. Oliver, and F. S. Schilt (1979). COCORP sesimic reflection studies of the Rio Grande rift, in Rio Grande rift: Tectonics and Magmatism, R. E. Riecker, Editor, 169-194, Am. Geophys. Un., Washington D. C.

Chapin, C. E., R. M. Chamberlin, G. R. Osburn, A. R. Sanford, and D. W. White (1975). Exploration framework of the Socorro Geothermal Area, New Mexico, N. Mex. Geol. Soc. Spec. Publ. 7, 114-129.

Drake, L. A. (1972). Rayleigh waves at a continental boundary by the finite element method, Bull. Seism. Soc. Am., 62, 1259-1268.

Drake, L. A. and B. A. Bolt (1980). Love waves normally incident at a continental boundary, Bull. Seism. Soc. Am., 70, 1103-1123.

Fuyuki, M. and Matsumoto, Y. (1980). Finite difference analysis of Rayleigh wave scattering at a trench, Bull. Seism. Soc. Am., 70, 2051-2069.

Hudson, J. A. and L. Knopoff (1967). Statistical properties of Rayleigh waves due to scattering by topography, Bull. Seism. Soc. Am., 57, 83-90.

Kuo, J. T. and G. A. Thompson (1963). Model studies on the effect of a sloping interface on Rayleigh waves, J. Geophys. Res., 68, 6187-6197.

Lysmer, J. (1970). Lumped mass method for Rayleigh waves, Bull. Seism. Soc. Am., 60, 89-104.

Lysmer, J. and L. A. Drake (1971). The propagation of Love waves across nonhorizontally layered structures, Bull. Seism. Soc. Am., 61, 1233-1251.

Lysmer, J. and L. A. Drake (1972). A finite element method for seismology, in Methods in Computational Physics, vol. 11, B. Bolt, Editor, 181-216, Academic Press, New York.

McGarr, A. and L. E. Alsop (1967). Transmission and reflection of Rayleigh waves at vertical boundaries, J. Geophys. Res., 72, 2169-2180.

Munasinghe, M. and G. W. Farnell (1973). Finite difference analysis of Rayleigh wave scattering at vertical discontinuities, J. Geophys. Res., 78, 463-477.

Rinehart, E. J. (1979). The determination of an upper crustal model for the Rio Grande rift near Socorro, New Mexico, employing S-wave reflections produced by local microearthquakes, Ph.D. Dissertation, New Mexico Institute of Mining and Technology, Socorro.

Sanford, A. R. (1968). Gravity survey in central Socorro County, New Mexico, Circ. 91, New Mexico Bureau of Mines and Minearal Resources.

Sanford, A. R. (1978). Characteristics of the Rio Grande rift in the vicinity of Socorro, New Mexico, from geophysical studies, in Guidebook to the Rio Grande rift in New Mexico and Colorado, Hawley, J. W., Editor,116-121, Circ. 163, New Mexico Bureau of Mines and Mineral Resources.

Sanford, A. R., R. P. Mott, Jr., P. J. Shuleski, E. J. Rinehart, F. J. Caravella, R. M. Ward, and T. C. Wallace (1977). Geophysical evidence for a magma body in the crust in the vicinity of Socorro, N. M., Am. Geophys. Un., Geophys. Mon. 20, 385-403.

Slavin, L. M. and B. Wolf (1970). Scattering of Love waves in a surface layer with an irregular boundary for the case of a rigid underlying half-space, Bull. Seism. Soc. Am., 60, 859-878.

Ward, R. M. (1980). Determination of three-dimensional velocity anomalies within the upper crust in the vicinity of Socorro, New Mexico, using first P-arrival times from local earthquakes, Ph.D. Dissertation, New Mexico Institute of Mining and Technology, Socorro.

Zienkiewicz, O. C. (1971). The Finite Element Method in Engineering Science, McGraw-Hill, New York.

Zienkiewicz, O. C. and Y. K. Cheung (1967). The Finite Element Method in Structural and Continuum Mechanics, McGraw-Hill, London.

APPENDIX 1
PROGRAM LISTINGS

TABLE OF CONTENTS

DISP

```
C   DISP--DISPERSION CURVE GRAPHING ROUTINE FOR LOVE AND
C         RAYLEIGH WAVES
C
C      SYMBOLS USED:
C
C                  C        PHASE VELOCITY
C                  G        GROUP VELOCITY
C                  IX       ABSCISSA LABEL
C                  IY       ORDINATE LABEL
C                  CMAX     MAXIMUM PHASE VELOCITY
C                  GMIN     MINIMUM GROUP VELOCITY
C                  T        PERIOD
C                  TMAX     MAXIMUM PERIOD
C                  TMIN     MINIMUM PERIOD
C                  TINC     PERIOD INCREMENT
C
C      OTHER ARRAYS ARE FOR ALPHANUMERIC SYMBOLS
C
C      THIS PROGRAM READS IN DATA FROM THE DISK FILE DISP.DAT
C      WHICH IS CONSTRUCTED WITH PHASE AND GROUP VELOCITIES
C      DERIVED FROM THE LAYERED ZONE ANALYSIS (LYLAT AND RYLAT).
C
        INTEGER IX(6),IY(6)
        INTEGER K(1),M(1),SLASH(1),S(1),SEC(7)
        REAL C(20),G(20)
C
C   FIND MAX AND MIN POINTS FOR CURVES
C
        OPEN(UNIT=2,DEVICE='DSK',FILE='DISP.DAT',ACCESS='SEQIN')
        READ(2,1000) N,TMIN,TMAX,TINC
        DO 10 I=1,N
        READ(2,1010) C(I)
10      READ(2,1010) G(I)
        CLOSE(UNIT=2,DEVICE='DSK',DISPOSE='SAVE')
        CMAX=C(1)
        GMIN=G(1)
C
C   FIND MAXIMUM PHASE VELOCITY AND MINIMUM GROUP VELOCITY
C
        DO 20 I=2,N
        IF(C(I) .GT. CMAX) CMAX=C(I)
20      IF(G(I) .LT. GMIN) GMIN=G(I)
C
C   ADD AND SUBTRACT 0.2 FOR SPACE AROUND CURVE
C
        CMAX=CMAX+0.2
        GMIN=GMIN-0.2
C
C   SET SCREEN WINDOW
C
        IXMAX=965
        IXMIN=115
        IYMAX=710
        IYMIN=76
        IWINX=IXMAX-IXMIN
        IWINY=IYMAX-IYMIN
        TINC=(TMAX-TMIN)/(N-1)
C
C   INITIALIZE TERMINAL
C
        CALL INITT(480)
C
C   SET VIRTUAL AND SCREEN WINDOW SIZES
C
        CALL DWINDO(TMIN,TMAX,GMIN,CMAX)
        CALL TWINDO(IXMIN,IXMAX,IYMIN,IYMAX)
C
C   DRAW AXES
C
        CALL MOVABS(IXMIN,IYMAX)
        CALL DRWREL(0,-IWINY)
        CALL DRWREL(IWINX,0)
C
C   DRAW CURVES--  PHASE VEL SOLID
C                  GROUP VEL DASHED
C
        T=TMIN
        CALL MOVEA(T,C(1))
        DO 30 I=2,N
        T=T+TINC
30      CALL DRAWA(T,C(I))
        T=TMIN
        CALL MOVEA(T,G(1))
        DO 40 I=2,N
        T=T+TINC
40      CALL DASHA(T,G(I),3)

C
C   CONSTRUCT VERTICAL SCALE
C
        CALL TWINDO(0,1023,0,780)
        INCX=IWINX/5
        XINC=(TMAX-TMIN)/5.
        INCY=IWINY/10
        YINC=(CMAX-GMIN)/10.
        CALL MOVABS(IXMIN,IYMAX)
        CALL MOVREL(0,-IWINY)
        CALL MOVREL(-LINWDT(7),0)
        CALL LABEL(GMIN,IY)
        CALL ANSTR(6,IY)
        CALL DRWREL(10,0)
        YSTRT=GMIN+YINC
        DO 50 Y=YSTRT,CMAX,YINC
        CALL MOVREL(-10,INCY)
        CALL MOVREL(-LINWDT(6),0)
        PAR=Y
        CALL LABEL(PAR,IY)
        CALL ANSTR(6,IY)
        CALL DRWREL(10,0)
50        CONTINUE
C
C   CONSTRUCT HORIZONTAL SCALE
C
        CALL MOVABS(IXMIN,IYMIN)
        CALL MOVREL(0,-15)
        CALL MOVREL(0,-LINHGT(1))
        CALL MOVREL(-LINWDT(4),0)
        CALL LABEL(TMIN,IX)
        CALL ANSTR(6,IX)
        CALL MOVREL(-LINWDT(3),0)
```

DISP (CONTINUED)

```
      CALL MOVREL(0,LINHGT(1))
      CALL MOVREL(0,5)
      CALL DRWREL(0,10)
      XSTRT=XMIN+XINC
      DO 60 X=XSTRT,XMAX,XINC
      CALL MOVREL(INCX,-15)
      CALL MOVREL(-LINWDT(4),0)
      PAR=X
      CALL MOVREL(0,-LINHGT(1))
      CALL LABEL(PAR,IX)
      CALL ANSTR(6,IX)
      CALL MOVREL(0,LINHGT(1))
      CALL MOVREL(-LINWDT(2),0)
      CALL MOVREL(0,5)

      CALL DRWREL(0,10)
60    CONTINUE
C
C  LABEL VERTICAL AXIS
C
      K(1)=75
      M(1)=77
      SLASH(1)=47
      S(1)=83
      CALL MOVABS(0,IWINT/2+IYMIN)
      CALL MOVREL(0,LINHGT(2))
      CALL ANSTR(1,K)
      CALL MOVREL(-LINWDT(1),0)
      CALL MOVREL(0,-LINHGT(1))
      CALL ANSTR(1,M)
      CALL MOVREL(-LINWDT(1),0)
      CALL MOVREL(0,-LINHGT(1))
      CALL ANSTR(1,SLASH)
      CALL MOVREL(-LINWDT(1),0)
      CALL MOVREL(0,-LINHGT(1))
      CALL ANSTR(1,S)
      CALL MOVREL(-LINWDT(1),0)
C
C  LABEL HORIZONTAL AXIS
C
      SEC(1)=83
      SEC(2)=69
      SEC(3)=67
      SEC(4)=79
      SEC(5)=78
      SEC(6)=68
      SEC(7)=83
      CALL MOVABS(505,0)
      CALL ANSTR(7,SEC)
      CALL MOVABS(0,780)
      CALL ANMODE
      WRITE(5,1020)
      READ(5,1030) CHAR
      CALL FINITT(0,0)
1000  FORMAT(I3,3F7.2)
1010    FORMAT(F7.2)
1020  FORMAT(37X,'PHASE AND GROUP VELOCITIES')
1030  FORMAT(A1)
      END
```

```
C
C
C  LABEL--ALPHANUMERIC CODE GENERATOR
C
C  SYMBOLS USED;
C
C        VAL      ARRAY CONTAINING ALPHANUMERIC CHARACTER
C                 CODES FOR EACH VALUE PRINTED ON THE SCREEN.
C
C        DIG      DIGITS TO RIGHT OF DECIMAL POINT
C
      SUBROUTINE LABEL(PAR,VAL)
      INTEGER VAL(6)
C
C  TRUNCATE DIGITS BEYOND HUNDRETHS
C
      N=INT(PAR*100.)
      PAR=N/100.
C
C  DETERMINE ASCII CHARACTER CODES FOR EACH VALUE.
C
      VAL(1)=INT(PAR/100)
      VAL(2)=INT((PAR-VAL(1)*100)/10)
      VAL(3)=INT(PAR-(VAL(1)*100+VAL(2)*10))
      DIG=INT(PAR*100-(VAL(1)*100+VAL(2)*10+VAL(3))*100)
      VAL(5)=INT(DIG/10.)
      VAL(6)=INT(DIG-VAL(5)*10)
      DO 10 I=1,6
10    VAL(I)=VAL(I)+48
C
C  IF FIRST TWO DIGITS ARE 0, MAKE BLANKS
C
      IF(VAL(1) .EQ. 48) VAL(1)=32
      IF(VAL(2) .EQ. 48 .AND. VAL(1) .EQ. 32) VAL(2)=32
C
C  VAL(4) IS DECIMAL POINT
C
      VAL(4)=46
      RETURN
      END
```

ENGY

```
C   ENGY-- ENERGY CURVE PLOTTING ROUTINE
C
C      SYMBOLS USED:
C
C                 TR       TRANSMITTED ENERGY
C                 RF       REFLECTED ENERGY
C                 IX       ABSCISSA LABEL
C                 IY       ORDINATE LABEL
C                 EMAX     MAXIMUM ENERGY VALUE
C                 EMIN     MINIMUM ENERGY VALUE
C                 T        PERIOD
C                 TMAX     MAXIMUM PERIOD
C                 TMIN     MINIMUM PERIOD
C                 TINC     PERIOD INCREMENT
C                 MODE     ALPHANUMERIC ARRAY CONTAINING TITLE
C
C      OTHER ARRAYS ARE ALPHANUMERIC STRINGS
C
C
C      THIS PROGRAM READS TRANSMITTED AND REFLECTED ENERGIES
C      FOR EACH MODE FROM ENGY.DAT WHICH IS CONSTRUCTED FROM THE
C      OUTPUT OF EITHER LYINR OR RTIPR.
C
        INTEGER IX(6),IY(6),MODE(50)
        INTEGER PLR(1),BLK(1),K(1),HH(1),M(1),G(1),YY(1),SEC(7)
        REAL TR(50),RF(50),T(50)
C
C   FIND MAX AND MIN POINTS FOR CURVES
C
        OPEN(UNIT=2,DEVICE='DSK',FILE='ENGY.DAT',ACCESS='SEQIN')
        READ(2,1000) N
          READ(2,1000) N
10      WRITE(5,1010)
        READ(5,1020) (MODE(I),I=1,30)
        DO 20 I=1,N
20      READ(2,1030) T(I),TR(I),RF(I)
        TMAX=6.0
        TMIN = 0.0
C
C   FIND MAXIMUM PERIOD IF T > 6 SECONDS
C
        IF(T(N) .GT. 6.0) TMAX=T(N)
        EMAX=100
        EMIN=0.
C
C   SET SCREEN WINDOW
C
        IXMAX=965
        IXMIN=115
        IYMAX=700
        IYMIN=70
        IWINX=IXMAX-IXMIN
        IWINY=IYMAX-IYMIN
C
C   INITIALIZE TERMINAL PARAMETERS
C
        CALL INITT(480)
C
C   SET VIRTUAL AND SCREEN WINDOW SIZES
C
        CALL DWINDO(TMIN,TMAX,EMIN,EMAX)
        CALL TWINDO(IXMIN,IXMAX,IYMIN,IYMAX)
C
C   DRAW AXES
C
        CALL MOVABS(IXMIN,IYMAX)
        CALL DRWREL(0,-IWINY)
        CALL DRWREL(IWINX,0)
C
C   DRAW CURVES--   TRANS ENERGY SOLID
C                   REFL ENERGY DASHED
C
        CALL MOVEA(T(1),TR(1))
        DO 30 I=2,N
30      CALL DRAWA(T(I),TR(I))
        CALL MOVEA(T(1),RF(1))
        DO 40 I=2,N
40      CALL DASHA(T(I),RF(I),3)

C
C   CONSTRUCT VERTICAL SCALE
C
        CALL TWINDO(0,1023,0,780)
        INCX=IWINX/5
        XINC=(TMAX-TMIN)/5.
        INCY=IWINY/10
        YINC=(EMAX-EMIN)/10.
        CALL MOVABS(IXMIN,IYMAX)
        CALL MOVREL(0,-IWINY)
        CALL MOVREL(-LINWDT(7),0)
        CALL LABEL(EMIN,IY)
        CALL ANSTR(6,IY)
        CALL DRWREL(10,0)
        YSTRT=EMIN+YINC
        DO 50 Y=YSTRT,EMAX,YINC
        CALL MOVREL(-10,INCY)
        CALL MOVREL(-LINWDT(6),0)
        PAR=Y
        CALL LABEL(PAR,IY)
        CALL ANSTR(6,IY)
        CALL DRWREL(10,0)
50        CONTINUE
C
C   CONSTRUCT HORIZONTAL SCALE
C
        CALL MOVABS(IXMIN,IYMIN)
        CALL MOVREL(0,-15)
        CALL MOVREL(0,-LINHGT(1))
        CALL MOVREL(-LINWDT(4),0)
        CALL LABEL(TMIN,IX)
        CALL ANSTR(6,IX)
        CALL MOVREL(-LINWDT(2),0)
        CALL MOVREL(0,LINHGT(1))
        CALL MOVREL(0,5)
        CALL DRWREL(0,10)
        XSTRT=TMIN+XINC
        DO 60 X=XSTRT,TMAX,XINC
        CALL MOVREL(INCX,-15)
        CALL MOVREL(-LINWDT(4),0)
        PAR=X
```

ENGY (CONTINUED)

```
      CALL MOVREL(0,-LINHGT(1))
      CALL LABEL(PAR,IX)
      CALL ANSTR(6,IX)
      CALL MOVREL(0,LINHGT(1))
      CALL MOVREL(-LINWDT(2),0)
      CALL MOVREL(0,5)

      CALL DRWREL(0,10)
60    CONTINUE
C
C  LABEL VERTICAL AXIS-- '% ENERGY'
C
      PER(1)=37
      BLK(1)=00
      E(1)=69
        NN(1)=78
        R(1)=82
      G(1)=71
      YY(1)=89
      CALL MOVABS(0,(WINY/2+IYMIN)
      CALL MOVREL(0,LINHGT(2))
      CALL ANSTR(1,PER)
      CALL MOVREL(-LINWDT(1),0)
      CALL MOVREL(0,-LINHGT(1))
      CALL ANSTR(1,BLK)
      CALL MOVREL(-LINWDT(1),0)
      CALL MOVREL(0,-LINHGT(1))
      CALL ANSTR(1,E)
      CALL MOVREL(-LINWDT(1),0)
      CALL MOVREL(0,-LINHGT(1))
      CALL ANSTR(1,NN)
      CALL MOVREL(-LINWDT(1),0)
      CALL MOVREL(0,-LINHGT(1))
      CALL ANSTR(1,E)
      CALL MOVREL(-LINWDT(1),0)
      CALL MOVREL(0,-LINHGT(1))
      CALL ANSTR(1,R)
      CALL MOVREL(-LINWDT(1),0)
      CALL MOVREL(0,-LINHGT(1))
      CALL ANSTR(1,G)
      CALL MOVREL(-LINWDT(1),0)
      CALL MOVREL(0,-LINHGT(1))
      CALL ANSTR(1,YY)
      CALL MOVREL(-LINWDT(1),0)
C
C  LABEL HORIZONTAL AXIS-- 'SECONDS'
C
      SEC(1)=83
      SEC(2)=69
      SEC(3)=67
      SEC(4)=79
      SEC(5)=78
      SEC(6)=68
      SEC(7)=83
      CALL MOVABS(505,0)
      CALL ANSTR(7,SEC)
      CALL MOVABS(0,780)
      CALL ANMODE
      IF(M .EQ. 1) WRITE(5,1040) (MODE(I),I=1,30)
      IF(M .EQ. 2) WRITE(5,1050) (MODE(I),I=1,30)
      IF(M .EQ. 3) WRITE(5,1060) (MODE(I),I=1,30)
      IF(M .EQ. 4) WRITE(5,1070) (MODE(I),I=1,30)
      IF(M .EQ. 5) WRITE(5,1080) (MODE(I),I=1,30)
      IF(M .EQ. 6) WRITE(5,1085) (MODE(I),I=1,30)
      READ(5,1090) CHAR
      CALL ERASE
      WRITE(5,1100)
      READ(5,1110) ANS
      IF(ANS .EQ. 'YES') GO TO 10
      CLOSE(UNIT=2,DEVICE='DSK',DISPOSE='SAVE')
      CALL FINITT(0,0)
1000  FORMAT(I1)
1010  FORMAT(/' WHICH MODE?')
1020  FORMAT(30A1)
1030    FORMAT(3F12.2)
1040  FORMAT(15X,'MAGDALENA MTNS.-- LOVE WAVES: ',30A1)
1050  FORMAT(15X,'MAGDALENA MTNS.-- RAYLEIGH WAVES: ',30A1)
1060  FORMAT(15X,'RIO GRANDE RIFT-- LOVE WAVES: ',30A1)
1070  FORMAT(15X,'RIO GRANDE RIFT-- RAYLEIGH WAVES: ',30A1)
1080  FORMAT(20X,'RIDGE A-- LOVE WAVES: ',30A1)
1085  FORMAT(20X,'RIDGE B-- LOVE WAVES: ',30A1)
1090  FORMAT(A1)
1100  FORMAT(/1X' ANOTHER PLOT?')
1110  FORMAT(A3)
      END

C
C
C  LABEL--ALPHANUMERIC CODE GENERATOR
C
C  SYMBOLS USED:
C
C          VAL      ARRAY CONTAINING ALPHANUMERIC CHARACTER
C                   CODES FOR EACH VALUE PRINTED ON THE SCREEN.
C
C          DIG      DIGITS TO RIGHT OF DECIMAL POINT
C
      SUBROUTINE LABEL(PAR,VAL)
      INTEGER VAL(6)
C
C  TRUNCATE DIGITS BEYOND HUNDRETHS
C
      N=INT(PAR*100.)
      PAR=N/100.
C
C  DETERMINE ASCII CHARACTER CODES FOR EACH VALUE.
C
      VAL(1)=INT(PAR/100)
      VAL(2)=INT((PAR-VAL(1)*100)/10)
      VAL(3)=INT(PAR-(VAL(1)*100+VAL(2)*10))
      DIG=INT(PAR*100-(VAL(1)*100+VAL(2)*10+VAL(3))*100)
      VAL(5)=INT(DIG/10.)
      VAL(6)=INT(DIG-VAL(5)*10)
      DO 10 I=1,6
10    VAL(I)=VAL(I)+48
C
C  IF FIRST TWO DIGITS ARE 0, MAKE BLANKS
C
      IF(VAL(1) .EQ. 48) VAL(1)=32
      IF(VAL(2) .EQ. 48 .AND. VAL(1) .EQ. 32) VAL(2)=32
```

ENGY (CONTINUED)

```
C
C  VAL(4) IS DECIMAL POINT
C
      VAL(4)=46
      RETURN
      END
```

GAUSL

```
      SUBROUTINE GAUSL(KM,MM,MU,RHO,COORD)
C
C   GAUSSIAN QUADRATURE OF ELEMENT MATRICES IN IRREGULAR ZONE
C
C        3-POINT GAUSSIAN QUADRATURE IS USED IN THIS ROUTINE TO
C   APPROXIMATE THE MASS AND STIFFNESS MATRICES FOR AN IRREGULAR
C   ELEMENT. DATA INPUTTED INCLUDES THE RECTANGULAR COORDINATES
C   OF THE NODES OF THE ELEMENT, IN CLOCKWISE ORDER, THE DENSITY
C   AND THE SHEAR MODULUS OF THE ELEMENT. THE EVALUATED MASS
C   AND STIFFNESS MATRICES ARE OUTPUT. SEE LYSMER AND DRAKE (1972)
C   FOR DIAGRAMS AND EXPLANATION.
C
C   VARIABLES USED IN THIS ROUTINE:
C
C                  P(II)          POINTS USED IN QUADRATURE IN
C                  XI,ETA         NATURAL COORDINATES
C                  W,WXI,WETA     WEIGHTING FACTORS FOR QUADRATURE
C                  XSHAPE         TERMS PRODUCING "SHAPE FUNCTIONS"
C                  ESHAPE         FOR THE ELEMENT
C                  H              SHAPE FUNCTIONS
C                  D              D MATRIX
C                  JAC            JACOBIAN MATRIX
C                  JACI           INVERSE JACOBIAN MATRIX
C                  DETJ           JACOBIAN
C                  COORD          X AND Z COORDINATE MATRIX
C                  MINT,KINT      INTEGRANDS (MASS AND STIFFNESS)
C                  MINT1,KINT1
C                  MM             MASS AND STIFFNESS
C                  KM             MATRICES
C                  RHO            DENSITY
C                  MU             SHEAR MODULUS
C
      REAL H(4,1),D(2,4),JAC(2,2),JACI(2,2),MINT(4,4),KINT(4,4),
     1MINT1(4,4),KINT1(4,4),MM(4,4),KM(4,4),P(3),MU,W(3),COORD(4,2),
     2B(2,4)
C
C   DEFINE POINTS AND WEIGHTS FOR QUADRATURE
C
      P(1)=-.774597
      P(2)=0.
      P(3)=-P(1)
      W(1)=.555556
      W(2)=.888889
      W(3)=W(1)
      DO 10 I=1,4
      DO 10 J=1,4
      MM(I,J)=0.
10    KM(I,J)=0.
C
C   START LOOP FOR 3-POINT QUADRATURE
C
      DO 200 II=1,3
      XI=P(II)
      WXI=W(II)
      DO 20 I=1,4
      DO 20 J=1,4
      MINT1(I,J)=0.
20    KINT1(I,J)=0.
      DO 180 JJ=1,3
      ETA=P(JJ)
      WETA=W(JJ)
C
C   DETERMINE TERMS OF SHAPE FUNCTIONS
C
      I=1
      X=-1.
      E=-1.
30    CONTINUE
      XSHAPE=1.+XI*X
      ESHAPE=1.+ETA*E
      H(I,1)=XSHAPE*ESHAPE*0.25
      D(1,I)=X*ESHAPE*0.25
      D(2,I)=E*XSHAPE*0.25
      I=I+1
      IF (X*E .GT. 0.) GO TO 40
      E=-E
      IF(X .LT. 0.) GO TO 50
      GO TO 30
40    X=-X
      IF(X .GT. 0.) GO TO 30
      IF(I .EQ. 4) GO TO 30
50    CONTINUE
C
C   DERIVE JACOBIAN MATRIX, INVERSE, AND DETERMINENT
C
      DO 70 I=1,2
      DO 70 J=1,2
      HIT=0.
      DO 60 K=1,4
60    HIT=HIT+D(I,K)*COORD(K,J)
70    JAC(I,J)=HIT
      DETJ=JAC(1,1)*JAC(2,2)-JAC(1,2)*JAC(2,1)
      IF (DETJ) 90,80,90
80    WRITE(6,1000)
      STOP
90    JACI(1,1)=JAC(2,2)
      JACI(1,2)=-JAC(1,2)
      JACI(2,1)=-JAC(2,1)
      JACI(2,2)=JAC(1,1)
      DENOM=1./DETJ
      DO 100 I=1,2
      DO 100 J=1,2
100   JACI(I,J)=JACI(I,J)*DENOM
C
C   COMPUTE MATRIX B
C
      DO 120 I=1,2
      DO 120 J=1,4
      HIT=0.
      DO 110 K=1,2
110   HIT=HIT+JACI(I,K)*D(K,J)
120   B(I,J)=HIT
C
C   COMPUTE INTEGRANDS
C
      DO 130 I=1,4
      DO 130 J=1,4
      MINT(I,J)=0.
130   KINT(I,J)=0.
      DO 140 I=1,4
```

