Geophysics Open-File Report 14
Geoscience Department
New Mexico Tech
Socorro, NM 87801

THE USE OF RELATIVE TRAVEL TIME RESIDUALS OF P PHASES FROM TELESEISMIC EVENTS TO STUDY THE CRUST IN THE SOCORRO, NEW MEXICO AREA

by

Jeffrey A. Fischer

Submitted in partial

fulfillment

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Geophysics 590

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The research described in this paper was sponsored jointly by the National Science Foundation (DES74-24187) and the State of New Mexico (Board of Educational Finance (BEF-6), Energy Resource Board (ERB-75-300), and New Mexico Energy Institute-New Mexico State University (EI-176-032)).

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Table of Contents

List of Figures

List of Tables

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Finally, I wish to thank Kristie and my family for suffering me through all this.

ABSTRACT

Relative travel time residuals calculated for 214 P phase arrivals from 46 teleseismic events were used to study the crust in the Socorro, New Mexico area. These events occurred between February 1975 and February 1977 and were recorded by an array of 19 short period seismometers in the Socorro area.

Mean relative travel time residuals were calculated at the various stations for arrivals from 5 general directions; NE, E, SE, SW, and NW. Grand means over all directions were also calculated at each station.

The relative travel time residuals increase in general for arrivals at stations to the south and southwest relative to those to the north and northeast, implying a relative delay of those arrivals in the south and southwest. This northeast - southwest distribution of travel time residuals is directly opposite to one that might be expected from a previously inferred crustal P velocity structure for the area. It may therefore be that teleseismic relative travel time residuals reflect the P velocity structure in the upper mantle

The directional analyses at many stations show that the relative travel time residuals are clear functions of the azimuth of teleseismic arrival. The complexity of the area, as reflected in its surface geology, appears to have expression deep into the crust and perhaps the upper mantle.

Chapter 1: Introduction

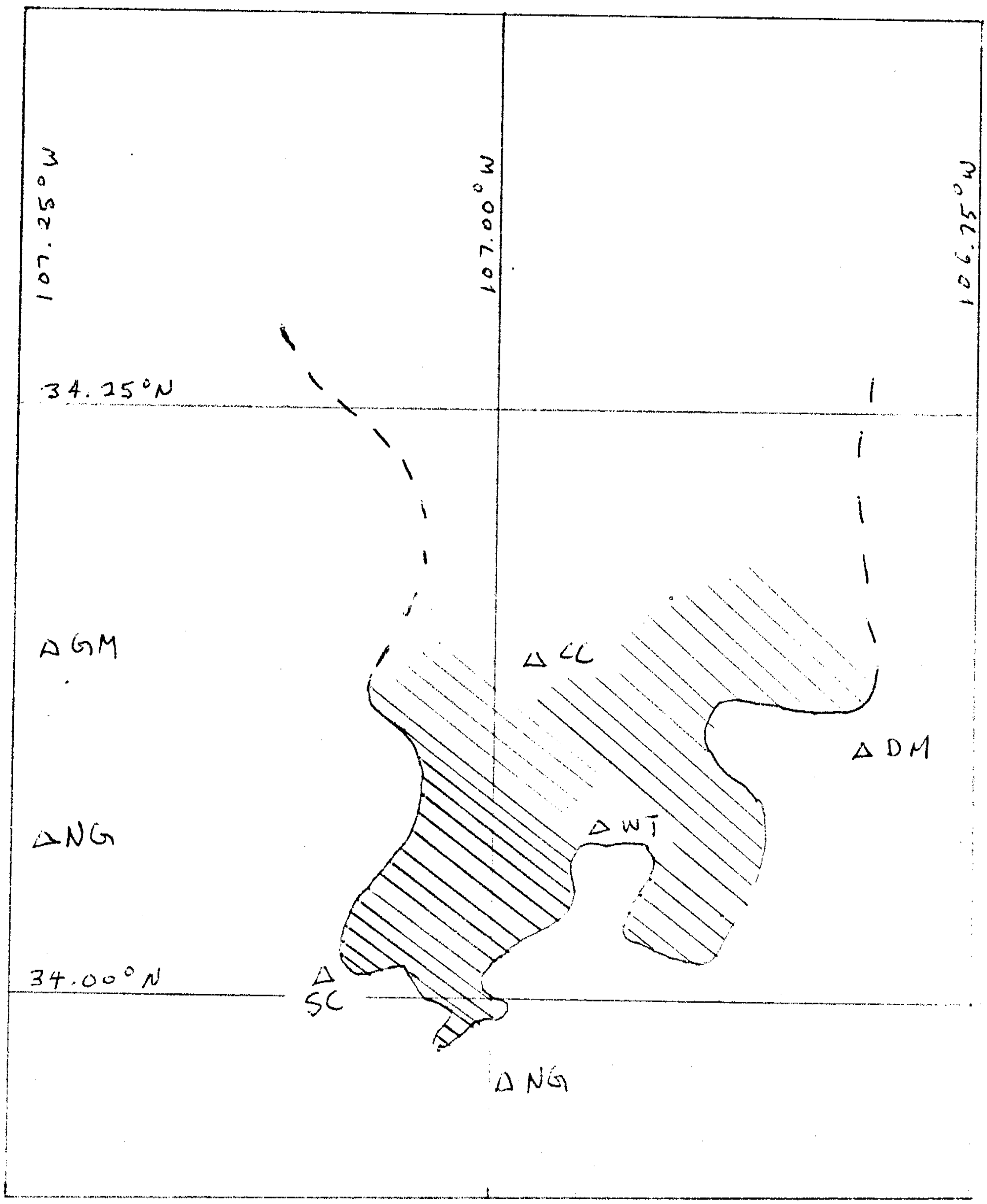
For the past several years the seismic research group at N.M.I.M.T. has been studying the properties and spatial distribution of a proposed magma body in the crust of the Socorro, New Mexico area. The main focus of this research has been the analysis of local microearthquakes and regional man-made events recorded by a moveable array of short period seismometers. A review of these studies is given by Sanford, et al. (1976).

The purposes of the study described in this paper were to 1) read the arrival times of the more distinct teleseismic events recorded by the above array, 2) calculate relative travel time residuals with respect to various base stations, 3) analyse these residuals as functions of both station location and azimuthal angle of arrival of the teleseismic energy, and 4) perform a preliminary interpretation of this analysis to evaluate the potential of using teleseismic travel time residuals to study the crust in the Socorro area.

Two recent studies are closely related to this study. By studying SxS reflections of microearthquake energy off points on the upper surface of the magma body, Rinehart (1976) was able to partially map the southern, eastern, and western edges of the body. He also determined some relief on the upper surface of the body; his findings were that the depth to this surface ranged between 18 and 20 km. Figure 1 is a map of the upper surface of the magma body from Rinehart's study. Yousef (1977) mapped relative travel time residuals with respect to station WT (Figure 1, Table 1) obtained from the arrivals of Pn phases from explosions occurring in the mining districts to the southwest of Socorro. He found a systematic variation in relative delay times, from southeast to northwest, with the total variance in delay being about

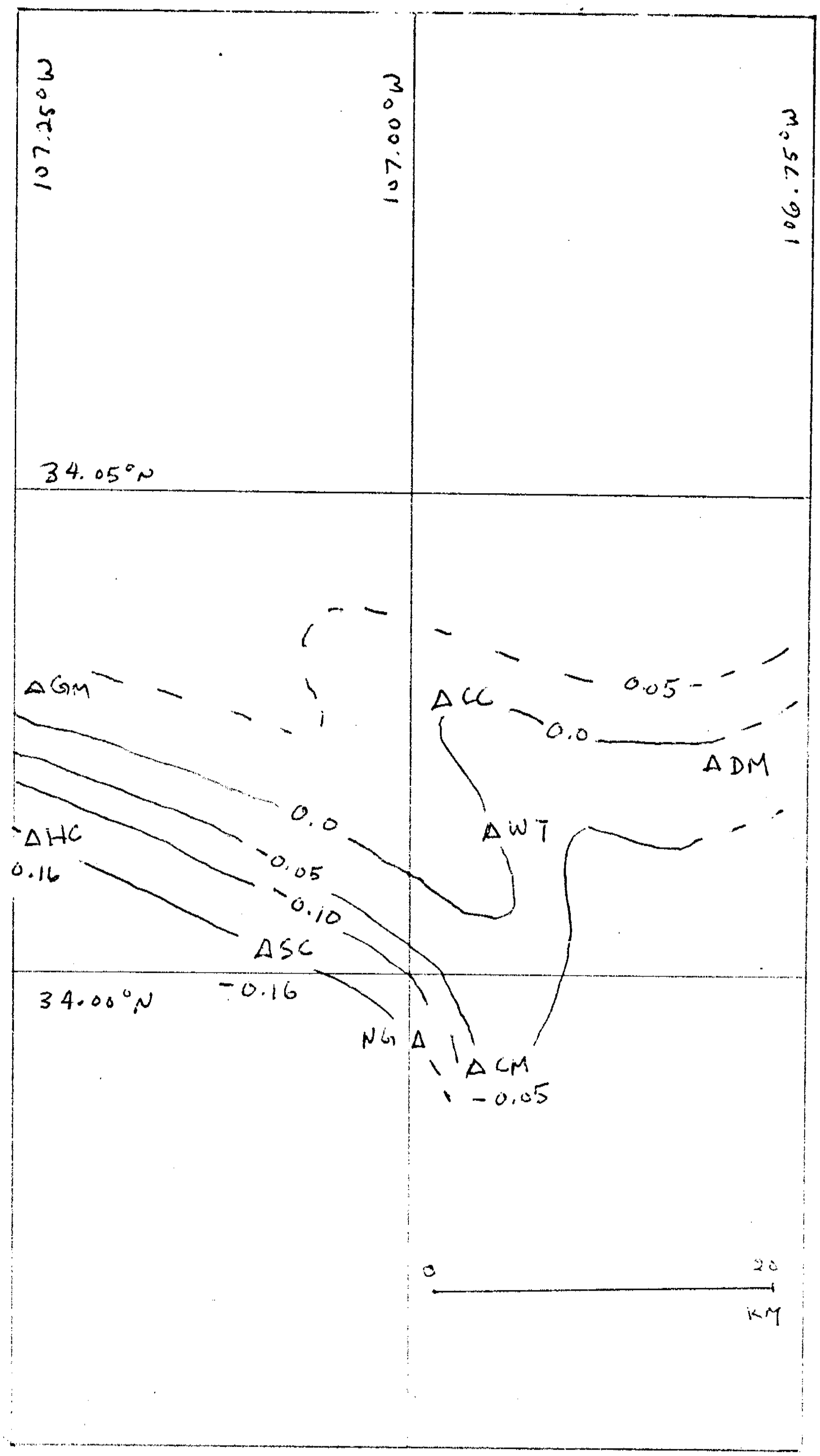
Figure 1: The Upper Surface of the

Magma Body From Rinehart (1976)



0 2 seconds. Figure 2 is his contour map of these residuals. Negative numbers mean a relative delay at the base station WT.

Figure 2: Contour Map of Pn Arrival Time Residuals with Respect to WT, from Mining Events at Santa Rita, New Mexico, from Yousef (1977)



Chapter 2: The Relative Travel Time Residuals

2.1: Collection of Teleseismic Arrival Time Data

The basic data for the study described in this paper were the arrival times of the initial P phases of 46 teleseisms. These arrivals were recorded by a portable array of Sprengnether MEQ-800 recording systems in the Socorro area between February 1975 and February 1977. The MEQ-800 system used either a Products L4-C (1.0 Hz) or a Willmore (1.5 Hz) vertical seismometer. The gain settings varied between 78 and 120 db. A filtering system to remove signals above 30 Hz was used. A description of the MEQ-800 system is given by Rinehart (1976). Of the 21 stations occupied by this array during the above time period, 19 recorded at least one arrival from the set of 46 teleseisms selected for the study. The relative locations of these 19 stations and their designations are shown in Figure 3; their latitude, longitude, and elevation are given in Table 1.

Of the hundreds of teleseisms recorded during the two year period from which the data was gathered only 46 had initial P phases impulsive enough to be measured with an uncertainty of 0.2 seconds or less. Figure 4 shows the epicentral locations for these 46 events. A total of 214 initial P-phase arrivals were measured at the various stations. Of these, 61 were considered to have been measured with an uncertainty of less than .1 second, 111 were measured with an uncertainty no larger than .1 second, 36 were in an interval no larger than .2 second, and the remaining 6 were in an interval no wider than .3 second. The measurements were made on seismograms written at a recording speed of 2mm per second using a Keuffel and Esser .1mm scale and hand lens. In all cases except the 61 most accurate measurements, an honest quotation of the interval of arrival was stressed more than a precise point at which one

Figure 3: Locations of the 19 Stations

Recording Teleseismic Arrivals Used in this Study

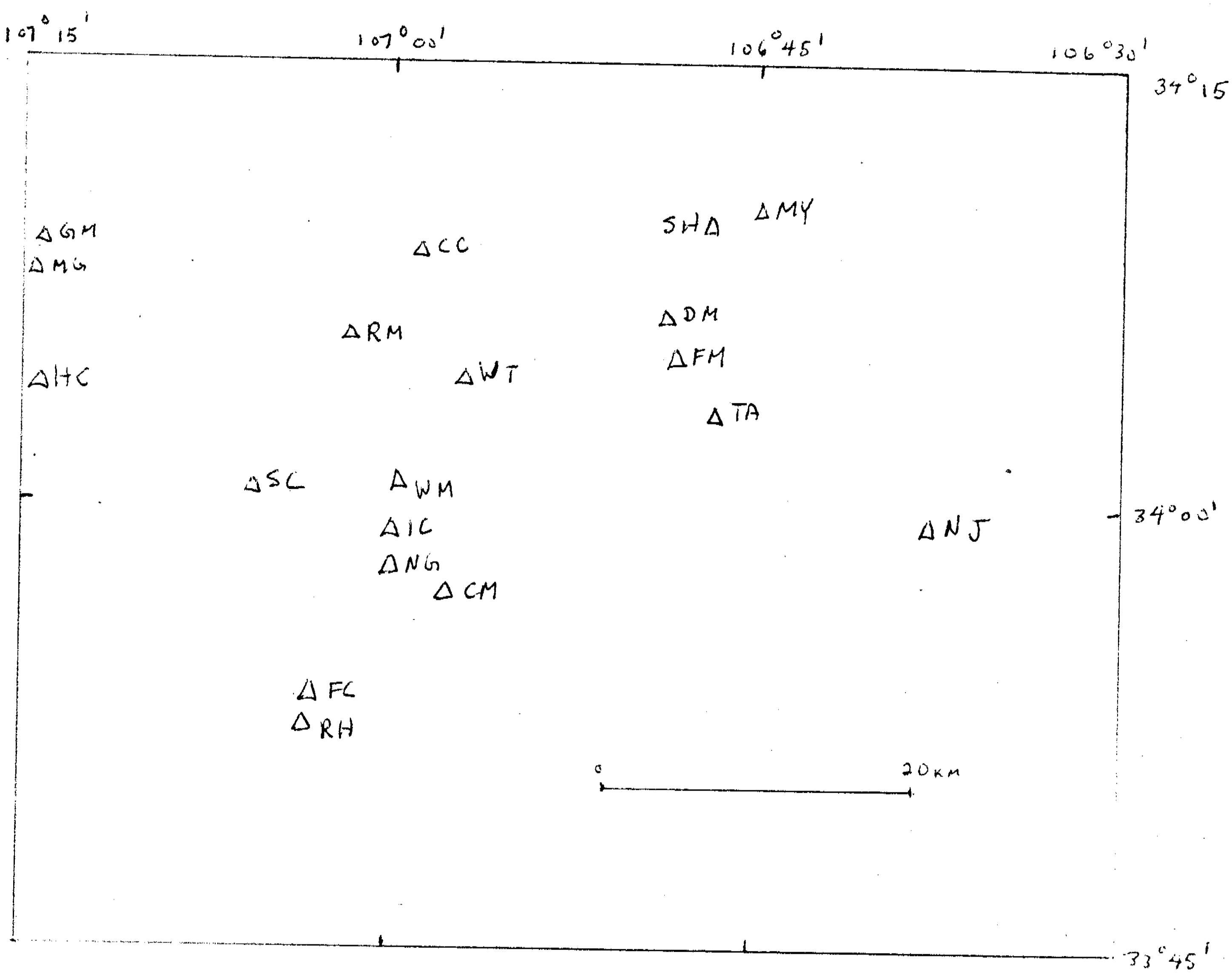
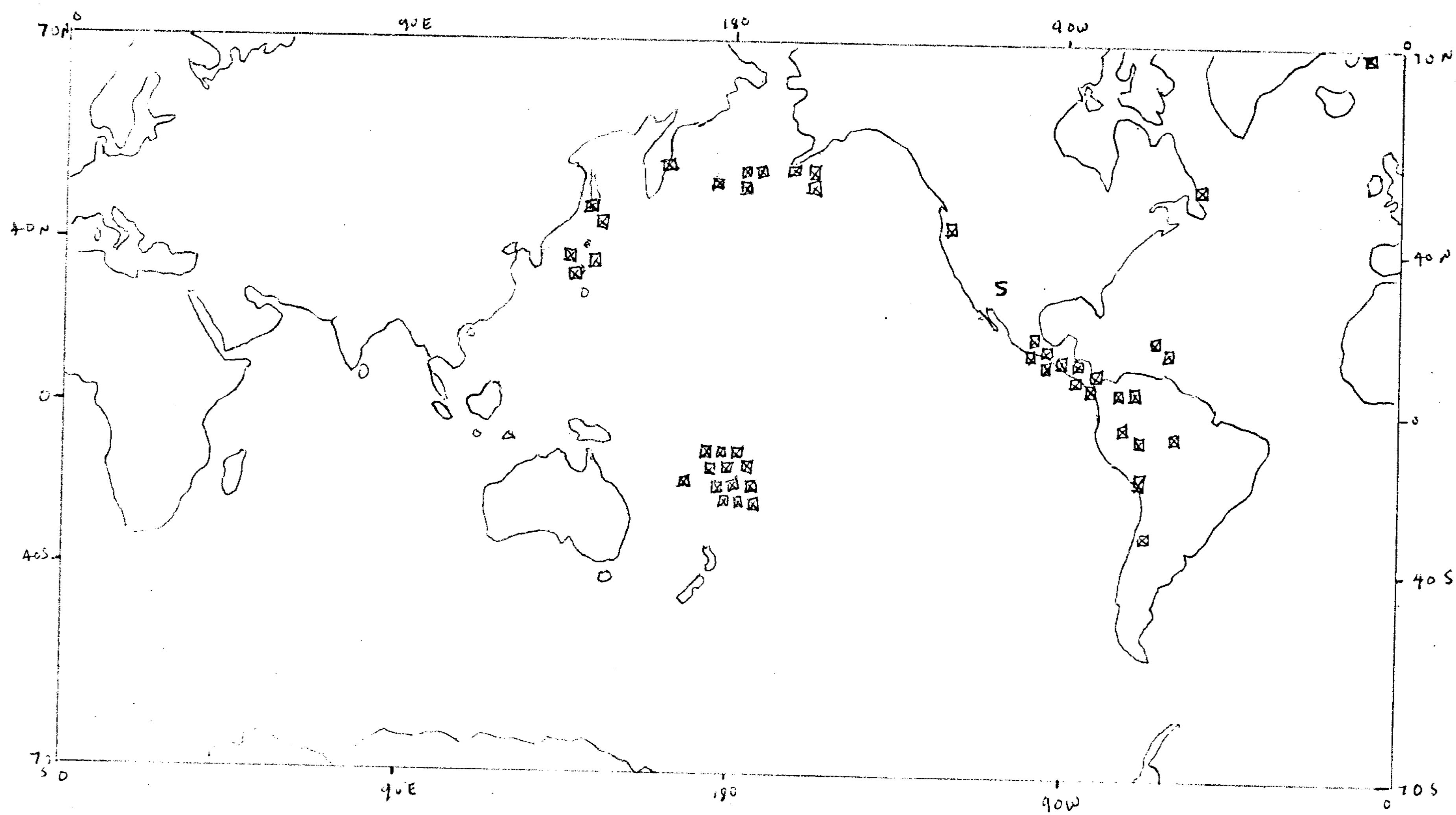


