A STOCHASTIC MANAGEMENT MODEL FOR THE OPERATION OF A STREAM-AQUIFER SYSTEM

By

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I dedicate this work to my parents, Lic. Luciano Z. Flores and Manuela W. de Z. Flores, to my wife, Olga L. de Z. Flores and daughter Lucia Z. Flores L. as well as to the other members of my family.

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ABSTRACT

The objective of this study is to develop and evaluate a simple management technique through which the cost of conjunctive operation of surface water and groundwater resources can be minimized under the effect of uncertainty.

A lumped parameter model represents the physics of the system and a linear outflow equation simulates the stream-aquifer flow. A subsurface outflow constant related to the response time of the aquifer proves to be an important concept in the simulation process. Furthermore, a drawdown correction is developed to compute the drawdown at wells.

In the developing of the management model, dynamics in the operation of the system is obtained by using linear decision rules. The nonlinear optimization problem (pumping cost dependent on the drawdown and the pumping volume) is solved by an iterative procedure which uses a standard linear programming package.

To study the effect of randomness in the system, uncertainties in the water demand, natural inputs and the physical properties of the system are considered. A stochastic differential equation governs the system and some of the statistics are obtained via spectral analysis. In addition, a conditional probability approach is followed to account for a random subsurface outflow constant. Chance constraints are introduced to include probabilities of satisfaction of constraints.

To test the reliability of the proposed model a comparative test with a previous study using a distributed parameter model is carried out; good agreement is obtained. An application to a basin in northwestern Mexico shows the capability of the proposed model in regional management problems involving hundreds of wells and large surface water facilities. A sensitivity analysis in the latter application shows a larger increase in the operational cost due to uncertainty in the water demand than to uncertainty in the aquifer parameters.

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a	subsurface outflow constant (1/T)
d	thickness of a restrictive layer (L)
$d(0.75)$	75 percentile of the water demand (L^3)
h	mean water table in the aquifer (L)
h'	mean water level at the wells (L)
h_o	initial water level
n	design period (T)
p_D	dimensionless pressure
q	stream-aquifer discharge per unit aquifer area (L/T)
r	nominal interest rate
r_w	average well radius (L)
$r(0.75)$	75 percentile of the streamflow (L^3)
$\overline{s}$	average drawdown (L)
s_w	drawdown at the wells (L)
t_{DA}	dimensionless time
t_h	response time of the system (L)
w	frequency (radians/T)
w_m	minimum dam storage fraction
w_o	initial dam storage fraction
y	fictitious net input to the system (L/T)
y_R	net inflow to the system (L/T)
A	aquifer area (L^2)
A_m	sine Fourier coefficient
A_w	average influence area of a well (L^2)

B	leakage factor (L)
C_P	unit cost of pumping ($\$/L^4$)
C_R	unit cost of spreading ($\$/L^3$)
C_S	unit cost of surface water ($\$/L^3$)
C_u	unit cost of water returned to the stream ($\$/L^3$)
CU	consumptive use (L^3)
C_v	coefficient of variation
D	demand of water (L^3)
DC	drawdown correction (L)
ET	evapotranspiration (L^3)
F(x)	gamma cumulative distribution
F1	dam freeboard (L^3)
H	mean stream stage (L)
HL	average initial lift (L)
K	hydraulic conductivity (L/T)
K1	downstream flow (L^3/T)
K_s	hydraulic conductivity of a restrictive layer (L/T)
K_y	autocovariance of $\underline{y}$
L	characteristic length (L)
N	design period (T)
N_r	recharge from precipitation (L^3)
N_s	seasons per year
OF	objective function (\$)
P	precipitation (L^3)
P'	percent error in the discounted expected cost in relation to the deterministic case
Q	average instantaneous pumping (L^3/T)