GAUSL (CONTINUED)

```
       DO 140 J=1,4
140    MINT(I,J)=MINT(I,J)+H(I,1)*H(J,1)*DETJ
       DO 160 I=1,4
       DO 160 J=1,4
       BIT=0.
       DO 150 K=1,2
150    BIT=BIT+B(K,I)*B(K,J)
160    KINT(I,J)=BIT*DETJ
C
C  MULTIPLY BY WEIGHTING FACTOR ASSOCIATED WITH ETA
C
       DO 170 I=1,4
       DO 170 J=1,4
       MINTI(I,J)=MINTI(I,J)+MINT(I,J)*HETA
170    KINTI(I,J)=KINTI(I,J)+KINT(I,J)*HETA
180    CONTINUE
C
C  MULTIPLY BY WEIGHTING FACTOR ASSOC. WITH XI
C
       DO 190 I=1,4
       DO 190 J=1,4
       MM(I,J)=MM(I,J)+HXI*MINTI(I,J)*RHO
190    KM(I,J)=KM(I,J)+HXI*KINTI(I,J)*MU
200    CONTINUE
       RETURN
1000   FORMAT(1X,'DETJ = 0, PROGRAM TERMINATED')
       END
```

GAUSR

```
      SUBROUTINE GAUSR(RKM,RMM,MU,LAMBDA,RHO,COORD)
C
C  GAUSSIAN QUADRATURE OF RAYLEIGH WAVE ELEMENT MATRICES
C  FROM THE IRREGULAR ZONE.
C
C        3-POINT GAUSSIAN QUADRATURE IS USED IN THIS ROUTINE TO
C  APPROXIMATE THE MASS AND STIFFNESS MATRICES FOR AN IRREGULAR
C  ELEMENT. DATA INPUTTED INCLUDES THE RECTANGULAR COORDINATES
C  OF THE NODES OF THE ELEMENT, IN CLOCKWISE ORDER, THE DENSITY
C  AND THE SHEAR MODULUS OF THE ELEMENT. THE EVALUATED MASS
C  AND STIFFNESS MATRICES ARE OUTPUT.
C
C  VARIABLES USED IN THIS ROUTINE:
C
C                 P(II)          POINTS USED IN QUADRATURE IN
C                 XI,ETA         NATURAL COORDINATES
C                 H              WEIGHTING FACTOR FOR QUADRATURE
C                 XSHAPE         TERMS REPRESENTING 'SHAPE FUNCTIONS'
C                 ESHAPE         FOR THE ELEMENT
C                 N              N MATRIX
C                 D              D MATRIX
C                 JAC            JACOBIAN MATRIX
C                 JACI           INVERSE JACOBIAN MATRIX
C                 DETJ           JACOBIAN
C                 COORD          X AND Z COORDINATE MATRIX
C                 RMINT,RKINT    INTEGRANDS (MASS AND STIFFNESS)
C                 RMINT1,RKINT1
C                 RMM            MASS AND STIFFNESS
C                 RKM            MATRICES
C                 RHO            DENSITY
C                 MU             SHEAR MODULUS
C                 LAMBDA         LAME CONSTANT
C                 C              ELASTIC MODULUS MATRIX
C                 S              MATRIX FOR RAYLEIGH INTEGRAND
C                 STC
C
      REAL N(4,1),D(2,4),JAC(2,2),JACI(2,2),
     1P(3),MU,H(3),COORD(4,2),
     2B(2,4),LAMBDA,RMM(8,8),RKM(8,8),RMINT1(8,8),RKINT1(8,8),
     3RMINT(8,8),RKINT(8,8),HR(2,8),S(3,8),STC(8,3),C(3,3)
C
C  CALCULATE C (ELASTIC MATRIX)
C
      DO 10 I=1,3
      DO 10 J=1,3
10    C(I,J)=0.
      TERM=LAMBDA+2*MU
      DO 20 I=1,2
      DO 20 J=1,2
      C(I,J)=LAMBDA
      IF(I .EQ. J) C(I,J)=TERM
20    CONTINUE
      C(3,3)=MU
C
C  DEFINE POINTS AND WEIGHTS FOR QUADRATURE
C
      P(1)=-.774597
      P(2)=0.
      P(3)=-P(1)
      H(1)=.555556
      H(2)=.888889
      H(3)=H(1)
      DO 30 I=1,8
      DO 30 J=1,8
      RMM(I,J)=0.
30    RKM(I,J)=0.
      DO 270 II=1,3
      XI=P(II)
      HXI=H(II)
      DO 40 I=1,8
      DO 40 J=1,8
      RMINT1(I,J)=0.
40    RKINT1(I,J)=0.
      DO 250 JJ=1,3
      ETA=P(JJ)
      HETA=H(JJ)
C
C  DETERMINE TERMS OF SHAPE FUNCTIONS
C
      I=1
      X=-1.
      E=-1.
50    CONTINUE
      XSHAPE=1.+XI*X
      ESHAPE=1.+ETA*E
      N(I,1)=XSHAPE*ESHAPE*0.25
      D(1,I)=X*ESHAPE*0.25
      D(2,I)=E*XSHAPE*0.25
      I=I+1
      IF (X*E .GT. 0.) GO TO 60
      E=-E
      IF(X .LT. 0.) GO TO 70
      GO TO 50
60    X=-X
      IF(X .GT. 0.) GO TO 50
      IF(I .EQ. 4) GO TO 50
70    CONTINUE
C
C  DERIVE JACOBIAN MATRIX, INVERSE, AND DETERMINENT
C
      DO 90 I=1,2
      DO 90 J=1,2
      BIT=0.
      DO 80 K=1,4
80    BIT=BIT+D(I,K)*COORD(K,J)
90    JAC(I,J)=BIT
      DETJ=JAC(1,1)*JAC(2,2)-JAC(1,2)*JAC(2,1)
      IF (DETJ) 110,100,110
100   WRITE(6,1000)
      STOP
110   JACI(1,1)=JAC(2,2)
      JACI(1,2)=-JAC(1,2)
      JACI(2,1)=-JAC(2,1)
      JACI(2,2)=JAC(1,1)
      DENOM=1./DETJ
      DO 120 I=1,2
      DO 120 J=1,2
120   JACI(I,J)=JACI(I,J)*DENOM
C
C  COMPUTE MATRIX B
```

GAUSR (CONTINUED)

```
C
      DO 140 I=1,2
      DO 140 J=1,4
      HIT=0.
      DO 130 K=1,2
130   HIT=HIT+JACI(I,K)*D(K,J)
140   H(I,J)=HIT
C
C  COMPUTE INTEGRANDS
C
      DO 150 I=1,8
      DO 150 J=1,8
      PMINT(I,J)=0.
150   PKINT(I,J)=0.
C
C  RAYLEIGH WAVE MATRICES COMPUTED
C
      DO 160 I=1,2
      DO 160 J=1,8
160   HR(I,J)=0.
      DO 170 J=0,3
      HR(1,1+2*J)=H(J+1,1)
      HR(2,2+2*J)=H(J+1,1)
      S(1,1+2*J)=H(1,J+1)
170   S(2,2+2*J)=H(2,J+1)
C
C  STIFFNESS INTEGRAND FOR RAYLEIGH WAVES
C
      S(3,1)=H(2,1)
      S(3,2)=H(1,1)
      S(3,3)=H(2,2)
      S(3,4)=H(1,2)
      S(3,5)=H(2,3)
      S(3,6)=H(1,3)
      S(3,7)=H(2,4)
      S(3,8)=H(1,4)
      DO 190 I=1,8
      DO 190 J=1,3
      HIT=0.
      DO 180 K=1,3
180   HIT=HIT+S(K,I)*C(K,J)
190   STC(I,J)=HIT
      DO 210 I=1,8
      DO 210 J=1,8
      HIT=0.
      DO 200 K=1,3
200   HIT=HIT+STC(I,K)*S(K,J)
210   PKINT(I,J)=HIT*DETJ
C
C  MASS INTEGRAND FOR RAYLEIGH WAVE MATRIX
C
      DO 230 I=1,8
      DO 230 J=1,8
      HIT=0.
      DO 220 K=1,2
220   HIT=HIT+HR(K,I)*HR(K,J)
230   PMINT(I,J)=HIT*DETJ
C
C  MULTIPLY BY WEIGHTING FACTOR ASSOCIATED WITH ETA
C
      DO 240 I=1,8
      DO 240 J=1,8
      RMINT1(I,J)=RMINT1(I,J)+PMINT(I,J)*HETA
240   RKINT1(I,J)=RKINT1(I,J)+PKINT(I,J)*HETA
250   CONTINUE
C
C  MULT BY WEIGHTING FACTOR ASSOC. WITH XI
C
      DO 260 I=1,8
      DO 260 J=1,8
      RMM(I,J)=RMM(I,J)+RMINT1(I,J)*HXI*RHO
260   RKM(I,J)=RKM(I,J)+RKINT1(I,J)*HXI
270   CONTINUE
      RETURN
1000  FORMAT(1X,'DETJ = 0, PROGRAM TERMINATED')
      END
```

GAUSSF

```
      SUBROUTINE GAUSSF(X,Y,RI,RK,N)
C
C             THIS SUBROUTINE SOLVES THE EQUATION  AX=Y  USING
C             GAUSS ELIMINATION.  THERE IS NO PIVOTING, AND
C             THE SUBROUTINE ASSUMES THE DIAGONAL ELEMENTS TO
C             BE NON-ZERO.  COMPLEX ARITHMETIC USED THROUGHOUT.
C
C                X    = VECTOR OF COMPLEX UNKNOWNS
C                Y    = VECTOR OF COMPLEX KNOWNS
C                RI   = COMPLEX VECTOR CONTAINING  I-TH  ROW
C                       OF A-MATRIX
C                RK   = COMPLEX VECTOR CONTAINING  K-TH  ROW
C                       OF A-MATRIX,  K > I
C                N    = ORDER OF A
C
C
C                UNIT3 = FILE CONTAINING INPUT MATRIX
C                UNIT4 = FILE CONTAINING OUTPUT MATRIX
C
C
      COMPLEX X(1),Y(1),RI(1),RK(1)
      COMPLEX CZERO,CONE,CTEMP,AIJ
      INTEGER UNIT3,UNIT4
       COMMON/INOUT1/UNIT3,UNIT4
C
      CZERO   = (0.0E0,0.0E0)
      CONE    = (1.0E0,0.0E0)
      NM1     = N - 1
C
C             OPEN INPUT AND OUTPUT TEMPORARY STORAGE DEVICES
C                    (UNIT3 HAS THE A-MATRIX)
C
C
C
C             LOOP OVER THE ROWS, ONE-BY-ONE
C
      DO 80 I=1,NM1
      IP1     = I + 1
      IM1     = I - 1
C
      REWIND UNIT3
      REWIND UNIT4
C
      IF(I.EQ.1) GO TO 20
C
C             SKIP (I-1) ROWS
C
      DO 10 ISKIP=1,IM1
      READ(UNIT3,1000) (RI(J),J=1,N)
10    WRITE(UNIT4,1000) (RI(J),J=1,N)
20    CONTINUE
C
C             READ IN  I-TH  ROW FROM UNIT3 AND NORMALIZE
C
      READ(UNIT3,1000) (RI(J),J=1,N)
      CTEMP   = CONE / RI(I)
      RI(I)   = CONE
      DO 30 J=IP1,N
30    RI(J)   = RI(J) * CTEMP
      Y(I)    = Y(I)  * CTEMP
C
C             WRITE  I-TH  ROW TO UNIT4
C
      WRITE(UNIT4,1000) (RI(J),J=1,N)
C
C             READ ROWS  K  (K=I+1,N) AND MODIFY
C
      DO 70 K=IP1,N
      READ(UNIT3,1000) (RK(J),J=1,N)
C
C             ...FIND FIRST NON-ZERO ELEMENT
C
      DO 40 J=1,N
      JFIRST = J
40    IF(RK(J).NE.CZERO) GO TO 50
50    IF(JFIRST.GT.I)     GO TO 70
      AIJ     = RK(JFIRST)
C
C             ...MULTIPLY  AIJ  TIMES  I-TH  ROW
C                AND SUBTRACT FROM  K-TH  ROW
C
      DO 60 J=JFIRST,N
60    RK(J) = RK(J) - RI(J) * AIJ
      Y(K)  = Y(K)  - Y(I)  * AIJ
C
C             ...WRITE  K-TH  ROW TO UNIT4
C
70    WRITE(UNIT4,1000) (RK(J),J=1,N)
C
C             SWAP UNIT DESIGNATIONS
C
      NTEMP  = UNIT3
      UNIT3  = UNIT4
      UNIT4  = NTEMP
80    CONTINUE
C
C             SOLVE BY BACK-SUBSTITUTION
C
      X(N)    = Y(N) / RK(N)
C
C             READ FROM UNIT3
C
      BACKSPACE UNIT3
C
      DO 100 I=1,NM1
      NMI     = N - I
      NMIP1   = NMI + 1
      BACKSPACE UNIT3
      CTEMP   = CZERO
      READ(UNIT3,1000) (RI(J),J=1,N)
      DO 90 J=NMIP1,N
90    CTEMP   = CTEMP + RI(J) * X(J)
      X(NMI) = Y(NMI) - CTEMP
      BACKSPACE UNIT3
100   CONTINUE
C
      REWIND UNIT3
      REWIND UNIT4
C
C
```

GAUSSF (CONTINUED)

```
C                  RETURN TO CALLING PROGRAM
C
       RETURN
1000   FORMAT(1000(2E15.8))
       END
```

LMTXA

```
      SUBROUTINE LMTXA(A,D,MU,NL,NL1,MAXDIM)
      REAL MU(1),A(MAXDIM,MAXDIM),D(1)
C
C     SUBROUTINE TO COMPUTE GLOBAL MATRIX A FOR LOVE WAVE CASE
C
C
C        A11      AN ELEMENT IN THE LIMIT STIFFNESS MATRIX
C        A(I,J)   AN ELEMENT IN THE GLOBAL STIFFNESS MATRIX
C        NL       NUMBER OF LAYERS
C        NL1      NUMBER OF LAYERS-1
C        MU       SHEAR MODULUS FOR EACH LAYER
C        D        THICKNESS OF EACH LAYER
C
C     CLEAR MATRIX A
C
      DO 5 I=1,NL
      DO 5 J=1,NL
    5 A(I,J)=0.
C
C     CALCULATE MATRIX FOR NTH LAYER
C
      DENOM=1.0/6.0
      DO 10 J=1,NL1
         A12=(MU(J)*D(J))*DENOM
         A11=2.0*A12
         A21=A12
         A22=A11
C
C     ARRANGE  MATRICES INTO GLOBAL NLXNL MATRIX
C
      JP1=J+1
      A(J,J)=A(J,J)+A11
      A(J,JP1)=A(J,JP1)+A12
      A(JP1,J)=A(JP1,J)+A21
      A(JP1,JP1)=A(JP1,JP1)+A22
   10 CONTINUE
C
C     COMPUTE MATRIX ENTRY FOR BOTTOM LAYER, ASSUME FIXED,
C
      A11=MU(NL)*D(NL)/3.0
      A(NL,NL)=A(NL,NL)+A11
      RETURN
      END
```

LMTXC

```
      SUBROUTINE LMTXC(C,M,NL,D,MU,RHO,OMEGA,NL1,MAXDIM)
C
C     SUBROUTINE TO COMPUTE GLOBAL MATRIX C FROM MATRICES
C     B AND M FOR LOVE WAVES
C
      DIMENSION C(MAXDIM,MAXDIM),D(1)
      DIMENSION RHO(1)
      REAL MU(1),M(MAXDIM,MAXDIM)
C
C        M(NL,NL)    GLOBAL MASS MATRIX
C        C(NL,NL)    GLOBAL MATRIX C
C        OMSQ        OMEGA**2
C        B11         ELEMENT OF MATRIX B
C        XM12        ELEMENT OF ELEMENT MASS MATRIX
C        D(NL)       THICKNESS OF EACH LAYER
C        MU(NL)      SHEAR MODULUS FOR EACH LAYER
C        RHO(NL)     DENSITY OF EACH ELEMENT
C
C     CLEAR MATRIX C
C
      DO 10 I=1,NL
      DO 10 J=1,NL
      M(I,J)=0.
 10   C(I,J)=0.
C
C     COMPUTE MATRIX B
C
      OMSQ=OMEGA**2
      DENOM1=1.0/6.0
      DO 20 J=1,NL1
         B11=MU(J)/D(J)
         B12=-B11
         B21=-B11
         B22=B11
C
C     COMPUTE OMSQ*M
C
         XM12=D(J)*RHO(J)*DENOM1
         XM11=2.0*XM12
         XM21=XM12
         XM22=XM11
C
C     COMPUTE C=B-(OMEGA**2)*M
C
         JP1=J+1
         C(J,J)=C(J,J)+B11-XM11*OMSQ
         C(J,JP1)=C(J,JP1)+B12-XM12*OMSQ
         C(JP1,J)=C(JP1,J)+B21-XM21*OMSQ
         C(JP1,JP1)=C(JP1,JP1)+B22-XM22*OMSQ
         M(J,J)=M(J,J)+XM11
         M(J,JP1)=M(J,JP1)+XM12
         M(JP1,J)=M(JP1,J)+XM21
         M(JP1,JP1)=M(JP1,JP1)+XM22
 20   CONTINUE
C
C     COMPUTE BOTTOM LAYER ELEMENT, ASSUMED FIXED.
C
      B11=MU(NL)/D(NL)
      XM11=D(NL)*RHO(NL)/3.0
      C(NL,NL)=C(NL,NL)+B11-XM11*OMSQ
      M(NL,NL)=M(NL,NL)+XM11
      RETURN
      END
```

LVGLB

```
      SUBROUTINE LVGLB(IRRX,IRRZ,RHOE,BETA,Z,OMSQ)
C
C  DETERMINING STIFFNESS AND MASS MATRIX FOR EACH ELEMENT AND ADD
C  INTO GLOBAL MATRIX
C
C           NFND      MAX # FREE NODES
C           NHND       "  " HORIZ NODES
C           NVND       "  " VERT NODES
C           LAY        "  " LAYERS
C           NCOL       "  " COLUMNS
C           Z         TEMPORARY GLOBAL MATRIX
C           MM        ELEMENT MASS MATRIX
C           KM        ELEMENT STIFFNESS MATRIX
C           NFN       # FREE NODES
C           L         ELEMENT COUNTER
C           NHN       # HORIZ NODES
C           NVN       # VERTICAL NODES
C           ALPHA     P WAVE VELOCITY FOR EACH ELEMENT
C           BETA      S WAVE    "       "    "     "
C           RHOE      DENSITY FOR EACH ELEMENT
C           OMSQ      OMEGA**2
C
C
C  SUBROUTINES USED: GAUSL-- ASSEMBLES ELEMENT MATRICES THROUGH
C                            GAUSSIAN QUADRATURE
C
C
      COMMON/SIZE/NL,NC,NFN,NHN,NVN,NFND,NHND,NVND,LAY,NCOL
      REAL MM(4,4),KM(4,4),MU,IRRX(NVND,NHND),IRRZ(NVND,NHND)
      REAL COORD(4,2)
      REAL RHOE(LAY,NCOL),BETA(LAY,NCOL)
      COMPLEX Z(NFND,NFND)
      L=1
      DO 10 I=1,NFN
      DO 10 J=1,NFN
10    Z(I,J)=0.
      DO 80 J=1,NC
      DO 70 I=1,NL
C
C  ASSEMBLE COORDINATE MATRIX FOR USE IN QUADRATURE ROUTINE
C
      COORD(1,1)=IRRX(I,J)
      COORD(1,2)=IRRZ(I,J)
      COORD(2,1)=IRRX(I,J+1)
      COORD(2,2)=IRRZ(I,J+1)
      COORD(3,1)=IRRX(I+1,J+1)
      COORD(3,2)=IRRZ(I+1,J+1)
      COORD(4,1)=IRRX(I+1,J)
      COORD(4,2)=IRRZ(I+1,J)
      RHO=RHOE(I,J)
      MU=RHO*BETA(I,J)**2
C
C  CALL QUADRATURE ROUTINE TO ASSEMBLE ELEMENT MATRICES
C
      CALL GAUSL(KM,MM,MU,RHO,COORD)
      L1=L+1
      LNL=L+NL
      LNL1=LNL+1
C
C  ADD IN FIXED ELEMENTS
C
      Z(L,L)=Z(L,L)+KM(1,1)-OMSQ*MM(1,1)
      Z(L,LNL)=Z(L,LNL)+KM(1,2)-OMSQ*MM(1,2)
      Z(LNL,LNL)=Z(LNL,LNL)+KM(2,2)-OMSQ*MM(2,2)
      IF(I .EQ. NL) GO TO 60
C
C  ADD IN FREE ELEMENTS
C
      Z(L,LNL1)=Z(L,LNL1)+KM(1,3)-OMSQ*MM(1,3)
      Z(L,L1)=Z(L,L1)+KM(1,4)-OMSQ*MM(1,4)
      Z(LNL,LNL1)=Z(LNL,LNL1)+KM(2,3)-OMSQ*MM(2,3)
      Z(L1,LNL)=Z(L1,LNL)+KM(2,4)-OMSQ*MM(2,4)
      Z(LNL1,LNL1)=Z(LNL1,LNL1)+KM(3,3)-OMSQ*MM(3,3)
      Z(L1,LNL1)=Z(L1,LNL1)+KM(3,4)-OMSQ*MM(3,4)
      Z(L1,L1)=Z(L1,L1)+KM(4,4)-OMSQ*MM(4,4)
60    L=L+1
70    CONTINUE
80    CONTINUE
      DO 90 I=1,NFN
      DO 90 J=1,NFN
90    Z(J,I)=Z(I,J)
      CONTINUE
      RETURN
      END
```

LVGLBX

```
      SUBROUTINE LVGLBX(IRRX,IRRZ,RHOE,BETA,NL,NC,NFN,NHN,NVN,
     1NFND,NHND,NVND,LAT,NCOL)
C
C  DETERMINE STIFFNESS AND MASS MATRIX FOR EACH ELEMENT AND ADD
C  INTO GLOBAL MATRIX. THE GLOBAL MATRIX IS WRITTEN OUT ON UNIT1
C  AT THE END OF THIS ROUTINE.
C  SYMBOLS USED:
C
C         NFND       MAX # FREE NODES
C         NHND        "  " HORIZ NODES
C         NVND        "  " VERT NODES
C         LAT         "  " LAYERS
C         NCOL        "  " COLUMNS
C         MM         ELEMENT MASS MATRIX
C         KM         ELEMENT STIFFNESS MATRIX
C         NFN        # FREE NODES
C         L          ELEMENT COUNTER
C         NHN        # HORIZ NODES
C         NVN        # VERTICAL NODES
C         RI         DUMMY ARRAY USED TO READ GLOBAL MATRIX
C                    ROWS AND TO WRITE OUT ROWS ON DISK
C         OMSQ       OMEGA**2
C
      INTEGER UNIT1,UNIT2
      REAL MM(4,4),KM(4,4),MU,IRRX(NVND,NHND),IRRZ(NVND,NHND)
      REAL COORD(4,2)
      REAL RHOE(LAT,NCOL),BETA(LAT,NCOL)
      REAL RI(1000)
      COMMON/INOUT/ UNIT1,UNIT2
      COMMON/CONTR/OMSQ
      L=1
      DO 10 JJ=1,NFN
10    RI(JJ)=0.
C
C  CLEAR DISK FILES
C
      DO 20 II=1,NFN
      WRITE(UNIT2,1000) (RI(JJ),JJ=1,NFN)
20    WRITE(UNIT1,1000) (RI(JJ),JJ=1,NFN)
      DO 50 J=1,NC
      DO 40 I=1,NL
C
C  ASSEMBLE COORDINATE MATRIX FOR USE IN QUADRATURE ROUTINE
C
      COORD(1,1)=IRRX(I,J)
      COORD(1,2)=IRRZ(I,J)
      COORD(2,1)=IRRX(I,J+1)
      COORD(2,2)=IRRZ(I,J+1)
      COORD(3,1)=IRRX(I+1,J+1)
      COORD(3,2)=IRRZ(I+1,J+1)
      COORD(4,1)=IRRX(I+1,J)
      COORD(4,2)=IRRZ(I+1,J)
      RHO=RHOE(I,J)
      MU=RHO*BETA(I,J)**2
C
C  CALL QUADRATURE ROUTINE TO ASSEMBLE ELEMENT MATRICES
C
      CALL GAUSL(KM,MM,MU,RHO,COORD)
      L1=L+1
      LNL=L+NL
      LNL1=LNL+1
C
C
C  ADD IN FIXED ELEMENTS
C
C
C
C  Z(L,L)=Z(L,L)+ZM(1,1)
C
      CALL RLOAD(NFN,L,L,KM(1,1),MM(1,1),RI)
C
C  Z(L,LNL)=Z(L,LNL)+ZM(1,2)
C
      CALL RLOAD(NFN,L,LNL,KM(1,2),MM(1,2),RI)
C
C  Z(LNL,LNL)=Z(LNL,LNL)+ZM(2,2)
C

      CALL RLOAD(NFN,LNL,LNL,KM(2,2),MM(2,2),RI)
      IF(I .EQ. NL) GO TO 30
C
C
C  ADD IN FREE ELEMENTS USING DISK ROUTINE RLOAD
C
C
C
C  Z(L,LNL1)=Z(L,LNL1)+ZM(1,3)
C
      CALL RLOAD(NFN,L,LNL1,KM(1,3),MM(1,3),RI)
C
C  Z(L,L1)=Z(L,L1)+ZM(1,4)
C
      CALL RLOAD(NFN,L,L1,KM(1,4),MM(1,4),RI)
C
C  Z(LNL,LNL1)=Z(LNL,LNL1)+ZM(2,3)
C
      CALL RLOAD(NFN,LNL,LNL1,KM(2,3),MM(2,3),RI)
C
C  Z(L1,LNL)=Z(L1,LNL)+ZM(2,4)
C
      CALL RLOAD(NFN,L1,LNL,KM(2,4),MM(2,4),RI)
C
C  Z(LNL1,LNL1)=Z(LNL1,LNL1)+ZM(3,3)
C
      CALL RLOAD(NFN,LNL1,LNL1,KM(3,3),MM(3,3),RI)
C
C  Z(L1,LNL1)=Z(L1,LNL1)+ZM(3,4)
C
      CALL RLOAD(NFN,L1,LNL1,KM(3,4),MM(3,4),RI)
C
C  Z(L1,L1)=Z(L1,L1)+ZM(4,4)
C
      CALL RLOAD(NFN,L1,L1,KM(4,4),MM(4,4),RI)
C
30    L=L+1
40    CONTINUE
50    CONTINUE
      CONTINUE
      REWIND UNIT1
      RETURN
1000  FORMAT(1000E15.8)
```