Table 1

Station	Latitude	Longitude	Elevation (m)
ALQ(ALB)	34.9425	106.4574	1849
CC	34.1442	106.9812	1649
CM	33.9501	106.9576	1640
DM	34.1075	106.8079	1536
FC	33.8950	107.0504	1850
FM	34.0829	106.8047	1537
GM	34.1454	107.2345	1945
HC	34.0658	107.2361	2240
IC	33.9870	106.9967	1730
MG	34.1305	107.2425	2024
MY	34.1667	106.7459	1645
NG	33.9648	106.9933	1730
NJ	33.9924	106.6253	1644
RH	33.9002	107.1135	2080
RM	84.0812	107.0069	1719
SC	34.0100	107.0894	2073
SH	34.1570	106.7802	1577
TA	34.0498	106.7751	1558
WM	34.0120	106.9929	1673
WT	34.0722	106.9459	1555

Figure 4: Epicenters of the 46 Teleseisms Used in This Study



might speculate the energy first arrived.

The events used ranged in magnitude (Mb scale) between 4.8 and 6.4 and had angular epicenter to station distances ranging from 14° to 95°. The accuracy of the measurement of the time of arrival was limited by 1) the combined distance and magnitude of the event as related to the amplification of the recording instruments, 2) the sharpness of the initial arrival, 3) the amount and character of the noise present on the record at the time of the teleseismic arrival, and 4) by the experience of the reader. It should be noted that all the readings basic to this study were made by the author, who was an absolute novice at reading seismic records at the beginning of this study. All of the readings were re-read at least once, and many received a third reading.

Table 2 lists the 46 events and the stations at which arrival times were measured. Event epicenter, focal depth, magnitude, and origin time were obtained from the Preliminary Determination of Epicenters, published by the U.S. Geological Survey. The azimuthal angles of arrival are measured clockwise from north at the station, and the angle of incidence, α , is measured from the vertical. The arrival times at the various stations are given with their associated uncertainty. A quotation of $\pm$ 0.0 means that the half-interval width is less than .05 second. These arrival times have been corrected for instrument clock drift and drum rotation error.

2.2 Calculation of Relative Travel Time Residuals

An observed travel time Δt_i^{obs} for a P phase is the difference between the arrival time at the ith station and the origin time, t_o, of the particular event:

$$\Delta t_i^{obs} = t_i^{obs} - t_o .$$

A travel time residual is defined as the difference between this travel time and an empirically determined theoretical travel time, Δt_i^t , for the particular epicentral distance Δ . This theoretical travel time can be written

Table 2

Date	Epicenter Lat.	Epicenter Long.	Focal Depth	(MB) Mag.	(GMT) Origin Time	Azimuth of Approach	Angle of Incidence	Sta	(GMT) Arrival Time	±
3/20/75	50.356N	175.999N	27	4.9	03:23.33.51	310°	24°	ALQ	03:32.38.0	-
								CC	03:32.40.71	.05
								FM	41.90	.05
								WT	41.34	.05
3/20-75	51.323N	179.556W	57	5.4	07:30.38.3	312°		ALQ	07:39.55.0	-
								CC	07:39.59.93	0.0
								FM	59.16	0.0
								WT	58.51	0.0
4/16/75	71.516N	10.427W	13	6.1	01:27.18.7	21	22	ALQ	01:37.20.1	-
								CC	01:37.26.68	0.0
								CM	28.23	0.0
								WT	27.14	0.5
4/16/75	10.456N	85.813N	69	5.4	04:46.21.38	136	28	ALQ	04:52.32.1	-
								CC	30.54	.15
								CM	29.43	.10
								WT	29.79	.10
5/20/75	25.091S	178.994W	362	5.1	14:31.50.4	240	15	ALQ	14:44.16.0	-
								CC	13.05	.05
								FM	13.46	.05
								SC	12.49	.05
5/28/75	37.726S	73.417W	24	5.8	13:57.34.5	153	17	ALQ	14:09.37.1	-
								CC	33.99	.05
								CM	32.99	.05
								FM	33.11	.05
								SC	33.74	.05
5/29/75	22.438S	179.527E	616	5.6	06:42.12.8	242	17	ALQ	06:54.12.1	-
								CC	8.76	.05
								CM	8.57	.05
								FM	9.41	.05
								SC	8.36	.05
6/24/65	20.611S	70.115W	33	5.6	00:39.47.4	142	21	ABQ	00.50.27.5	-
								CC	24.96	.05
								CM	24.09	.05
								FM	23.93	.05
								SC	24.89	.05
6/25/75	14.792N	104.766W	90	4.8	05:59.16.2	172	40*	ABQ	06:02.48.7	-
								CC	39.96	.05
								CM	36.50	.05
								FM	37.72	.05
								SC	37.59	0.0
6/25/75	16.172N	94.436W	66	4.8	06:05.13.0	145	35	ABQ	06:10.01.9	--
								CC	09.56.55	.05
								CM	54.78	.05
								FM	55.57	.05
								SC	56.04	.05

Date	Epicenter Lat.	Long.	Focal Depth	(MB) Mag	(GMT) Origin Time	Azimuth of Approach	Angle of Incidence	Sta	(GMT) Arrival Time	±
6/26/75	13.995N	90.770W	76	4.8	06:45.17.6	140	30	ABQ	06:50.38.7	-
								CC	35.47	.1
								CM	34.14	.05
								FM	33.84	.1
								SC	35.24	.05
7/24/75	23.476S	179.775W	579	5.6	19:01.42.6	241	15	ABQ	19:13.45.5	-
								CC	42.39	0.0
								FM	42.61	0.0
								SC	41.80	0.0
7/25/75	55.055N	160.377W	17	5.8	10:40.25.0	317	26	ABQ	10:48.16.3	-
								CC	19.12	.05
								CM	19.92	.1
								FM	19.93	.05
								SC	19.35	.05
7/29/75	43.687N	126.103W	33	5.2	01:48.16.2	309	38*	ABQ	01:52.19.5	-
								CC	22.32	.05
								CM	24.29	.05
								FM	24.24	.05
								SC	22.48	.05
								WT	23.02	.1
8/1/75	56.140N	162.918E	13	5.0	02:14.46.0	320	21	ABQ	02:25.09.4	-
								CC	12.07	0.0
								CM	13.23	.1
								FM	13.12	0.0
								SC	12.57	0.0
								WT	12.53	0.0
8/6/75	43.900N	139.263E	230	5.6	21:37.39.7	318	16	ABQ	21:49.32.9	-
								CC	34.77	.05
								CM	35.80	.1
								FM	35.76	.05
								SC	35.04	.05
								WT	34.80	.05
8/7/75	22.836S	178.912E	626	5.4	20:12.15.2	243	14	ABQ	20:24.15.8	-
								CC	12.80	.05
								CM	12.81	.1
								FM	13.32	0.0
								SC	12.13	.05
8/12/75	19.0435	175.542W	197	4.9	01:12.05.8	242	16	ABQ	01:24.19.8	-
								CC	16.54	.05
								CM	16.38	.1
								FM	17.14	.05
								MY	16.34	.1
								SC	16.14	.1
								WT	16.46	.1

Date	Epicenter Lat	Long.	Focal Depth	(MB) Mag	(GMT) Origin Time	Azimuth of Approach	Angle of INCIDENCE	Sta	(GMT) Arrival Time	±
8/12/75	32.042N	137.715E	391	5.7	14:21.14.7	310	14	ALQ	14:33.24.0	-
								CC	24.87	0.0
								FM	25.84	0.0
								MY	24.46	.05
								SC	25.36	.05
8/15/75	54.877N	167.845E	4	6.0	07:28.18.9	318	22	ABQ	07.38.28.2	-
								CC	30.47	.05
								CM	31.63	.1
								SC	30.92	0.0
								WT	30.97	.1
8/21/75	51.112N	177.830E	33	5.0	07:24.19.8	312	23	ALQ	07:33.51.2	-
								CC	53.24	0.0
								FM	54.41	.05
								SC	53.78	0.0
9/16/75	22.276N	140.125E	374	5.2	11:09.07.8	305	14	ALQ	11:21.35.5	-
								CM	36.92	.05
								HC	36.17	0.0
								MG	35.64	0.0
								NG	37.94	.05
								WT	36.44	.05
10/28/75	22.862S	70.508W	38	5.9	06:54.22.4	143	20	ABQ	07:05.12.3	-
								CC	10.02	0.0
								CM	9.04	.05
								NJ	8.34	.05
								MY	9.43	.05
								SC	9.90	0.0
								WT	9.55	0.0
10/29/75	17.229N	95.507W	35	5.2	04:54.007	157	38	ALQ	04:58.19.1	-
								CC	12.76	.05
								CM	10.68	.15
								NJ	9.75	.05
								MY	11.87	.05
								SC	11.79	.1
								WT	11.80	.15
11/4/75	54.355N	167.542E	24	5.5	12:05.56.9	318	22	ALQ	12:16.06.0	-
								CC	8.19	0.0
								CM	9.30	.05
								NJ	10.39	0.0
								SC	8.73	.05
								WT	8.65	.05
11/5/75	6.253N	76.918W	44	5.4	01:58.54.4	128	26	ALQ	02:06.23.1	-
								CM	20.46	.1
								NJ	18.68	.05
								MY	20.27	0.0
								SC	21.63	0.0
								WT	20.86	0.0

Date	Epicenter Lat.	Epicenter Long.	Focal Depth	(MB) Mag.	(GMT) Origin Time	Azimuth of Approach	Angle of Incidence	Sta	(GMT) Arrival Time	±
1/20/76	21.921S	174.104W	33	5.4	19:16.12.0	239	16	CM	19:28.44.60	.05
								DM	45.40	.05
								TA	45.55	.05
								WT	44.78	.05
2/3/76	25.136S	179.693E	477	5.8	12:27.30.1	240	14	ALQ	12:39.48.9	-
								CC	45.47	0.0
								DM	45.84	.05
								SC	44.86	0.0
								SH	46.20	0.0
								WT	45.45	0.0
2/3/76	18.108S	175.032W	212	5.7	18:03.52.0	243	16	ALQ	18:15.59.8	-
								CC	56.05	.05
								CM	55.68	.1
								DM	56.69	.1
								SC	55.32	.05
								SH	56.83	.05
								WT	55.90	.05
2/5/76	21.702S	68.222W	98	5.8	09:53.11.7	141	20	CC	10:03.53.84	.05
								DM	52.97	.05
								SC	53.58	0.0
								WT	53.10	0.0
2/6/75	18.174S	178.402W	590	5.2	00:02.32.5	245	15	ALQ	00:14.14.7	-
								CC	11.24	.05
								CM	11.04	.05
								DM	11.71	.05
								WT	11.16	.05
2/18/76	51.573N	178.676W	139	4.9	08:00.58.6	312	23	ALQ	08:10.13.3	
								CC	15.24	.05
								CM	16.48	.05
								IC	16.12	0.0
								SC	15.71	0.0
								WM	16.08	.05
								WT	15.75	.1
2/19/76	15.935N	95.076W	33	5.1	18:31.31.1	147	36	ALQ	18:36.20.0	-
								CC	16.13	0.0
								CM	14.13	0.0
								IC	14.63	0.0
								SC	15.41	0.0
								WM	14.97	.05
								WT	15.02	.05
3/24/76	29.887S	177.873W	33	6.4	04:46.04.4	235	15	CM	04:59.13.70	.1
								DM	14.30	.1
								FR	14.13	.1
								IC	13.62	.1
								TA	14.42	.1
								WT	14.01	.1

Date	Epicenter Lat.	Epicenter Long	Focal Depth	(MB) Mag.	Origin Time (GMT)	Azimuth	Angle Incidence	Sta	Arrival Time (GMT)	±
4/13/76	18 546N	104.803W	33	5.3	20:17.48.2	173	40*	ALQ	20:21.40.8	-
								CC	28.97	.05
								IC	27.14	.05
								SC	28.75	.15
								WT	27.91	.05
5/27/76	4.940N	82.561W	33	4.8	14:46.10.1	137	28	ALQ	14:53.20.4	-
								CM	16.37	.05
								DM	17.32	.15
								FR	14.84	.05
								NJ	15.35	.05
								TA	15.95	.05
6/18/76	24.814S	175.356W	33	5.6	01:45.37.3	238	15	ALQ	01:58.27.4	-
								GM	23.30	.1
								NG	23.24	0.0
								RM	23.76	.15
								SC	23.07	.1
								WM	23.59	.05
								WT	23.46	.05
7/1/76	16.597N	61.241W	36	5.1	03:38.12.1	101	26	ALQ	03:46.21.0	-
								GM	24.56	.1
								HC	24.73	0.0
								NG	22.52	.05
								RM	23.27	.05
								SC	23.68	0.0
								WT	22.51	.05
7/15/76	19.218N	64 134W	33	5.0	00:07.56.6	100	26	ALQ	00:15.35.3	-
								HC	38.10	.05
								NG	37.02	.1
								RM	37.38	.1
								SC	37.74	.05
								WT	36.94	.05
7/29/76	47.782N	48.120E	0	5.9	04:59.57.7	343	15	GM	05:13.26.90	.05
								HC	27.60	0.0
								NG	27.52	.05
								RH	28.02	.05
								SC	27.41	0.0
								WT	26.70	0.0
8/4/76	30.159N	138 470E	435	5.4	23:21.44.2	308	14	ALQ	23:34.02.5	-
								GM	2.67	.05
								HC	3.09	0.0
								NG	3.87	.05
								SC	3.43	0.0
								WT	3.37	.05
8/10/76	2.136N	79.02W	33	5.5	00:10.26.9	135	26	ALQ	00:18.14.0	-
								FC	11.23	0.0
								HC	12.94	0.0
								NG	11.07	0.0
								SC	11.65	.05
								WT	11.12	.1

Date	Epicenter Lat.	Epicenter Long.	Focal Depth	(MB) Mag.	Origin Time (GMT)	azimuth	angle incidence	Sta	Arrival Time (GMT)	±
8/12/76	46.535N	142.200E	301	5.0	23:29.08.9	319	17	ALQ	23:40.36.9	-
								FC	39.66	0.0
								GM	38.01	0.0
								HC	38.31	.05
								NG	39.25	.05
								WT	38.84	0.0
8/20/76	45.048N	149.781E	47	5.5	03:56.00.6	314	18	ALQ	04:07.375	-
								FC	39.97	.05
								GM	38.31	.05
								HC	39.35	.1
								NG	39.82	.05
								SC	39.23	.05
								WT	39.30	.1
8/20/76	20 412S	69.993W	81	5.6	06:54.11.3	141	20	ALQ	07:04:45.1	-
								FC	42.39	0.0
								GM	44.12	.05
								HC	44.25	.05
								NG	42.16	.05
								SC	42.92	.05
								WT	42.33	0.0
1/21/77	18.014S	178.379W	604	5.8	06:11.05.6	245	15	CC	06:23.42.55	0.0
								CM	42.29	.05
								DM	43.06	0.0
								GM	41.85	.1
								SC	42.11	0.0
								WT	42.60	0.0

$\Delta t_i^t = t_i^t - t_o$,

so that a theoretical travel time implies a theoretical arrival time. The travel time residual is thus,

$$(2.1) \quad \Delta t_i^a = \Delta t_i^{obs} - \Delta t_i^t = (t_i^{obs} - t_o) - (t_i^t - t_o) = t_i^{obs} - t_i^t$$

where the superscript on Δt_i^a denotes absolute travel time residual. It is seen that a travel time residual does not involve the event origin time.