Q'	capacity of water facilities (L^3)
Q_C	conveyance loss (L^3)
Q_i	subsurface inflow (L^3)
Q_{ou}	water release from the dam (L^3)
Q_P	quantity of water pumped from the aquifer (L^3)
Q_R	amount of water recharged to the aquifer (L^3)
Q_{ret}	irrigation return flow (L^3)
Q_{RS}	surface drainage (L^3)
Q_S	stream-aquifer discharge (L^3/T)
Q_{SD}	quantity of water diverted from the stream (L^3)
Q_{ST}	streamflow (L^3/T)
Q_u	amount of water returned to the stream (L^3)
R_o, R_2	radial dimension (L)
S	storage coefficient
S'	dam storage (L^3)
S_{ff}	spectral density function of <u>f</u>
T	transmissivity (L^2/T)
V	volume (L^3)
WN	number of wells
Z	ground surface level (L)
Z(w)	random process with orthogonal increments
$Z^*(w)$	complex conjugate of Z(w)
α^2	hydraulic diffusivity (L^2/T)
α_1	fraction of precipitation that actually recharges the aquifer
α_1	fraction of developed water for return to the stream
α_2	fraction of water applied that infiltrates

α_2	fraction of developed water for spreading
α_4	fraction of water infiltrated that actually recharges the aquifer
β	dimensionless constant, shape of the gamma distribution
β_1	fraction of precipitation that helps to satisfy the demand
γ_B	dam operation decision variable (L^3)
γ_P	pumping decision variable
γ_R	artificial recharge decision variable
γ_{R_1}	size of the dam (L^3)
γ_S	surface water decision variable
δ	R_o / R_2
$\delta(u)$	Dirac delta function
ϵ	natural recharge (L/T)
θ	angle (radians)
λ	probability level
μ	expected value
ρ	autocorrelation function
ρ_{hQ_p}	cross correlation function of head and pumping
σ	standard deviation
σ^2	variance
τ	lag time (T)
χ	chi-square distribution
Δh	aquifer head difference (L)
Δt	time increment (T)

CHAPTER 1 INTRODUCTION

1.1 Background

There is urgent need in many areas of the world to develop or allocate water resources in an optimal manner. Aquifers important reservoirs provided by nature and able to store and convey water, and to improve its quality often are not used properly by planners; instead, emphasis is given to the development of surface water resources by constructing large reservoirs and ignoring the dynamic connection between stream and aquifer. Management of the conjunctive use of groundwater and surface water is an amenable solution to the problem.

In recent years, the use of distributed models in groundwater hydrology has been widespread. The trend has been favored by the development of numerical techniques and electronic computers. Regardless of field information available, the trend has been biased toward more elegant and detailed techniques.

With the previous ideas in mind the present study was oriented toward developing simple models capable of simulating the behavior of an interconnected stream-aquifer system and managing it in an economically optimal way. There was also interest in the study of the effects of uncertainties induced by nature and man on the operation of the system.

1.2 Previous Work

Optimal management of the conjunctive use of surface water and groundwater is a complex problem widely discussed in the literature but not yet exhausted.

Table 1.1 presents a review of the literature on optimal management of the conjunctive use of surface water (SW) and groundwater (GW); it also lists papers related to the research subject in terms of concepts and techniques.

Dynamic Programming is a widely used optimization technique (see Table 1.1). It is favored mainly because of the dynamic characteristics of the management problem. Therefore, it is extensively used for scheduling purposes. Probabilities of events can easily be included in its recursive equation (Buras, 1963; Saleem and Jacob, 1971) which is based on the optimality principle (Bellman, 1957, p.83). In general, a computer program has to be written for solving a specific problem. See Aron (1969) for advantages and disadvantages of the technique.

Linear Programming has fewer advantages than dynamic programming mainly because of nonlinear pumping costs and the dynamics of the system; see Table 1.1.