LVGLBX (CONTINUED)

```
      END
C
C  PLOAD-- LOADS ELEMENTS FROM ELEMENT MASS AND STIFFNESS
C          MATRICES INTO THEIR RESPECTIVE POSITIONS IN THE
C          GLOBAL MATRIX.
C
C      NFN      # FREE NODES
C      IR       ROW COUNTER FOR GLOBAL MATRIX
C      IC       COLUMN COUNTER FOR GLOBAL MATRIX
C      ELEK     ELEMENT OF ELEMENT STIFFNESS MATRIX
C      ELEM     ELEMENT OF ELEMENT MASS MATRIX
C      RI       DUMMY ARRAY USED TO READ AND WRITE ROWS TO DISK
C
      SUBROUTINE PLOAD(NFN,IR,IC,ELEK,ELEM,RI)
      INTEGER UNIT1,UNIT2
      REAL RI(1)
      COMMON/INOUT/ UNIT1,UNIT2
      COMMON/CONTR/OMSQ
      KOUNT=0
C
C  REWIND DATA (DISK) UNITS
C
10    REWIND UNIT1
      REWIND UNIT2
      IRM1=IR-1
      IRP1=IR+1
      IF(IR .EQ. 1) GO TO 30
C
C  SKIP OVER I-1 ROWS
C
      DO 20 I=1,IRM1
      READ(UNIT1,1000) (RI(J),J=1,NFN)
20    WRITE(UNIT2,1000) (RI(J),J=1,NFN)
30    READ(UNIT1,1000) (RI(I),I=1,NFN)
C
C  ADD ELEMENT OF ELEMENT MATRIX INTO ROW OF GLOBAL MATRIX
C
      RI(IC)=RI(IC)+ELEK-OMSQ*ELEM
      WRITE(UNIT2,1000) (RI(J),J=1,NFN)
      IF(IR .EQ. NFN) GO TO 50
C
C  SKIP OVER REMAINING ROWS AND WRITE MODIFIED GLOBAL MATRIX TO UNIT2
C
      DO 40 I=IRP1,NFN
      READ(UNIT1,1000) (RI(J),J=1,NFN)
40    WRITE(UNIT2,1000) (RI(J),J=1,NFN)
C
C  SWITCH ROWS AND COLUMNS
C
50    ITEMP=IC
      IC=IR
      IR=ITEMP
C
C  SWAP UNIT DESIGNATIONS
C
      ITEMP=UNIT1
      UNIT1=UNIT2
      UNIT2=ITEMP
      KOUNT=KOUNT+1
C
C  ADD IN SYMMETRIC ELEMENTS IF NOT ON DIAGONAL
C
      IF(KOUNT .EQ. 1 .AND. IC .NE. IR) GO TO 10
      RETURN
1000  FORMAT(1000E15.8)
      END
```

LVIRR

```
C
C   IRREGULAR ZONE FINITE ELEMENT ANALYSIS-- LOVE WAVES
C
C       IN THIS PROGRAM THE IRREGULAR ZONE MODES OF VIBRATION ARE
C   COMPUTED WITH ENERGY TRANSFER AND AVERAGE PHASE VELOCITIES.
C   ENERGY IS IMPARTED TO THE IRREGULAR STRUCTURE VIA AN INCIDENT
C   PLANE WAVE AS SPECIFIED BY THE INCIDENT DISPLACEMENTS AND THE
C   MATRIX R1. MATRIX R2 REPRESENTS THE BOUNDARY CONDITIONS ON THE
C   RIGHT END OF THE IRREGULAR ZONE. THERE IS NO ENERGY LOSS ACROSS
C   THE IRREGULAR ZONE.
C
C   Bibliography:
C
C   Lysmer, J. and  L. A. Drake (1971). The propagation of Love waves
C       across nonhorizontally layered structures, Bull. Seis. Soc. Am.,
C       v. 61, 1233-1251.
C
C
C   SUBROUTINES USED: LVGLB-- LOADS GLOBAL MATRIX
C                     LEQTIC-- IMSL LINEAR SYSTEM OF EQUATIONS SOLVER
C
C
C   DATA FILES:  IRR.DAT--CONTAINS COORDINATES OF NODES, NUMBERS OF
C                         FUNDAMENTAL MODES (FROM LAYERED ZONES),
C                         AND ELEMENT PARAMETERS.
C
C                R1.DAT-- CONTAINS R MATRIX AS WELL AS DISPLACEMENTS AND
C                         NUMBERS FROM LEFT LAYERED ZONE. ALSO PERIODS.
C
C                R2.DAT-- CONTAINS R MATRIX, WAVENUMBER, AND DISPLACEMENTS
C                         FROM RIGHT LAYERED ZONE.
C
C
C   SYMBOLS USED IN THIS PROGRAM:
C
C           FNDS            MAXIMUM # FREE NODES (315)
C           HNOD,HNMAX      MAX # HORIZ NODES (21)
C           VNOD,VNMAX      MAX # VERTICAL NODES (20)
C           LAT,LMAX        MAX # LAYERS (22)
C           COL,CMAX        MAX # COLUMNS (20)
C
C           NOTE--IF THESE NUMBERS ARE EXCEEDED THE PARAMETER STATEMENT
C                 MUST BE CHANGED.
C
C           FN              NUMBER OF FREE NODES
C           VN,HN           VERT AND HORIZ NUMBER OF NODES IN IRR ZONE
C           IRRX            MATRICES CONTAINING COORDINATES (X,Z) OF
C           IRRZ            NODES IN IRREGULAR ZONE
C           MU              SHEAR MODULUS
C           BETA            S WAVE VELOCITY FOR EACH ELEMENT
C           RHOE,RHO        DENSITY OF EACH ELEMENT
C           K               GLOBAL STIFFNESS MATRIX FOR IRR ZONE
C           M               GLOBAL MASS MATRIX FOR IRR ZONE
C           R1,R2           BOUNDARY CONDITION MATRICES
C           IDISPL          INCIDENT DISPLACEMENT
C           OMSQ            FREQUENCY OF INCIDENT WAVEFORM**2
C           NL              NUMBER OF LAYERS
C           NC              NUMBER OF COLUMNS IN IRR ZONE
C           Y               DISPLACEMENTS OF IRR ZONE
C           YR              REFLECTED DISPLACEMENTS
C           U               LEFT LAYERED ZONE MODES OF DISPLACEMENT
C           UINV1,UINV2     INVERSE OF U
C           NUM1            INCIDENT FUNDAMENTAL MODE NUMBER
C           NUM2            TRANSMITTED FUNDAMENTAL MODE NUMBER
C           MAXDIM          NODAL STRUCTURE ORDER
C           PART            MODE PARTICIPATION FACTOR (MPF)
C           RPART           REFLECTED MODE PART. FACTOR
C           CAVE            PHASE VELOCITY FOR EACH PERIOD
C           X               COEFFICIENT MATRIX FOR LEQTIC
C           WORK            DUMMY ARRAY FOR LEQTIC
C           WN1             WAVE NUMBER MATRIX FROM LEFT LAYERED ZONE
C           WN2             WAVE NUMBER MATRIX FROM RIGHT LAYERED ZONE
C           TEST            USED TO DETERMINE ZERO IN ROUNDOFF CASES
C           ENGY            TRANSMITTED ENERGY IN EACH MODE
C           RENGY           REFLECTED ENERGY
C           ESUM            SUM OF TRANSMITTED ENERGY
C           RESUM           SUM OF REFLECTED ENERGY
C           PHI,RPHI        PHASE OF TRANSMITTED AND REFLECTED WAVES
C           AMPL,RAMPL      TRANSMITTED AND REFLECTED AMPLITUDES
C           TMIN            MINIMUM PERIOD
C           TMAX            MAXIMUM PERIOD
C           TINC            PERIOD INCREMENT
C
C
C
C           --OTHER SYMBOLS ARE DEFINED IN THEIR RESPECTIVE ROUTINES
C
      PARAMETER FNDS=315,LAT=22,COL=20,HNOD=21,VNOD=20
      INTEGER FN,HN,VN,HNMAX,VNMAX,CMAX,NUM1(100),NUM2(100)
      INTEGER THUM,HM1
      REAL IRRX(VNOD,HNOD),IRRZ(VNOD,HNOD),IK),(K)
        REAL RHOE(LAT,COL),BETA(LAT,COL),WORK(FNDS)
      COMPLEX R1(LAT,LAT),R2(LAT,LAT),YR(LAT),UINV2(LAT,LAT),WN2(LAT)
      COMPLEX IDISPL(FNDS),X(FNDS,FNDS),Y(FNDS,1),WN1(LAT)
      COMPLEX U(LAT,LAT),UINV1(LAT,LAT),CNIT,CZERO,CTWO,PART,RPART
      COMPLEX DISPL,CNORM
      COMMON/SIZE/NL,NC,FN,HN,VN,MAXDIM,HNMAX,VNMAX,LMAX,CMAX
      MAXDIM=FNDS
      LMAX=LAT
      CMAX=COL
      HNMAX=HNOD
      VNMAX=VNOD
      ZERO=0.0
      ONE=1.00
      TWO=2.00
      CENT=100.00
      CZERO=(0.00E00,0.00E00)
      CTWO=(2.0E00,0.0E00)
      TEST=1.00E-05
      TPI=2.0*3.14159265
C
C   READ IN COORDINATES, DENSITIES, VELOCITIES, AND PERIODS OFF OF DISK
C
      OPEN(UNIT=5,DEVICE='DSK',FILE='IRR.DAT',ACCESS='SEQIN')
      OPEN(UNIT=6,DEVICE='LPT',ACCESS='SEQOUT')
      OPEN(UNIT=24,DEVICE='DSK',FILE='R1.DAT',ACCESS='SEQIN')
      OPEN(UNIT=25,DEVICE='DSK',FILE='R2.DAT',ACCESS='SEQIN')
C
C   READ IN DIMENSIONS OF DATA
C
      READ(5,1020) VN,HN
```

```
      READ(24) TMIN,TMAX,TINC
      TNUM=(TMAX-TMIN)/TINC+1
C
C  CHECK FOR DATA FILE ERROR
C
      IF(VN .LE. 1 .OR. HN .LE. 1) GO TO 250
      IF(VN .GT. VNOD .OR. HN .GT. HNOD) GO TO 260
C
C  SET STRUCTURE PARAMETERS
C
10    NL=VN-1
      NC=HN-1
      FN=NN+NL
      ND=FN-NL
C
C  READ IN CARTESIAN COORDINATES OF NODES
C
      DO 20 I=1,VN
      DO 20 J=1,HN
20    READ(5,1030) INRX(I,J),INRZ(I,J)
C
C  COMPUTE DISTANCE TRAVELLED BY WAVE
C
      DIST=0.
      HN1=HN-1
      DO 30 I=1,HN1
30    DIST=DIST+SQRT((INRX(1,I+1)-INRX(1,I))**2+
      ((INRZ(1,I+1)-INRZ(1,I))**2)
C
C  READ IN FUNDAMENTAL MODE NUMBERS FOR EACH PERIOD
C
      READ(5,1000) (NUM1(I),I=1,TNUM)
      READ(5,1000) (NUM2(I),I=1,TNUM)
C
C  READ IN ELASTIC CONSTANTS AND DENSITIES OF EACH ELEMENT
C
      DO 40 I=1,NL
      DO 40 J=1,NC
40    READ(5,1050) BETA(I,J),RHOE(I,J)
      CLOSE(UNIT=5,DEVICE='DSK',FILE='INR.DAT',DISPOSE='SAVE')
C
C  CHECK DATA BY PRINTING IT OUT

C
      WRITE(6,1040)
      WRITE(6,1060) NL,NC
      WRITE(6,1070)
      DO 50 I=1,VN
50    WRITE(6,1090) (INRX(I,J),J=1,HN)
      WRITE(6,1080)
      DO 60 I=1,VN
60    WRITE(6,1090) (INRZ(I,J),J=1,HN)
      WRITE(6,1100)
      WRITE(6,1110)
      DO 70 I=1,NL
      DO 70 J=1,NC
70    WRITE(6,1120) I,J,BETA(I,J),RHOE(I,J)
C
C  LOOP OVER PERIODS
C
      II=0
      DO 240 PER=TMIN,TMAX,TINC
      OMSQ=(TPI/PER)**2
      II=II+1
      WRITE(6,1130) PER
C
C  CALL MATRIX ASSEMBLER SUBROUTINE TO COMPUTE GLOBAL MASS AND
C  STIFFNESS MATRICES
C
      CALL LVGLB(INRX,INRZ,RHOE,BETA,X,OMSQ)
C
C  READ IN R1 MATRIX, INCIDENT DISPLACEMENTS, U, UINV1 FROM LEFT
C  SIDE OF STRUCTURE,
C

      DO 80 I=1,FN
      IDISPL(I)=CZERO
80    T(I,1)=CZERO
      DO 90 I=1,NL
90    READ(24) (R1(I,J),J=1,NL)
      DO 100 I=1,NL
      READ(24) WN1(I)
      DO 100 J=1,NL
      READ(24) U(I,J)
100   READ(24) UINV1(I,J)
C
C  READ IN R2 BOUNDARY CONDITION MATRIX FOR RIGHT SIDE OF STRUCTURE
C
      DO 110 I=1,NL
      READ(25) WN2(I)
      READ(25) (UINV2(I,J),J=1,NL)
110   READ(25) (R2(I,J),J=1,NL)
      CLOSE(UNIT=5,DEVICE='DSK',FILE='R1.DAT',DISPOSE='SAVE')
C
C  CHOOSE PROPER INCIDENT DISPLACMENT AND WAVE NUMBER
C
      DO 120 I=1,NL
120   IDISPL(I)=U(I,NUM1(II))
C
C  COMBINE MATRICES TO FORM (K-OMEGA**2*M-R1-R2)*X AND
C  -2*R1*IDISPL=Y
C
      DO 140 I=1,NL
      CRIT=CZERO
      DO 130 J=1,NL
      X(I,J)=X(I,J)-R1(I,J)
      X(I+ND,J+ND)=X(I+ND,J+ND)-R2(I,J)
130   CRIT=CRIT+R1(I,J)*IDISPL(J)
140   T(I,1)=-CRIT*CTWO
C
C  SOLVE LINEAR EQUATION XU=Y
C
      CALL LEQTIC(X,FN,MAXDIM,T,1,MAXDIM,0,WORK,IER)
      WRITE(6,1140) IER
C
C  OUTPUT RESULTS ON LINE PRINTER
C
C
C     WRITE(6,1130)
C
C
C  PRINT R MATRIX FOR LEFT HAND SIDE
```

LVIRR (CONTINUED)

```
C
      DO 150 I=1,NL
      WRITE(6,1230) I
150   WRITE(6,1240) (R1(I,J),J=1,NL)
C
C  PRINT P MATRIX FOR RIGHT HAND SIDE
C
      WRITE(6,1160)
      DO 160 I=1,NL
      WRITE(6,1230) I
160   WRITE(6,1240) (P2(I,J),J=1,NL)
C
C  PRINT OUT INCIDENT DISPLACEMENTS (NORMALIZED TO 1)
C
      WRITE(6,1170)
      CNORM=IDISPL(I)
      DO 170 I=1,NL
      DISPL=IDISPL(I)/CNORM
170   WRITE(6,1180) IDISPL(I)
C
C  PRINT OUT DISPLACEMENTS OF IRREGULAR ZONE
C
      WRITE(6,1190)
      CNORM=T(1,1)
      DO 180 I=1,FM
      DISPL=T(I,1)/CNORM
180   WRITE(6,1250) J,T(I,1)
C
C  CALCULATE REFLECTED DISPLACEMENTS
C
      DO 190 I=1,NL
190   TR(I)=T(I,1)-IDISPL(I)
C
C  INITIALIZE VARIABLES
C
      WRITE(6,1210)
      WRITE(6,1220)
      ENORM=ONE/WN1(NUM1(II))
      ESUM=0.
      RESUM=0.
      DO 230 I=1,NL
      CAVE=ZERO
      PART=CZERO
      RPART=CZERO
      ENGY=ZERO
      RENGY=ZERO
C
C  FLAG FUNDAMENTAL MODES
C
      FLAG=' '
      IF(I .EQ. NUM1(II)) FLAG=' *'
      IF(I .EQ. NUM2(II)) FLAG=' #'
      IF(I .EQ. NUM1(II) .AND. I .EQ. NUM2(II)) FLAG='*#'
C
C  COMPUTE TRANSMITTED AND REFLECTED HPP'S
C
      DO 200 J=1,NL
      PART=PART+UINV2(I,J)*Y(ND+J,1)
C
C  INTRODUCE "-" SIGN TO SIGNIFY MOTION IN -X DIRN.
C
200   RPART=RPART+UINV1(I,J)*(-TR(J))
C
C  COMPUTE PHASE VELOCITIES, AND ENERGY IN
C  TRANSMITTED AND REFLECTED MODES AND WRITE OUT
C
      P1=1.*REAL(PART)
      P2=1.*AIMAG(PART)
      RP1=REAL(RPART)
      RP2=AIMAG(RPART)
      IF(ABS(P1) .LT. TEST .AND. ABS(P2) .LT. TEST) GO TO 220
      AMPL=SQRT(P1**2+P2**2)
      RAMPL=SQRT(RP1**2+RP2**2)
C
C  CALCULATE PHASE FROM ARGUMENT OF HPP
C
      PHI=ATAN2(P2,P1)
      RPHI=ATAN2(RP2,RP1)
      IF(PHI .LT. ZERO) PHI=TPI+PHI
      IF(RPHI .LT. ZERO) RPHI=TPI+RPHI
C
C  COMPUTE AVERAGE PHASE VELOCITY FROM PHASE AND DISTANCE TRAVELLED
C
      CAVE=DIST*TPI/(PER*PHI)
C
C  COMPUTE TRANSMITTED AND REFLECTED ENERGY %
C
      RK1=REAL(WN1(I))
      RK2=REAL(WN2(I))
      IK1=AIMAG(WN1(I))
      IK2=AIMAG(WN2(I))
      IF(ABS(RK2) .LT. TEST .OR. ABS(IK2) .GT. TEST) GO TO 210
      ENGY=ABS(RK2)*CABS(PART)**2*ENORM*CENT
      ESUM=ESUM+ENGY
210     IF(ABS(RK1) .LT. TEST .OR. ABS(IK1) .GT. TEST) GO TO 220
      RENGY=ABS(RK1)*CABS(RPART)**2*ENORM*CENT
      RESUM=RESUM+RENGY
220   WRITE(6,1200) I,FLAG,AMPL,RAMPL,PHI,RPHI,CAVE,ENGY,RENGY
230   CONTINUE
      WRITE(6,1010) ESUM,RESUM
240   CONTINUE
      GO TO 280
C
C  ERROR MESSAGES
C
250   WRITE(6,1260)
      GO TO 270
260   WRITE(6,1270)
C
C  CLOSE OUTPUT DEVICES
C
280   CLOSE(UNIT=6,DEVICE='LPT',DISPOSE='PRINT')
      CLOSE(UNIT=24,DEVICE='DSK',FILE='R1.DAT',DISPOSE='SAVE')
      CLOSE(UNIT=25,DEVICE='DSK',FILE='R2.DAT',DISPOSE='SAVE')
      STOP
1000. FORMAT(I3)
1010    FORMAT(//10X'TOTAL TRANSMITTED ENERGY =',F7.2,' %'33X,
     1'*  INCIDENT MODE',/9X,' TOTAL REFLECTED ENERGY =',
     2F7.2,' %',25X,'#  TRANSMITTED FUNDAMENTAL MODE')
1020  FORMAT(2I3)
```

LVIRR (CONTINUED)

```
1030     FORMAT(2F12.5)
1040     FORMAT(1X,'LOVE WAVE ANALYSIS')
1050     FORMAT(3X,3F10.5)
1060     FORMAT(//,' IRREGULAR STRUCTURE CONSISTING OF',I3,
         1' VERTICAL ELEMENTS AND',I3,' HORIZONTAL ELEMENTS:'//)
1070     FORMAT(//1X'HORIZONTAL COORDINATES ='/)
1080     FORMAT(//1X'VERTICAL COORDINATES ='/)
1090     FORMAT(1X,15F4.2)
1100     FORMAT(//6X,'ELEMENT',6X,'BETA',7X,'RHO'6)
1110     FORMAT(50('_')/)
1120     FORMAT(4X,'(',I3,',',I3,')',3X,F7.2,3X,F7.2)
1130     FORMAT(1H1'PERIOD =',F7.4)
1140     FORMAT(/1X'GROUP PARAMETER FROM LEUTIC =',I5/)
1150     FORMAT(//1X,'BOUNDARY CONDITION MATRICES'//,1H1,'MATRIX R1 =')
1160     FORMAT(1H1,'MATRIX R2 =')
1170     FORMAT(1H1,1X'INCIDENT DISPLACEMENT =')
1180     FORMAT(25X,2E12.5)
1190     FORMAT(1H1,'DISPLACEMENTS OF IRREGULAR NODES ='/)
1200     FORMAT(1X,I3,A2,3X,4(E12.5,5X),F12.5,2(5X,F12.2))
1210     FORMAT(1H1,9X,'TRANS AMPL',8X,'REFL AMPL',7X,'TRANS PHASE',6X,
         1'REFL PHASE',7X,'PHASE VELOCITY',5X,'% TRANS ENERGY',
         24X,'% REFL ENERGY')
1220     FORMAT(4X,120('_')/)
1230     FORMAT(///I10/)
1240     FORMAT(5(1X,2E12.5))
1250     FORMAT(7X,I3,10X,2(E12.5,2X))
1260     FORMAT(//1X'STRUCTURE SIZE INPUT ERROR'/)
1270     FORMAT(/' STRUCTURE EXCEEDS DIMENSIONS OF PROGRAM')
         END
```