The theoretical travel times used in this study were taken from the tables of Herrin (1968). The use of these tables demanded the knowledge of event focal depth and epicentral distance, Δ , in degrees. The epicentral distances were calculated by a program donated by my advisor. The ellipticity of the earth was corrected for in this calculation (Schlue, personal communication).

In the analysis of travel time residuals, any deviation from the theoretical travel time can be attributed to anomalous features either in the area of the event OR along the travel path or the region of the recording station. To study anomalous features in the crust or upper mantle beneath the recording station, Eaton et al. (1975), suggested the use of relative travel time residuals, which are defined as the difference between the travel time residual at the observing station, Δt_i^a, and the travel time residual at some given base station Δt_b^a , where,

$$(2.2) \quad \Delta t_b^a = t_b^{obs} - t_b^t .$$

The relative travel time residual for a particular event at a particular station with respect to base station b is therefore,

$$(2.3) \quad \Delta t_i = \Delta t_i^a - \Delta t_b^a .$$

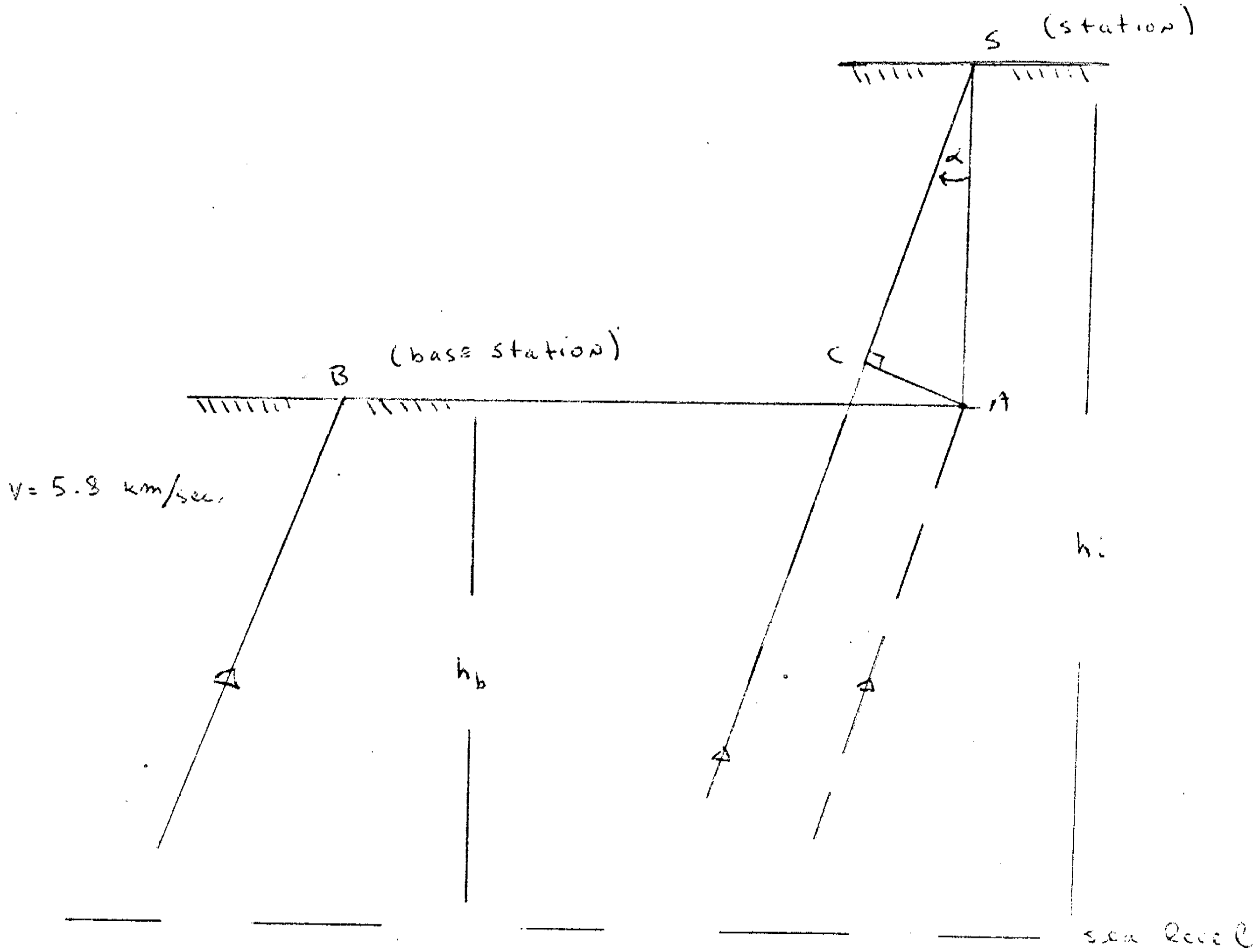
By substituting equations (2.1) and (2.2) into (2.3) and rearranging, this residual becomes

$$(2.4)\quad \Delta t_i = \left(t_i^{obs} - t_b^{obs} \right) - \left(t_i^{t} - t_b^{t} \right),$$

where the first term to the right in parentheses is just the difference in observed arrival times between the station and the base, and the second term is the associated theoretical difference. When the base station is in the immediate vicinity of the area being studied, it is seen that this definition of a relative travel time residual leads to ambiguities in interpretation. For example, a relative delay at station i with respect to station b can mean either anomalously low P velocity material traversed by the raypath to i, or anomalously high P velocity material traversed by the raypath to p, or both, or for that matter neither. When relative travel time residuals are used, all knowledge of absolute travel time with respect to the theoretical at a particular station is lost. To make the interpretation of the residuals in the area of study less ambiguous, Eaton et al.(1975) suggested the choice of a reference station outside the area of interest with crustal properties known or assumed different from those within the area of study. In this study the Albuquerque, New Mexico stations ALQ and ABQ were used as this outside reference station. Relative travel time residuals with respect to stations WT and CC, within the area of interest, were also calculated because 1) the accuracy of the Albuquerque arrival times are unknown and 2) all of the 46 events were recorded at either WT or CC.

All of the relative travel time residuals were also corrected for the difference in travel time caused by the difference in elevation between the station and the reference station. Figure 5 illustrates this correction.

Figure 5: Illustration of the Elevation Correction



In this figure, B is the base station and S is the station at which the relative travel time residual is being calculated. This residual is calculated by equation (2.4) as if the station S were located at point A (at an elevation equal to that of B). In the local area, where parallel straight ray paths can be used, the travel time to A is equivalent to the travel time to point C. Therefore the difference in travel path between B and S due to their difference in elecation is $\overline{CS}$. The correction for elevation is therefore

$$\Delta t^h = \frac{\overline{CS}}{V} = \frac{\overline{AS}\cos\alpha}{V} = |h_i - h_b| \cos\alpha / V,$$

where h_i and h_b are the elevations of the recording station and the base station respectively. In practice the absolute value is dropped; then Δt^h is always subtracted and the sign of the correction is correct. The crustal velocity, V, used was 5.8 km/sec, an appropriate choice for the Socorro area (Sanford, personal communication). The final relative travel time residuals were then computed,

$$(2.5) \quad \Delta t_i = \Delta t^{obs} - \Delta t^t - \Delta t^h$$

where $\Delta t^{obs} = (t_i^{obs} - t_b^{obs})$ and $\Delta t^t = (t_i^t - t_b^t)$

The angles of incidence were read from the tables of Pho and Behe (1972).

Table 3 presents the calculated relative travel time residuals with respect to Albuquerque, WT and CC, for the 46 events. $\Delta t_{(b)}^{obs}$ is with respect to ALQ or ABQ unless otherwise noted. With the relative travel time residuals with respect to WT and CC appear the capital letters A-F. These are accuracy ratings for the residuals. They were assigned primarily by comparing the size of the half intervals of arrival of the base with

Table 3

DATE (AT)	STA.	Δt^{obs} (h)	$-\Delta t^{t}$	$-\Delta t^{h}$	Δt(A&B)	Δt(WT)	Δt(CC)
3/20/75	CC	2.71	-1.37	.03	1.37	-.14C	0
	FM	3.90	-2.49	.05	1.46	-.05C	.09C
(310)	WT	3.34	-1.88	.05	1.51	0	-
3/20/75	CC	2.93	-1.53	.03	1.43	.01B	0
	FM	4.16	-2.62	.05	1.59	.17B	.16B
(312)	WT	3.41	-2.04	.05	1.42	0	-
4/16/75	CC	6.68	-6.15	.03	.56	.01A	0
	CM	8.13	-6.60	.03	1.56*	.01A	0.0A
(21)	WT	7.04	-6.54	.05	.55	0	-
4/16/75	CC	-1.56	2.48	.03	.95	.10E	0
	CM	-2.67	3.83	.03	1.19	.34E	.24F
(136)	WT	-2.31	3.12	.04	.85	0	-
5/20/75	CC	-2.95	3.60	.03	..68		0
	FM	-2.54	3.16	.05	.67		-.01C
(240)	SC	-3.51	4.28	-.04	.73		.05C
5/28/75	CC	-3.11	2.89	.03	-.19		0
	CM	-4.12	3.90	.03	-.19		0.0D
	FM	-3.99	3.56	.05	-.38		-.19D
(153)	SC	-3.36	3.33	-.04	-.07		.12D
5/29/75	CC	-3.34	3.47	.03	.16		0
	CM	-3.43	3.81	.03	.41		.25C
(243)	FM	-2.69	3.00	.05	.36		.20C
	SC	-3.34	4.13	-.04	.35		.19C
6/24/75	CC	-2.54	2.36	03	-.15		0
	CM	-3.41	3.43	.03	.05		-.09C
	FM	-3.57	3.26	.05	-.26		.20C
(142)	SC	-2.61	2.68	-.04	.03		.18C
6/25/75	CC	-9.74	9.51	.03	-.20		0
	CM	-12.20	12.02	.03	-.15		.05C
	FM	-10.98	10.56	.04	-.38		-.18C
(172)	SC	-11.11	11.05	-.03	-.09		.11B
6/25/75	CC	-5.35	4.35	.03	-.97		0
	CM	-7.12	6.11	.03	-.98		-.01C
	FM	-6.33	5.75	.05	-.53		.44D
(144)	SC	-5.86	4.94	-.03	-.95		.02D

DATE (AZ)	STA.	obs $\Delta t(o)$	$-\Delta t^{t}$	$-\Delta t^{h}$	$\Delta t(AIB)$	$\Delta t(WT)$	$\Delta t(CC)$
6/26/75	CC	-3.33	3.21	.03	-.09		0
	CM	-4.56	4.70	.03	.17		.26D
	FM	-4.86	4.52	.05	-.29		-.20E
(140)	SC	-3.46	3.62	-.03	.13		.22E
7/24/76	CC	-3.11	3.53	.03	.45		0
	FM	-2.89	3.07	.05	.23		-.22A
(241)	SC	-3.70	4.19	-.04	..45		0.0A
7/25/75	CC	2.83	-2.24	.03	.61		0
	CM	3.62	-3.50	.03	.15		-.46D
	FM	3 63	-3.42	.05	.26		-.23C
(317)	SC	3.05	-2.53	-.03	.49		-.12C
7/29/75	CC	2 82	-1.65	.03	1.20	.11D	0
	CM	4.79	-3.31	.03	1.51	.42D	.31C
	FM	4.74	-3.51	.04	1.27	.18D	.07C
	SC	2.98	-1.81	-.03	1.04	-.05D	-.16C
(309)	WT	3.52	-2.47	.04	1.09	0	-
8/1/75	CC	2.67	-2.30	.03	.40	.01B	0
	CM	3.83	-3.38	.03	.48	.09C	.08C
	FM	3.72	-3.23	05	.54	.15B	.14B
	SC	3.17	-2.61	-.04	.52	.13B	.12B
(321)	WT	3.13	-2.79	05	.39	0	-
8/6/75	CC	1.87	-1.58	.03	.32	.33C	0
	CM	2.90	-2.40	.03	.53	.54D	.21D
	FM	2.86	-2.33	.05	.58	.59C	.26C
	SC	2.14	-1.79	-.04	.31	.32C	-.01C
(318)	WT	1.90	-1.96	.05		0	-
8/7/75	CC	-3.00	3.47	.03	.52		0
	CM	-2.99	3.80	.03	.84		.32D
	FM	-2 48	3.00	.05	.57		.05D
(243)	SC	-3 67	4.12	-.04	.41		-.11C
8/12/75	CC	-3 26	3.78	.03	.55	.02D	0
	CM	-3 42	4.14	.03	.75	.22E	.20D
	FM	-2.66	3.27	.05	.66	.13D	.11C
	MY	-3.46	2.85	.03	-.58*	-.11E	-.13D
	SC	-3 66	4.49	-.04	.79	.26E	.24D
(242)	WT	-3.34	3.82	.05	.53	0	-
8/12/75	CC	.87	- .84	.03	.06		0
	FM	1.84	-1.54	.05	.35		.29A
(310)	MY	.46	- .92	.03	-.98*		-.04B
	SC	1 36	-1.47	-.04	.40		.34A

DATE (AZ)	STA.	Δt(obs)	$-\Delta t^t$	$-\Delta t^h$	Δt(ALB)	Δt(WT)	Δt(CC)
8/15/75	CC	2.27	-2.11	.03	.19	-.02D	0
	CM	3.43	-3.19	.03	.27	.06E	.08D
(318)	SC	2.72	-2.39	-.04	.26	.05C	.07B
	WT	2.77	-2.61	.05	.21	0	-
8/21/75	CC	2.04	-1.53	.03	.54		0
	FM	3.21	-2.60	.05	.66		.12A
(312)	SC	2.58	-1.70	-.04	.84		.21A
9/16/75	CM	1.42	-1.06	.03	.39	.18C	
	HC	.67	.12	-.07	.72	.51A	
(305)	MG	.14	.31	-.03	.42	.21B	
	NG	2.44	- .91	.02	.54	.33B	
	WT	.94	- .78	.05	.21	0	
10/28/75	CC	-2.28	2.47	.03	.22	-.05A	0
(143)	CM	-3.26	3.55	.03	.32	.05B	.10B
	MY	-2.87	3.11	.03	.27	0.0A	.05A
	NJ	-3.96	4.39	.03	.46	.19C	.24C
	SC	-2.40	2.82	-.04	.38	.11A	.16A
	WT	-2.75	2.96	.05	.27	0	-
10/29/75	CC	-6.34	6.92	.03	.61	.01E	0
(157)	CM	-8.42	9.16	.03	.77	.17F	.16D
	MY	-7.23	7.60	.03	.40	-.20E	-.19D
	NJ	-9.35	10.00	.03	.68	.08E	.07D
	SC	-7.31	7.96	-.03	.62	.02F	.01D
	WT	-7.31	7.86	.04	.60	0	-
11/4/75	CC	2.19	-2.05	.03	.17	.01A	0
	CM	3.30	-3.11	.03	.22	.06B	.05A
(318)	NJ	4.39	-4.16	.03	.26	.10A	.09A
	SC	2.73	-2.31	-.04	.38	.22B	.21A
	WT	2.65	-2.54	.05	.16	0	-
11/5/75	CM	-2.64	2.48	.03	-.13	.14C	
	MY	-2.83	2.52	.03	-.29	-.02A	
(128)	NJ	-4.42	4.07	.03	-.32	-.05A	
	SC	-1.47	1.45	-.03	-.05	.22A	
	WT	-2.24	1.92	.05	-.27	0	
1/20/76	CM	- .18(WT)	.35	-.01		.16D	
(239)	DM	.62	- .58	0		.04D	
	TA	.77	- .55	0		.22D	
	WT	0	0	0		0	