A significant contributor to a better understanding of the economics of groundwater resources has been O. Burt. A sequence of his papers (Table 1.1) discusses intensively the problem of optimal water allocation where random streamflows and conditional probabilities for storages have been included. Simple decision rules were developed and Burt (1970) worked

TABLE 1.1 Review of Literature Relating to Management of Conjunctive Use of Groundwater and Surface Water

Reference	Objective	Optimization Technique	Type of System	Physical Model	Randomness	Remark
Buras (1963)	optimal planning, design and operation of the system	dynamic programming	SW-GW	continuity equation	random streamflow	variable pumping costs
Burt (1964a)	optimal allocation of a single resource	dynamic programming	GW	continuity equation	deterministic	a functional equation is obtained to derive approximate decision rules
Burt (1964b)	optimal management of groundwater and surface water	dynamic programming	SW-GW	continuity equation	random streamflow and conditional probability for storages	variable pumping cost; detail field data computations
Burt (1966)	develop a sequential decision model	dynamic programming	GW	continuity equation	random recharge	the variance of the net output is considered
Dracup (1966)	optimum use of ground-water and surface water	parametric linear programming	SW-GW	continuity equation	deterministic	an application is given; constant pumping cost
Burt (1967a)	a simple dynamic model for allocation of ground-water	marginal analysis	GW	continuity equation	random recharge	a linear decision rule is obtained
Burt (1967b)	temporal allocation of groundwater under quadratic criterion functions	marginal analysis	GW	continuity equation	random recharge	a linear decision rule is given
Bear and Levin (1967)	optimal operation of a groundwater system	dynamic programming	GW	lumped linear outflow model	random recharge	the aquifer outflow is linear; variable pumping cost
Domenico et.al. (1968)	optimal ground-water mining	marginal analysis	GW	continuity equation	deterministic	an optimal mining yield volume is found; variable pumping cost

TABLE 1.1 (Continued)

Reference	Objective	Optimization Technique	Type of System	Physical Model	Randomness	Remark
Aron (1969) and Aron and Scott (1971)	optimization of a complex system with subsystem preoptimization	dynamic programming	SW-GW	continuity equation	random streamflow	discussion of other optimization techniques; variable pumping cost
Revelle et.al.	optimal design and operation of a dam	linear programming	SW	continuity equation	random streamflow	use of a linear decision rule; use of chance constraints
Longenbaugh (1970)	optimal operation for conjunctive use of groundwater and surface water	linear programming	SW-GW	distributed model	deterministic	constant pumping cost; the physical model is tied to the management model, through the constraints
Taylor (1970)	optimal conjunctive use of water	linear programming	SW-GW	distributed model	deterministic	the stream depletion is optimized; only 3 cells were used
Milligan (1970)	optimal conjunctive use of water	linear programming	SW-GW	continuity equation	random streamflow and natural recharge	constant pumping cost for a head range
Bredehoeft and Young (1970)	optimization of temporal allocation of groundwater	simulation	GW	distributed model	deterministic	part of the problem is solved by linear programming
Cummings and Winkelman (1970)	water resource management in arid regions	marginal analysis and lagrangian multiplier	SW-GW	continuity equation	random streamflow	the decision rules are interpreted but application is not given; the aquifer is considered to be closed to outside recharge
Saleem and Jacob (1971)	optimal utilization of several leaky aquifers and surface water	dynamic programming	SW-GW	continuity equation	random streamflow and natural recharge	optimiz. ion in time and space is performed

TABLE 1.1 (Continued)