LVIRRX

```
C
C  TOTAL FINITE ELEMENT ANALYSIS-- LOVE WAVES
C
C      IN THIS PROGRAM THE IRREGULAR ZONE MODES OF VIBRATION ARE
C  COMPUTED WITH ENERGY TRANSFER AND AVERAGE PHASE VELOCITIES.
C  ENERGY IS IMPARTED TO THE IRREGULAR STRUCTURE VIA AN INCIDENT
C  PLANE WAVE AS SPECIFIED BY THE INCIDENT DISPLACEMENTS AND THE
C  MATRIX R1 FROM LVLAT. MATRIX R2 REPRESENTS THE BOUNDARY CONDITIONS ON
C  RIGHT END OF THE IRREGULAR ZONE.
C      TRANSFER OF GLOBAL MATRICES AND SOLVING OF
C  GLOBAL LINEAR EQUATIONS ARE ACCOMPLISHED THROUGH READING AND
C  WRITING OFF OF DISK.
C
C  SYMBOLS USED IN THIS PROGRAM:
C
C          FNDS            MAXIMUM # FREE NODES (48)
C          HNOD,VNOD       MAX # HORIZ AND VERT NODES (3,16)
C          LAY,COL         MAX # COLUMNS AND LAYERS (2,15)
C
C          NOTE--IF THIS NUMBER IS EXCEEDED THE PARAMETER STATEMENT
C                MUST BE CHANGED.
C
C          FN              NUMBER OF FREE NODES
C          VN,HN           VERT AND HORIZ NUMBER OF NODES IN IRR ZONE
C          IRRX            MATRICES CONTAINING COORDINATES (X,Z) OF
C          IRRZ            NODES IN IRREGULAR ZONE
C          MU              SHEAR MODULUS
C          BETA            S WAVE VELOCITY FOR EACH ELEMENT
C          RHOE,RHO        DENSITY OF EACH ELEMENT
C          R1,R2           BOUNDARY CONDITION MATRICES
C          IDISPL          INCIDENT DISPLACEMENT
C          OMSQ            FREQUENCY OF INCIDENT WAVEFORM**2
C          NL              NUMBER OF LAYERS
C          NC              NUMBER OF COLUMNS IN IRR ZONE
C          Y               DISPLACEMENTS OF IRR ZONE
C          YR              REFLECTED DISPLACEMENTS
C          Z               KNOWN DISPLACEMENTS OF LAYERED ZONE
C          U               REGULAR ZONE MODES
C          UINV            INVERTED DISPLACEMENT MATRIX
C          SUM             THE DESIRED INCIDENT DISPLACEMENT AND WN
C          PART            MODE PARTICIPATION FACTOR
C          RPART           REFLECTED MODE PART. FACTOR
C          CPI             ROWS OF GLOBAL MATRIX FROM DISK
C          CWORK                       "
C          SUM                         "
C          WN              WAVE NUMBER MATRIX FROM LAYERED ZONE
C          CV              PHASE VELOCITY ARRAY FOR DISPERSION CURVES
C          TEST            USED TO DETERMINE ZERO IN ROUNDOFF CASES
C
C  SUBROUTINES USED: LVGLRX, GAUSSF
C
C          **OTHER SYMBOLS ARE DEFINED IN THEIR RESPECTIVE ROUTINES
C
      PARAMETER FNDS=48,LAY=15,COL=2,HNOD=3,VNOD=16
      INTEGER FN,HN,VN,UNIT1,UNIT2,UNIT3,UNIT4,NUM1(100),NUM2(100)
      INTEGER TNUM
      REAL IRRX(VNOD,HNOD),IRRZ(VNOD,HNOD)
      REAL  MT(FNDS),IR1,IR2
      REAL RHOE(LAY,COL),BETA(LAY,COL),CV(200)
      COMPLEX R1(LAY,LAY),R2(LAY,LAY),YR(LAY),UINV2(LAY,LAY)
      COMPLEX IDISPL(FNDS),Y(FNDS),CWORK(FNDS),WN(LAT)
      COMPLEX U(LAY,LAY),UINV1(LAY,LAY),CNIT,CZERO,CTWO,PART,RPART
      COMPLEX SUM(FNDS),Z(FNDS),CPI(FNDS),DISPL,CNORM,WN2(LAY)
      COMMON/INOUT/ UNIT1,UNIT2
      COMMON/INOUT1/UNIT3,UNIT4
      COMMON/CONTR/OMSQ
C
C  DESIGNATE UNITS FOR READING AND WRITING OFF OF DISK
C
      UNIT1=20
      UNIT2=21
      UNIT3=22
      UNIT4=23
      ZERO=0.0
      ONE=1.00
      TWO=2.00
      CENT=100.00
      CZERO=(0.00E00,0.00E00)
      CTWO=(2.0E00,0.0E00)
      TEST=1.00E-05
      TPI=2.0*3.14159265
C
C  READ IN COORDINATES, DENSITIES, VELOCITIES, PERIOD OFF OF DISK
C
      OPEN(UNIT=5,DEVICE='DSK',FILE='INP.DAT',ACCESS='SEQIN')
      OPEN(UNIT=6,DEVICE='LPT',ACCESS='SEQOUT')
      OPEN(UNIT=20,DEVICE='DSK',FILE='LEQ1.DAT',ACCESS='SEQINOUT',
     1RECORD SIZE=LRECL)
      OPEN(UNIT=21,DEVICE='DSK',FILE='LEQ2.DAT',ACCESS='SEQINOUT',
     1RECORD SIZE=LRECL)
      OPEN(UNIT=22,DEVICE='DSK',FILE='X1.DAT',ACCESS='SEQINOUT',
     1RECORD SIZE=LRECL1)
      OPEN(UNIT=23,DEVICE='DSK',FILE='X2.DAT',ACCESS='SEQINOUT',
     1RECORD SIZE=LRECL1)
      OPEN(UNIT=24,DEVICE='DSK',FILE='R1.DAT',ACCESS='SEQIN')
      OPEN(UNIT=25,DEVICE='DSK',FILE='R2.DAT',ACCESS='SEQIN')
C
C  READ IN DIMENSIONS OF DATA
C
      READ(5,1010) VN,HN
      READ(24) TMIN,TMAX,TINC
      TNUM=(TMAX-TMIN)/TINC+1
C
C  CHECK FOR DATA FILE ERROR
C
      IF(VN .LE. 1 .OR. HN .LE. 1) GO TO 310
      IF(VN .GT. VNOD .OR. HN .GT. HNOD) GO TO 320
C
C  SET STRUCTURE SIZE VARIABLES
C
10    NL=VN-1
      NC=HN-1
      FN=HN*NL
      ND=FN-NL
C
C  SET RECORD LENGTH FOR DISK
C
      LRECL=15*FN
      LRECL1=30*FN
```

```
C
C  READ IN CARTESIAN COORDINATES OF NODES
C
      DO 20 I=1,VN
      DO 20 J=1,HN
20    READ(5,1020) IRRX(I,J),IRRZ(I,J)
C
C  COMPUTE DISTANCE TRAVELED BY WAVE
C
      DIST=0.
      HN1=HN-1
      DO 21 I=1,HN1
21    DIST=DIST + SQRT((IRRX(1,I+1)-IRRX(1,I))**2 + (IRRZ(1,I+1) -
     IRRZ(1,I))**2)
C
C  READ IN NUMBERS OF FUNDAMENTAL MODES
C
      READ(5,1000) (NUM1(I),I=1,TNUM)
      READ(5,1000) (NUM2(I),I=1,TNUM)
1000  FORMAT(I5)
C
C  READ IN ELASTIC CONSTANTS AND DENSITIES OF EACH ELEMENT
C
      DO 30 I=1,NL
      DO 30 J=1,NC
30    READ(5,1060) BETA(I,J),RHOE(I,J)
      CLOSE(UNIT=5,DEVICE='DSK',FILE='IRR.DAT',DISPOSE='SAVE')
C
C  CHECK DATA BY WRITING OUT
C
      WRITE(6,1030)
      WRITE(6,1070) NL,NC
      WRITE(6,1080)
      DO 40 I=1,VN
      WRITE(6,1250) I
40    WRITE(6,1100) (IRRX(I,J),J=1,HN)
      WRITE(6,1090)
      DO 50 I=1,VN
      WRITE(6,1250) I
50    WRITE(6,1100) (IRRZ(I,J),J=1,HN)
      WRITE(6,1110)
      WRITE(6,1120)
      DO 60 I=1,NL
      DO 60 J=1,NC
60    WRITE(6,1130) I,J,BETA(I,J),RHOE(I,J)
C
C  LOOP OVER PERIODS
C
      II=0
      DO 290 PER=TMIN,TMAX,TINC
      II=II+1
      OMSQ=(TPI/PER)**2
      WRITE(6,1140) PER
C
C. CALL MATRIX ASSEMBLER SUBROUTINE TO COMPUTE GLOBAL MASS AND
C  STIFFNESS MATRICES, COMPUTE GLOBAL MATRIX K-OMSQ*M AND WRITE OUT ON DISK
C
      CALL LVGLOB(IRRX,IRRZ,RHOE,BETA,NL,NC,FN,HN,VN,FNOD,NNOD,
     IVNOD,LAY,COL)
      DO 80 I=1,FN
C
C  READ GLOBAL MATRIX FROM DISK AND WRITE TO UNIT4 AS COMPLEX
C
      READ(UNIT3,1300) (RI(J),J=1,FN)
      DO 70 J=1,FN
70    SUM(J)=CMPLX(RI(J),0.)
80    WRITE(UNIT4,1310) (SUM(J),J=1,FN)
C
C  READ IN R1 MATRIX, INCIDENT DISPLACEMENTS, U, UINV FROM LEFT
C  SIDE OF STRUCTURE.
C
      DO 90 I=1,FN
      IDISPL(I)=CZERO
90    Z(I)=CZERO
      DO 100 I=1,NL
100   READ(24) (R1(I,J),J=1,NL)
      DO 110 I=1,NL
      READ(24) WN1(I)
      DO 110 J=1,NL
      READ(24) U(I,J)
110   READ(24) UINV1(I,J)
      ND=FN-NL
C
C  READ IN R2 BOUNDARY CONDITION MATRIX FOR RIGHT SIDE OF STRUCTURE
C
      DO 120 I=1,NL
      READ(25) WN2(I)
      READ(25) (UINV2(I,J),J=1,NL)
120   READ(25) (R2(I,J),J=1,NL)
C
C  CHOOSE PROPER INCIDENT DISPLACEMENT AND WAVE NUMBER
C
      DO 130 I=1,NL
130   IDISPL(I)=U(I,NUM1(II))
C
C  COMBINE MATRICES TO FORM (K-OMEGA**2*M-R1-R2)*SUM AND
C  -2*R1*IDISPL=Z
C
      REWIND UNIT3
      REWIND UNIT4
      DO 150 I=1,NL
      READ(UNIT4,1310) (SUM(J),J=1,FN)
      DO 140 J=1,NL
C
C  SUBTRACT R1 MATRIX FROM UPPER LEFT HAND CORNER OF GLOBAL MATRIX
C
140   SUM(J)=SUM(J)-R1(I,J)
150   WRITE(UNIT3,1310) (SUM(J),J=1,FN)
C
C  SKIP OVER MIDDLE OF GLOBAL MATRIX
C
      DO 160 I=NL+1,ND
      READ(UNIT4,1310) (SUM(J),J=1,FN)
160   WRITE(UNIT3,1310) (SUM(J),J=1,FN)
      DO 190 I=1,NL
      READ(UNIT4,1310) (SUM(J),J=1,FN)
C
C  SUBTRACT R2 FROM LOWER RIGHT CORNER OF GLOBAL MATRIX
C
```

```
      ND1=ND+1
      DO 170 J=ND1,FN
      JND=J-ND
170   SUM(J)=SUM(J)-H2(I,JND)
180   WRITE(UNIT),1310) (SUM(J),J=1,FN)
      REWIND UNIT)
C
C  FORM RIGHT HAND SIDE OF GLOBAL LINEAR EQUATION
C
      DO 200 I=1,NL
      CHIT=CZERO
      DO 190 J=1,NL
190   CHIT=CHIT+P1(I,J)*IDISPL(J)
200   Z(I)=-CHIT*CTWO
C
C  SOLVE LINEAR EQUATION SUM*Y=Z
C
      CALL GAUSSF(Y,Z,CHI,CWORK,FN)
C
C  CLOSE DATA FILES
C
      WRITE(6,1170)
C
C  PRINT P MATRIX FOR LEFT HAND SIDE
C
      DO 210 I=1,NL
      WRITE(6,1250) I
210   WRITE(6,1260) (P1(I,J),J=1,NL)
C
C  PRINT P MATRIX FOR RIGHT HAND SIDE
C
      WRITE(6,1180)
      DO 220 I=1,NL
      WRITE(6,1250) I
220   WRITE(6,1260) (H2(I,J),J=1,NL)
C
C  PRINT OUT INCIDENT DISPLACEMENTS
C
      WRITE(6,1190)
      CNORM=IDISPL(1)
      DO 230 I=1,NL
      DISPL=IDISPL(I)/CNORM
230   WRITE(6,1300) DISPL
C
C  PRINT OUT DISPLACEMENTS OF IRREGULAR ZONE
C
      WRITE(6,1210)
      CNORM=Y(1)
      DO 240 I=1,FN
      DISPL=Y(I)/CNORM
240   WRITE(6,1270) I,DISPL
C
C  CALCULATE REFLECTED DISPLACEMENTS
C
      DO 250 I=1,NL
250   TR(I)=Y(I)-IDISPL(I)
C
C  INITIALIZE VARIABLES
C
      WRITE(6,1230)
      WRITE(6,1240)
      ESUM=0.
      RESUM=0.
      ENORM=ONE/WN1(NUM1(II))
      DO 280 I=1,NL
      CAVE=ZERO
      PART=CZERO
      RPART=CZERO
      ENGT=ZERO
      RENGT=ZERO
      FLAG=' '
      IF(I .EQ. NUM1(II)) FLAG=' *'
      IF(I .EQ. NUM2(II)) FLAG=' #'
      IF(I .EQ. NUM1(II) .AND. I .EQ. NUM2(II)) FLAG='*#'
C
C  COMPUTE TRANSMITTED AND REFLECTED MPF/S
C
      DO 260 J=1,NL
      PART=PART+UINV2(I,J)*Y(ND+J)
260   RPART=RPART+UINV1(I,J)*TR(J)
C
C  COMPUTE PHASE VELOCITIES, AND ENERGY IN
C  TRANSMITTED AND REFLECTED MODES AND WRITE OUT
C
      P1=REAL(PART)
      P2=AIMAG(PART)
      RP1=REAL(RPART)
      RP2=AIMAG(RPART)
      AMPL=SQRT(P1**2 + P2**2)
      RAMPL=SQRT(RP1**2 + RP2**2)
      IF(ABS(P1) .LT. TEST .AND. ABS(P2) .LT. TEST) GO TO 270
C
C  FIND PHASE FROM ARGUMENT OF MPF
C
      PHI=ATAN2(P2,P1)
      RPHI=ATAN2(RP2,RP1)
      IF(PHI .LT. ZERO) PHI = TPI + PHI

      IF(RPHI .LT. ZERO) RPHI = RPHI + TPI
C
C  COMPUTE AVERAGE PHASE VELOCITY FROM PHASE AND DISTANCE
C
      CAVE=DIST*TPI/(PER*ABS(PHI))
C
C  COMPUTE TRANSMITTED AND REFLECTED ENERGY %
C
      RK1=REAL(WN1(I))
      RK2=REAL(WN2(I))
      IK1=AIMAG(WN1(I))
      IK2=AIMAG(WN2(I))
      IF(ABS(RK2) .LT. TEST .OR. ABS(IK2) .GT. TEST) GO TO 263
      ENGT=ABS(RK2)*CABS(PART)**2*ENORM*CENT
      ESUM=ESUM+ENGT
 263  IF(ABS(RK1) .LT. TEST .OR. ABS(IK1) .GT. TEST) GO TO 270
      RENGT=ABS(RK1)*CABS(RPART)**2*ENORM*CENT
      RESUM=RESUM+RENGT
270   WRITE(6,1220) I,FLAG,AMPL,RAMPL,PHI,RPHI,CAVE,ENGT,RENGT
280   CONTINUE
      WRITE(6,1222) ESUM,RESUM
 1222 FORMAT(//10X,'TOTAL TRANSMITTED ENERGY =',F7.2,' %',33X,
```

LVIRRX (CONTINUED)

```
     1'%  INCIDENT MODE',/9X,' TOTAL REFLECTED ENERGY =',
     2F7.2,' %',25X,'%  TRANSMITTED FUNDAMENTAL MODE')
290        CONTINUE
      GO TO 340
C
C  ERROR MESSAGES
C
310   WRITE(5,1290)
      GO TO 340
330   WRITE(5,1320)
C
C  CLOSE OUTPUT DEVICES
C
340   CLOSE(UNIT=6,DEVICE='LPT',DISPOSE='PRINT')
      CLOSE(UNIT=20,DEVICE='DSK')
      CLOSE(UNIT=21,DEVICE='DSK')
      CLOSE(UNIT=22,DEVICE='DSK')
      CLOSE(UNIT=23,DEVICE='DSK')
      CLOSE(UNIT=24,DEVICE='DSK')
      CLOSE(UNIT=25,DEVICE='DSK')
      STOP
1010  FORMAT(2I3)
1020  FORMAT(2F12.5)
1030  FORMAT(1X,'LOVE WAVE ANALYSIS')
1040  FORMAT(2F12.5)
1050  FORMAT(F10.5)
1060  FORMAT(I4,3F10.5)
1070  FORMAT(//,' IRREGULAR STRUCTURE CONSISTING OF',I3,
     1' VERTICAL ELEMENTS AND',I3,' HORIZONTAL ELEMENTS:'//)
1080  FORMAT(//1X'HORIZONTAL COORDINATES ='//)
1090  FORMAT(1H1X'VERTICAL COORDINATES ='//)
1100  FORMAT(1X,10F12.2/)
1110  FORMAT(1H1,4X,'ELEMENT',6X,'BETA',7X,'RHO'/)
1120  FORMAT(50('-')/)
1130  FORMAT(4X,'(',I3,',',I3,')',3X,F7.2,3X,F7.2)
1140  FORMAT(1H1'PERIOD =',F7.2)
1150  FORMAT(1X,9(2E7.1))
1160  FORMAT(1X10E6.1)
1170  FORMAT(//1X,'BOUNDARY CONDITION MATRICES'//,1H1,'MATRIX R1 =')
1180  FORMAT(1H1,'MATRIX R2 =')
1190  FORMAT(1H1,'INCIDENT DISPLACEMENT =')
1200  FORMAT(25X,2E12.5)
1210  FORMAT(1H1,'DISPLACEMENTS OF IRREGULAR NODES ='/)
1220  FORMAT(1X,I3,A2,3X,4(E12.5,5X),F12.5,2(5X,F12.2))
1230  FORMAT(1H1,9X,'TRANS AMPL',8X,'REFL AMPL',7X,'TRANS PHASE',6X,
     1'REFL PHASE',7X,'PHASE VELOCITY',5X,'% TRANS ENERGY',
     24X,'% REFL ENERGY')
1240  FORMAT(4X,120('-')/)
1250  FORMAT(///I10/)
1260  FORMAT(5(1X,2E12.5))
1270  FORMAT(7X,I3,10X,2(E12.5,7X))
1290  FORMAT(/' YOU CANNOT HAVE < 2 NODES IN ANY DIRECTION')
1300  FORMAT(1000E15.8)
1310  FORMAT(1000(2E15.8))
1320  FORMAT(/' # OF NODES EXCEED DIMENSIONS OF PROGRAM')
      END
```

LVLAY

```
C     LVLAY-- LAYERED STRUCTURE LOVE WAVES
C
C     LVLAY IS DESIGNED TO SIMULATE LOVE WAVE PROPAGATION
C     THROUGH A LAYERED MEDIUM USING FINITE ELEMENT TECHNIQUES.
C     THIS PROGRAM CALLS TWO SUBROUTINES, LMTXA AND LMTXC TO ASSEMBLE
C     GLOBAL MATRICES A AND C RESPECTIVELY. THESE MATRICES ARE
C     USED IN THE EIGENVALUE EQUATION (C-K**2(A))X=0 WHICH
C     IS SOLVED USING THE IMSL ROUTINE EIGZF. EIGZF CALCULATES
C     EIGENVALUES (K**2) AND EIGENVECTORS (X), CORRESPONDING TO
C     TO WAVE NUMBERS AND DISPLACEMENTS OF THE REGULAR ZONE
C     FOR EACH PERIOD DESIRED.
C
C     Bibliography:
C
C     Lysmer, J. and L. A. Drake (1972), A finite element method for
C        seismology, in Methods in Computational Physics, vol. 11,
C        B. Bolt, Ed., 181-216, Acad. Press, N. Y.
C
C
C     SUBROUTINES USED: LMTXA-- ASSEMBLES GLOBAL MATRIX A
C                       LMTXC-- ASSEMBLES GLOBAL MATRIX C
C                       EIGZF-- IMSL ROUTINE TO SOLVE EIGENVALUE EQN.
C
C
C     DATA FILES:       LAY.DAT-- THICKNESS AND PARAMETERS FOR LAYERS
C                       R1.DAT--  R MATRIX, DISPLACEMENTS, WAVE NUMBERS,
C                                 AND PERIODS WRITTEN OUT HERE IF LEFT
C                                 SIDE IS BEING ANALYZED.
C                       R2.DAT--  R MATRIX, DISPLACEMENTS, AND WAVE
C                                 NUMBERS WRITTEN OUT IF RIGHT SIDE
C                                 IS BEING EXAMINED.
C
C
C     THE SYMBOLS USED IN THIS PROGRAM ARE AS FOLLOWS:
C
C           VN      MAX # OF VERTICAL NODES
C           LYS     MAX # OF LAYERS
C
C     WARNING------VN AND LYS MUST BE CHANGED IF MORE THAN 40  LAYERS
C                  ARE DESIRED.
C
C           NL      NUMBER OF LAYERS (FROM DATA)
C           A       NLxNL GLOBAL DERIVED FROM STIFFNESS MATRIX
C           C       NLxNL MATRIX DERIVED FROM MASS AND STIFFNESS
C           M       GLOBAL MASS MATRIX
C           K       MATRIX OF WAVE NUMBERS
C           R       BOUNDARY CONDITION MATRICES
C           BETA    SHEAR WAVE VELOCITY
C           D       THICKNESS OF EACH LAYER
C           DEPTH   DEPTH OF EACH NODE BELOW THE SURFACE
C           MU      SHEAR MODULUS OF EACH LAYER
C           RHO     DENSITY OF EACH LAYER
C           PER     PERIOD (SEC)
C           OMEGA   FREQUENCY (2*PI/T)
C           CVEL    PHASE VELOCITY
C           GVEL    GROUP VELOCITY
C           WORK    DUMMY PARAMETER FOR ROUTINE EIGZF AND USED
C                   IN COMPUTING GROUP VELOCITY
C           R       MATRIX CONTAINING THE BOUNDARY CONDITIONS OF
C                   THE LAYERED ZONE
```

LVLAY (CONTINUED)

```
C          PTEMP  USED TO CONSTRUCT MATRIX R
C          U      MATRIX OF EIGENVECTORS (MODE SHAPES)
C          UINV   INVERSE OF U (DISPLACEMENT MATRIX)
C          DISPL  DISPLACEMENTS NORMALIZED TO 1 FOR PRINTING
C          TEST   USED TO FIND FUNDAMENTAL MODES WHERE ALL
C                 DISPLACEMENTS ARE > 0
C          TMIN   MINIMUM PERIOD
C          TMAX   MAXIMUM PERIOD
C          TINC   PERIOD INCREMENT
C
C  NOTE-- THIS PROGRAM MUST BE RUN AGAIN IF DIFFERENT BOUNDARY
C         CONDITIONS ARE DESIRED ALONG THE RIGHT SIDE OF THE
C         IRREGULAR ZONE,BOUNDARY CONDITION MATRIX R1 OR R2 CAN
C         BE WRITTEN TO DISK BY RESPONDING TO THE INITIAL QUERY.
C
C
C
C
         PARAMETER LTS=10,YN=41
         REAL A(LTS,LTS),C(LTS,LTS),D(LTS),RHO(LTS),KR,KI,M(LTS,LTS)
         REAL BETA(LTS),ALPHA(LTS),FREAL(LTS),MU(LTS),PER,DEPTH(YN)
         COMPLEX XCOMP(LTS),U(LTS,LTS),K(LTS),EIK,CZERO,R(LTS,LTS),CTEMP
         COMPLEX PTEMP(LTS,LTS),UINV(LTS,LTS),KSQ,CONE
         COMPLEX WORK(LTS),CNORM,DISPL
C
C     OPEN INPUT FILE AND OUTPUT DEVICES
C
         OPEN(UNIT=5,DEVICE='DSK',FILE='LAT.DAT',ACCESS='SEQIN')
         OPEN(UNIT=6,DEVICE='LPT',ACCESS='SEQOUT')
C
C     READ IN # OF LAYERS
C
         READ(5,1050) NL
C
C     CHECK FOR INPUT SIZE ERROR
C
         IF(NL .LE. 1) GO TO 300
         IF(NL .GT. LTS) GO TO 310
C
C     INITIALIZE PARAMETERS
C
10       MAXDIM=LTS
         PI=3.14159265
         CZERO=(0.0E0,0.0E0)
         CONE=(1.0E0,0.0E0)
         EYE=(0.0E0,1.0E0)
         TEST=1.0E-04
         NL1=NL-1
C
C     READ IN LAYER PARAMETERS
C
         DO 20 I=1,NL
         READ(5,1060) D(I),BETA(I),RHO(I)
         MU(I)=(BETA(I)**2)*RHO(I)
20       DEPTH(I+1)=DEPTH(I)+D(I)
C
C     CLOSE INPUT FILE
C
         CLOSE(UNIT=5,DEVICE='DSK',FILE='LAT.DAT',DISPOSE='SAVE')
C
C     READ RANGE OF PERIODS
C
         WRITE(5,1210)
         READ(5,*) TMIN,TMAX,TINC
C
C     DECIDE IF R1 OR R2 IS TO BE COMPUTED IN THIS RUN
C
         WRITE(5,1020)
         READ(5,1040) R1ANS
         WRITE(5,1030)
         READ(5,1040) R2ANS
C
C     OPEN APPROPIATE DISK FILES
C
         IF(R1ANS .EQ. 'NO')GO TO 30
         OPEN(UNIT=24,DEVICE='DSK',FILE='R1.DAT',ACCESS='SEQOUT')
         WRITE(24) TMIN,TMAX,TINC
30       IF(R2ANS .EQ. 'NO') GO TO 40
         OPEN(UNIT=25,DEVICE='DSK',FILE='R2.DAT',ACCESS='SEQOUT')
C
C     CHECK DATA BY PRINTING OUT
C
40       WRITE(6,1070) NL
         DO 50 I=1,NL
         WRITE(6,1080) DEPTH(I)
         WRITE(6,1100) D(I), BETA(I), RHO(I)
50       CONTINUE
         WRITE (6,1090) DEPTH(NL+1)
C
C  LOOP OVER PERIODS
C
         II=1
         DO 290 PER=TMIN,TMAX,TINC
         JJ=II
         WRITE(6,1110) PER
         OMEGA=2.0*PI/PER
C
C     IMPLEMENT SUBROUTINES TO COMPUTE MATRICES A,M, AND C
C
         CALL LMTXA(A,D,MU,NL,NL1,MAXDIM)
C
C     MAKE 'A' NEGATIVE FOR EIGZF (MOVE TO OTHER SIDE OF EQUATION)
C
         DO 60 I=1,NL
         DO 60 J=1,NL
60       A(I,J)=-A(I,J)
         CALL LMTXC(C,M,NL,D,MU,RHO,OMEGA,NL1,MAXDIM)
C
C  SOLVE (C-(K**2)*A)*U=0 USING EIGZF
C
C     SYMBOLS USED IN EIGZF:
C        KSQ       K**2 (K=WAVE NUMBER)
C        XCOMP     COMPLEX COMPONENT OF EIGENVALUE, KSQ
C        FREAL     REAL COMPONENT OF EIGENVALUE, KSQ
C        U         DISPLACEMENT VECTORS (EIGENVECTORS)
C        CVEL      CURRENT PHASE VELOCITY
C        MAXDIM    MAX # OF LAYERS AS DUMMY VARIABLE FOR SUBROUTINES
C
         CALL EIGZF(C,MAXDIM,A,MAXDIM,NL,1,XCOMP,FREAL,U,MAXDIM,WOR
         K,IER)
         WRITE(6,1120) IER
```

```
C
C     COMPUTE EIGENVALUES (WAVE NUMBERS) AND EIGENVECTORS
C     (DISPLACEMENTS) CORRESPONDING TO DIFFERENT MODES, IF
C     EIGENVALUES IMAGINARY, SET FLAG.
C
        DO 160 IL=1,NL
        KCHEK='    '
        KSQ=FCMPP(IL)/FREAL(IL)
        IF (REAL(KSQ) .LT. 0.) KCHEK='IMAG'
C
C     CHOOSE EIGENVALUES ASSUMING IMAGINARY KSQ DOES NOT PROPAGATE
C
        KR=ABS(REAL(CSQRT(KSQ)))
        KI=ABS(AIMAG(CSQRT(KSQ)))
C
C     FORM MATRIX OF EIGENVALUES (K) FROM REAL AND NEGATIVE IMAGINARY
C     PARTS
C
        K(IL)=CMPLX(KR,-KI)
        IF(KR .EQ. 0.) KR=KI
C
C     FIND PHASE VELOCITY
C
        CVEL=OMEGA/KR
C
C     WRITE OUT EIGENVALUES, PHASE VELOCITIES, AND FLAG
C
        WRITE(6,1130) IL,KSQ
        WRITE(6,1140) IL,CVEL,KCHEK
        DO 80 I=1,NL
        CTEMP=CZERO
C
C  NORMALIZE DISPLACEMENT ACCORDING TO UT*A*U=1 (UT=U TRANSPOSE)
C
C     REGENERATE 'A' MATRIX
C
        CALL LMTXA(A,D,HU,NL,NL1,MAXDIM)
        DO 70 J=1,NL
70      CTEMP=CTEMP+U(J,IL)*CMPLX(A(J,I),0.)
80      WORKC(I)=CTEMP
        CNORM=CZERO
        DO 90 I=1,NL
90      CNORM=CNORM+WORKC(I)*U(I,IL)
        CTEMP=CONE/CSQRT(CNORM)
        DO 100 J=1,NL
100     U(J,IL)=U(J,IL)*CTEMP
C
C     COMPUTE GROUP VELOCITIES = 1/OMEGA*UT*M*U
C
        DO 120 I=1,NL
        CTEMP=CZERO
        DO 110 J=1,NL
110     CTEMP=CTEMP+M(J,IL)*CMPLX(M(J,I),0.)
120     WORKC(I)=CTEMP
        CNORM=CZERO
        DO 130 I=1,NL
130     CNORM=CNORM+WORKC(I)*U(I,IL)
        GVEL=KR/(OMEGA*CNORM)
        IF(IANS .EQ. 'NO') GO TO 140
C
C     WRITE OUT GROUP VELOCITIES AND DISPLACEMENTS
C
140     WRITE(6,1190) IL,GVEL
        WRITE(6,1150)
C
C  NORMALIZE DISPLACEMENTS TO 1 FOR OUTPUT
C
        CNORM=U(1,IL)
        DO 150 J=1,NL
        DISPL=U(J,IL)/CNORM
150     WRITE(6,1160) DISPL
160     CONTINUE
        IF(R1ANS .EQ. 'NO' .AND. R2ANS .EQ. 'NO') GO TO 240
C
C  COMPUTE R=A*U*K*UINV FOR TRANSMISSION OF FORCES TO
C  IRREGULAR ZONE
C
C     COMPUTE A*U (U = V)
C
        DO 180 I=1,NL
        DO 180 J=1,NL
        CTEMP=CZERO
        DO 170 L=1,NL
170     CTEMP=CTEMP+CMPLX(A(I,L),0.)*U(L,J)
180     RTEMP(I,J)=CTEMP
C
C     MULTIPLY BY DIAGONAL MATRIX OF WAVE NUMBERS
C
        DO 190 I=1,NL
        DO 190 J=1,NL
        RTEMP(I,J)=RTEMP(I,J)*K(J)
190     CONTINUE
C
C     FORM UINV
C
        DO 210 I=1,NL
        DO 210 J=1,NL
        CTEMP=CZERO
        DO 200 L=1,NL
200     CTEMP=CTEMP+U(L,I)*CMPLX(A(L,J),0.)
210     UINV(I,J)=CTEMP
C
C     FORM R MATRIX CONTAINING BOUNDARY VALUES
C
        DO 230 I=1,NL
        DO 230 J=1,NL
        CTEMP=CZERO
        DO 220 L=1,NL
220     CTEMP=CTEMP+RTEMP(I,L)*UINV(L,J)
230     R(I,J)=CTEMP*EIC
C
C     PRINT OUT MATRIX R ON LINE PRINTER
C
        WRITE(6,1170)
        DO 240 I=1,NL
        WRITE(6,1200) I
240     WRITE(6,1180) (R(I,J),J=1,NL)
C
C     WRITE R,K,U, AND UINV OUT ON DISK TO BE READ IN
C     BY LVIRR AS BOUNDARY CONDITIONS FOR LAYERED ZONES
```