DATE (A°z)	STA.	obs $\Delta t(b)$	$-\Delta t^t$	$-\Delta t^h$	$\Delta t(ALD)$	$\Delta t(WT)$	$\Delta t(CC)$
2/3/76	CC	-3.43	3.55	.03	.15	-.05A	0
	DM	-3.06	3.06	.05	.05	-.15B	-.10B
(240)	SC	-4.04	4.21	-.04	.13	-.07A	-.02A
	SH	-2.70	2.85	.05	.20	0.0A	.05A
	WT	-3.45	3.60	.05	.20	0	-
2/3/76	CC	-3.74	3.81	.03	.10	.11C	0
	CM	-4.12	4.17	.03	.08	.09D	-.02D
(242)	DM	-3.11	3.24	.05	.18	.19D	.08D
	SC	-4.48	4.53	-.04	.01	.02C	-.09C
	SH	-2.97	3.02	.05	.10	.11C	0.0C
	WT	-3.90	3.84	.05	-.01	0	-
2/5/76	CC	.74(WT)	-.47	-.02		.25C	0
	DM	- .13	.29	0		.16B	-.09B
(141)	SC	.48	-.18	-.08		.22B	-.03B
	WT	0	0	0		0	-
2/6/76	CC	-3.46	3.49	.03	.06	.04C	0
(245)	CM	-3.66	3.80	.03	.17	.15C	.11C
	DM	-2.99	2.94	.05	0.0	-.02C	-.06C
	WT	-3.54	3.51	.05	.02	0	-
2/18/76	CC	1.94	-1.57	.03	.40	-.02D	0
(312)	CM	3.18	-2.63	.03	.58	.16D	.18C
	IC	2.82	-2.27	.02	.57	.15C	.17B
	SC	2.41	-1.74	-.04	.63	.21C	.23B
	WM	2.78	-2.16	.03	.65	.23D	.25C
	WT	2.45	-2.08	.05	.42	0	-
2/19/76	CC	-3.87	4.69	.03	.88	.30A	0
	CM	-5.87	6.54	.03	.70	.15A	-.15A
(146)	IC	-5.37	6.01	.02	.66	.11A	-.19A
	SC	-4.59	5.34	-.03	.72	.17A	-.13A
	WT	-4.98	5.49	.04	.55	0	-
	WM	-5.03	5.80	.02	.79	.24B	-.06A
3/24/76	CM	- .31(WT)	.36	-.01		.04E	
	DM	.29	- .53	0		-.24E	
(235)	FR	.12	- .17	0		-.05E	
	IC	- .39	.38	-.03		-.04E	
	TA	.41	- .48	0		-.07E	
	WT	0	0	0		0	
4/13/76	CC	-11.83	9.59	.03	-2.21	.09C	0
(173)	IC	-13.66	11.57	.02	-2.07	.23C	.14C
	SC	-12.05	11.14	-.03	- .94*	.36E	.25E
	WT	-12.89	10.55	.04	-2.30	0	-
5/27/76	CM	- 4.03	3.80	.03	- .20		
	DM	- 3.08	3.56	.05	.53		
(136)	FR	- 5.56	5.39	.04	- .13		
	NJ	- 5.05	5.15	.03	.08		
	TA	- 4.45	4.07	.04	- .31		

DATE (AZ)	STA.	obs Δt (h)	$-\Delta t^t$	$-\Delta t^h$	Δt(ALE)	Δt(WT)	Δt(CC)
6/18/76	GM	-4 10	4.66	-.02	.54	.56D	
	NG	-4 16	4.31	.02	.17	.19C	
(238)	RM	-3 64	4.05	.01	.42	.44E	
	SC	-4.33	4 52	-.04	.15	.17C	
	WM	-3 81	4.19	.03	.41	.43D	
	WT	-3 94	3 87	.05	-.02	0	
7/1/76	GM	3 56	-3.70	-.02	-.16	.01D	
	HC	3.73	-3 59	-.06	.08	.23B	
(101)	NG	1 52	-1 85	.02	-.13	-.16C	
	RM	2 27	-2.12	.02	.17	.32C	
	SC	2 68	-2.54	-.03	.11	.26B	
	WT	1 51	-1.71	05	-.15	0	
7/15/76	HC	2.80	-3.87	-.06	-1.13*	.10C	
	NG	1 72	-2 09	.02	-.35	-.12D	
	RM	2.08	-2.34	.05	-.21	.02D	
(100)	SC	2 44	-2.80	-.03	-.39	-.16C	
	WT	1.64	-1.92	.05	-.23	0	
7/29/76	GM	.20(WT)	.01	-.07		.12B	
	HC	.90	- .34	-.11		.45A	
(343)	NG	.82	- .52	-.03		.27B	
	RH	1.32	- .93	-.09		.30B	
	SC	.71	- .43	-.08		.20B	
	WT	0	0	0		0	
8/4/76	GM	.17	- .06	-.02	.09	.18B	
	HC	.59	- .15	-.06	.38	.47A	
(308)	NG	1 37	-1.18	.02	.21	.30B	
	SC	.93	- .76	-.04	.13	.21A	
	WT	.87	-1.01	.05	-.09	0	
8/10/76	FC	-2.77	3 24	0	.47	.59C	
	HC	-1 06	1.36	-.06	.24	.35C	
(125)	NG	-2.93	3 12	.02	.21	.32C	
	SC	-2 35	2.39	-.04	0.0	.11D	
	WT	-2.88	2 73	.04	-.11	0	
8/12/76	FC	2 76	2.53	0	.23	.36A	
	GM	1.11	- .96	-.02	.13	.26A	
(319)	HC	1 41	-1.28	-.06	.07	.20B	
	NG	2.35	-2.41	.02	-.04	.09B	
	WT	1 94	-2.12	.05	-.13	0	
8/20/76	FC	2.47	-2.21	0	.26	.26D	
(314)	GM	.81	- .57	-.02	.22	.22D	
	HC	1 85	- .88	-.06	.91	.91E	
	NG	2.32	-2.12	.02	.18	.18D	
	SC	1 73	-1.61	-.04	.08	.08D	
	WT	1.80	-1.85	.05	0.0	0	

DATE (?7)	STA.	$\Delta t^{obs}_{(b)}$	$-\Delta t^{t}$	$-\Delta t^{h}$	Δt(ALB)	Δt(WT)	Δt(CC)
8/20/76	FC	-2.71	3.36	0	.66	.56C	
	GM	- 98	1 47	-.02	.47	.37C	
(141)	HC	- 85	1 87	-.06	.96	.87C	
	NG	-2 94	3.20	.02	.28	.18C	
	SC	-2 18	2.65	-.04	.33	.23C	
	WT	-2 77	2.82	.05	.10	0	
1/21/77	CC	- .05(WT)	-.02	-.02		-.09A	
	CM	- 31	29	-.02		-.04B	
(245)	DM	.46	-.57	-.02		-.13B	
	GM	- .75	.89	-.07		.07C	
	SC	- 49	.64	-.09		.06A	
	WT	0	0	0	0	0	-

*These residuals have been adjusted by 1.0 second as seemed appropriate.

the station, as discussed in the preceding section and tabulated in Table 2. The sum of these half intervals is a measure of the possible reading error in the difference $t_i^{obs} - t_b^{obs}$. Table 4 illustrates the rating criteria. $\frac{1}{2}\ I_i$ and $\frac{1}{2}\ I_b$ are half intervals of arrival for the station and the base respectively as quoted in Table 2. Some of these ratings were further adjusted, somewhat subjectively, by consideration of 1) clearness (lack of noise) of the record, 2) consistency of the wave form between stations for a particular event, and 3) amplitude of the arrival. For example, if one record from a set of relatively small amplitude records of a particular event showed an initial break with a signature different from the rest, then doubt was placed on the entire set, and the entire set would be given a lower rating. This, I believe, is the most significant error which has found its way into some of the quoted time residuals. It seems quite possible, because of the small amplitude of an event, to miss the initial break of that event even on the clearest of records. On the other hand, I did at times raise B and C ratings to A and B for particularly strong events for which I felt the above error did not affect the measurements.

Table 4

$\frac{1}{2} I_i + \frac{1}{2} I_b$	RATING
0.0	A
0.5	B
1.0	C
1.5	D
2.0	E
>2.0	F

Chapter 3: Analysis of the Relative Travel Time Residuals

3.1: Estimated Variance of the Measured Relative Travel Time Residuals

The A-F ratings of the relative travel time residuals are only a qualitative guide to the magnitude of the errors present in these residuals. To find a more quantitative measure of this error, and to find an estimate of the difference in error between measurements of different ratings, the following analysis was done.

For each rating A-D, at each station an estimate of the variance was computed by the equation

$$s_i^2 = \frac{1}{N_i} \sum_{j=1}^{N_i} (\Delta t_{ij} - \overline{\Delta t_i})^2$$

for arrivals from each of five directions (NE(20°-21°), E(100°-101°), SE(128°-173°), SW(235°-245°), and NW(305°-343°)), and for residuals with respect to both WT and CC. N_i is the number of residuals in the ith. group, the Δt_{ij}'s are the residuals within that group, and $\overline{\Delta t_i}$ is the average relative travel time residual for that group. Each of these estimates was made using only residuals with respect to one base station, with arrivals from a single general direction, within a single recording station, and for a given rating. In this fashion all the variability caused by these factors was eliminated and it was assumed that the variances calculated were only those of the reading error. These errors do not depend upon station, azimuth, or base station, therefore the variance estimates for a given rating can be assumed to be estimates of the variance of a single population. It is therefore justifiable to pool these variances (see for example Hays and Winkle(1971)) using the equation

$$s_p^2 = \sum_{i=1}^{p} N_i s_i^2 \Big/ \left[\left(\sum_{i=1}^{p} N_i \right) - p \right],$$

where S_i^2 is the estimate within each of the P groups being pooled. The pooling for each rating took place over all stations, all directions, and with respect to both WT and CC.

The pooled estimates were as follows: for A residuals, $S_A^2 = .0126$; for B residuals, $S_B^2 = .0096$; for C residuals, $S_C^2 = .0409$; and for D residuals, $S_D^2 = .0396$. Since the estimates S_A^2 and S_B^2 were very nearly equal, it seemed reasonable to consider A and B residuals to have the same variability and to find the variance of that combined group. The result was that $S_{AB}^2 = .0116$. The same reasoning applied to C and D estimates resulted in $S_{CD}^2 = .0308$. These estimates were used in the weighted analyses which will be described below.

There were not enough E and F rated residuals available to estimate their variances, however the a priori knowledge of their inaccuracy seemed reason enough for not using them in the following analyses.

3.2: Weighted Directional Analysis with Respect to WT

At each station, the relative travel time residuals with respect to station WT were grouped according to direction described above. Some stations had several residuals in each group, and some had but one representing a single direction. At any station, the model was

$$\Delta t_{ij} = \beta_i + \varepsilon_{ij}$$

where Δt_{ij} is the jth. residual in the ith. direction group; i=1,P(the number of direction groups represented at that particular station) and j=1,N_i(the number of residuals in the ith. direction group). A β_i represents the population mean relative travel time residual for the ith. direction at that station, and an ε_{ij} is a random error associated with the residual Δt_{ij} away from β_i. ε_{ij} is assumed to be a normal random variable with mean zero and variance σ_{ij}^2 .

The analysis was cast into the form of a linear regression for the b_i's, which are the estimates of the mean relative travel time residuals with respect to WT. The details of this calculation are shown in Appendix I; only the essence of the analysis is given here. A weighted mean relative travel time residual for each direction group was calculated from

$$b_i = \sum_{j=1}^{N_i} \frac{\Delta t_{ij}}{S_{ij}^2} \Big/ \sum_{j=1}^{N_i} \frac{1}{S_{ij}^2},$$

where S_{ij} is the estimated variance of Δt_{ij}, and b_i is the weighted least squares estimate of β_i. The weighting factors S_{ij}^2 only take on two values, S_{AB}^2 and S_{CD}^2, which were calculated in the preceding section.

Once the estimated mean travel time residuals, b_i, were calculated, the variances of these estimates were estimated by

$$S_i^2 = S_p^2 \Big/ \sum_{j=1}^{N_i} \frac{1}{S_{ij}^2},$$

where S_p is a doubly-pooled estimate of $\sigma_{ij}^2 / S_{ij}^2 \equiv \sigma_p^2$, over both stations and directions for the entire array of stations. Note that if the previous estimates S_{AB}^2 and S_{CD}^2 were accurate, then $S_p^2 = 1$. At each station a pooled estimate over directions (a consequence of the regression calculation) is

$$S_k^2 = \sum_{i=1}^{P} \sum_{j=1}^{N_i} (\Delta t_{ij} - b_i)^2 / S_{ij}^2 \, (M - P - 1),$$

where $M = \sum_{i=1}^{P} N_i$ (the total number of residuals at station k), and $M-P-1 = df_k$ is the degrees of freedom of this estimate. S_p^2 is then a pooled estimate over all stations, calculated

$$S_p^2 = \sum_{k=1}^{\ell} df_k \cdot S_k^2 \Big/ \sum_{k=1}^{\ell} df_k .$$

This estimate has $\sum_{k=1}^{\ell} df_k$ degrees of freedom, where l is the total number of stations being pooled.

The advantage of this pooling of variances is that all of the data are used to calculate the variability in the estimate of the mean relative travel time residuals. This is valid since the factors which make residuals at one station and direction imprecise are the same factors operating at any other station and direction. For any station and direction, then, S_p^2 is a constant and S_i^2 will have $\sum_{k=1}^{\ell} df_k$ degrees of freedom, no matter the number of residuals being used in the particular estimate of β_i. The variances are not equal, however, since the factor dividing S_p^2, $\sum_{j=1}^{N_i} \frac{1}{s_{ij}^2}$, still depends heavily on the number of observations, N_i, used in calculating b_i.

The final step in this analysis is the calculation of 95% confidence intervals for the mean relative travel time residuals with respect to WT. Once the estimates of β_i and σ_{ij}^2 are made, this is a standard procedure involving the formation of a T random variable from the estimates b_i and S_i^2. This confidence interval is calculated from

$$b_i - t_{.025}\left(\sum_{k=1}^{\ell} df_k\right) S_i \leq \beta_i \leq b_i + t_{.025}\left(\sum_{k=1}^{\ell} df_k\right) S_i ,$$

where $t_{.025}\left(\sum_{k=1}^{\ell} df_k\right)$ is the tabulated value for the T distribution with degrees of freedom above which 2.5% of the probability mass resides.

A summary of this analysis is presented in Table 5. The total number of stations at which the analysis was performed was 18, of which 14 allowed estimates of S_k^2 by having more than one residual in a given direction group. The total degrees of freedom of S_p^2 was $\sum_{k=1}^{14} df_k = 63$ and $t_{.025}(63) \simeq 2.00$. S_p^2=1.0199, which is close enough to one to indicate that the original estimates S_{AB}^2 and S_{CD}^2 of σ_{AB}^2 and σ_{CD}^2 were probably fairly good, i.e., the data were weighted reasonably.

The results for stations CC, CM, SC, NG, GM, and HC are also given in Figure 6 with graphs of the various means and 95% confidence intervals. These results will be discussed in a later chapter.