Reference	Objective	Optimization Technique	Type of System	Physical Model	Randomness	Remark
Nieswand and Granstrom (1971)	optimal planning of conjunctive use of surface water and groundwater	linear programming	SW-GW	continuity equation	random streamflow	lag 3 linear regression models of streamflow were used; use of chance constraints and a zero order decision rule
Young and Bredehoeft (1972)	optimal conjunctive use of groundwater and surface water	simmulation	SW-GW	distributed model	deterministic	no natural recharge; a sequential linear programming approach is followed to solve the problem; constant pumping cost
Brown and Deacon (1972)	optimal economic use of an aquifer under several conditions	lagrange multiplier and marginal analysis	GW	continuity equation	deterministic	emphasis on economic aspects
Maddock III (1972a)	coupling of a distributed physical model to a management model	quadratic programming	GW	distributed model	deterministic	the effect of each well is considered in the pumping cost; variable pumping cost
Moody and Maddock III (1972)	developing of a water resources planning model to design a data collection network	mixed-integer programming	-	-	deterministic	separable programming techniques are used in nonlinear functions
Maddock III (1972b)	planning and operation of a groundwater system	mixed-integer programming	GW	distributed model	deterministic	a separable programming procedure was used to manage a quadratic pumping cost; a sensitivity analysis was performed to study the system performance to changes in economical and hydrological factors

TABLE 1.1 (Continued)

Reference	Objective	Optimization Technique	Type of System	Physical Model	Randomness	Remark
Maddock III (1973)	planning and operation of a ground-water system	quadratic programming	GW	distributed model	deterministic	there is not natural recharge; the effect of economics, hydrologic, and physical factors on the management is studied
Maddock III (1974)	optimal operation of a stream-aquifer system under random demands	quadratic programming	SW-GW	distributed model	random streamflow and demand	the demand persistence is included in the problem
Yu and Haimes (1974)	multilevel optimization for conjunctive use of groundwater and surface water	conjugate gradient method	SW-GW	distributed model	deterministic	the overall regional problem is decomposed to two levels; variable pumping cost; an example is given; a penalty function method is used to solve the first level
Morey-Seytoux (1975)	conjunctive surface ground-water management	linear programming	SW-GW.	distributed model	deterministic	three examples are given; an analytical procedure to compute well drawdown for a regular boundary geometry is given

out a situation in which institutional constraints were important.

Few cases with variable pumping cost, dependent on the well drawdown, are found in the literature (Buras, 1963; Aron, 1969; Maddock, 1972a; Yu and Haimes, 1974).

A linear decision rule in connection with chance constraints was used by Revelle et.al. (1969) in the optimal design and operation of a surface reservoir. Much controversy arose concerning the use of the linear decision rule (see e.g., Lockus, 1970; Eisel, 1970; Kirby et.a., 1970; Nayak and Arora, 1970). A zero decision rule was used by Nieswand and Granston (1971) to find the deterministic equivalent of the chance constraint in the management of the conjunctive use of surface water and groundwater.

Maddock (1974) emphasizes the stochastic nature of the problem and deals with a stochastic process represented by its mean, variance and autocovariance (persistence). Bear and Levin (1967) deal explicitly with a lumped model which includes a linear outflow, though the system consists of only a groundwater reservoir.

The optimal management of the conjunctive use of groundwater and surface water is a problem to which much attention has been given in the literature. However, there are only a few cases which have dealt with stochastic models, and none of these have included uncertainty in the properties of the stream-aquifer system.

The representation of physical systems by distributed models in the management model is a recent advance. Three different couplings between the physical and management model have been noted. One includes the distributed aquifer model in the objective function and/or in the constraints of the management model (Taylor, 1970; Longenbaugh, 1970). Another considers only the distributed effect at the wells, with part of the drawdown computation done outside of the management model (Maddock, 1972; Morel Seytoux, 1975). Thirdly, the drawdown may be computed entirely outside of the management model. Young and Bredehoeft (1972) followed the latter approach; however instead of a mathematical programming technique a simulation technique was used to approach and optimum.

Despite the common use of linear reservoirs or lumped parameter models to describe surface runoff phenomena (Chow 1964, Section 14), little importance has been given to this type of model in groundwater hydrology. Kraijenhoff Van de Leur (1958),Dooge (1960), Eriksson (1970), Eliasson (1971), Dowming et.a.(1974), Klemes (1974) and Gelhar and Wilson (1974) deal with lumped model applications to groundwater hydrology.