LVLAY (CONTINUED)

```
C       FOR EACH PERIOD,
C
C       WRITE OUT BOUNDARY CONDITION MATRIX R1 ON DISK (R1,DAT) IF DESIRED
C
          IF(P1ANS .EQ. 'NO') GO TO 270
          DO 250 I=1,NL
250       WRITE(24) (R(I,J),J=1,NL)
C
C       WRITE OUT K MATRIX (WAVE NUMBERS) AND U,UINV ON DISK (R1,DAT)
C       WITH R1 MATRIX
C
          DO 260 I=1,NL
          WRITE(24) K(I)
          DO 260 J=1,NL
          WRITE(24) U(I,J)
260       WRITE(24) UINV(I,J)
C
C       WRITE R2 OUT ON DISK (R2,DAT) IF DESIRED
C
270       IF (P2ANS .EQ. 'NO') GO TO 290
          DO 280 I=1,NL
          WRITE(25) K(I)
            WRITE(25) (UINV(I,J),J=1,NL)
280       WRITE(25) (H(I,J),J=1,NL)
290       CONTINUE
          GO TO 120
C
C  ERROR NOTES
C
300       WRITE(5,1000)
          GO TO 320
310         WRITE(5,1010)
C
C
C  CLOSE OUTPUT DEVICES
C
320       CLOSE(UNIT=6,DEVICE='LPT',ACCESS='SEQOUT',DISPOSE='PRINT')
          IF(P1ANS .EQ. 'NO') GO TO 330
          CLOSE(UNIT=24,DEVICE='DSK',FILE='R1,DAT',DISPOSE='SAVE')
330       IF(P2ANS .EQ. 'NO') GO TO 340
          CLOSE(UNIT=25,DEVICE='DSK',FILE='R2,DAT',DISPOSE='SAVE')
340       STOP
1000      FORMAT(/1X'# OF LAYERS CANNOT BE < 2')
1010      FORMAT(/' # OF LAYERS EXCEED PROGRAM DIMENSION')
1020      FORMAT(' WILL MATRIX R1 BE COMPUTED USING THIS STRUCTURE?')
1030      FORMAT(' WILL R2 BE COMPUTED?')
1040      FORMAT(A3)
1050      FORMAT(I5)
1060      FORMAT(3F12.5)
1070      FORMAT(10X,'FINITE ELEMENT LOVE WAVE MODEL: ',3X,I5
          1,' LAYERS'//12X,'THICKNESS',11X,'SHEAR VEL.',7X,'DENSITY'/)
1080      FORMAT(1X,F7.2,60('-'))
1090      FORMAT(1X,F7.3,60('-'))
1100      FORMAT(10X,F9.2,9X,F10.2,9X,F7.2)
1110      FORMAT(1H1,8X,'PERIOD = ',F7.4/)
1120      FORMAT(9X,'ERROR PARAMETER FROM EIGZF =',I4/)
1130      FORMAT(1H1,8X,'EIGENVALUE',I4,' =',2E12.4)
1140      FORMAT(10X,'PHASE VELOCITY',I4,' =',E12.4,4X,A4)
1150      FORMAT(10X,'CORRESPONDING DISPLACEMENTS =')
1160      FORMAT(4(1X,2E12.5))
1170      FORMAT(1H1,2X,'MATRIX R ='/)
1180      FORMAT(5(2E12.5,1X))
1190      FORMAT(10X,'GROUP VELOCITY',I4,' =',E12.4//)
1200      FORMAT(///I10/)
1210      FORMAT(/1X'ENTER MINIMUM AND MAXIMUM PERIOD, AND PERIOD
          1 INCREMENT, SEPERATED BY COMMAS, IF ONLY ONE PERIOD IS DESIRED,
          2 MAKE MAX AND MIN THE SAME.'/)
          END
```

PLOT

```
C   PLOT--PLOTS OUT 2-D FINITE ELEMENT MESH
C
C   VARIABLES USED IN THIS PROGRAM:
C
C         VNODES            # VERT NODES
C         HNODES            # HORIZ NODES
C         IDEPTH            ASCII CHARACTER CODE ARRAYS
C         IWIDTH            REPRESENTING DEPTHS AND WIDTHS
C         XMIN              MIN HORIZ SCREEN COORD
C         XMAX              MAX    "      "      "
C         ZMIN              MIN VERT SCREEN COORD
C         ZMAX              MAX    "      "      "
C         HMAX              MAX WINDOW HEIGHT
C         INR1.DAT          INPUT FILE
C
C   NOTE: PROGRAM WILL HALT AFTER PLOTTING OUT STRUCTURE.
C          TYPE 'CR' TO FINISH PROGRAM.
C
C   ALSO, BE SURE XWIN AND ZWIN ARE EVENLY DIVISIBLE BY 5.
C
          PARAMETER VN=100,HN=100
          REAL INRX(VN,HN),INRZ(VN,HN),NAME(30)
          INTEGER VNODES,HNODES,IDEPTH(6),IWIDTH(6),UNITS(2)
          INTEGER XMIN,XMAX,ZMIN,ZMAX,XWIN,ZWIN
          OPEN(UNIT=2,DEVICE='DSK',FILE='INR.DAT',ACCESS='SEQIN')
C
C   READ IN DATA AND CHECK
C
          WRITE(5,1000)
          READ(5,1010) (NAME(I),I=1,30)
          WRITE(5,1020)
          READ(5,1030) ANS
            READ(2,1040) VNODES,HNODES
          IF(ANS .EQ. 'NO') GO TO 10
          WRITE(5,1050)
          READ(5,*) VNODES
10          DO 20 I=1,VNODES
            DO 20 J=1,HNODES
          READ(2,1060) INRX(I,J),INRZ(I,J)
C
C   INVERT Z AXIS
C
20        INRZ(I,J)=1000.-INRZ(I,J)
          CLOSE(UNIT=2,DEVICE='DSK',FILE='INR.DAT',DISPOSE='SAVE')
C
C   FIND MAX Z VALUE FOR VIRTUAL WINDOW
C
          HMAX=INRZ(1,1)
          DO 30 I=2,HNODES
30        IF(INRZ(1,I) .GT. HMAX) HMAX=INRZ(1,I)
C
C   INITIALIZE SCREEN AND BAUD RATE
C
          CALL INITT(480)
C
C   SET UP VIRTUAL WINDOW SIZE
C
          CALL DWINDO(INRX(1,1),INRX(1,HNODES),INRZ(VNODES,1),HMAX)
C
C   SET SIZE OF SCREEN WINDOW IN SCREEN COORDS
C
          XMIN=115
          XMAX=965
          ZMIN=70
          ZMAX=730
          CALL TWINDO(XMIN,XMAX,ZMIN,ZMAX)
          XWIN=XMAX-XMIN
          ZWIN=ZMAX-ZMIN
C
C   PLOT OUT FINITE ELEMENT MESH
C
          DO 40 I=1,VNODES
          CALL MOVEA(INRX(I,1),INRZ(I,1))
          DO 40 J=1,HNODES
40        CALL DRAWA(INRX(I,J),INRZ(I,J))
          DO 50 J=1,HNODES
          CALL MOVEA(INRX(1,J),INRZ(1,J))
          DO 50 I=1,VNODES
50        CALL DRAWA(INRX(I,J),INRZ(I,J))
          CALL MOVEA(INRX(1,1),HMAX)
          DEP=1000.-INRZ(VNODES,1)
          WID=INRX(1,HNODES)-INRX(1,1)
C
C   RESET GRAPHICS WINDOW TO COVER ENTIRE SCREEN
C
          CALL TWINDO(0,1023,0,780)
C
C   FIND PLOTTING INCREMENTS IN VIRTUAL AND SCREEN
C   COORDS--ASSUME 5 TIC MARKS WANTED.
C
          DINC=DEP/5.
          WINC=WID/5.
          INCX=XWIN/5
          INCZ=ZWIN/5
C
C   CONSTRUCT VERTICAL SCALE
C
          CALL MOVREL(-20,0)
          CALL DRWREL(0,-ZWIN)
          CALL MOVREL(0,ZWIN)
          CALL MOVREL(-LINWDT(7),0)
          CALL LABEL(0.,IDEPTH)
          CALL ANSTR(6,IDEPTH)
          CALL DRWREL(10,0)
          DO 60 Z=DINC,DEP,DINC
          CALL MOVREL(-10,-INCZ)
          CALL MOVREL(-LINWDT(6),0)
          PAR=Z
          CALL LABEL(PAR,IDEPTH)
          CALL ANSTR(6,IDEPTH)
          CALL DRWREL(10,0)
60        CONTINUE
C
C      CONSTRUCT HORIZONTAL SCALE
C
          CALL MOVABS(XMIN,ZMAX)
          CALL MOVREL(0,20)
          CALL DRWREL(XWIN,0)
          CALL MOVREL(-XWIN,0)
          CALL MOVREL(0,15)
```

PLOT (CONTINUED)

```
      CALL MOVREL(-LINWDT(4),0)
      CALL LABEL(V,,IWIDTH)
      CALL ANSTR(6,IWIDTH)
      CALL MOVREL(-LINWDT(2),0)
      CALL MOVREL(0,-5)
      CALL DRWREL(0,-10)
      DO 70 X=XINC,XID,XINC
      CALL MOVREL(IVCX,15)
      CALL MOVREL(-LINWDT(4),0)
      PAR=X
      CALL LABEL(PAR,IWIDTH)
      CALL ANSTR(6,IWIDTH)
      CALL MOVREL(-LINWDT(2),0)
      CALL MOVREL(0,-5)
      CALL DRWREL(0,-10)
70    CONTINUE
      CALL MOVREL(0,15)
      CALL MOVREL(LINWDT(3),0)
C
C  LABEL WITH PROPER UNITS
C
      UNITS(1)=75
      UNITS(2)=77
      CALL ANSTR(2,UNITS)
C
C  SIGNIFY FIXED BASE
C
      IF(ANS .EQ. 'YES') GO TO 90
      CALL MOVABS(XMIN,ZMIN)
      BINC=XMIN/20.
      DO 80 I=1,IFIX(BINC)
      CALL MOVREL(20,0)
      CALL DRWREL(-20,-20)
80    CALL MOVREL(20,20)
C
C  POSITION POINTER FOR TITLE
C
90       CALL MOVABS(0,35)
C
C  WRITE TITLE
C
      CALL ANMODE
      WRITE(5,1070) (NAME(I),I=1,30)
C
C  READ IN 'CR' TO CONTINUE
C
      READ(5,1080) CHAR
      CALL FINITT(0,0)
1000  FORMAT(/' ENTER NAME OF STRUCTURE')
1010    FORMAT(30A1)
1020    FORMAT(/1X'BLOWUP?')
1030    FORMAT(A1)
1040    FORMAT(2I)
1050    FORMAT(1X'ENTER # VERTICAL NODES IN BLOWUP')
1060   FORMAT(2F12.5)
1070   FORMAT(20X,'FINITE ELEMENT SIMULATION--',30A1)
1080   FORMAT(A1)
       END
C
C
```

```
C  LABEL--ALPHANUMERIC CODE GENERATOR
C
C  SYMBOLS USED:
C
C          VAL      ARRAY CONTAINING ALPHANUMERIC CHARACTER
C                   CODES FOR EACH VALUE PRINTED ON THE SCREEN.
C
C          DIG      DIGITS TO RIGHT OF DECIMAL POINT
C
      SUBROUTINE LABEL(PAR,VAL)
      INTEGER VAL(6)
C
C  TRUNCATE DIGITS BEYOND HUNDRETHS
C
      N=INT(PAR*100.)
      PAR=N/100.
C
C  DETERMINE ASCII CHARACTER CODES FOR EACH VALUE.
C
      VAL(1)=INT(PAR/100)
      VAL(2)=INT((PAR-VAL(1)*100)/10)
      VAL(3)=INT(PAR-(VAL(1)*100+VAL(2)*10))
      DIG=INT(PAR*100-(VAL(1)*100+VAL(2)*10+VAL(3))*100)
      VAL(5)=INT(DIG/10.)
      VAL(6)=INT(DIG-VAL(5)*10)
      DO 10 I=1,6
10    VAL(I)=VAL(I)+48
C
C  IF FIRST TWO DIGITS ARE 0, MAKE BLANKS
C
      IF(VAL(1) .EQ. 48) VAL(1)=32
      IF(VAL(2) .EQ. 48 .AND. VAL(1) .EQ. 32) VAL(2)=32
C
C  VAL(4) IS DECIMAL POINT
C
      VAL(4)=46
      RETURN
      END
```

RMTXA

```
      SUBROUTINE RMTXA(A,D,ALPHA,BETA,RHO,MAXDIM,NL)
C
C     GLOBAL MATRIX A ASSEMBLER FOR RAYLEIGH WAVE CASE
C
      DIMENSION A(MAXDIM,MAXDIM),D(1),ALPHA(1),BETA(1)
      REAL LAMBDA,RHO(1),MU
C
C     A11      ELEMENT IN LAYER STIFFNESS MATRIX
C     A(I,I)   ELEMENT IN GLOBAL STIFFNESS MATRIX
C     NL2      NUMBER OF LAYERS*2
C     MU       SHEAR MODULUS
C     D        THICKNESS OF EACH LAYER
C
C  CLEAR A
C
      NL2=NL*2
      DO 5 I=1,NL2
      DO 5 J=1,NL2
    5 A(I,J)=0.0
C
C  CALCULATE MATRIX FOR NTH LAYER
C
      J=1
      JSTOP=NL-1
      DO 10 I=1,JSTOP
      MU=RHO(I)*(BETA(I)**2)
      G=MU*D(I)/6.0
      LAMBDA=((ALPHA(I)**2)*RHO(I)-2.0*MU)*D(I)/6.0
      A11=2.0*(2.0*G+LAMBDA)
      A12=0.
      A13=A11/2.0
      A14=0.
      A21=0.
      A22=2.0*G
      A23=0.
      A24=G
      A31=A13
      A32=0.
      A33=A11
      A34=0.
      A41=0.
      A42=A24
      A43=0.
      A44=A22
C
C  ARRANGE INTO GLOBAL NL2 X NL2 MATRIX
C
      J1=J+1
      J2=J+2
      J3=J+3
C
      A(J,J)=A(J,J)+A11
      A(J,J1)=A(J,J1)+A12
      A(J,J2)=A(J,J2)+A13
      A(J,J3)=A(J,J3)+A14
C
      A(J1,J)=A(J1,J)+A21
      A(J1,J1)=A(J1,J1)+A22
      A(J1,J2)=A(J1,J2)+A23
      A(J1,J3)=A(J1,J3)+A24
C
      A(J2,J)=A(J2,J)+A31
      A(J2,J1)=A(J2,J1)+A32
      A(J2,J2)=A(J2,J2)+A33
      A(J2,J3)=A(J2,J3)+A34
C
      A(J3,J)=A(J3,J)+A41
```

RMTXA (CONTINUED)

```
      A(J3,J1)=A(J3,J1)+A42
      A(J3,J2)=A(J3,J2)+A43
      A(J3,J3)=A(J3,J3)+A44
      J=J+2
 10   CONTINUE
C
C  ADD IN BOTTOM LAYER
C
      MU=(BETA(NL)**2)*RHO(NL)
      G=MU*D(NL)/6.0
      LAMBDA=((ALPHA(NL)**2)*RHO(NL)-2.0*MU)*D(NL)/6.0
      A11=2.0*(2.0*G+LAMBDA)
      A12=0.
      A21=0.
      A22=2.0*G
C
      A(J,J)=A(J,J)+A11
      A(J,J+1)=A(J,J+1)+A12
      A(J+1,J)=A(J+1,J)+A21
      A(J+1,J+1)=A(J+1,J+1)+A22
      RETURN
      END
```

RMTXB

```
      SUBROUTINE RMTXB(B,ALPHA,BETA,RHO,MAXDIM,NL)
C
C     COMPUTE GLOBAL MATRIX B FOR RAYLEIGH WAVE CASE
C
      DIMENSION B(MAXDIM,MAXDIM),ALPHA(1),BETA(1)
      REAL RHO(1),MU,LAMBDA
C
C          B(I,J)  GLOBAL MATRIX B (NL2 X NL2)
C          BIJ     ELEMENT OF MATRIX B (4 X 4)
C          MU      SHEAR MODULUS
C          LAMBDA  LAME CONSTANT
C
C  CLEAR B
C
      NL2=NL*2
      DO 10 I=1,NL2
      DO 10 J=1,NL2
   10 B(I,J)=0.0
C
C  COMPUTE MATRIX FOR NTH LAYER
C
      J=1
      JSTOP=NL-1
      DO 20 I=1,JSTOP
      MU=(BETA(I)**2)*RHO(I)
      G=MU/2.0
      LAMBDA=((ALPHA(I)**2)*RHO(I)-2.0*MU)/2.0
      B11=0.0
      B12=G-LAMBDA
      B13=0.0
      B14=G+LAMBDA
      B21=-B12
      B22=0.0
      B23=B14
      B24=0.0
      B31=0.0
      B32=-B14
      B33=0.0
      B34=-B12
      B41=-B14
      B42=0.0
      B43=B12
      B44=0.0
C
C  ARRANGE INTO GLOBAL NL2 X NL2 MATRIX
C
      J1=J+1
      J2=J+2
      J3=J+3
C
      B(J,J)=B(J,J)+B11
      B(J,J1)=B(J,J1)+B12
      B(J,J2)=B(J,J2)+B13
      B(J,J3)=B(J,J3)+B14
C
      B(J1,J)=B(J1,J)+B21
      B(J1,J1)=B(J1,J1)+B22
      B(J1,J2)=B(J1,J2)+B23
      B(J1,J3)=B(J1,J3)+B24
C
      B(J2,J)=B(J2,J)+B31
      B(J2,J1)=B(J2,J1)+B32
      B(J2,J2)=B(J2,J2)+B33
      B(J2,J3)=B(J2,J3)+B34
C
      B(J3,J)=B(J3,J)+B41
      B(J3,J1)=B(J3,J1)+B42
      B(J3,J2)=B(J3,J2)+B43
      B(J3,J3)=B(J3,J3)+B44
      J=J+2
   20 CONTINUE
C
C  ADD IN BOTTOM LAYER
C
      MU=(BETA(NL)**2)*RHO(NL)
      G=MU/2.0
      LAMBDA=((ALPHA(NL)**2)*RHO(NL)-2.0*MU)/2.0
      B11=0.0
      B12=G-LAMBDA
      B21=-B12
      B22=0.0
C
      B(J,J)=B(J,J)+B11
      B(J,J+1)=B(J,J+1)+B12
      B(J+1,J)=B(J+1,J)+B21
      B(J+1,J+1)=B(J+1,J+1)+B22
      RETURN
      END
```

RMTXC

```
      SUBROUTINE RMTXC(C,D,ALPHA,BETA,RHO,MAXDIM,OMSQ,NL,M)
      DIMENSION C(MAXDIM,MAXDIM),D(1),ALPHA(1)
      REAL RHO(1),BETA(1),LAMBDA,MU,M(MAXDIM,MAXDIM)
C
C   COMPUTE GLOBAL MATRIX C FOR RAYLEIGH WAVE CASE
C
C          C(I,J)       GLOBAL MATRIX C (NL2 X NL2)
C          M(I,J)       GLOBAL MASS MATRIX
C          GG11         ELEMENT OF TEMPORARY GG MATRIX
C          XM11         ELEMENT OF MASS MATRIX*OMEGA**2
C          OMSQ         OMEGA**2
C          MU           SHEAR MODULUS
C          LAMBDA       LAME CONSTANT
C
      ZERO=0.0
      TWO=2.0
      THREE=3.0
C
C   CLEAN C
C
      NL2=NL*2
      DO 10 I=1,NL2
      DO 10 J=1,NL2
      M(I,J)=ZERO
 10   C(I,J)=ZERO
C
C   COMPUTE MATRIX FOR NTH LAYER
C
      J=1
      JSTOP=NL-1
      DO 20 I=1,JSTOP
         MU=(BETA(I)**2)*RHO(I)
         G=MU/D(I)
         LAMBDA=((ALPHA(I)**2)*RHO(I)-TWO*MU)/D(I)
         GG11=G
         GG12=ZERO
         GG13=-GG11
         GG14=ZERO
         GG21=ZERO
         GG22=TWO*G+LAMBDA
         GG23=ZERO
         GG24=-GG22
         GG31=-GG11
         GG32=ZERO
         GG33=GG11
         GG34=ZERO
         GG41=ZERO
         GG42=-GG22
         GG43=ZERO
         GG44=GG22
C
C   COMPUTE OMSQ*M
C
          XM11=RHO(I)*D(I)/THREE
          XM12=ZERO
          XM13=XM11/TWO
          XM14=ZERO
          XM21=ZERO
          XM22=XM11
          XM23=ZERO
          XM24=XM11/TWO
          XM31=XM11/TWO
          XM32=ZERO
          XM33=XM11
          XM34=ZERO
          XM41=ZERO
          XM42=XM11/TWO
          XM43=ZERO
          XM44=XM11
C
C   ARRANGE INTO GLOBAL MATRIX C AND M
C
          J1=J+1
          J2=J+2
          J3=J+3
C
          C(J,J)=C(J,J)+GG11-OMSQ*XM11
          C(J,J1)=C(J,J1)+GG12-OMSQ*XM12
          C(J,J2)=C(J,J2)+GG13-OMSQ*XM13
          C(J,J3)=C(J,J3)+GG14-OMSQ*XM14
C
          C(J1,J)=C(J1,J)+GG21-OMSQ*XM21
          C(J1,J1)=C(J1,J1)+GG22-OMSQ*XM22
          C(J1,J2)=C(J1,J2)+GG23-OMSQ*XM23
          C(J1,J3)=C(J1,J3)+GG24-OMSQ*XM24
C
          C(J2,J)=C(J2,J)+GG31-OMSQ*XM31
          C(J2,J1)=C(J2,J1)+GG32-OMSQ*XM32
          C(J2,J2)=C(J2,J2)+GG33-OMSQ*XM33
          C(J2,J3)=C(J2,J3)+GG34-OMSQ*XM34
C
          C(J3,J)=C(J3,J)+GG41-OMSQ*XM41
          C(J3,J1)=C(J3,J1)+GG42-OMSQ*XM42
          C(J3,J2)=C(J3,J2)+GG43-OMSQ*XM43
          C(J3,J3)=C(J3,J3)+GG44-OMSQ*XM44
C
          M(J,J)=M(J,J)+XM11
          M(J,J1)=M(J,J1)+XM12
          M(J,J2)=M(J,J2)+XM13
          M(J,J3)=M(J,J3)+XM14
C
          M(J1,J)=M(J1,J)+XM21
          M(J1,J1)=M(J1,J1)+XM22
          M(J1,J2)=M(J1,J2)+XM23
          M(J1,J3)=M(J1,J3)+XM24
C
          M(J2,J)=M(J2,J)+XM31
          M(J2,J1)=M(J2,J1)+XM32
          M(J2,J2)=M(J2,J2)+XM33
          M(J2,J3)=M(J2,J3)+XM34
C
          M(J3,J)=M(J3,J)+XM41
          M(J3,J1)=M(J3,J1)+XM42
          M(J3,J2)=M(J3,J2)+XM43
          M(J3,J3)=M(J3,J3)+XM44
          J=J+2
 20   CONTINUE
C
C   ADD IN BOTTOM LAYER
C
```