Table 5: Results of Weighted Directional Analysis

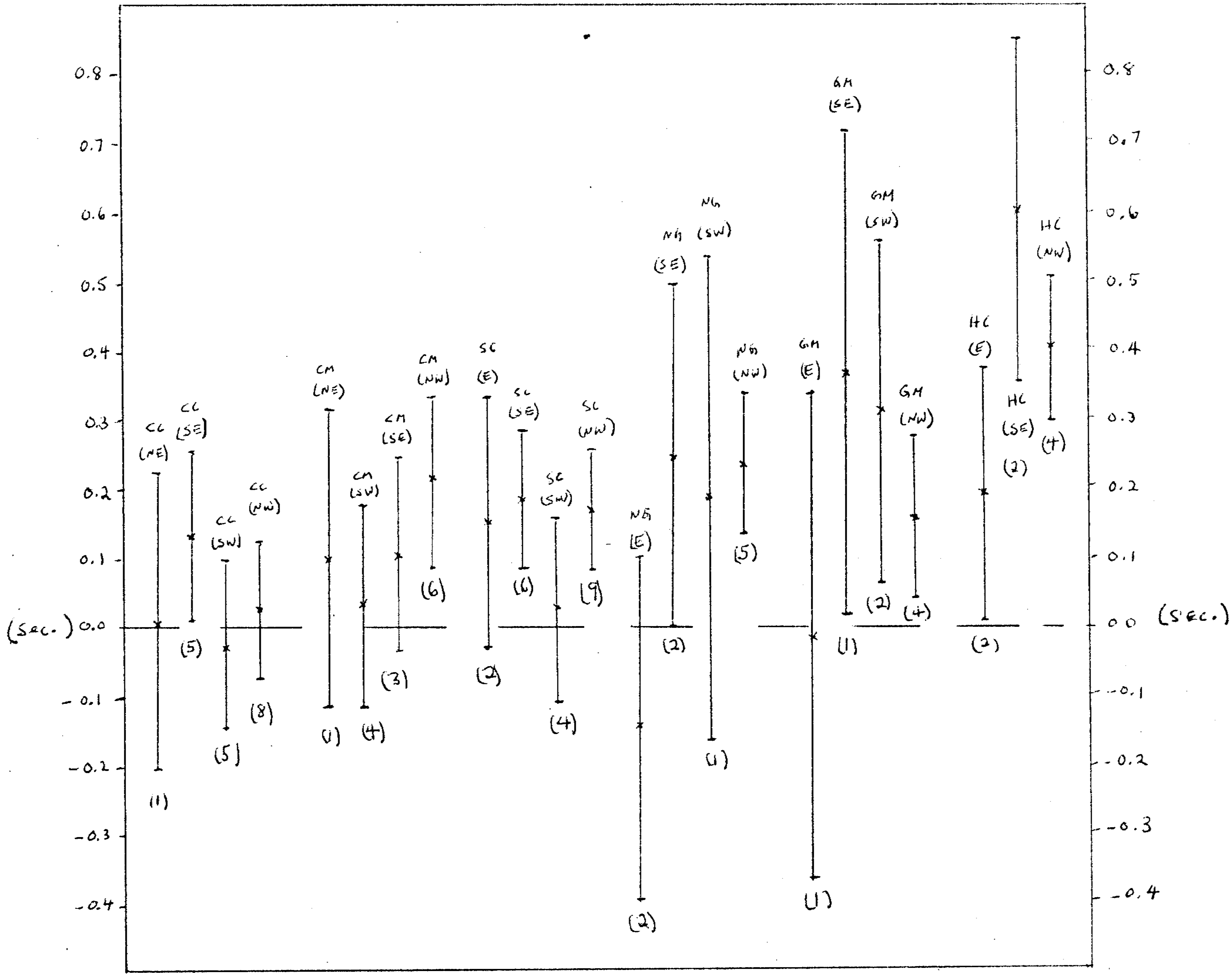
With Respect to WT

Station	Dir	N_i	S_i	$b_i \pm \frac{1}{2}(95\%\ C.I.)_i$
CC	NE	1	.108	.010 ± .215
	SE	5	.061	.134 ± .122
	SW	5	.061	-.024 ± .122
	NW	8	.049	.026 ± .097
CM	NE	1	.108	.100 ± .215
	SE	3	.070	.106 ± .139
	SW	4	.074	.032 ± .147
	NW	6	.064	.209 ± .127
DM	SE	1	.108	.160 ± .215
	SW	5	.061	-.064 ± .122
FC	SE	2	.124	.575 ± .248
	NW	2	.092	.333 ± .183
FM	SW	1	.176	.130 ± 351
	NW	5	.061	.189 ± .122
GM	E	1	.176	-.010 ± .351
	SE	1	.176	.370 ± .351
	SW	2	.124	.315 ± .248
	NW	4	.059	.161 ± .117
HC	E	2	.092	.195 ± .183
	SE	2	.124	.605 ± .248
	NW	4	.054	.408 ± .107
IC	SE	2	.092	.143 ± .183
	NW	1	.176	.150 ± .351
NJ	SE	2	.092	.015 ± .183
	NW	1	.108	.100 ± .215
NG	E	2	.124	-.140 ± .248
	SE	2	.124	.250 ± .248
	SW	1	.176	.190 ± .351
	NW	5	.052	.239 ± .103

Table 5, Continued

Station	Dir	N_i	S_i	$b_i \pm \frac{1}{2}(95\%\ C.I.)_i$
MG	NE	1	.108	.210 ± .215
MY	SE	2	076	-.010 ± .152
RH	NW	1	108	.300 ± .215
RM	E	2	.124	.170 ± .248
SC	E	2	.092	.145 ± .183
	SE	6	.050	.178 ± .099
	SW	4	.065	.022 ± .130
	NW	9	.045	.162 ± .089
SH	SW	2	092	.030 ± .183
TA	SW	1	.176	.220 ± .351
WM	SE	1	.120	240 ± .215
	SW	1	.215	.430 ± .351
	NW	1	115	.230 ± .351

Figure 6: Results of Directional Analysis With Respect to WT



3.3: Weighted Directional Analysis With Respect to CC

This analysis is identical in form to that of the previous section. A weighted least squares regression was done for the estimated mean of the relative travel time residuals within direction groups at stations SC, CM, and FM. The residuals were, in this case, with respect to station CC. Only these stations were analysed, because these stations had by far the largest number of samples in the various directions, and it was felt that the analysis of the other stations would produce no new information. The residuals of station WT with respect to CC were not analysed since the information gained would be almost (the variance estimates would be slightly different) identical to the analysis of CC with respect to WT. The pooled estimate, S_p^2, did, however, use all the information from all stations including WT. This estimate was found to be S_p^2=1.1396 and had 67 degrees of freedom.

The results of this analysis are tabulated in Table 6 and the mean residuals and their associated 95% confidence intervals are graphed in Figure 7.

3.4: Weighted Analysis of Lumped Station Residuals With Respect to WT

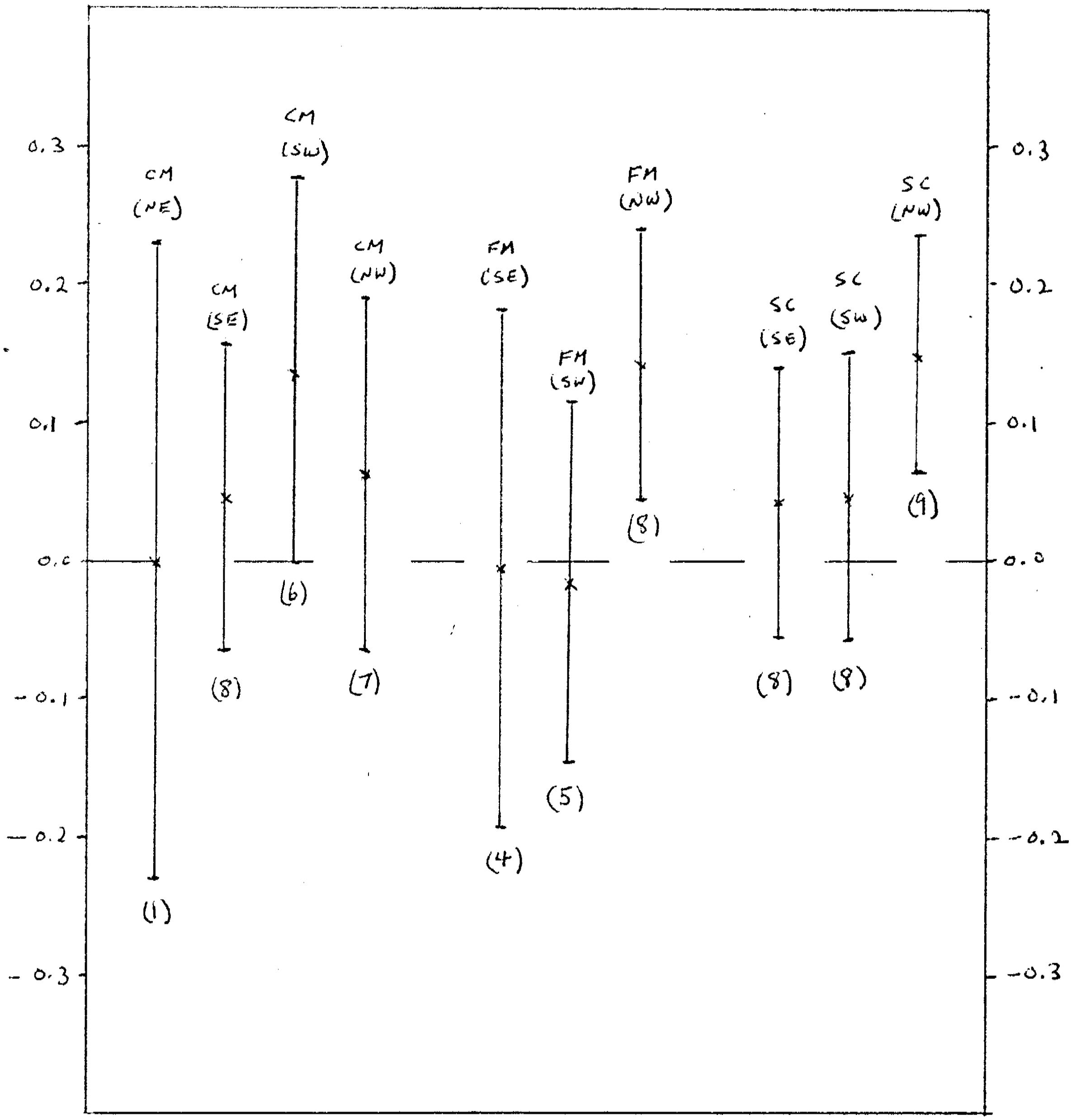
A weighted regression for the estimated mean relative travel time residual with respect to WT at each station was also done. This weighted average was taken over all residuals at each station without regard to direction. The analysis of the directional means at each station showed that for any station except NG the hypothesisthat all the directional means are equal cannot be rejected at the 95% confidence level. So in this respect the analysis of this section is reasonable. I do, however, believe that there is

Table 6

Station	Dir.	N_i	S_i	$b_i \pm \frac{1}{2}(95\%\ C.I.)_i$
CM	NE	1	.115	0.0 ± .229
	SE	8	.056	.046 ± .111
	SW	6	.068	.133 ± .135
	NW	7	.064	.062 ± .127
FM	SE	4	.094	-.005 ± .187
	SW	5	.065	-.016 ± .130
	NW	8	.049	.142 ± .098
SC	SE	8	.049	.042 ± .098
	SW	8	.052	.048 ± .104
	NW	9	.043	.150 ± .086

Figure 7: Results of Directional Analysis

With Respect to CC



a directional difference in mean relative travel times at some of the stations. This anisotropy is strongly suggested in Figure 6 for stations CC, CM, SC, GM, and HC as well as NG. One reason that statistical results do not support this conclusion is due to the width of the intervals - essentially there are not enough measurements in this experiment to detect, at many of the stations, the kinds of differences that are practically important. If, on the other hand, the directional differences are small, then there is still information to be gained by the analysis.

The form of this analysis is identical to the preceding two. In this case, however, the groups are the stations, there are P=18 means to be calculated. The number of residuals to be averaged is N_i, which is the total number of residuals at the ith. station. In this analysis, the second pooling of variance over stations is done automatically by the regression. The pooled estimate of σ_p^2 thus becomes simply

$$S_p^2 = \sum_{i=1}^{18} \sum_{j=1}^{N_i} \frac{(\Delta t_{ij} - b_i)^2}{S_{ij}^2} \Big/ M - P - 1 ,$$

and is found to have a value of 1.033. The total number of residuals over the entire array is M=115. There are P=18 groups, and therefore the estimate of the variance at every station has M-P-1=96 degrees of freedom.

Table 7 summarizes the results of this analysis by listing at each station the number of residuals used in estimating the weighted mean of the relative travel time residual, the standard deviation associated with the estimate, and the estimated mean b_i and associated 95% confidence interval for β_i. Figure 8 displays the graphs of the estimated mean values and the 95% confidence intervals.

3.5: Weighted Analysis of Lumped Station Residuals With Respect to CC

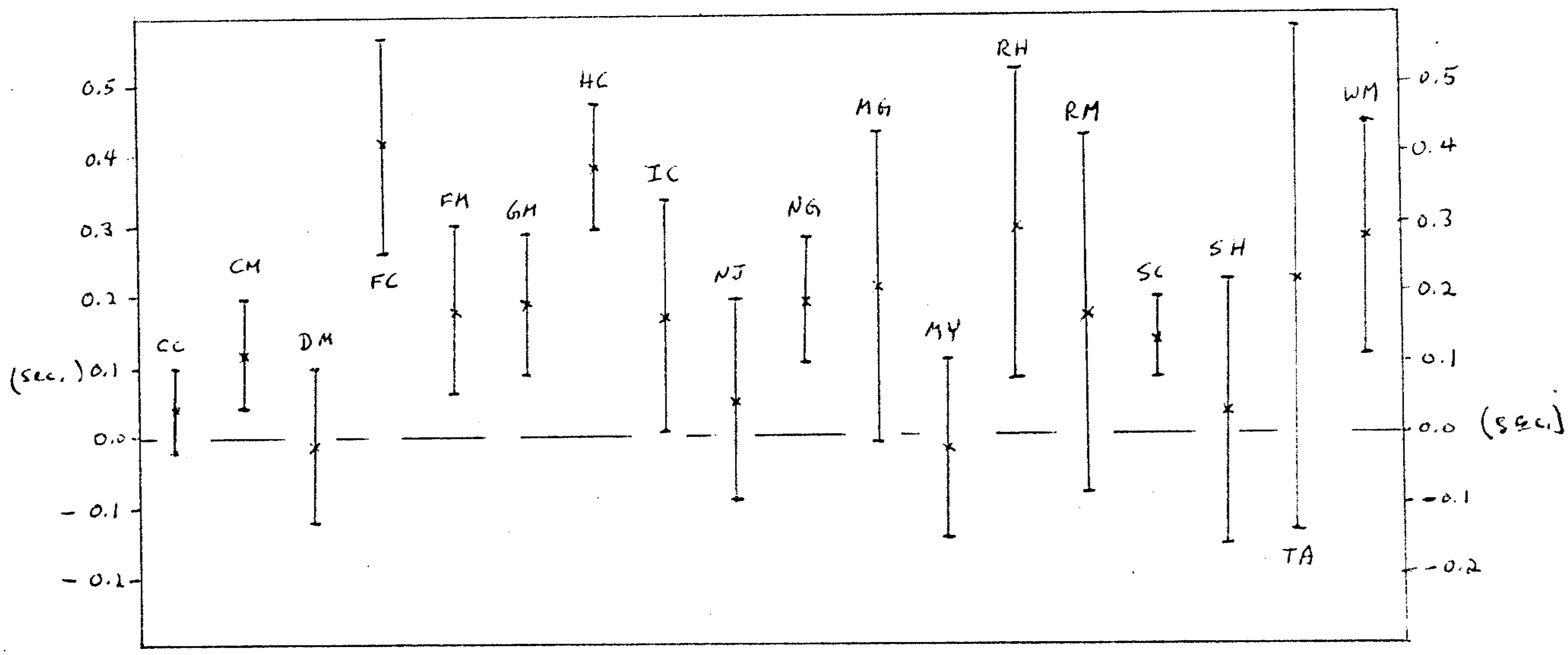
This analysis is identical to the previous analysis except that the

Table 7: Results of Lumped Weighted Analysis

With Respect to WT

Station	N_i	S_i	$b_i \pm \frac{1}{2}$ (95% C.I.)
CC	19	.031	.040 ± .063
CM	14	.038	.122 ± .076
DM	6	.055	-.010 ± .112
FC	4	.075	.418 ± .152
FM	6	.058	.182 ± .118
GM	8	.050	.187 ± .100
HC	8	.044	.384 ± .089
IC	3	.083	.168 ± .167
NJ	3	.071	.051 ± .143
NG	10	.043	.192 ± .088
MG	1	.109	.210 ± .221
MY	3	.063	-.020 ± .127
RH	1	.109	.300 ± .221
RM	2	.126	.170 ± .255
SC	21	.028	.139 ± .058
SH	2	.093	.030 ± .188
TA	1	.178	.220 ± .361
WM	3	.083	.279 ± .167

Figure 8: Results of Lumped Analysis With Respect to WT



residuals are taken with respect to CC. The analysis yielded a S_p^2=1.119 with 92 degrees of freedom. Eleven stations were represented by a total of 102 relative travel time residuals. Again, no analysis of WT with respect to CC was done since only redundant information would be obtained.

Table 8 and Figure 9 give the results of this analysis.

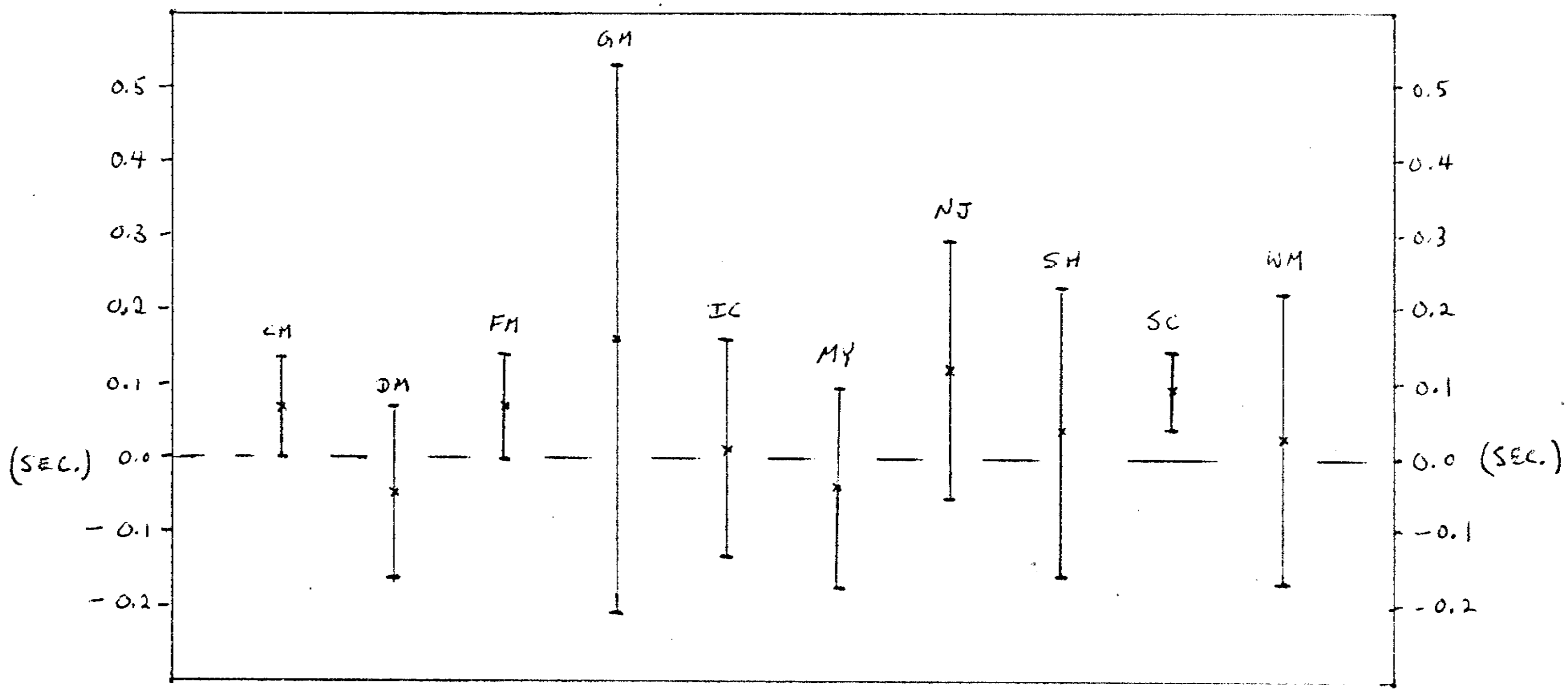
3.6: Unweighted Analysis of Residuals with Respect to Albuquerque

To analyze the relative travel time residuals with respect to Albuquerque, the stations ALQ and ABQ were assumed equivalent, since they are located at the same latitude, longitude and elevation, and thus have the same theoretical travel time for all events. An unweighted analysis was performed to find the estimated mean residual at each station (all directions lumped together) and the associated 95% confidence interval. A weighted analysis seemed unreasonable, since no estimates of the variability of the Albuquerque measurements were known. This analysis was essentially equivalent to the weighted analysis with both weighting factors S_{AB}^2 and S_{CD}^2 equal to unity. The mean values were estimated by the simple average of all the residuals at that station. An estimate was made of the variance; this again was a pooled estimate over the entire array. The assumption that all relative travel time residuals are members of populations having the same variance structure (the random errors affecting the measurements at one station are the same as those at any other station) was the essential assumption for all the analyses of this study. If the standard deviations of many of the average residuals had not been calculated under this assumption, then they would have been very large due to the small number of residuals used in the calculations. The estimates of the variance and the mean value were used to form a T random

Table 8: Results of the Weighted Analysis of Lumped Station Residuals with Respect to CU

STATION	N_i	S_i	$b_i \pm \frac{1}{2}(95\% \text{ C.I.})_i$
CM	22	.034	.069 ± .067
DM	5	.058	-.044 ± .116
FM	17	.038	.071 ± .071
GM	1	.186	.160 ± .369
IC	3	.074	.014 ± .148
MY	4	.068	-.040 ± .138
NJ	3	.086	.118 ± .171
SH	2	.097	.036 ± .193
SC	25	.027	.088 ± .054
NM	2	.097	.025 ± .193

Figure 9: Results of Lumped Analysis With Respect to CC



variable and confidence intervals were found for the various population mean relative travel time residuals.

This analysis, however, was not actually carried out in the form of a linear regression. Instead, it was done in the form of a standard one-way analysis of variance. A discussion of this analysis can be found in Draper and Smith (1966). I will only outline the method and present the results.

The model for this analysis is

$$\Delta t_{ij} = \mu + \alpha_i + \varepsilon_{ij} ,$$

where Δt_{ij} is the jth. residual at the ith. station. In this analysis there were 17 stations at which mean values were found, so i ran from 1 to 17. At each station there were N_i residuals, so at the ith. station, j runs from 1 to N_i. μ is the grand population mean residual of the entire array of stations, α_i is the deviation from that mean due to the ith. station (note that in the previous analyses μ and α_i were combined into a single population mean $\beta_i = \mu + \alpha_i$), and ε_{ij} is again a random error associated with the jth. relative travel time residual at the ith. station. These ε_{ij}'s are assumed to be normal random variables with mean zero and variance σ_ρ^2. In this analysis these variances are assumed equal.

The one-way analysis of variance proceeds by looking at a measure of the total variability about an estimate of the grand mean

$$\hat{\mu} = \sum_{i=1}^{17} \sum_{j=1}^{N_i} \Delta t_{ij} / M$$

where $M = \sum_{i=1}^{17} N_i$ is the total number of residuals used in the entire analysis. This measure is called SST (total sum of squares) and is calculated from the equation

$$SST = \sum_{i=1}^{17} \sum_{j=1}^{N_i} (\Delta t_{ij} - \hat{\mu})^2$$

The difference $\Delta t_{ij} - \hat{\mu}$ is written as

$$\Delta t_{ij} - \hat{\mu} = (\Delta t_{ij} - b_i) + (b_i - \hat{\mu})$$

where b_i is the unweighted average residual at each station,

$$b_i = \sum_{j=1}^{n_i} \Delta t_{ij} / n_i .$$

When SST is expanded in this partitioned form, it is seen that SST is the sum of two other sums of squares, i.e., SST= SSA + SSE, where

$$SSA = \sum_{i=1}^{17} \sum_{j=1}^{n_i} (b_i - \hat{\mu})^2$$

and

$$SSE = \sum_{i=1}^{17} \sum_{j=1}^{n_i} (\Delta t_{ij} - b_i)^2$$

SSA is a measure of the variability of the station means about the grand mean, and SSE is a measure of the variability within the station samples between the residuals and the station mean relative travel time residual. SSE/M-17 is in fact an unbiased estimate of σ_p^2 and is analogous to S_p^2 in the previous weighted analyses. This estimate has M - 17 degrees of freedom, and a 95% confidence interval for β_i is

$$b_i - t_{.025}(M-17)\, S_p/\sqrt{n_i} \leq \beta_i \leq b_i + t_{.025}(M-17)\, S_p/\sqrt{n_i}$$

$\sqrt{n_i}$ is analogous to $\left(\sum_{j=1}^{n_i} \frac{1}{S_{ij}^2} \right)^{1/2}$ in the previous weighted analyses. In this analysis, the total number of residuals was M= 170, S_p = .568 (note the large variability in these measurements), and $t_{.025}(153) = 1.97$.

The entire analysis of variance procedure is generally summarized in an analysis of variance (ANOVA) table. The general form of this table is illustrated in Table 9.

Table 9
ANOVA TABLE

Source	SS	d.f.	MS	EMS	F-Ratio
Between Groups	SSA	P-1	SSA/P-1	$\sigma_p^2 + \sum_{i=1}^{p} \frac{n_i \alpha_i^2}{p-1}$	$\frac{SSA/P-1}{SSE/M-P}$
Within Groups	SSE	M-P	SSE/M-P	σ_p^2	
Total	SST	M-1			

The source column is a description of the variability of which the sum of squares (SS) is a measure. The next column is the degrees of freedom of that measure, and MS(mean square) is the estimate divided by its degrees of freedom. EMS stands for the expected value of the mean square. In the case of SSE/M-1, this is an unbiased estimate of σ_ρ^2 so that $E[MSE] = \sigma_\rho^2$. The F-ratio is an added attraction of the analysis of variance calculation. In this model the hypothesis is that all the α_i's are zero (equivalently all the β_i's are equal; that is, all the mean residuals at the various stations are equal) can be tested. If this hypothesis is true, $\frac{SSA/P-1}{SSE/M-P}$ is an F random variable with P-1 and M-P degrees of freedom. The basic interpretation of the F-ratio is that to be confident of the hypothesis at some probability level, then the F-ratio should be less than the value of the F probability distribution function (a tabulated number) corresponding to the probability level of the test. If the F-ratio is greater than the tabulated value, then at that level of probability, one cannot be sure that $\frac{SSA/P-1}{SSE/M-P}$ is actually a F-random variable, and thus should reject the hypothesis.

The analysis of variance table for the analysis of the relative travel time residuals with respect to ALQ and ABQ is shown in Table 10.

Table 10
ANOVA TABLE

Source	SS	d.f	MS	EMS	F-Ratio
Between Stations	2.878	16	.180	$\sigma_\rho^2 + \sum_{i=1}^{P} \frac{n_i \alpha_i^2}{P-1}$	.3169
Within Stations	49.431	153	.323	σ_ρ^2	
Total	52.309	169			

The tabulated $F_{.05}$ $(16,153) \simeq 1.7$ and is greater than MSA/MSE. Therefore the hypothesis that all the station mean relative travel time residuals are equal cannot be rejected. This fact is illustrated in Figure 10 with graphs of the station mean relative travel time residuals, and their 95% confidence intervals. The results of this analysis are also summarized in Table 11.

Figure 10: Results of the Analysis With Respect to Albuquerque

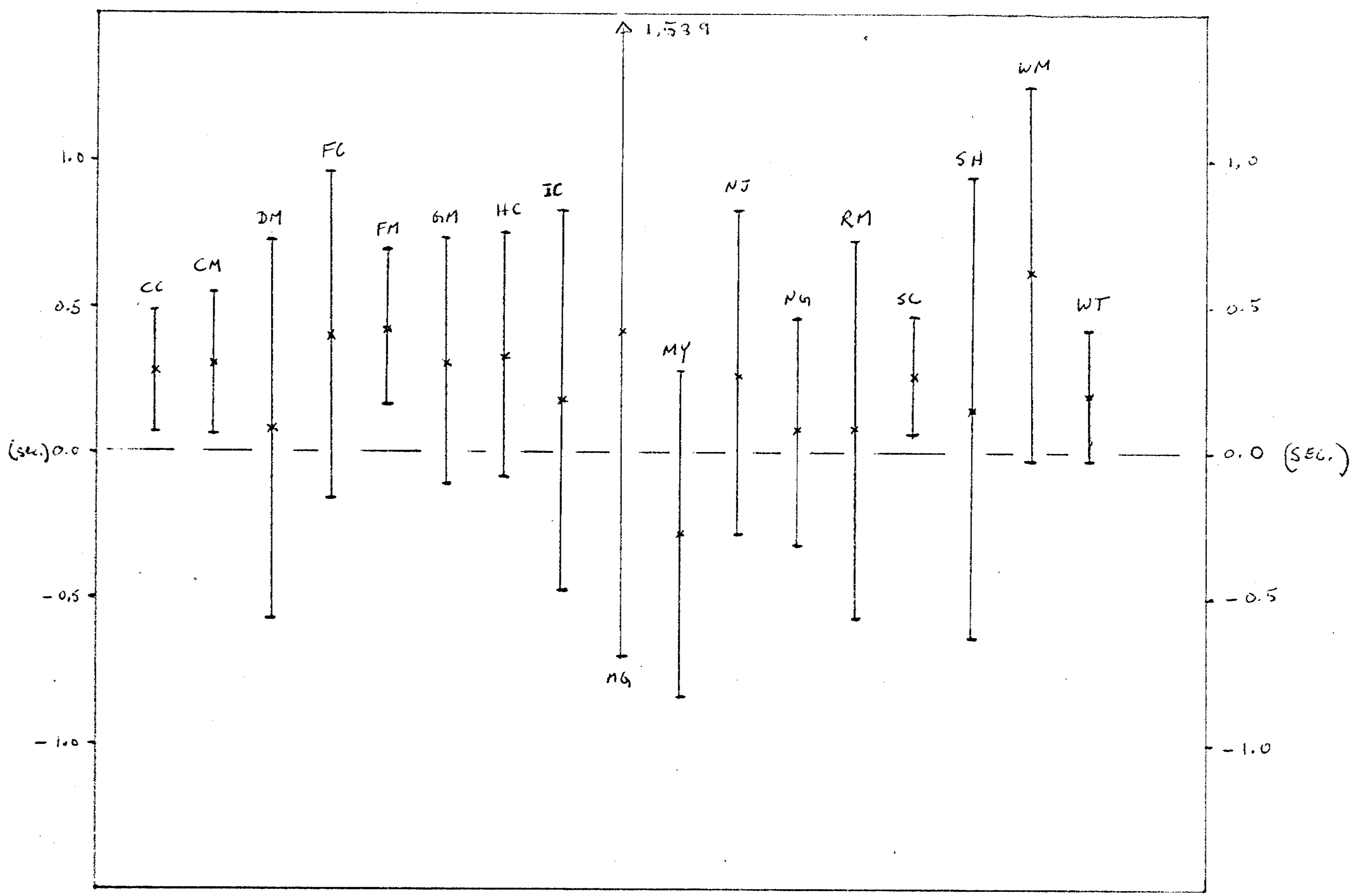


Table 11: Results of the Unweighted Analysis with Respect to the Albuquerque Stations

Station	N_i	$b_i \pm \frac{1}{2}(95\%\ C.I.)_i$
CC	28	.278 ± .211
CM	21	.306 ± .244
DM	3	.077 ± .646
FC	4	.405 ± .560
FM	17	.431 ± .271
GM	7	.314 ± .423
HC	7	.331 ± .423
IC	3	.183 ± .646
MG	1	.420 ±1.119
MY	4	-.280 ± .560
NJ	4	.270 ± .560
NG	8	.076 ± .396
RM	3	.080 ± .646
SC	30	.265 ± .204
SH	2	.150 ± .791
NM	3	.617 ± .646
WT	25	.196 ± .224

Chapter 4: Summary of Results

4.1: Summary of the Lumped Analyses with Respect to WT and CC

A study of Figures 8 and 9 reveals that at the 95% confidence level there exist differences between the directionally averaged travel time residuals calculated with respect to WT and CC. If the 95% confidence intervals of two stations overlap only slightly, then there is a suggestion that the two stations have differing average travel time residuals, although this is not required by the data. When the confidence intervals have large overlap, there is no difference between residuals, to within the stated error bounds, on the average When two stations differ in travel time residuals, the station with the larger residual is delayed.

For the following discussion let the symbolism,

$\text{Station Designation}_1 > \text{Station Designation}_2$

mean station 1 has a larger relative travel time residual than station 2, and thus station 1 is delayed. Let,

$\{ \ldots \text{Station Designation}_k \ldots \} > \{ \ldots \text{Station Designation}_i \ldots \}$

mean each station in set k has a larger travel time residual than each station in set i.

From the analysis with respect to WT it is seen that at the 95% confidence level

$\{CM,FC,FM,GM,HC,IC,NG,SC,WM\} > WT$

i.e , all these stations are delayed relative to WT; while it is suggested that

$\{CC,RM,TA,MG\} > WT$

although this is not required at the 95% confidence level.

At the 95% confidence level

FC > {CC,CM,DM,NJ,MY,SC,SH,WT}

and,

HC > {CC,CM,DM,NJ,MY,SC,SH,WT,GM,NG}.