RMTXC (CONTINUED)

```
      MU=(BETA(NL)**2)*RHO(NL)
      G=MU/D(NL)
      LAMBDA=((ALPHA(NL)**TWO)*RHO(NL)-TWO*MU)/D(NL)
      GG11=G
      GG12=ZERO
      GG21=ZERO
      GG22=TWO*G+LAMBDA
C
      EM11=RHO(NL)*D(NL)/THREE
      EM12=ZERO
      EM21=ZERO
      EM22=EM11
C
      C(J,J)=C(J,J)+GG11-OMSQ*EM11
      C(J,J+1)=C(J,J+1)+GG12-OMSQ*EM12
      C(J+1,J)=C(J+1,J)+GG21-OMSQ*EM21
      C(J+1,J+1)=C(J+1,J+1)+GG22-OMSQ*EM22
C
      M(J,J)=M(J,J)+EM11
      M(J,J+1)=M(J,J+1)+EM12
      M(J+1,J)=M(J+1,J)+EM21
      M(J+1,J+1)=M(J+1,J+1)+EM22
      RETURN
      END
```

RMTXE

```
      SUBROUTINE RMTXE(E,ALPHA,BETA,RHO,MAXDIM,NL)
      DIMENSION E(MAXDIM,MAXDIM),ALPHA(1),BETA(1),RHO(1)
      REAL LAMBDA,MU
C
C     COMPUTE GLOBAL MATRIX E FOR RAYLEIGH WAVE CASE
C
C           E(I,J) AN ELEMENT IN GLOBAL E (NL2 X NL2)
C           E11    AN ELEMENT IN LAYER MATRIX (4 X 4)
C           MU     SHEAR MODULUS
C           LAMBDA LAME CONSTANT
C
C  CLEAR E
C
      NL2=NL*2
      DO 5 I=1,NL2
      DO 5 J=1,NL2
    5 E(I,J)=0.0
C
C  COMPUTE LAYER MATRIX
C
      J=1
      JSTOP=NL-1
      DO 10 I=1,JSTOP
         MU=RHO(I)*(BETA(I)**2)
         G=MU
         LAMBDA=((ALPHA(I)**2)*RHO(I)-2.0*MU)
         E11=0.0
         E12=LAMBDA/2.0
         E13=E11
         E14=-E12
         E21=G/2.
         E22=0.0
         E23=-E21
         E24=0.0
         E31=0.0
         E32=E12
         E33=0.0
         E34=-E12
         E41=E21
         E42=0.0
         E43=E23
         E44=0.0
C
C  COMPUTE GLOBAL MATRIX
C
         J1=J+1
         J2=J+2
         J3=J+3
C
         E(J,J)=E(J,J)+E11
         E(J,J1)=E(J,J1)+E12
         E(J,J2)=E(J,J2)+E13
         E(J,J3)=E(J,J3)+E14
C
         E(J1,J)=E(J1,J)+E21
         E(J1,J1)=E(J1,J1)+E22
         E(J1,J2)=E(J1,J2)+E23
         E(J1,J3)=E(J1,J3)+E24
C
         E(J2,J)=E(J2,J)+E31
         E(J2,J1)=E(J2,J1)+E32
         E(J2,J2)=E(J2,J2)+E33
         E(J2,J3)=E(J2,J3)+E34
C
         E(J3,J)=E(J3,J)+E41
         E(J3,J1)=E(J3,J1)+E42
         E(J3,J2)=E(J3,J2)+E43
         E(J3,J3)=E(J3,J3)+E44
         J=J+2
   10 CONTINUE
C
C  ADD IN BOTTOM LAYER
C
      MU=(BETA(NL)**2)*RHO(NL)
      G=MU
      LAMBDA=((ALPHA(NL)**2)*RHO(NL)-2.0*MU)
      E11=0.0
      E12=LAMBDA/2.0
      E21=G/2.0
      E22=0.0
C
      E(J,J)=E(J,J)+E11
      E(J,J+1)=E(J,J+1)+E12
      E(J+1,J)=E(J+1,J)+E21
      E(J+1,J+1)=E(J+1,J+1)+E22
      RETURN
      END
```

RYGLB

```
      SUBROUTINE RYGLB(IRRX,IRRZ,RHOE,ALPHA,BETA,RZ,OMSQ)
C
C  RAYLEIGH WAVE GLOBAL LOADER
C
C  DETERMINE STIFFNESS AND MASS MATRIX FOR EACH ELEMENT AND ADD
C  INTO GLOBAL MATRIX
C
C     SYMBOLS:
C               NFND2  MAX # FREE NODES + 2
C               LT     MAX # OF LAYERS
C               NCOL   MAX # OF COLUMNS
C               NHND   MAX # OF HORIZ NODES
C               NVND   MAX # OF VERT NODES
C               NFN    # OF FREE NODES
C               NFN2   # OF FREE NODES * 2
C               COORD  COORDINATES OF VERTICES OF ELEMENTS
C               RMM    ELEMENT MASS MATRIX
C               RKM    ELEMENT STIFFNESS MATRIX
C               RZ     GLOBAL MATRIX
C               L      ELEMENT COUNTER
C
      COMMON/SIZE/NL,NC,NFN,NFN2,NHN,NVN,NFND2,LT,NCOL,NHND,NVND
      REAL RMM(8,8),RKM(8,8),MU,IRRX(NVND,NHND),IRRZ(NVND,NHND)
      REAL RHOE(LT,NCOL),BETA(LT,NCOL),LAMBDA
      REAL ALPHA(LT,NCOL),COORD(4,2)
      COMPLEX RZ(NFND2,NFND2)
      L=1
      DO 50 I=1,NFN2
      DO 50 J=1,NFN2
 50   RZ(I,J)=0.
      DO 101 J=1,NC
      DO 100 I=1,NL
C
C  ASSEMBLE COORDINATE MATRIX FOR USE IN QUADRATURE ROUTINE
C
      COORD(1,1)=IRRX(I,J)
      COORD(1,2)=IRRZ(I,J)
      COORD(2,1)=IRRX(I,J+1)
      COORD(2,2)=IRRZ(I,J+1)
      COORD(3,1)=IRRX(I+1,J+1)
      COORD(3,2)=IRRZ(I+1,J+1)
      COORD(4,1)=IRRX(I+1,J)
      COORD(4,2)=IRRZ(I+1,J)
      RHO=RHOE(I,J)
      MU=RHO*BETA(I,J)**2
      LAMBDA=RHO*ALPHA(I,J)**2-2.*MU
C
C  ASSEMBLE ELEMENT MATRICES THROUGH 3-POINT GAUSSIAN QUADRATURE
C
      CALL GAUSR(RKM,RMM,MU,LAMBDA,RHO,COORD)
      L1=L+1
      L2=L+2
      L3=L+3
      NL2L=NL*2+L
      NL2L1=NL2L+1
      NL2L2=NL2L+2
      NL2L3=NL2L+3
      NL2L4=NL2L+4
      NL2L5=NL2L+5
C
C  ADD FIXED ELEMENTS INTO GLOBAL MATRIX
C
      RZ(L,L)=RZ(L,L)+RKM(1,1)-OMSQ*RMM(1,1)
      RZ(L,L1)=RZ(L,L1)+RKM(1,2)-OMSQ*RMM(1,2)
      RZ(L,NL2L)=RZ(L,NL2L)+RKM(1,3)-OMSQ*RMM(1,3)
      RZ(L,NL2L1)=RZ(L,NL2L1)+RKM(1,4)-OMSQ*RMM(1,4)
C
      RZ(L1,L1)=RZ(L1,L1)+RKM(2,2)-OMSQ*RMM(2,2)
      RZ(L1,NL2L)=RZ(L1,NL2L)+RKM(2,3)-OMSQ*RMM(2,3)
      RZ(L1,NL2L1)=RZ(L1,NL2L1)+RKM(2,4)-OMSQ*RMM(2,4)
C
      RZ(NL2L,NL2L)=RZ(NL2L,NL2L)+RKM(3,3)-OMSQ*RMM(3,3)
      RZ(NL2L,NL2L1)=RZ(NL2L,NL2L1)+RKM(3,4)-OMSQ*RMM(3,4)
      RZ(NL2L1,NL2L1)=RZ(NL2L1,NL2L1)+RKM(4,4)-OMSQ*RMM(4,4)
      IF(I .EQ. NL) GO TO 95
C
C  ADD IN FREE ELEMENTS
C
      RZ(L,NL2L2)=RZ(L,NL2L2)+RKM(1,5)-OMSQ*RMM(1,5)
      RZ(L,NL2L3)=RZ(L,NL2L3)+RKM(1,6)-OMSQ*RMM(1,6)
      RZ(L,L2)=RZ(L,L2)+RKM(1,7)-OMSQ*RMM(1,7)
      RZ(L,L3)=RZ(L,L3)+RKM(1,8)-OMSQ*RMM(1,8)
C
      RZ(L1,NL2L2)=RZ(L1,NL2L2)+RKM(2,5)-OMSQ*RMM(2,5)
      RZ(L1,NL2L3)=RZ(L1,NL2L3)+RKM(2,6)-OMSQ*RMM(2,6)
      RZ(L1,L2)=RZ(L1,L2)+RKM(2,7)-OMSQ*RMM(2,7)
      RZ(L1,L3)=RZ(L1,L3)+RKM(2,8)-OMSQ*RMM(2,8)
C
      RZ(NL2L,NL2L2)=RZ(NL2L,NL2L2)+RKM(3,5)-OMSQ*RMM(3,5)
      RZ(NL2L,NL2L3)=RZ(NL2L,NL2L3)+RKM(3,6)-OMSQ*RMM(3,6)
      RZ(L2,NL2L)=RZ(L2,NL2L)+RKM(3,7)-OMSQ*RMM(3,7)
      RZ(L3,NL2L)=RZ(L3,NL2L)+RKM(3,8)-OMSQ*RMM(3,8)
C
      RZ(NL2L1,NL2L2)=RZ(NL2L1,NL2L2)+RKM(4,5)-OMSQ*RMM(4,5)
      RZ(NL2L1,NL2L3)=RZ(NL2L1,NL2L3)+RKM(4,6)-OMSQ*RMM(4,6)
      RZ(L2,NL2L1)=RZ(L2,NL2L1)+RKM(4,7)-OMSQ*RMM(4,7)
      RZ(L3,NL2L1)=RZ(L3,NL2L1)+RKM(4,8)-OMSQ*RMM(4,8)
C
      RZ(NL2L2,NL2L2)=RZ(NL2L2,NL2L2)+RKM(5,5)-OMSQ*RMM(5,5)
      RZ(NL2L2,NL2L3)=RZ(NL2L2,NL2L3)+RKM(5,6)-OMSQ*RMM(5,6)
      RZ(L2,NL2L2)=RZ(L2,NL2L2)+RKM(5,7)-OMSQ*RMM(5,7)
      RZ(L3,NL2L2)=RZ(L3,NL2L2)+RKM(5,8)-OMSQ*RMM(5,8)
C
      RZ(NL2L3,NL2L3)=RZ(NL2L3,NL2L3)+RKM(6,6)-OMSQ*RMM(6,6)
      RZ(L2,NL2L3)=RZ(L2,NL2L3)+RKM(6,7)-OMSQ*RMM(6,7)
      RZ(L3,NL2L3)=RZ(L3,NL2L3)+RKM(6,8)-OMSQ*RMM(6,8)
C
      RZ(L2,L2)=RZ(L2,L2)+RKM(7,7)-OMSQ*RMM(7,7)
      RZ(L2,L3)=RZ(L2,L3)+RKM(7,8)-OMSQ*RMM(7,8)
C
      RZ(L3,L3)=RZ(L3,L3)+RKM(8,8)-OMSQ*RMM(8,8)
 95   L=L+2
 100  CONTINUE
 101  CONTINUE
C
C  REPLACE SYMMETRIC ELEMENTS
C
      DO 175 I=1,NFN2
      DO 175 J=1,NFN2
 175  RZ(J,I)=RZ(I,J)
      RETURN
      END
```

```
C
C  RAYLEIGH WAVE IRREGULAR ZONE FINITE ELEMENT ANALYSIS
C
C     IN THIS PROGRAM THE IRREGULAR ZONE MODES OF VIBRATION ARE
C  COMPUTED WITH ENERGY TRANSFER AND AVERAGE PHASE VELOCITIES.
C  ENERGY IS IMPARTED TO THE IRREGULAR STRUCTURE VIA AN INCIDENT
C  PLANE WAVE AS SPECIFIED BY THE INCIDENT DISPLACEMENTS AND THE
C  MATRIX R1 AS COMPUTED IN RYLAY. MATRIX R2 CONTAINS THE BOUNDARY
C  CONDITIONS FOR THE RIGHT END OF THE IRREGULAR ZONE.
C
C  Bibliography:
C
C  Drake, L. A. (1972). Rayleigh waves at a continental boundary by the
C     finite element method, Bull. Seis. Soc. Am., 62, 1259-1268.
C
C
C  DISK FILES USED:
C
C                          IRR--   COORDINATES OF NODES, FUNDAMENTAL MODE
C                                  NUMBERS, AND ELEMENT PARAMETERS
C                          RR1--   R MATRICES, DISPLACEMENTS, AND PERIODS
C                                  FROM LEFT LAYERED ZONE
C                          RR2--   R MATRICES AND DISPLACEMENTS FROM RIGHT
C                                  LAYERED ZONE
C
C  SUBROUTINES USED:
C
C                          RYGLB-- ASSEMBLES GLOBAL MATRIX X WHICH CONTAINS
C                                  NODAL INTERACTIONS FOR IRREGULAR ZONE
C
C
C  SYMBOLS USED IN THIS PROGRAM:
C
C          NFN2            MAX # FREE NODES * 2
C          LY,LMAX          "  "  "  LAYERS
C          LY2              "  "  "     "      * 2
C          COL,CMAX         "  "  "  COLUMNS
C          HNOD,HNMAX       "  "  "  HORIZ NODES
C          VNOD,VNMAX       "  "  "  VERT     "
C
C  NOTE----CHANGE PARAMETER STATEMENTS IF ANY OF THE
C          ABOVE ARE EXCEEDED.
C
C          FN2             2*(# OF FREE NODES)
C          VN,HN           VERT AND HORIZ NUMBER OF NODES IN IRR ZONE
C          NL2             NUMBER OF LAYERS*2
C          NC              NUMBER OF COLUMNS IN IRR ZONE
C          VNOD            # OF VERT NODES
C          IRRX            MATRICES CONTAINING COORDINATES (X,Z) OF
C          IRRZ            NODES IN IRREGULAR ZONE
C          MU              SHEAR MODULUS
C          LAMBDA          LAME CONSTANT
C          BETA            S WAVE VELOCITY FOR EACH ELEMENT
C          ALPHA           P WAVE VELOCITY FOR EACH ELEMENT
C          RHOE,RHO        DENSITY OF EACH ELEMENT
C          R1,R2,S         BOUNDARY CONDITION MATRICES
C          IDISPL          INCIDENT DISPLACEMENT
C          VN,HN           VERT AND HORIZ NUMBER OF NODES IN IRR ZONE
C          OMSQ            FREQUENCY OF INCIDENT WAVEFORM**2
C          NL2             NUMBER OF LAYERS*2
C          NC              NUMBER OF COLUMNS IN IRR ZONE
C          Y               DISPLACEMENTS OF IRR ZONE
C          YR              REFLECTED DISPLACEMENTS
C          V               REGULAR ZONE MODES
C          VINV1,VINV2     INVERSE DISPLACEMENTS
C          IDISPL          INCIDENT DISPLACEMENTS FROM REGULAR ZONE
C          NUM             THE # OF DESIRED INCIDENT DISPLACEMENT AND WN
C          MAXDIM          NODAL STRUCTURE ORDER
C          PART            MODE PARTICIPATION FACTOR
C          RPART           REFLECTED MODE PART. FACTOR
C          Z               REAL DUMMY GLOBAL MATRIX
C          X               COMPLEX GLOBAL MATRIX USED IN LEQTIC
C          WN1,WN2         WAVE NUMBERS OF DESIRED INCIDENT DISPL'S
C          CAVE            PHASE VELOCITY
C          AMPL            AMPLITUDE OF TRANSMITTED WAVES
C          RAMPL               "        "   REFLECTED      "
C          PHI             PHASE OF TRANSMITTED WAVES
C          RPHI              "   "   REFLECTED    "
C          ENGY            ENERGY TRANSMITTED FOR EACH MODE
C          RENGY             "      REFLECTED   "   "    "
C          ESUM            SUM OF TRANSMITTED ENERGY
C          RESUM            "  "  REFLECTED     "
C          TMIN            MINIMUM PERIOD
C          TMAX            MAXIMUM PERIOD
C          TINC            PERIOD INCREMENT
C          WORK            DUMMY ARRAY FOR USE IN LEQTIC
C          TEST            ROUNDOFF "ZERO"
C
C  SUBROUTINES CALLED: RYGLB, IMSL:LEQTIC
C
C          --OTHER SYMBOLS ARE DEFINED IN THEIR RESPECTIVE ROUTINES
C
      PARAMETER NFN2=310,LY=14,LY2=28,COL=10,HNOD=11,VNOD=15
      INTEGER VN,HN,FN,FN2,CMAX,HNMAX,VNMAX,TNUM,NUM1(100),NUM2(100)
      INTEGER HN1
      REAL IRRX(VNOD,HNOD),IRRZ(VNOD,HNOD)
      REAL RHOE(LY,COL),BETA(LY,COL),ALPHA(LY,COL)
      REAL WORK(NFN2),IR,IR2
      COMPLEX R1(LY2,LY2),YR(LY2),S(LY2,LY2),R2(LY2,LY2)
      COMPLEX IDISPL(NFN2),X(NFN2,NFN2),Y(NFN2,1),WN1(LY2),WN2(LY2)
      COMPLEX V(LY2,LY2),VINV1(LY2,LY2),CRIT,CZERO,CTWO,PART,RPART
      COMPLEX CONE,DENOM,DISPU,DISPR,CNORM,VINV2(LY2,LY2)
      COMPLEX CTEMP1,CTEMP2,C1,C2,T1,T2
      COMMON/SIZE/NL,NC,FN,FN2,HN,VN,MAXDIM,LMAX,CMAX,HNMAX,VNMAX
C
C  INITIALIZE PARAMETERS
C
      MAXDIM=NFN2
      LMAX=LY
      CMAX=COL
      HNMAX=HNOD
      VNMAX=VNOD
      ZERO=0.0
      ONE=1.0
      TPI=3.1415927*2.0000000
      CENT=100.0
      CZERO=(0.0E00,0.0E00)
      CONE=(1.0E00,0.0E00)
      CTWO=(2.0E00,0.0E00)
      EYE=(0.0E00,1.0E00)
```

RYIRR (CONTINUED)

```
      TEST=1.0E-5
C
C  READ IN COORDINATES, DENSITIES, VELOCITIES, PERIOD OFF OF DISK
C
      OPEN(UNIT=5,DEVICE='DSK',FILE='INN.DAT',ACCESS='SEQIN')
      OPEN(UNIT=6,DEVICE='LPT',ACCESS='SEQOUT')
      OPEN(UNIT=24,DEVICE='DSK',FILE='RR1.DAT',ACCESS='SEQIN')
      OPEN(UNIT=25,DEVICE='DSK',FILE='RR2.DAT',ACCESS='SEQIN')
C
C  INPUT DIMENSIONS OF DATA
C
      READ(5,1110) YN,HN
C
C  CHECK FOR DATA INPUT ERROR
C
      IF(YN .LE. 1 .OR. HN .LE. 1) GO TO 250
      IF(YN .GT. YMOD .OR. HN .GT. HMOD) GO TO 260
C
C  SET SIZE CONTROL VARIABLES
C
      NL=YN-1
      NL2=NL*2
      NC=HN-1
      FN=NL*NY
      FN2=FN*2
      NO=FN2-NL2
C
C  READ IN COORDINATES
C
      DO 10 I=1,YN
      DO 10 J=1,HN
10    READ(5,1120) IRRX(I,J),IRRZ(I,J)
C
C  COMPUTE DISTANCE TRAVELED BY WAVE
C
      DIST=0.
      HN1=HN-1
      DO 11 I=1,HN1
 11   DIST=DIST+SQRT((IRRX(1,I+1)-IRRX(1,I))**2+(IRRZ(1,I+1)
     1-IRRZ(1,I))**2)
C
C  READ IN NUMBERS OF FUNDAMENTAL MODES
C
      READ(24) TMIN,TMAX,TINC
      TNUM=(TMAX-TMIN)/TINC+1
      READ(5,1349) (NUM1(I),I=1,TNUM)
      READ(5,1349) (NUM2(I),I=1,TNUM)
 1349 FORMAT(I1)
C
C  READ IN ELEMENT PARAMETERS
C
      DO 20 I=1,NL
      DO 20 J=1,NC
20    READ(5,1040) BETA(I,J),RHOE(I,J),ALPHA(I,J)
      CLOSE(UNIT=5,DEVICE='DSK',FILE='INN.DAT',DISPOSE='SAVE')
C
C  CHECK DATA BY PRINTING OUT
C
      WRITE(6,1050)
      WRITE(6,1060) NL,NC
      WRITE(6,1140)
      DO 30 I=1,YN
30    WRITE(6,1160) (IRRX(I,J),J=1,HN)
      WRITE(6,1150)
      DO 40 I=1,YN
40    WRITE(6,1160) (IRRZ(I,J),J=1,HN)
      WRITE(6,1070)
      WRITE(6,1080)
      DO 50 I=1,NL
      DO 50 J=1,NC
50    WRITE(6,1090) I,J,BETA(I,J),RHOE(I,J),ALPHA(I,J)
C
C  LOOP OVER PERIODS
C
      II=0
      DO 230 PER=TMIN,TMAX,TINC
      OMSQ=(TPI/PER)**2
      II=II+1
      WRITE(6,1100) PER
C
C  COMPUTE GLOBAL MATRIX Z=K-OMEGA**2*M
C
      CALL RYGLB(IRRX,IRRZ,RHOE,ALPHA,BETA,Z,OMSQ)
C
C  READ IN R AND S MATRICES, INCIDENT DISPLACEMENTS,WAVE NUMBERS,
C  AND V,VINV1 FROM LEFT SIDE OF IRREGULAR ZONE.
C
C     READ(24) NUM2
      DO 70 I=1,FN2
      IDISPL(I)=CZERO
70    T(I,1)=CZERO
C     READ(24) NUM1
      DO 80 I=1,NL2
80    READ(24) (R1(I,J),J=1,NL2)
      DO 90 I=1,NL2
      READ(24) WN1(I)
      DO 90 J=1,NL2
      READ(24) S(I,J)
      READ(24) V(I,J)
90    READ(24) VINV1(I,J)
C
C  READ IN R2 AND VINV FROM RIGHT SIDE
C
      DO 100 I=1,NL2
      READ(25) WN2(I)
      READ(25) (VINV2(I,J),J=1,NL2)
100   READ(25) (R2(I,J),J=1,NL2)
C
C  COMBINE MATRICES TO FORM (K-OMEGA**2*M+S-R2)*X AND
C  (S-R1)*IDISPL=T
C
C
C  GET DESIRED INCIDENT DISPLACEMENT
C
      DO 110 I=1,NL2
110   IDISPL(I)=V(I,NUM1(II))
C
C  SET X AND T FOR USE IN LEQTIC
C
      DO 130 I=1,NL2
```