There is also the suggestion that

{HC,FC} > {FM,IC}

and that

{CM,FM,GM,SC,WM} > {CC,DM,NJ,MY}

At the 95% confidence level one would probably say that DM, MY, and SH have absolute travel time residuals which cannot be distinguished from that of WT on the average.

Figure 11 summarizes these relationships with a possible ranking of the stations by size of relative travel time residual with respect to WT. A question mark in parentheses indicates that the station's relative ranking is somewhat in doubt.

The analysis with respect to CC is not independent of that with respect to WT since redundant data is used (this occured whenever a particular teleseismic event was recorded at both WT and CC). There were, however, enough arrivals at just CC to allow further information to be gained. From this analysis it is now confident at the 95% level that,

{CM,FM,SC} > CC.

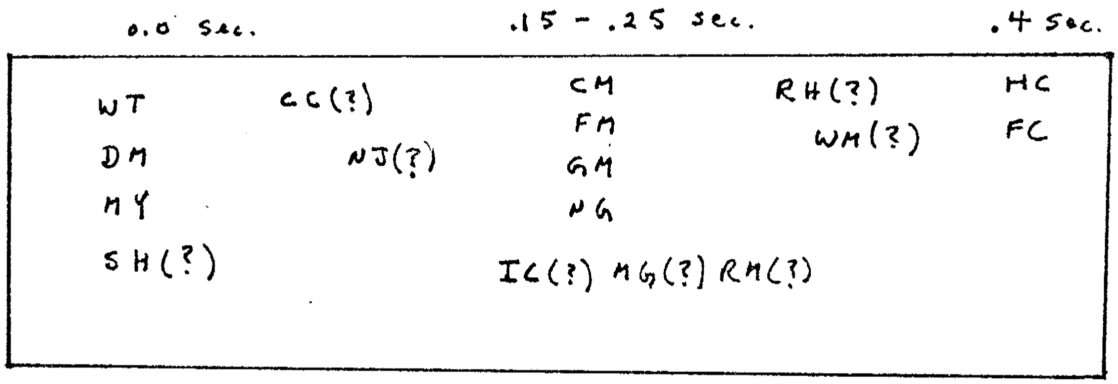
This was only suggested in the WT analysis. It is also suggested strongly that

NJ > CC,

which cannot be seen in the WT analysis. With this added information one

Figure 11: Relative Size of Travel Time Residuals

With Respect to WT



might be tempted to push NJ forward into the third group of Figure 11.

Combining the analyses with respect to WT and CC the following inequality is strongly suggested.

$\{HC, FC\} > \{CM, FM, GM, NJ, NG, SC\} > \{CC, WT, DM, MY\}$

Assuming these sets have somewhat arbitrary average group relative travel time residuals with respect to WT of .4, .2, and 0.0 second respectively, a partial contour map illustrated in Figure 12 is suggested.

4.2: Summary of the Directional Analyses with Respect to WT and CC

To facilitate the interpretation of the relative travel time residuals with respect to WT, the results tabulated in Table 5 are displayed for azimuths in the SE, SW, and NE quadrants in Figures 13, 14, and 15 respectively. The numbers in parentheses are the number of residuals used in estimating the means.

For arrivals from the southeast (Figure 13), a consideration of the confidence intervals and the number of observations used in calculating the means suggests that the only significant associations which can be made are,

$\{FC, HC\} > \{CC, CM, SC\} > WT.$

If the groups are given average travel time residuals with respect to WT of .6, .15, and 0.0 respectively (as is suggested by Figure 13), then a contour like that seen in Figure 16 is appropriate. The only significant difference between this map and that of Figure 12 is that CC is now clearly grouped with SC and CM rather than with WT.

For arrivals from the southwest (Figure 14), it appears that perhaps CC, CM, and SC should be grouped with WT, and that