RYIRR (CONTINUED)

```
      CRIT=CZERO
      DO 120 J=1,NL2
      X(I,J)=X(I,J)+S(I,J)
      X(I+ND,J+ND)=X(I+ND,J+ND)-R2(I,J)
120   CRIT=CRIT+(S(I,J)-R1(I,J))*IDISPL(J)
130   T(I,1)=CRIT
C
C  SOLVE LINEAR EQUATION X*T (T=RESULTANT NODAL DISPLACEMENTS)
C
      CALL LEQT1C(X,FN2,MAXDIM,T,1,MAXDIM,0,WORK,IER)
      WRITE(6,1190) IER
C
C  OUTPUT RESULTS

      WRITE(6,1200)

C  ECHO CHECK BOUNDARY CONDITION MATRICES R1,R2,AND S
C
      DO 140 I=1,NL2
      WRITE(6,1210) I
 140  WRITE(6,1320) (R1(I,J),J=1,NL2)
      WRITE(6,1210)
      DO 150 I=1,NL2
      WRITE(6,1210) I
 150  WRITE(6,1320) (R2(I,J),J=1,NL2)
      WRITE(6,1220)
      DO 160 I=1,NL2
      WRITE(6,1210) I
 160  WRITE(6,1320) (S(I,J),J=1,NL2)
      WRITE(6,1240)
      WRITE(6,1250)
C
C  NORMALIZE DISPLACEMENTS TO 1 FOR PRINTING
C
      ISTOP=NL2-1
      CNORM=IDISPL(2)
      DO 170 I=1,ISTOP,2
      DISPU=IDISPL(I)/CNORM
      DISPW=IDISPL(I+1)/CNORM
170   WRITE(6,1260) IDISPL(I),IDISPL(I+1)
      WRITE(6,1280)
      WRITE(6,1250)
C
C  WRITE OUT NODAL DISPLACEMENTS FOR IRR ZONE
C
      ISTOP=FN2-1
      CNORM=T(2,1)
      DO 180 I=1,ISTOP,2
C
C  NORMALIZE TO 1 FOR PRINTING
C
      DISPU=T(I,1)/CNORM
      DISPW=T(I+1,1)/CNORM
180   WRITE(6,1340) I,T(I,1),T(I+1,1)
C
C  COMPUTE REFLECTED DISPLACEMENTS
C
C
C
C  CHANGE SIGN ON HORIZ DISPLACEMENTS TO ACCOUNT
C  FOR LEFT-MOVING REFLECTED WAVE
C
      DO 190 I=1,NL2,2
      TR(I)=-T(I,1)+IDISPL(I)
190   TR(I+1)=T(I+1,1)-IDISPL(I+1)
      WRITE(6,1300)
      WRITE(6,1310)
      ENORM=CABS(CONE/WN1(NUM1(II)))
      ESUM=0.
      RESUM=0.
      DO 220 I=1,NL2
C
C  INITIALIZE FINAL VARIABLES
C
      CAVE=ZERO
      PART=CZERO
      RPART=CZERO
      ENGY=ZERO
      RENGY=ZERO
C
C  SET FLAGS FOR TRANSMITTED AND INCIDENT
C  (REFLECTED) FUNDAMENTAL MODES
C
      FLAG=' '
      IF(I .EQ. NUM1(II)) FLAG='* '
      IF(I .EQ. NUM2(II)) FLAG='$ '
      IF(I .EQ. NUM1(II) .AND. I .EQ. NUM2(II)) FLAG='*$'
C
C  COMPUTE TRANSMITTED AND REFLECTED MODE PART. FACTORS
C  USING A HIGH-ACCURACY TECHNIQUE FOR COMPLEX MANIPULATION
C
      PART=CZERO
      RPART=CZERO
      C1=CZERO
      C2=CZERO
      DO 200 J=1,NL2
      CTEMP1=C1+VINV2(I,J)*T(ND+J,1)
      CTEMP2=C2+VINV1(I,J)*TR(J)
      T1=PART+CTEMP1
      T2=RPART+CTEMP2
      C1=(PART-T1)+CTEMP1
      C2=(RPART-T2)+CTEMP2
      PART=T1
 200  RPART=T2
      PART=PART+C1
      RPART=RPART+C2
C
C  COMPUTE PARTICIPATION PHASE VELOCITIES, AND ENERGY IN
C  TRANSMITTED AND REFLECTED MODES--WRITE OUT.
C
      P1=REAL(PART)
      P2=AIMAG(PART)
      RP1=REAL(RPART)
      RP2=AIMAG(RPART)
      IF(ABS(P1) .LT. TEST .AND. ABS(P2) .LT. TEST) GO TO 210
      AMPL=SQRT(P1**2+P2**2)
      RAMPL=SQRT(RP1**2+RP2**2)
C
C  DETERMINE PHASE FROM ARGUMENT OF RPF
C
```

RYIRR (CONTINUED)

```
      PHI=ATAN2(P2,P1)
      RPHI=ATAN2(RP2,RP1)
      IF(PHI .LT. 0.) PHI=PHI+TPI
      IF(RPHI .LT. 0.) RPHI=RPHI+TPI
C
C  FIND AVERAGE PHASE VELOCITIES FROM PHASE AND DISTANCE
C
      CAVE=DIST*TPI/(PER*PHI)
C
C  FIND % TRANSMITTED AND REFLECTED ENERGY
C
      RF1=REAL(RH1(I))
      RF2=REAL(RH2(I))
      IF1=AIMAG(RH1(I))
      IF2=AIMAG(RH2(I))
      IF(ABS(RF2) .LT. TEST .OR. ABS(IK2) .GT. TEST) GO TO 209
      ENGY=ABS(RF2)*CABS(PART)**2*ENORM*CENT
      ESUM=ESUM+ENGY
209   IF(ABS(RF1) .LT. TEST .OR. ABS(IK1) .GT. TEST) GO TO 210
      RENGY=ABS(RF1)*CABS(RPART)**2*ENORM*CENT
      RESUM=RESUM+RENGY
C
C  WRITE OUT WPF'S, PHASE VEL, AND % TRANS AND REFL ENERGY,
C
210   WRITE(6,1290) I,FLAG,AMPL,RAMPL,PHI,RPHI,CAVE,ENGY,RENGY
220   CONTINUE
      WRITE(6,1300) ESUM,RESUM
 1300 FORMAT(//10X,'TOTAL TRANSMITTED ENERGY =',F7.2,' %',23X,
     1'*  INCIDENT MODE',/10X,'TOTAL REFLECTED ENERGY =',
     2F7.2,' %',25X,'#  TRANSMITTED FUNDAMENTAL MODE')
230   CONTINUE
C
C  ERROR NOTES
C
      GO TO 270
250   WRITE(5,1000)
      GO TO 270
260   WRITE(5,1010)
C
C  CLOSE DEVICES
C
270   CLOSE(UNIT=2,DEVICE='DSK')
280   CLOSE(UNIT=5,DEVICE='DSK')
      CLOSE(UNIT=6,DEVICE='LPT',DISPOSE='PRINT')
      CLOSE(UNIT=24,DEVICE='DSK')
      CLOSE(UNIT=25,DEVICE='DSK')
      STOP
1000  FORMAT(/' YOU CANNOT HAVE < 2 NODES IN ANY DIRECTION')
1010  FORMAT(/' DIMENSIONS OF PROGRAM EXCEEDED')
1030  FORMAT(2E12.5)
1040  FORMAT(1X,3F10.5)
1050  FORMAT(1X,'RAYLEIGH WAVE ANALYSIS'//)
1060  FORMAT(//,' IRREGULAR STRUCTURE CONSISTING OF',I3,
     1' VERTICAL ELEMENTS AND',I3,' HORIZONTAL ELEMENTS:'//)
1070  FORMAT(//6X,'ELEMENT',6X,'BETA',7X,'RHO',6X,'ALPHA'4)
1080  FORMAT(50('_')/)
1090  FORMAT(4X,'(',I1,',',I3,')',3X,F7.2,3X,F7.2,3X,F7.2)
1100  FORMAT(1H1'PERIOD =',F7.2)
1110  FORMAT(2I3)
1120  FORMAT(2F12.5)
1130  FORMAT(F10.5)
1140  FORMAT(//1X'HORIZONTAL COORDINATES ='/)
1150  FORMAT(//1X'VERTICAL COORDINATES ='/)
1160  FORMAT(1X,10F8.2)
1170  FORMAT(1X,9(2E7,1))
1180  FORMAT(1X10F6.1)
1190  FORMAT(/1X'ERROR PARAMETER FROM LEOTIC =',I5/)
1200  FORMAT(1H1,'BOUNDARY CONDITION MATRICES'///,1X,'MATRIX R1 =')
1210  FORMAT(1H1,'MATRIX R2 =')
1220  FORMAT(1H1,'MATRIX S =')
1230  FORMAT(//I10/)
1240  FORMAT(1H1,'INCIDENT DISPLACEMENT =')
1250  FORMAT(///23X,'UR',12X,'UI',16X,'WR',12X,'WI'//)
1260  FORMAT(16X,2E14.6,4X,2E14.6)
1270  FORMAT(25X,2E12.5)
1280  FORMAT(1H1,'DISPLACEMENTS OF IRREGULAR NODES ='/)
1290  FORMAT(1X,I3,A2,1X,4(E12.5,5X),F12.5,2(5X,F12.2))
1300  FORMAT(1H1,9X'TRANS AMPL',8X,'REFL AMPL',3X,
     1'TRANS PHASE',6X,'REFL PHASE',7X,'PHASE VELOCITY',5X,
     2'% TRANS ENERGY',4X'% REFL ENERGY')
1310  FORMAT(8X,120('_')/)
1320  FORMAT(5(2X,2E12.5))
1330  FORMAT(7X,I3,10X,2(F12.5,7X))
1340  FORMAT(7X,I3,6X,2F14.6,4X,2F14.6)
      END
```

RYIRRX

```
C
C     RAYLEIGH WAVE TOTAL FINITE ELEMENT ANALYSIS
C
C         IN THIS PROGRAM THE IRREGULAR ZONE MODES OF VIBRATION ARE
C     COMPUTED WITH ENERGY TRANSFER AND AVERAGE PHASE VELOCITIES.
C     ENERGY IS IMPARTED TO THE IRREGULAR STRUCTURE VIA AN INCIDENT
C     PLANE WAVE AS SPECIFIED BY THE INCIDENT DISPLACEMENTS AND THE
C     MATRIX R1. MATRIX R2 REPRESENTS THE BOUNDARY CONDITIONS ON THE
C     RIGHT END OF THE IRREGULAR ZONE.
C         TRANSFER OF GLOBAL MATRICES AND SOLVING
C     OF GLOBAL LINEAR EQUATIONS ARE ACCOMPLISHED THROUGH READING
C     AND WRITING OFF OF DISK.
C
C     SYMBOLS USED IN THIS PROGRAM:
C
C             NFN2            MAX # FREE NODES * 2
C             LY               "  "  "   LAYERS
C             LY2              "  "  "     "       * 2
C             COL              "  "  "   COLUMNS
C             HNOD             "  "  "   HORIZ NODES
C             VNOD             "  "  "   VERT    "
C
C     NOTE----CHANGE PARAMETER STATEMENTS IF ANY OF THE
C             ABOVE ARE EXCEEDED.
C
C             FN2             2*(# OF FREE NODES)
C             VN,HN           VERT AND HORIZ NUMBER OF NODES IN IRR ZONE
C             NL2             NUMBER OF LAYERS*2
C             NC              NUMBER OF COLUMNS IN IRR ZONE
C             VNOD            # OF VERT NODES
C             IRRX            MATRICES CONTAINING COORDINATES (X,Z) OF
C             IRRZ            NODES IN IRREGULAR ZONE
C             MU              SHEAR MODULUS
C             LAMBDA          LAME CONSTANT
C             BETA            S WAVE VELOCITY FOR EACH ELEMENT
C             ALPHA           P WAVE VELOCITY FOR EACH ELEMENT
C             RHOE,RHO        DENSITY OF EACH ELEMENT
C             R1,R2,S         BOUNDARY CONDITION MATRICES
C             IDISPL          INCIDENT DISPLACEMENT
C             VN,HN           VERT AND HORIZ NUMBER OF NODES IN IRR ZONE
C             OMSQ            FREQUENCY OF INCIDENT WAVEFORM**2
C             NL2             NUMBER OF LAYERS*2
C             NC              NUMBER OF COLUMNS IN IRR ZONE
C             IDISPL          INCIDENT DISPLACEMENTS FROM REGULAR ZONE
C             Y               DISPLACEMENTS OF IRR ZONE
C             Z               KNOWN DISPLACEMENTS OF LAYERED ZONE
C             YR              REFLECTED DISPLACEMENTS
C             V               REGULAR ZONE MODES
C             VINV1,VINV2     INVERSE DISPLACEMENTS
C             NUM1            THE DESIRED INCIDENT DISPLACEMENT NUMBER
C             NUM2            FUNDAMENTAL TRANSMITTED MODE NUMBER
C             PART            MODE PARTICIPATION FACTOR
C             RPART           REFLECTED MODE PART. FACTOR
C             CHI,RI          ARRAY CONTAINING SUM OF GLOBAL MATRIX
C             CNORM           DUMMY PARAMETER FOR GAUSSF
C             SUM             ROWS OF LHS OF GLOBAL LINEAR EQUATION
C             WN1             INCIDENT WAVE NUMBER
C             WN2             TRANSMITTED WAVE NUMBER
C             CV,CAVE         PHASE VELOCITY
C
C     SUBROUTINES CALLED: RYGLOC, GAUSSF
C
C            --OTHER SYMBOLS ARE DEFINED IN THEIR RESPECTIVE ROUTINES
C
      PARAMETER NFN2=100,LY=6,LY2=12,COL=3,HNOD=1,VNOD=7
      INTEGER VN,HN,FN,FN2,UNIT1,UNIT2,UNIT3,UNIT4,NUM1(100),NUM2(100)
        INTEGER TNUM
      REAL IRRX(VNOD,HNOD),IRRZ(VNOD,HNOD),R(NFN2),CV(200)
      REAL RHOE(LY,COL),BETA(LY,COL),ALPHA(LY,COL)
      COMPLEX R1(LY2,LY2),R2(LY2,LY2),YR(LY2),S(LY2,LY2)
      COMPLEX IDISPL(NFN2),Y(NFN2),CNORM(NFN2),WN1(LY2),WN2(LY2)
      COMPLEX V(LY2,LY2),VINV1(LY2,LY2),CSLT,CZERO,CTWO,PART,RPART
      COMPLEX CONE,DENOM,Z(NFN2),CHI(NFN2),SUM(NFN2)
      COMPLEX DISPU,DISPW,CNORM,VINV2(LY2,LY2),IK1,IK2
      COMMON/INOUT/UNIT1,UNIT2
      COMMON/INOUT1/UNIT3,UNIT4
      COMMON/CONTR/OMSQ
C
C     INITIALIZE VARIABLES
C
      UNIT1=20
      UNIT2=21
      UNIT3=22
      UNIT4=23
      ZERO=0.0
      ONE=1.0
      TPI=3.1415927*2.0000000
      CENT=100.0
      CZERO=(0.0E00,0.0E00)
      CONE=(1.0E00,0.0E00)
      CTWO=(2.0E00,0.0E00)
      EYE=(0.0E00,1.0E00)
      TEST=1.0E-5
C
C     READ IN COORDINATES, DENSITIES, VELOCITIES, PERIOD OFF OF DISK
C
      OPEN(UNIT=5,DEVICE='DSK',FILE='IRR.DAT',ACCESS='SEQIN')
      OPEN(UNIT=6,DEVICE='LPT',ACCESS='SEQOUT')
        OPEN(UNIT=20,DEVICE='DSK',FILE='LEQ1.DAT',ACCESS='SEQINOUT',
      1RECORD SIZE=LRECL)
        OPEN(UNIT=21,DEVICE='DSK',FILE='LEQ2.DAT',ACCESS='SEQINOUT',
      1RECORD SIZE=LRECL)
        OPEN(UNIT=22,DEVICE='DSK',FILE='X1.DAT',ACCESS='SEQINOUT',
      1RECORD SIZE=LRECL)
        OPEN(UNIT=23,DEVICE='DSK',FILE='X2.DAT',ACCESS='SEQINOUT',
      1RECORD SIZE=LRECL)
      OPEN(UNIT=24,DEVICE='DSK',FILE='WN1.DAT',ACCESS='SEQIN')
      OPEN(UNIT=25,DEVICE='DSK',FILE='WN2.DAT',ACCESS='SEQIN')
C
C     INPUT DIMENSIONS OF DATA
C
C
       READ(5,1110) VN,HN
C
C     CHECK FOR DATA ERROR
C
        IF(VN .LE. 1 .OR. HN .LT. 1) GO TO 390
       IF(VN .GT. VNOD .OR. HN .GT. HNOD) GO TO 390
C
C     SET STRUCTURE SIZE VARIABLES
C
```

RYIRRX (CONTINUED)

```
      NL=YN-1
      NL2=NL*2
      NC=NN-1
      FN=NL*NN
      FN2=FN*2
      ND=FN2-NL2
C
C  SET RECORD LENGTH FOR DISK
C
        LRECL=15*FN2
        LRECL1=30*FN2
C
C  READ IN COORDINATES OF NODES
C
      DO 10 I=1,YN
      DO 10 J=1,NN
10    READ(5,1120) IRRX(I,J),IRRZ(I,J)
C
C  COMPUTE DISTANCE TRAVELED BY WAVE
C
        DIST=0.
        NN1=NN-1
        DO 20 I=1,NN1
20      DIST=DIST+SQRT((IRRX(1,I+1)-IRRX(1,I))**2+(IRRZ(1,I+1)
     1-IRRZ(1,I))**2)
C
C  READ IN FUNDAMENTAL MODE NUMBERS
C
        READ(5,1000) (NUM1(I),I=1,TNUM)
        READ(5,1000) (NUM2(I),I=1,TNUM)
C
C  READ IN ELEMENT PARAMETERS
C
      DO 30 I=1,NL
      DO 30 J=1,NC
30    READ(5,1040) BETA(I,J),RHOE(I,J),ALPHA(I,J)
      CLOSE(UNIT=5,DEVICE='DSK',FILE='IRR.DAT',DISPOSE='SAVE')
C
C  READ RANGE OF PERIODS
C
        READ(24) TMIN,TMAX,TINC
        TNUM=(TMAX-TMIN)/TINC + 1
C
C  CHECK DATA BY WRITING OUT
C
      WRITE(6,1050)
      WRITE(6,1060) NL,NC
      WRITE(6,1140)
      DO 40 I=1,YN
        WRITE(6,1360) I
40    WRITE(6,1160) (IRRX(I,J),J=1,NN)
      WRITE(6,1150)
      DO 50 I=1,YN
      WRITE(6,1360) I
50    WRITE(6,1160) (IRRZ(I,J),J=1,NN)
      WRITE(6,1070)
      WRITE(6,1080)
      DO 60 I=1,NL
      DO 60 J=1,NC
60    WRITE(6,1090) I,J,BETA(I,J),RHOE(I,J),ALPHA(I,J)
C
C  LOOP OVER DESIRED PERIODS
C
      II=0
      DO 200 PER=TMIN,TMAX,TINC
      II=II+1
      OMSQ=(TPI/PER)**2
      WRITE(6,1100) PER
C
C  OPEN DISK FILES FOR INPUT AND OUTPUT
C
C
C
C  COMPUTE GLOBAL MATRIX K-OMEGA**2*M AND WRITE TO DISK
C
C
      CALL RYGLBX(IRRX,IRRZ,RHOE,ALPHA,BETA,NL,NC,FN,FN2,NN,
     1YN,NFN2,LT,COL,NNOD,VNOD)
C
C  READ GLOBAL MATRIX FROM DISK AND WRITE TO UNIT4 AS COMPLEX
C
      DO 80 I=1,FN2
        READ(UNIT1,1370) (RI(J),J=1,FN2)
        DO 70 J=1,FN2
70      SUM(J)=CMPLX(RI(J),0.)
80      WRITE(UNIT4,1380) (SUM(J),J=1,FN2)
C
C  READ IN R AND S MATRICES, INCIDENT DISPLACEMENTS,WAVE NUMBERS,
C  AND V,VINV FROM LEFT SIDE OF IRREGULAR ZONE.
C
      DO 90 I=1,FN2
      IDISPL(I)=CZERO
90    Y(I)=CZERO
      DO 100 I=1,NL2
100   READ(24) (R1(I,J),J=1,NL2)
      DO 110 I=1,NL2
      READ(24) WN1(I)
      DO 110 J=1,NL2
      READ(24) S(I,J)
      READ(24) V(I,J)
110   READ(24) VINV1(I,J)
C
C  READ IN R2 BOUNDARY CONDITIONS FOR RIGHT SIDE
C
      DO 120 I=1,NL2
      READ(25) WN2(I)
      READ(25) (VINV2(I,J),J=1,NL2)
120   READ(25) (R2(I,J),J=1,NL2)
C
C  GET DESIRED INCIDENT DISPLACEMENT
C
      DO 130 I=1,NL2
130   IDISPL(I)=V(I,NUM1(II))
C
C
C  COMBINE MATRICES TO FORM (K-OMEGA**2*M+S-R2)*R AND
C  (S-R1)*IDISPL*Y
C
C
C
```

```
C   ADD MATRIX S TO UPPER LEFT HAND CORNER OF GLOBAL MATRIX
C
            REWIND UNIT3
            REWIND UNIT4
            DO 150 I=1,NL2
            READ(UNIT4,1380) (SUM(J),J=1,FM2)
          DO 140 J=1,NL2
140         SUM(J)=SUM(J)+S(I,J)
150       WRITE(UNIT3,1380) (SUM(J),J=1,FM2)
C
C   SKIP OVER MIDDLE OF GLOBAL MATRIX
C
          NL21=NL2+1
          DO 160 I=NL21,ND
          READ(UNIT4,1380) (SUM(J),J=1,FM2)
160       WRITE(UNIT3,1380) (SUM(J),J=1,FM2)
            DO 180 I=1,NL2
            READ(UNIT4,1380) (SUM(J),J=1,FM2)
C
C   SUBTRACT R2 FROM LOWER RIGHT CORNER OF GLOBAL MATRIX
C
          ND1=ND+1
          DO 170 J=ND1,FM2
          JND=J-ND
170         SUM(J)=SUM(J)-R2(I,JND)
180         WRITE(UNIT3,1380) (SUM(J),J=1,FM2)
            REWIND UNIT3
          DO 200 I=1,NL2
          CRIT=CZERO
          DO 190 J=1,NL2
190       CRIT=CRIT+(S(I,J)-R1(I,J))*IDISPL(J)
200       Z(I)=CRIT
C
C   SOLVE LINEAR EQUATION SUM*T=Z TWO LINES AT A TIME
C
            CALL GAUSSF(T,Z,CPI,CWORK,FM2)
C
C   OUTPUT RESULTS
C
C         WRITE(6,1170)
C
C   ECHO CHECK BOUNDARY CONDITION MATRICES R1,R2,AND S
C
C         DO 200 I=1,NL2
C         WRITE(6,1200) I
C200      WRITE(6,1290) (R1(I,J),J=1,NL2)
C         WRITE(6,1180)
C         DO 210 I=1,NL2
C         WRITE(6,1200) I
C210      WRITE(6,1290) (R2(I,J),J=1,NL2)
C         WRITE(6,1190)
C         DO 220 I=1,NL2
C         WRITE(6,1200) I
C220      WRITE(6,1290) (S(I,J),J=1,NL2)
          WRITE(6,1230)
          WRITE(6,1240)
          ISTOP=NL2-1
C
C   CHECK INCIDENT DISPLACEMENTS
C
          CNORM=IDISPL(2)
          DO 210 I=1,ISTOP,2
          DISPU=IDISPL(I)/CNORM
          DISPW=IDISPL(I+1)/CNORM
210       WRITE(6,1250) DISPU,DISPW
          WRITE(6,1270)
          WRITE(6,1240)
C
C   WRITE OUT NODAL DISPLACEMENTS FOR IPR ZONE
C
          ISTOP=FM2-1
          CNORM=T(2)
          DO 220 I=1,ISTOP,2
          DISPU=T(I)/CNORM
          DISPW=T(I+1)/CNORM
220       WRITE(6,1330) I,DISPU,DISPW
C
C   COMPUTE REFLECTED DISPLACEMENTS
C
          DO 230 I=1,NL2
230       TR(I)=T(I)-IDISPL(I)
          WRITE(6,1290)
          WRITE(6,1300)
          CNORM=CABS(CONE/WN(NUM1(IL)))
          ESUM=0.
          RESUM=0.
          DO 270 I=1,NL2
C
C   INITIALIZE FINAL VARIABLES
C
          CAVE=ZERO
          PART=CZERO
          RPART=CZERO
          ENGY=ZERO
          RENGY=ZERO
          FLAG=' '
          IF(I .EQ. NUM1(IL)) FLAG='* '
          IF(I .EQ. NUM2(IL)) FLAG='$ '
          IF(I .EQ. NUM1(IL) .AND. I .EQ. NUM2(IL)) FLAG='*$'
C
C   COMPUTE TRANSMITTED AND REFLECTED MODE PART. FACTORS
C
          DO 240 J=1,NL2
          PART=PART+YINV2(I,J)*T(ND+J)
240       RPART=RPART+YINV1(I,J)*TR(J)
C
C   COMPUTE PARTICIPATION PHASE VELOCITIES, AND ENERGY IN
C   TRANSMITTED AND REFLECTED MODES--WRITE OUT.
C
          P1=REAL(PART)
          P2=AIMAG(PART)
            RP1=REAL(RPART)
            RP2=AIMAG(RPART)
          IF(ABS(P1) .LT. TEST .AND. ABS(P2) .LT. TEST) GO TO 260
C
C   FIND PHASE FROM ARGUMENT OF RPF
C
          PHI=ATAN2(P2,P1)
            RPHI=ATAN2(RP2,RP1)
            IF(PHI .LT. 0) PHI = PHI + TPI
```

RYIRRX (CONTINUED)

```
          IF(PPHI .LT. 0) PPHI = PPHI + TPI
C
C   FIND AVERAGE PHASE VELOCITIES FROM PHASE AND DISTANCE
C
        CAVE=DIST*TPI/(PER*PHI)
C
C   COMPUTE TRANSMITTED AND REFLECTED ENERGY
C
        RF1=REAL(WN1(I))
        RF2=REAL(WN2(I))
        IF1=AIMAG(WN1(I))
        IF2=AIMAG(WN2(I))
        IF(ABS(RF2) .LT. TEST .OR. ABS(IK2) .GT. TEST) GO TO 250
        ENGY=ABS(RF2)*CABS(PART)**2*ENORM*CENT
        ESUM=ESUM+ENGY
250     IF(ABS(RK1) .LT. TEST .OR. ABS(IK1) .GT. TEST) GO TO 260
        RENGY=ABS(RK1)*CABS(RPART)**2*ENORM*CENT
        RESUM=RESUM+RENGY
C
C   WRITE OUT RPP'S, PHASE VEL., AND % TRANS AND REFL ENERGY.
C
260     WRITE(6,1280) I,FLAG,PART,CAVE,ENGY,RENGY
270     CONTINUE
        WRITE(6,1010) ESUM,RESUM
280       CONTINUE
        GO TO 310
C
C   ERROR NOTES
C
290       WRITE(5,1340)
          GO TO 310
300       WRITE(5,1350)
C
C   CLOSE DEVICES
C
310     CLOSE(UNIT=6,DEVICE='LPT',DISPOSE='PRINT')
        CLOSE(UNIT=20,DEVICE='DSK')
        CLOSE(UNIT=21,DEVICE='DSK')
        CLOSE(UNIT=22,DEVICE='DSK')
        CLOSE(UNIT=23,DEVICE='DSK')
        CLOSE(UNIT=24,DEVICE='DSK')
        CLOSE(UNIT=25,DEVICE='DSK')
        STOP
1000      FORMAT(I3)
1010      FORMAT(//10X,'TOTAL TRANSMITTED ENERGY =',F7.3,' %',23X,
        1'%  INCIDENT MODE',/9X,' TOTAL REFLECTED ENERGY =',
        2F7.3,' %',25X,'%  TRANSMITTED FUNDAMENTAL MODE')
1020      FORMAT(/' ENTER MINIMUM AND MAXIMUM PERIOD, AND PERIOD',
        1' INCREMENT, EXACTLY AS IN HYLAY')
1030    FORMAT(3E12.5)
1040    FORMAT(1X,3F10.5)
1050      FORMAT(1X'RAYLEIGH WAVE ANALYSIS'//)
1060    FORMAT(//,' IRREGULAR STRUCTURE CONSISTING OF',I3,
        1' VERTICAL ELEMENTS AND',I3,' HORIZONTAL ELEMENTS:'//)
1070    FORMAT(1H1,4X,'ELEMENT',6X,'BETA',7X,'RHO',6X,'ALPHA'/)
1080    FORMAT(50('_')/)
1090    FORMAT(4X,'(',I3,',',I3,')',1X,F7.2,3X,F7.2,3X,F7.3)
1100    FORMAT(1H('PERIOD =',F7.3)
1110    FORMAT(2I3)
1120    FORMAT(2F12.5)
1130    FORMAT(F10.5)
1140    FORMAT(//1X'HORIZONTAL COORDINATES ='//)
1150    FORMAT(1H1,1X'VERTICAL COORDINATES ='//)
1160    FORMAT(1X,16F8.2)
1170    FORMAT(1X,9(2E7.1))
1180    FORMAT(1X18F6.1)
1190    FORMAT(1H1,'BOUNDARY CONDITION MATRICES'///,1X,'MATRIX P1 =')
1200    FORMAT(1H1,'MATRIX P2 =')
1210    FORMAT(1H1,'MATRIX A =')
1220    FORMAT(//I10/)
1230    FORMAT(1H1,'INCIDENT DISPLACEMENT =')
1240    FORMAT(///23X,'UR',12X,'UI',16X,'WR',12X,'WI'//)
1250    FORMAT(16X,2E14.6,4X,2E14.6)
1260    FORMAT(25X,2E12.5)
1270    FORMAT(1H1,'DISPLACEMENTS OF IRREGULAR NODES ='/)
1280    FORMAT(1X,I3,A2,1X,4(E12.5,5X),F12.5,2(5X,F12.3))
1290    FORMAT(1H1,9X,'TRANS AMPL',8X,'REFL AMPL',7X,'TRANS PHASE',6X,
        1'REFL PHASE',7X,'PHASE VELOCITY',5X,'% TRANS ENERGY',4X,
        2'% REFL ENERGY')
1300    FORMAT(8X,120('_')/)
1310    FORMAT(5(3X,2F12.5))
1320    FORMAT(7X,I3,10X,2(E12.5,2X))
1330    FORMAT(7X,I3,6X,2E14.6,4X,2E14.6)
          FORMAT(/1X,'INPUT DATA ERROR--JOB TERMINATED')
          FORMAT(/' PROGRAM DIMENSIONS EXCEEDED BY STRUCTURE')
          FORMAT(///I10/)
          FORMAT(1000E15.8)
          FORMAT(1000(2E15.8))
        END
```

RYLAY

```
C       RYLAY -- LAYERED STRUCTURE RAYLEIGH WAVES
C
C       RYLAY IS DESIGNED TO SIMULATE RAYLEIGH WAVE MOTION
C       IN A TWO-DIMENSIONAL LAYERED MEDIUM USING FINITE
C       ELEMENT TECHNIQUES AS DEVELOPED BY LYSMER (1970)
C       AND LYSMER AND DRAKE (1972). THE PROGRAM ASSEMBLES GLOBAL
C       MATRICES A, B, C, AND E FROM SUBROUTINES OF THE SAME
C       NAME WHICH  ARE THEN USED TO SOLVE THE ALGEBRAIC
C       EIGENVALUE PROBLEM
C
C               ([A]K**2 + I[B]K + [C])[V] = [0]
C
C       WHERE K'S ARE WAVE NUMBERS. THE PROCEDURE USED
C       IN SOLVING THIS EQUATION WAS DEVISED BY PETERS AND WILKENSON
C       (1970).
C
C       THE MATRIX [R]=[I]*[A]*[V]*[K]*[VINV]+[E] CONTAINS STRESSES
C       THAT ARE TRANSMITTED TO THE IRREGULAR ZONE. THESE ARE
C       TRANSFERRED TO THE IRREGULAR ZONE ANALYSIS PROGRAM, RYIRR,
C       VIA A DISK FILE. DISPLACEMENTS (MODE SHAPES OR EIGENVECTORS),
C       WAVE NUMBERS, AND THE PERIOD RANGE ARE TRANSFERRED TO RYIRR AS WELL.
C
C
C       Bibliography:
C
C       Lysmer, J. (1970). Lumped mass method for Rayleigh waves, Bull.
C          Seis. Soc. Am., 60, 89-104.
C
C       Lysmer, J. and L. A. Drake (1972). A finite element method for
C          seismology, in Methods in Computational Physics, vol. 11,
C          B. Bolt, Ed., 181-216, Acad. Press, N.Y.
C
C       Peters, G. and J. H. Wilkenson (1970). The generalized eigenproblem,
C         SIAM J. Numer. Anal., v. 7, 479-492.
C
C
C       THE SUBROUTINES CALLED ARE:    RMTXA-- COMPUTES GLOBAL MATRIX A
C                                      RMTXB--    "       "      "   B
C                                      RMTXC--    "       "      "   C
C                                      RMTXE--    "       "      "   E
C                                      EIGRF-- IMSL EIGENVALUE EQUATION SOLV
C                                      VMULFF--MATRIX PRODUCT ROUTINE (IMSL)
C                                      LINV1F--LINEAR EQUATION SOLVER (IMSL)
C                                      LEQT1C--COMPLEX LIN. EQN. SOLVER (INS
C
C
C       DATA FILES:
C                                      LAY.DAT-- PARAMETERS OF LAYERED ZONE
C                                      RB1.DAT-- OUTPUT FOR LEFT BOUNDARY CON
C                                      RB2.DAT--   "      "  RIGHT    "
C
C
C       THE SYMBOLS USED IN THIS PROGRAM ARE AS FOLLOWS:
C
C             LY          MAX # OF LAYERS
C             VN          MAX # OF VERTICAL NODES
C
C       NOTE-IF MORE THAN 15 LAYERS ARE NEEDED THE PARAMETER
C             STATEMENT MUST BE CHANGED.
C
C             NL          NUMBER OF LAYERS
C             NL2         2*NL (USED AS  MATRIX DIMENSION)
C             NL4         4*NL (USED AS MATRIX DIM. FOR EIGRF)
C             MAXDIM      MAXIMUM MATRIX SIZE
C             ALPHA       P-WAVE VELOCITY
C             BETA        S-WAVE VELOCITY
C             D           THICKNESS OF EACH LAYER
C             DEPTH       DEPTH OF EACH NODE
C             RHO         DENSITY OF EACH LAYER
C             PER         PERIOD IN SEC
C             X(I,J)      4NL*4NL GLOBAL MATRIX USED FOR EIGEN-EQUATION
C             RHO         DENSITY OF EACH LAYER
C             OMEGA       FREQUENCY (2*PI/T)
C             A,B,C,E     GLOBAL MATRICES USED IN FORMULATING THE EIGEN-
C                         VALUE PROBLEM. THEY CONTAIN STIFFNESS AND MASS
C                         MATRICES.
C             M           GLOBAL MASS MATRIX FOR LAYERED ZONE
C             WORK        DUMMY ARRAY
C             AINV        A**-1
C             AINVB       AINV*B
C             AINVC       AINV*C
C             DISPL,V     NORMALIZED EIGENVECTORS (DISPLACEMENTS)
C             VINV        V**-1
C             VINV2       VINV SAVER ARRAY
C             R           BOUNDARY CONDITION MATRIX
C             S           REFLECTED BOUNDARY CONDITION MATRIX
C             RTEMP       MATRIX USED IN FORMULATING R
C             CVEL        PHASE VELOCITY
C             GVEL        GROUP VELOCITY
C             TEST        ZERO WITH ROUNDOFF ERRORS
C             TMIN        MINIMUM PERIOD
C             TMAX        MAXIMUM PERIOD
C             TINC        PERIOD INCREMENT
C             EIGEN       EIGENVALUES (WAVE NUMBERS)
C
C     NOTE-- THIS PROGRAM MUST BE RUN AGAIN IF DIFFERENT BOUNDARY
C            CONDITIONS ARE DESIRED ALONG THE RIGHT MARGIN OF THE
C            IRREGULAR ZONE. TO DO THIS ANSWER THE QUERY AS TO WHETHER
C            THIS PROGRAM IS BEING RUN TO GENERATE R1 (RIGHT SIDE)
C            OR TO GENERATE R2 (LEFT SIDE).
C
      PARAMETER LY=15,LY2=30,LY4=60,VN=16,LY8=120
      REAL A(LY2,LY2),B(LY2,LY2),C(LY2,LY2),D(LY),RHO(LY)
      REAL AINB(LY2,LY2),AINC(LY2,LY2),AINV(LY2,LY2),K1,X(LY4,LY4)
      REAL M(LY2,LY2),GV(200),CV(200)
      REAL ALPHA(LY),BETA(LY),WORK(LY8),DEPTH(VN),KR,E(LY2,LY2)
      COMPLEX DISPL(LY4,LY4),FTP,EIGEN(LY4),CZERO,CONE,CTWO,DENOM
      COMPLEX VINV(LY2,LY2),R(LY2,LY2),FSQ,WORKC(LY4),CTEMP,CNORM
      COMPLEX RTEMP(LY2,LY2),V(LY2,LY2)
      COMPLEX VINV2(LY2,LY2),DISPU,DISPW
      EQUIVALENCE (AINB(1,1),DISPL(1,1)),(AINC(1,1),DISPL(1,1))
C
C  OPEN INPUT FILE AND OUTPUT DEVICE
C

      OPEN(UNIT=5,DEVICE='DSK',FILE='LAY.DAT',ACCESS='SEQIN')
      OPEN(UNIT=6,DEVICE='LPT',ACCESS='SEQOUT')
      READ(5,1000) NL
C
C  CHECK NUMBER OF LAYERS
```

```
C
      IF(NL .LE. 1) GO TO 430
      IF(NL .GT. LT) GO TO 440
C
C  INITIALIZE VARIABLES
C
      DEPTH(1)=0.
      NL2=NL*2
      NL4=NL*4
      MAXDIM=LT2
      MAX2=MAXDIM*2
      TPI=2.000000D0*3.14159265
      TEST=1.0E-5
      OVF=1.00
      EYE=(0.0E0,1.0E0)
      CZERO=(0.0E0,0.0E0)
      CONE=(1.0E0,0.0E0)
      CTWO=(2.0E0,0.0E0)
      JSTOP=NL2-1
      DO 10 I=1,NL
      READ(5,1090) D(I),BETA(I),RHO(I),ALPHA(I)
      DEPTH(I+1)=DEPTH(I)+D(I)
10    CONTINUE
C
C  CLOSE INPUT FILE
C
      CLOSE(UNIT=5,DEVICE='DSK',FILE='LAY.DAT',DISPOSE='SAVE')
C
C  READ RANGE OF PERIODS
C
      WRITE(5,1020)
      READ(5,*) TMIN,TMAX,TINC
C
C  DECIDE IF R1 OR R2 IS TO BE COMPUTED IN THIS RUN
C
      WRITE(5,1050)
      READ(5,1040) R1ANS
      WRITE(5,1060)
      READ(5,1040) R2ANS
C
C  OPEN DESIRED OUTPUT FILES
C
      IF(R1ANS .EQ. 'NO') GO TO 13
      OPEN(UNIT=24,DEVICE='DSK',FILE='RR1.DAT',ACCESS='SEQOUT')
13    IF(R2ANS .EQ. 'NO') GO TO 15
      OPEN(UNIT=25,DEVICE='DSK',FILE='RR2.DAT',ACCESS='SEQOUT')
C
C  CHECK DATA BY WRITING OUT
C
15    IF(R1ANS .EQ. 'YES') WRITE(24) TMIN,TMAX,TINC
      WRITE(6,1100) NL
      DO 20 I=1,NL
      WRITE(6,1110) DEPTH(I)
      WRITE(6,1130) D(I),ALPHA(I),BETA(I),RHO(I)
20    CONTINUE
      WRITE (6,1120) DEPTH(NL+1)
C
C  LOOP OVER PERIODS
C
      II=1
      DO 390 PER=TMIN,TMAX,TINC
      JJ=II
      WRITE(6,1140) PER
      OMEGA=TPI/PER
      OMSQ=OMEGA**2
C
C  CONSTRUCT GLOBAL MATRICES A, AINV, B, AND C
      CALL RMTXA(A,D,ALPHA,BETA,RHO,MAXDIM,NL)
      CALL LINV1F(A,NL2,MAXDIM,AINV,0,WORK,IER)
      WRITE(6,1150) IER
      CALL RMTXB(B,ALPHA,BETA,RHO,MAXDIM,NL)
      CALL RMTXC(C,D,ALPHA,BETA,RHO,MAXDIM,OMSQ,NL,K)
C
C  COMPUTE AINV*C
C
      CALL VMULFF(AINV,C,NL2,NL2,NL2,MAXDIM,MAXDIM,AINC,MAXDIM,IER)
      WRITE(6,1160) IER
C
C  PUT AINC IN LOWER LEFT OF X
C
      DO 30 I=1,NL2
      IPNL=I+NL2
      DO 30 J=1,NL2
30    X(IPNL,J)=AINC(I,J)
C
C  COMPUTE AINV*B
C

      CALL VMULFF(AINV,B,NL2,NL2,NL2,MAXDIM,MAXDIM,AINB,MAXDIM,IER)
      WRITE(6,1170) IER
C
C  PUT -AINB IN LOWER RIGHT OF X
C
      DO 40 I=1,NL2
      IPNL=I+NL2
      DO 40 J=1,NL2
      JPNL=J+NL2
40    X(IPNL,JPNL)=-AINB(I,J)
C
C  ZERO OUT UPPER HALF OF X
C
      DO 50 J=1,NL4
      DO 50 I=1,NL2
50    X(I,J)=0.
C
C  MAKE UNIT MATRIX IN UPPER RIGHT OF X
C
      DO 60 I=1,NL2
      J=I+NL2
60    X(I,J)=1.0
C
C  SOLVE EIGENVALUE PROBLEM  ([A]K**2 + I[B]K + [C]){V} = {0}
C  WHICH IS IN THE FORM [X] {DISPL} = K{DISPL} = {0}, SEE
C  PETERS AND WILKINSON (1970).
C
      CALL EIGRF(X,NL4,MAX2,1,EIGEN,DISPL,MAX2,WORK,IER)
      WRITE (6,1180) IER
C
C  CORRECT EIGENVALUES TO WAVE NUMBERS
C
```

RYLAY . (CONTINUED)

```
      DO 70 I=1,NL4
70    EIGEN(I)=CYE*EIGEN(I)
C
C  NORMALIZE EIGENVECTORS
C
      CALL MATEA(A,D,ALPHA,BETA,RHO,MAXDIM,NL)
      CALL MATEC(C,D,ALPHA,BETA,RHO,MAXDIM,OMSQ,NL,M)
      DO 130 IL=1,NL4
      KSQ=EIGEN(IL)**2
      DO 90 I=1,NL2
      CTEMP=CZERO
      DO 80 J=1,NL2
80    CTEMP=CTEMP+(A(I,J)*KSQ-C(I,J))*DISPL(J,IL)
90    WORKC(I)=CTEMP
C
C  HORZ EIGENVECTOR ELEMENTS ARE NEGATIVE, VERT ARE POSITIVE
C
      CNORM=CZERO
      DO 100 I=1,JSTOP,2
100   CNORM=CNORM-DISPL(I,IL)*WORKC(I)
      DO 110 I=2,NL2,2
110   CNORM=CNORM+DISPL(I,IL)*WORKC(I)
      CTEMP=CSQRT(CTWO*KSQ/CNORM)
C
C  MULTIPLY BY NORMALIZATION FACTOR
C
      DO 120 J=1,NL2
120   DISPL(J,IL)=DISPL(J,IL)*CTEMP
130   CONTINUE
C
C  CHOOSE APPROPIATE EIGENVALUES ON BASIS THAT EXPONENTIAL
C  MODES DO NOT PROPAGATE
C
      I=0
      DO 170 J=1,NL4
      KR=REAL(EIGEN(J))
      KI=AIMAG(EIGEN(J))
      IF(ABS(KI) .LT. TEST) GO TO 140
      IF(KI .LT. 0.0) GO TO 150
      GO TO 170
C
C  KR REAL--CHOOSE SIGN BASED ON EIGENVECTORS;
C                    IF VERT DISPL REAL, K
C                    IF VERT DISPL IMAG, -K
C                    IF BOTH DISPL AND KR=0, FALL THROUGH LOOP
C
140   VERR=ABS(REAL(DISPL(2,J)))
      IF((VERR .GT. TEST) .AND. (KR .GT. 0.)) GO TO 150
      VERI=ABS(AIMAG(DISPL(2,J)))
      IF((VERI .GT. TEST) .AND. (KR .LT. 0.)) GO TO 150
      GO TO 170
150   I=I+1
      EIGEN(I)=EIGEN(J)
      DO 160 K=1,NL2
160   DISPL(K,I)=DISPL(K,J)
170   CONTINUE
      IF(I .NE. NL2) WRITE(6,1190) I,NL2
C
C  WRITE OUT ACCEPTED EIGENVALUES AND EIGENVECTORS
C
      WRITE(6,1200)
      DO 250 L=1,I
C
C  COMPUTE PHASE VELOCITY
C
      CVEL=0.0
      GVEL=0.0
      KR=ABS(REAL(EIGEN(L)))
      IF(KR .EQ. 0.) GO TO 210
      CVEL=TPI/(KR*PER)
C
C  COMPUTE GROUP VELOCITY
C
      DO 190 J=1,NL2
      CTEMP=CZERO
      DO 180 K=1,NL2
180   CTEMP=CTEMP+DISPL(K,L)*CMPLX(H(K,J),0.)
190   WORKC(J)=CTEMP
      CNORM=CZERO
      DO 200 K=1,NL2
200   CNORM=CNORM+WORKC(K)*DISPL(K,L)
      GVEL=KR/(OMEGA*CNORM)
210   WRITE(6,1220) L,EIGEN(L)
      WRITE(6,1210) L,CVEL
      WRITE(6,1230) L,GVEL
C
C  NORMALIZE DISPLACEMENTS TO 1 FOR PRINTING: DISPU = HORIZ. DISPLACEMENT
C                                             DISPW = VERT. DISPLACEMENT
C
      CNORM=DISPL(2,L)
      DO 240 K=1,JSTOP,2
      DISPU=DISPL(K,L)/CNORM
      DISPW=DISPL(K+1,L)/CNORM
240   WRITE(6,1240) DISPU,DISPW
250   CONTINUE
C
C  SEE IF R MATRICES ARE DESIRED
C
      IF(R1ANS .EQ. 'NO' .AND. R2ANS .EQ. 'NO') GO TO 450
C
C  COMPUTE BOUNDARY CONDITION MATRIX OF FORCES R=I*A*V*K*VINV+D
C  AND WRITE OUT ON DISK IN REMAINING PART OF PROGRAM.
C
C  LET V=DISPL
C
C
C  FORM IDENTITY MATRIX FOR LEQTIC
C
      DO 260 I=1,NL2
      DO 260 J=1,NL2
      VINV(I,J)=CZERO
      IF(I .EQ. J) VINV(I,J)=CONE
260   CONTINUE
C
C  SAVE V
C
      DO 270 I=1,NL2
      DO 270 J=1,NL2
270   V(I,J)=DISPL(I,J)
C
```

RYLAY (CONTINUED)

```
C   OBTAIN INVERSE OF V
C
        CALL LEQTIC(DISPL,NL2,MAX2,VINV,NL2,MAXDIM,0,WORK,IER)
        WRITE(6,1250) IER
C
C   MULTIPLY VINV BY "DIAGONAL MATRIX" OF EIGENVALUES, EIGEN
C
        DO 290 I=1,NL2
        CTEMP=EIGEN(I)
        DO 280 J=1,NL2
        VINV2(I,J)=VINV(I,J)
280     VINV(I,J)=VINV(I,J)*CTEMP
290     CONTINUE
C
C
C   MULTIPLY BY V
C
        DO 300 I=1,NL2
        DO 300 J=1,NL2
        CTEMP=CZERO
        DO 300 K=1,NL2
        CTEMP=CTEMP+V(I,K)*VINV(K,J)
300     RTEMP(I,J)=CTEMP
C
C
C   FORM MATRICES A AND E
C
        CALL RMTXA(A,D,ALPHA,BETA,RHO,MAXDIM,NL)
        CALL RMTXE(E,ALPHA,BETA,RHO,MAXDIM,NL)
C
C   MULTIPLY BY I * A AND ADD E
C
        DO 320 I=1,NL2
        DO 320 J=1,NL2
        CTEMP=CZERO
        DO 310 K=1,NL2
310     CTEMP=CTEMP+A(I,K)*RTEMP(K,J)
C
C   COMPUTE R AND S BOUNDARY CONDITION MATRICES
C
320     R(I,J)=-CYE*CTEMP-E(I,J)
C
C   WRITE OUT MATRIX R ON LPT
C
        WRITE(6,1270)
        DO 330 I=1,NL2
        WRITE(6,1300) I
        WRITE(6,1290) (R(I,J),J=1,NL2)
 330    CONTINUE
        WRITE(6,1280)
C
C   WRITE  R,S,V,VINV,AND WAVE NUMBERS OUT ON DISK (RN1.DAT) IF DESIRED
C
        IF(R1ANS .EQ. 'NO') GO TO 353
        DO 350 I=1,NL2
350     WRITE(24) (R(I,J),J=1,NL2)
C
C   WRITE OUT R2 MATRIX ON RN2.DAT AS WELL AS ALL EIGENVALUES/VECTORS
C
        IF(R2ANS .EQ. 'NO') GO TO 357
353     DO 355 I=1,NL2
        WRITE(25) EIGEN(I)
        WRITE(25) (VINV2(I,J),J=1,NL2)
 355    WRITE(25) (R(I,J),J=1,NL2)
 357    IF(P1ANS .EQ. 'NO') GO TO 390
        DO 360 I=1,NL2
        WRITE(24) EIGEN(I)
C
C   COMPUTE S MATRIX AND SUBSTITUTE INTO R TO SAVE SPACE
C
        DO 360 J=1,NL2
        R(I,J)=-R(I,J)*(-ONE)**(I+J)
C
C   WRITE OUT ON DISK RN2.DAT
C
        WRITE(24) R(I,J)
        WRITE(24) V(I,J)
360     WRITE(24) VINV2(I,J)
C
C   PRINT MATRIX S
C
        DO 340 I=1,NL2
        WRITE(6,1300) I
340     WRITE(6,1290) (R(I,J),J=1,NL2)
390     CONTINUE
        GO TO 450
C
C   ERROR NOTES
C
430     WRITE(6,1000)
        GO TO 450
        GO TO 450
440     WRITE(6,1010)
C
C   CLOSE DEVICES
C
450     CLOSE(UNIT=6,DEVICE='LPT',DISPOSE='PRINT')
        IF(P1ANS .EQ. 'NO') GO TO 460
        CLOSE(UNIT=24,DEVICE='DSK')
460     IF(P2ANS .EQ. 'NO') GO TO 470
        CLOSE(UNIT=25,DEVICE='DSK')
470     STOP
1000    FORMAT(/' # OF LAYERS TOO SMALL--2 IS MINIMUM')
1010    FORMAT(/' # OF LAYERS EXCEEDS PROGRAM DIMENSIONS')
1020    FORMAT(/' ENTER MINIMUM AND MAXIMUM PERIOD, AND PERIOD INCREMENT
       1, SEPERATED BY COMMAS')
1040    FORMAT(A1)
1050    FORMAT(/' WILL MATRIX R1 BE COMPUTED USING THIS STRUCTURE?')
1060    FORMAT(/' WILL MATRIX R2 BE COMPUTED?')
1080    FORMAT(I5,I4,F10.5)
1090    FORMAT(4F13.5)
1100    FORMAT(10X,'FINITE ELEMENT RAYLEIGH WAVE MODEL:',I4,I5,' LAY
       1ERS'////10X,'THICKNESS',9X,'ALPHA',9X,'BETA',10X,'RHO'/)
1110    FORMAT(1X,F7.2,60('-'))
1120    FORMAT(1X,F7.2,60('-'))
1130    FORMAT(10X,4(F9.2,4X))
1140    FORMAT(1H1,'PERIOD = ',F7.2,' SEC'/)
1150    FORMAT(///' ERROR PARAMETER FROM LEQTIC =',I4)
1160    FORMAT(' ERROR PARAMETER FROM VMULFF(AINC) =',I4)
1170    FORMAT(' ERROR PARAMETER FROM VMULFF(AINB) =',I4)
```

RYLAY (CONTINUED)

```
1180      FORMAT(' ERROR PARAMETER FROM EIGRF =',I4)
1190      FORMAT(//' >>ERROR<<',3X,'I =',I4,3X,'NL2 =',I4)
1200      FORMAT(1H1,9X,'ACCEPTED VELOCITIES AND DISPLACEMENTS')
1210      FORMAT(10X,'PHASE VELOCITY',I4,' =',E12.4)
1220      FORMAT(1H1,9X,'EIGENVALUE',I4,' =',2E14.3)
1230      FORMAT(10X,'GROUP VELOCITY',I4,' =',E12.4/,10X,
         1'NORMALIZED DISPLACEMENTS ='///23X,'UR',12X,'UI',
         21AX,'WR',12X,'WI'//)
1240      FORMAT(1AX,2E14.6,4X,2E14.6)
1250      FORMAT(//' ERROR PARAMETER FOR LEOTIC = ',I4)
1260      FORMAT(1X,6(F9.2,1X))
1270      FORMAT(1H1,'MATRIX R = ')
1280      FORMAT(1H1,'MATRIX S =')
1290      FORMAT(5(2X,2E17.5))
1300      FORMAT(///110/)
          END
```