$\{CC, CM, SC, WT\} > DM.$

Figure 12: Contour Map of Lumped

Relative Travel Time Residuals With Respect to WT

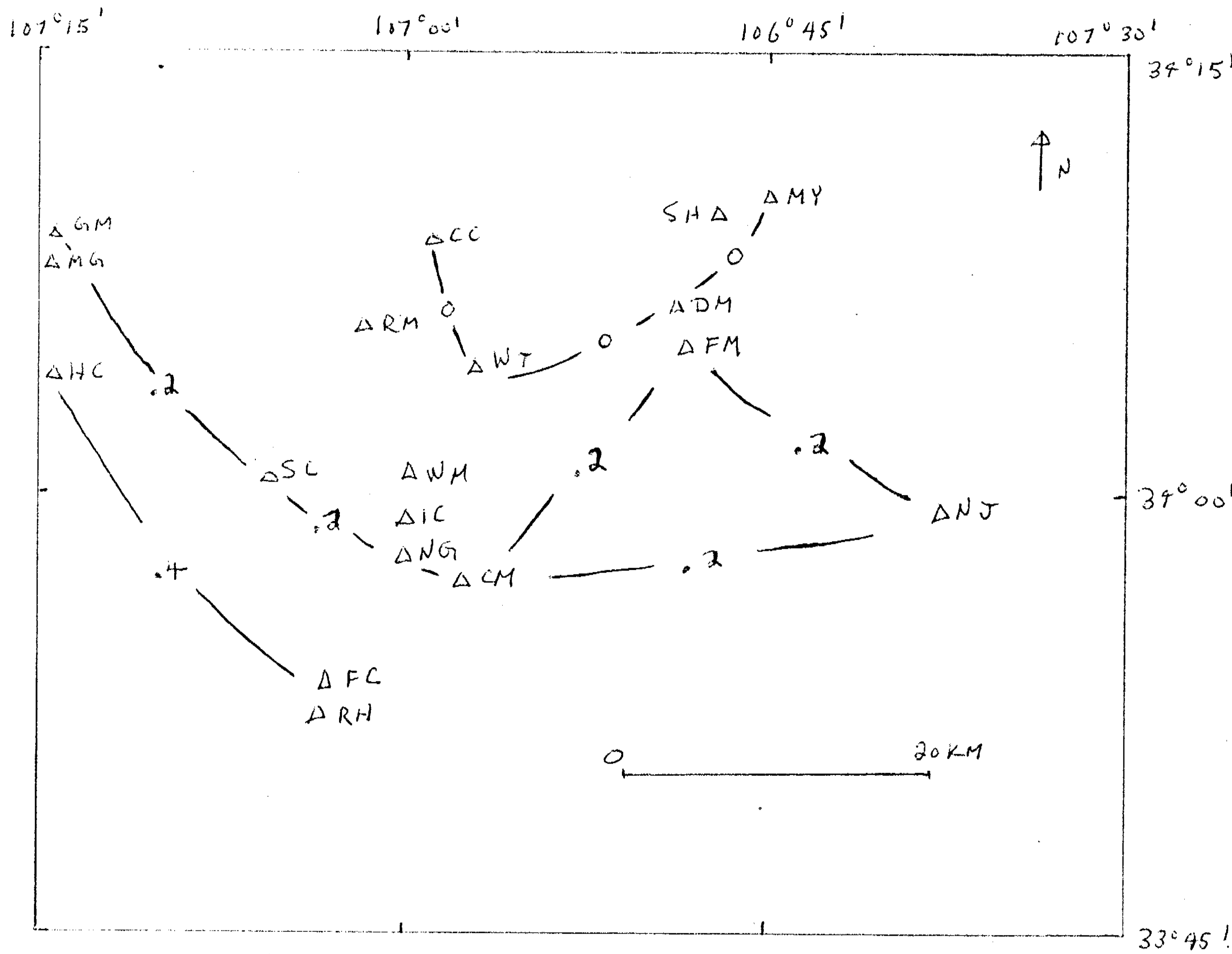


Figure 13: Relative Travel Time Residuals
With Respect to WT, SE Arrivals

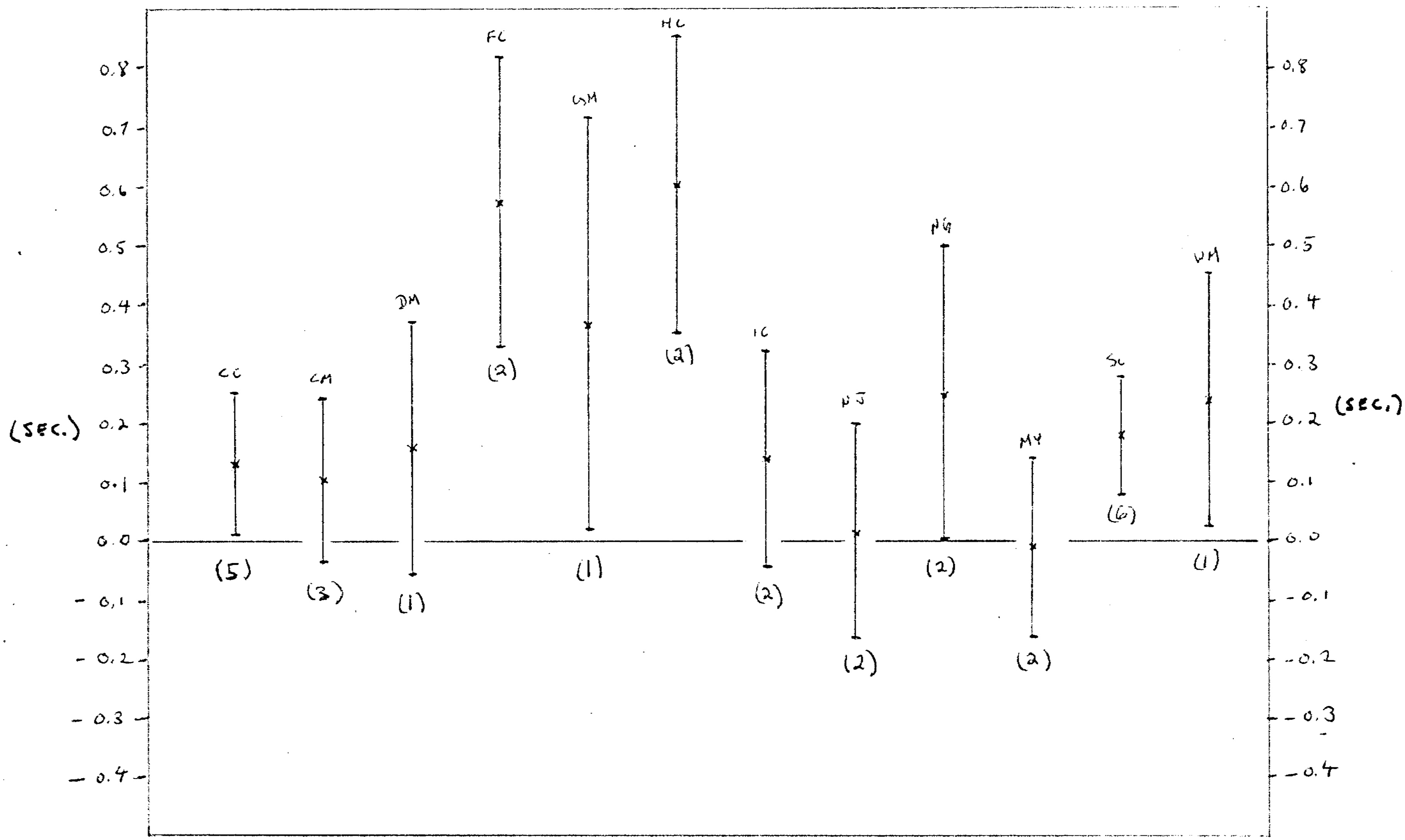


Figure 14: Relative Travel Time Residuals

With Respect to WT, SW Arrivals

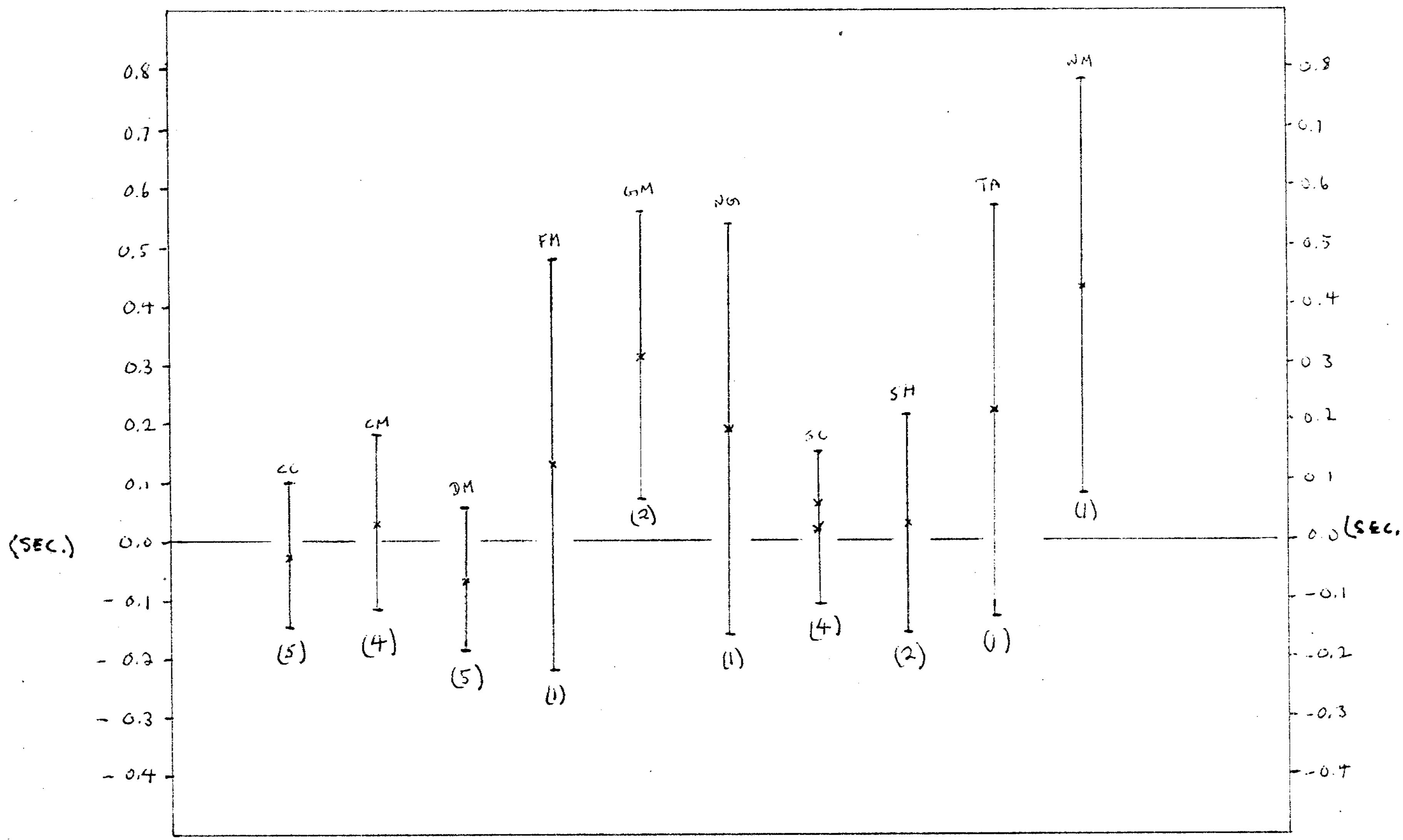


Figure 15: Relative Travel Time Residuals

With Respect to WT, NW Arrivals

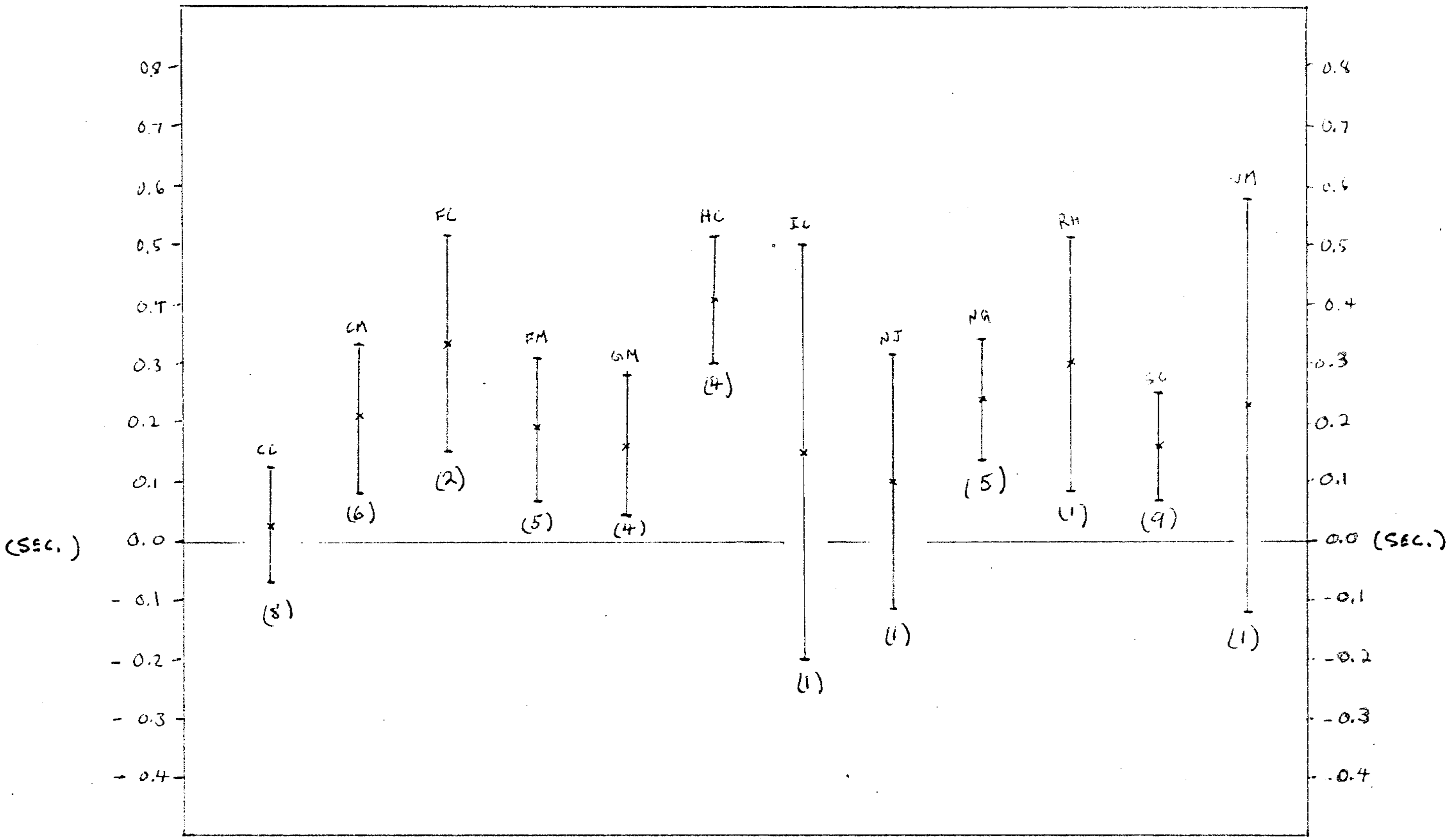
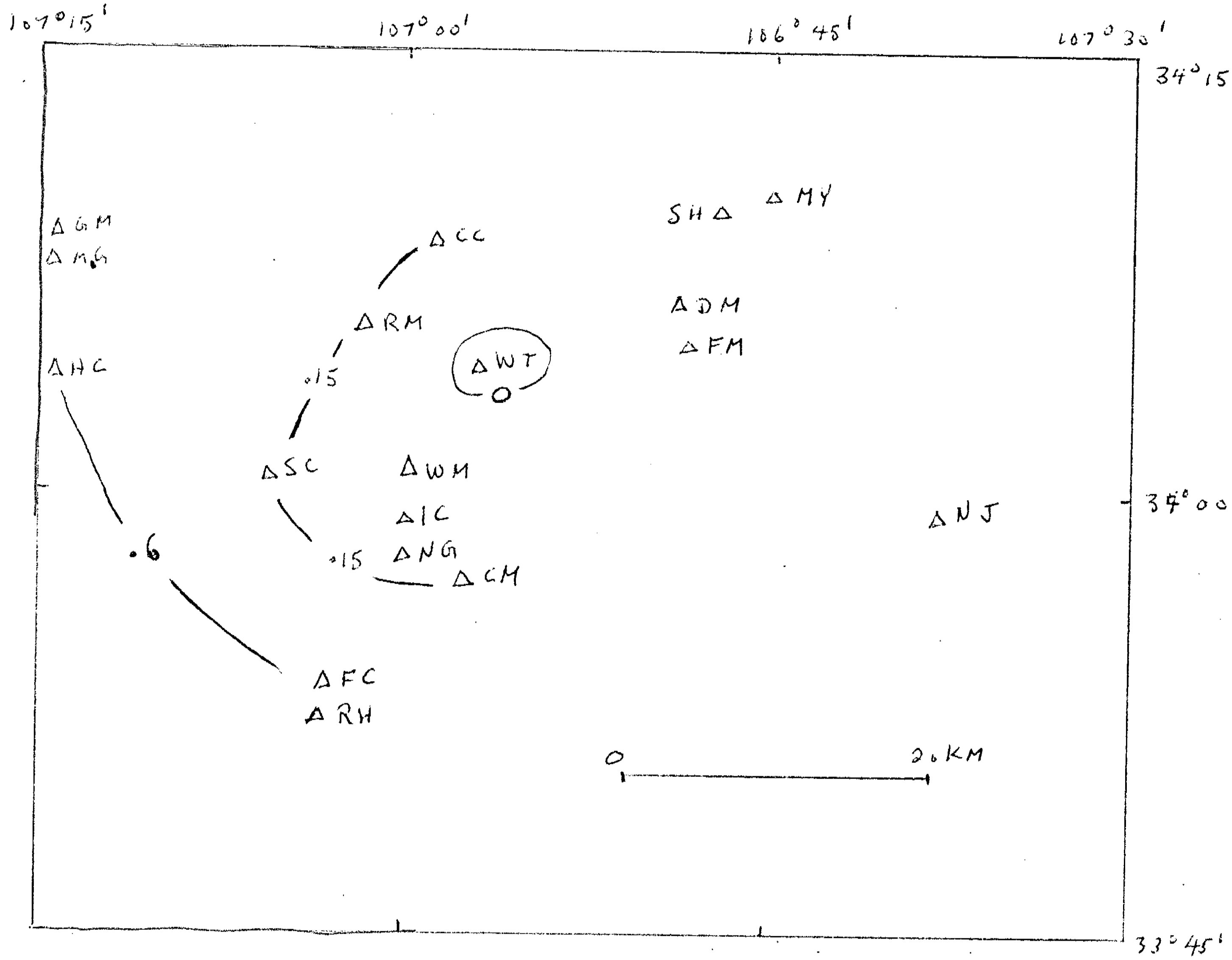


Figure 16: Contour Map of Relative Travel Time Residuals With Respect to WT, SE Arrivals



Assigning these relative travel time residuals the values 0.0 and -0.05 respectively, a possible contour is given in Figure 17.

For arrivals from the northwest (Figure 15), the groupings would suggest a contour map similar to Figure 12, which is for all the directions lumped together. An obvious explanation for this is that most of the teleseismic events occurred to the sorthwest and therefore Figure 12 reflects this direction more than any of the others.

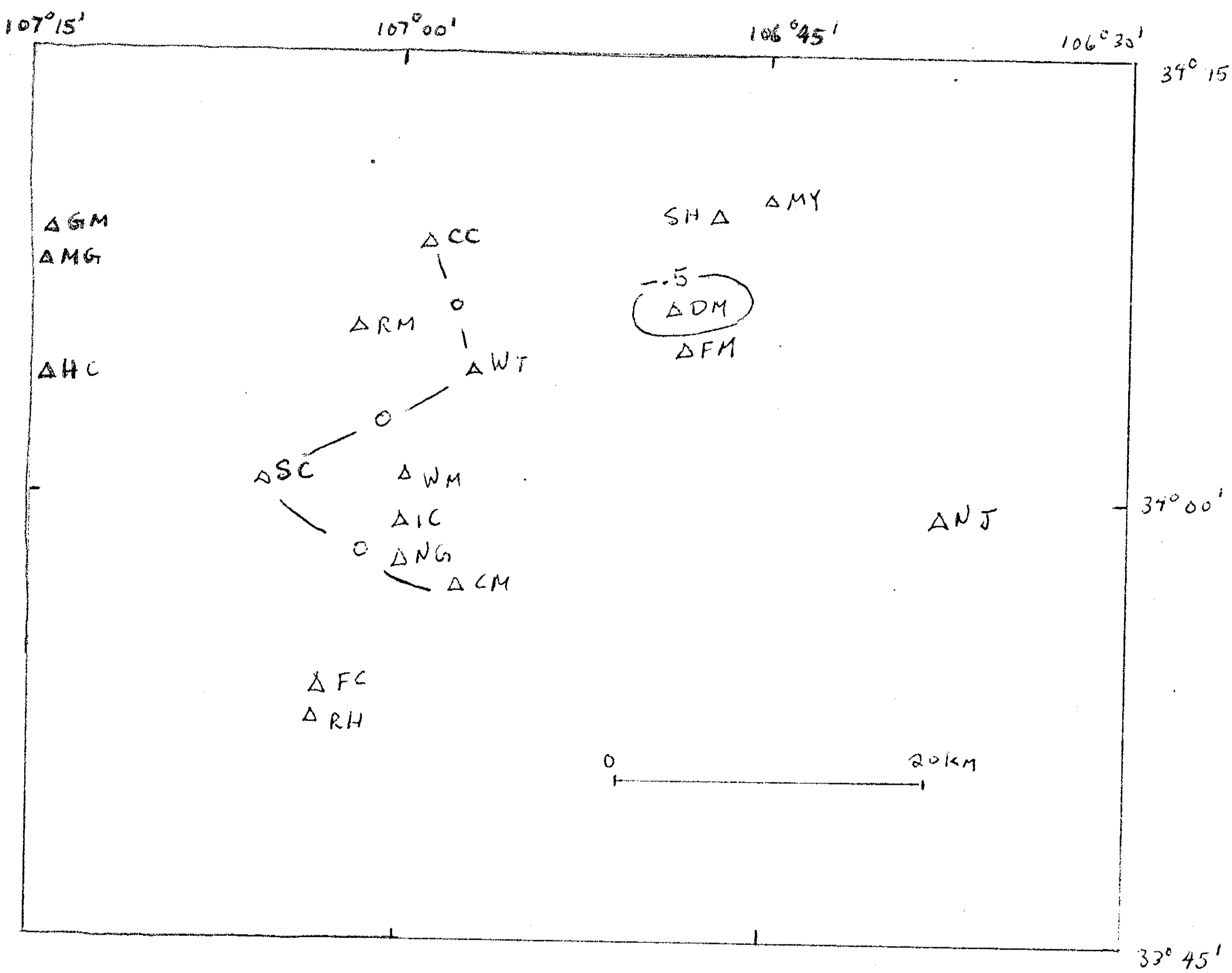
For further evidence of anisotropic relative travel time residuals for various stations within the Socorro array one should see Figures 6 and 7.

4.3: Summary of the Analysis with Respect to the Albuquerque Stations

Table 11 and Figure 10 display the results of the analysis of relative travel time residuals with respect to either ALQ or ABQ. The 95% confidence intervals on all the mean values are large, reflecting a rather great amount of variability in the Albuquerque data. The only trend seen is that the travel times to the Socorro stations are generally delayed with respect to Albuquerque. Crustal thickening under Albuquerque as compared to Socorro, (Toppozada and Sandord (1976)), would cause relative delays at Albuquerque, so that these results cannot be explained by differences in crustal thickness.

Figure 17: Contour Map of Relative Travel Time Residuals

With Respect to WT, SW Arrivals



Chapter 5: Conclusion

Since the relative travel time residuals for teleseismic arrivals are larger in absolute value and mainly of a different sign than the travel time residuals for Yousef's (1977) study, it appears that the teleseismic travel time residuals reflect upper mantle P velocity structure more than that of the crust. In this respect, the data of this study perhaps will not be very usefull in the study of the crustal anomaly (magma body) which supplied the original interest in the study. The data may, however, be of aid in understanding the deeper and more gross features of the area.

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APPENDIX I

The model is

$$\Delta t_{ij} = \beta_i + \varepsilon_{ij}$$

where i=1,P, P being the number of groups whose means β_i are to be estimated; and j=1,N_i, N_i being the number of quantities Δt_{ij} being averaged to estimate β_i. The ε_{ij}'s are assumed to be normal random variables with mean

$$E[\varepsilon_{ij}] = 0$$

and variance

$$\mathrm{VAR}(\varepsilon_{ij}) = \sigma_{ij}^2$$

The model can be put is the form of a linear regression by forming the matrix equation,

$$(1) \quad \underline{\Delta t} = \underline{a}\,\underline{\beta} + \underline{\varepsilon}$$

where $\underline{\Delta t}$ is a column vector of the data Δt_{ij} listed as shown below.

$$\underline{\Delta t} = \begin{Bmatrix} \Delta t_{11} \\ \vdots \\ \Delta t_{1N_1} \\ \Delta t_{21} \\ \vdots \\ \Delta t_{2N_2} \\ \vdots \\ \Delta t_{P1} \\ \vdots \\ \Delta t_{PN_P} \end{Bmatrix} \quad M = \sum_{i=1}^{P} N_i \text{ components}$$

The column matrices $\underline{\beta}$ and $\underline{\varepsilon}$ have definitions corresponding to that of $\underline{\Delta t}$, and are illustrated below.

$$\underline{\beta} = \begin{Bmatrix} \beta_1 \\ \vdots \\ \beta_P \end{Bmatrix} \quad P\text{- components} \qquad \underline{\varepsilon} = \begin{Bmatrix} \varepsilon_{11} \\ \vdots \\ \varepsilon_{1N_1} \\ \vdots \\ \varepsilon_{P1} \\ \vdots \\ \varepsilon_{PN_P} \end{Bmatrix}$$

The matrix $\underline{a}$ is the MxP matrix which combines the P estimations of the β_i's into a single regression for the estimates. It has the form,

$$\underline{a} = \begin{pmatrix} \left.\begin{matrix} 1 \\ \vdots \\ 1 \end{matrix}\right\} N_1 & & & 0 \\ & \left.\begin{matrix} 1 \\ \vdots \\ 1 \end{matrix}\right\} N_2 & & \\ & & \ddots & \\ 0 & & & \left.\begin{matrix} 1 \\ \vdots \\ 1 \end{matrix}\right\} N_p \end{pmatrix}$$

Equation (1) would be in the form of a standard linear regression except for the variances of the $\underline{\varepsilon}$ being unequal, i.e.

$$\mathrm{VAR}(\underline{\varepsilon}) = \underline{V}\,\sigma^2$$

where

$$\underline{V} = \begin{pmatrix} V_{11} & & & & & 0 \\ & \ddots & & & & \\ & & V_{1N_1} & & & \\ & & & \ddots & & \\ & & & & V_{p1} & \\ 0 & & & & & \ddots \quad V_{pN_p} \end{pmatrix}$$

Draper and Smith (1966) show that if a diagonal matrix $\underline{P}$ can be found such that

$$\underline{P}^T\underline{P} = \underline{P}\,\underline{P}^T = \underline{P}^2 = \underline{V}$$

then equation (1) can be put in the standard form of a least squares regression. This done by weighting equation (1) by $\underline{P}^{-1}$,

$$(2) \quad \underline{P}^{-1}\underline{\Delta t} = \underline{P}^{-1}\underline{a}\,\underline{\beta} + \underline{P}^{-1}\underline{\varepsilon}$$

Equation (2) defines a new regression equation for $\underline{\beta}$, i.e.,

$$(3) \quad \underline{\Delta t}' = \underline{a}'\,\underline{\beta} + \underline{\varepsilon}'$$

where

$$\underline{\Delta t}' = \underline{P}^{-1}\underline{\Delta t}, \qquad \underline{a}' = \underline{P}^{-1}\underline{a} \quad \text{and} \quad \underline{\varepsilon}' = \underline{P}^{-1}\underline{\varepsilon}.$$

The mean value of $\underline{\varepsilon}'$ is

$$E[\underline{\varepsilon}'] = E[\underline{P}^{-1}\underline{\varepsilon}] = \underline{P}^{-1}E[\underline{\varepsilon}] = 0$$

and the variance of is

$$\mathrm{var}(\underline{\varepsilon}') = E[\underline{\varepsilon}'\underline{\varepsilon}'^{T}] - E[\underline{\varepsilon}']E[\underline{\varepsilon}'^{T}] = E[\underline{P}^{-1}\underline{\varepsilon}\,\underline{\varepsilon}^{T}P^{-1}]$$

$$= \underline{P}^{-1}E[\underline{\varepsilon}\,\underline{\varepsilon}^{T}]\underline{P}^{-1}$$

$$= \underline{P}^{-1}\mathrm{var}(\underline{\varepsilon})\underline{P}^{-1} = \underline{P}^{-1}\underline{V}\sigma^{2}P^{-1}$$

$$= \underline{P}^{-1}\underline{P}^{2}\underline{P}^{-1}\sigma^{2} = \underline{I}\sigma^{2}$$

Equation (3) is now in the form of a simple linear regression. The solution is standard, and is

$$\underline{b} = (\underline{a}'^{T}\underline{a}')^{-1}\underline{a}'^{T}\underline{\Delta t},$$

where $\underline{b}$ are the least squares estimates of $\underline{\beta}$. The variances of these estimates are,

$$\mathrm{var}(\underline{b}) = (\underline{a}'^{T}\underline{a})^{-1}\sigma^{2}.$$