1.3 Purpose and Scope of this Study

The main purpose of this research is to develop simple and reliable models which can be used to manage a regional system. The physical prototype under consideration is composed of a stream which is hydraulically connected to an aquifer; uncertainties exist in its properties and inputs.

The principal objectives of the study are:

1. Development of a simple model capable of representing the physics of a stream-aquifer system, simulating the head in the aquifer and at the wells, and being naturally connected to a management model.
2. Inclusion of uncertainties into the operation of the system.
3. Testing of the developed models against suitable work obtained from the literature and application to a real example.

Scope of the Study

A lumped model formed by an aquifer water balance and a linear outflow term represents the stream-aquifer system. The subsurface outflow constant and the response time are important concepts in the understanding and modeling of the system. The mean water levels of the aquifer are computed by a convolution integral. To obtain an average head at pumping wells, a drawdown correction is included in the physical model. A link between the physical and the management models was obtained through the mean head of the aquifer. Because of the nature of this connection it is possible to solve a nonlinear optimization problem with an iterative procedure that uses a linear programming package.

When randomness is included in the system, a stochastic differential equation represents the physical model, and the principal statistics of the head are obtained. The uncertainty in the head is described by its standard deviation. By analysis of conditional probability, the uncertainty in the outflow constant is included in the problem. The cross correlation function between head and pumping, was found via spectral analysis. In the stochastic representation of the management model, the expected value of the objective function was used as an economic indicator; uncertainties in the demand of water and future availability of water facilities were included through chance constraints.

To examine the reliability of the proposed models, the results are compared with results obtained from a management model connected to a distributed type of model (Maddock, 1974). The sensitivity of the system to uncertainties is illustrated by an application of the models to a real basin in northwestern Mexico.

CHAPTER 2 DEVELOPMENT OF THE PHYSICAL MODEL

2.1 Introduction

In recent years many authors have used distributed models with the purpose of reproducing natural systems. Some of this work can be misleading in that very detailed models are not consistent with the field information available. In groundwater hydrology limited attention has been given to lumped parameter models (Kraijenhoff Van de Leur, 1958; Dooge, 1960; Van Schilfgaarde, 1965; Eriksson, 1970; Eliasson, 1971; Gelhar and Wilson, 1974) but none used this type of model in stochastic management of groundwater systems.

A lumped parameter model consisting of an aquifer water balance and a linear stream-aquifer flow will be developed in this work. This model is defined as the physical model, since it will deal with the physics of the groundwater flow system. The stream-aquifer interaction and the mean head in the aquifer are governed by this model. The output of the physical model will serve as a link to a management model.

In general, a system can be defined as a set of inter-related objects which can respond to one or several inputs producing one or a series of outputs. Many definitions of a system exist in several disciplines. Two interesting discussions in the hydrological literature can be found in Dooge (1973, p.3) and Chow (1973, p.17). A simplified representation of nature which tries to clarify its behavior by simulation is called a model. Chorafa (1965) defines simulation simply as a working analogy. A common practice is to use a mathematical model to simulate a complex system. A mathematical model is a set of mathematical equations used to describe a model. A proposed classification of hydrological models with reference to applications is given by Clarke (1973a, p.10; 1973b).

Two main types of models can be used to represent a hydrological system. The distributed model describes the spatial structure of a system and considers the inputs of the system as distributed in time and space. In general, a partial differential equation governs its behavior. The lumped parameter model groups inputs, and deals with a system in which temporal variation of the parameters is treated by an ordinary differential equation while spatial variation of the parameters is not considered. Black box is another name for a lumped model, because inputs and outputs can be measured, although the process which governs the system is masked, distorted or averaged. In other words no detailed description of interrelated processes is observed (Domenico, 1972, p.8).

2.2 Mathematical Representation of the Lumped Model

Description of the System

The system studied in this work is formed by two interconnected subsystems, an aquifer and a stream (Figure 2.1). An aquifer is defined as a saturated and permeable bed, formation or group of formations able to yield a

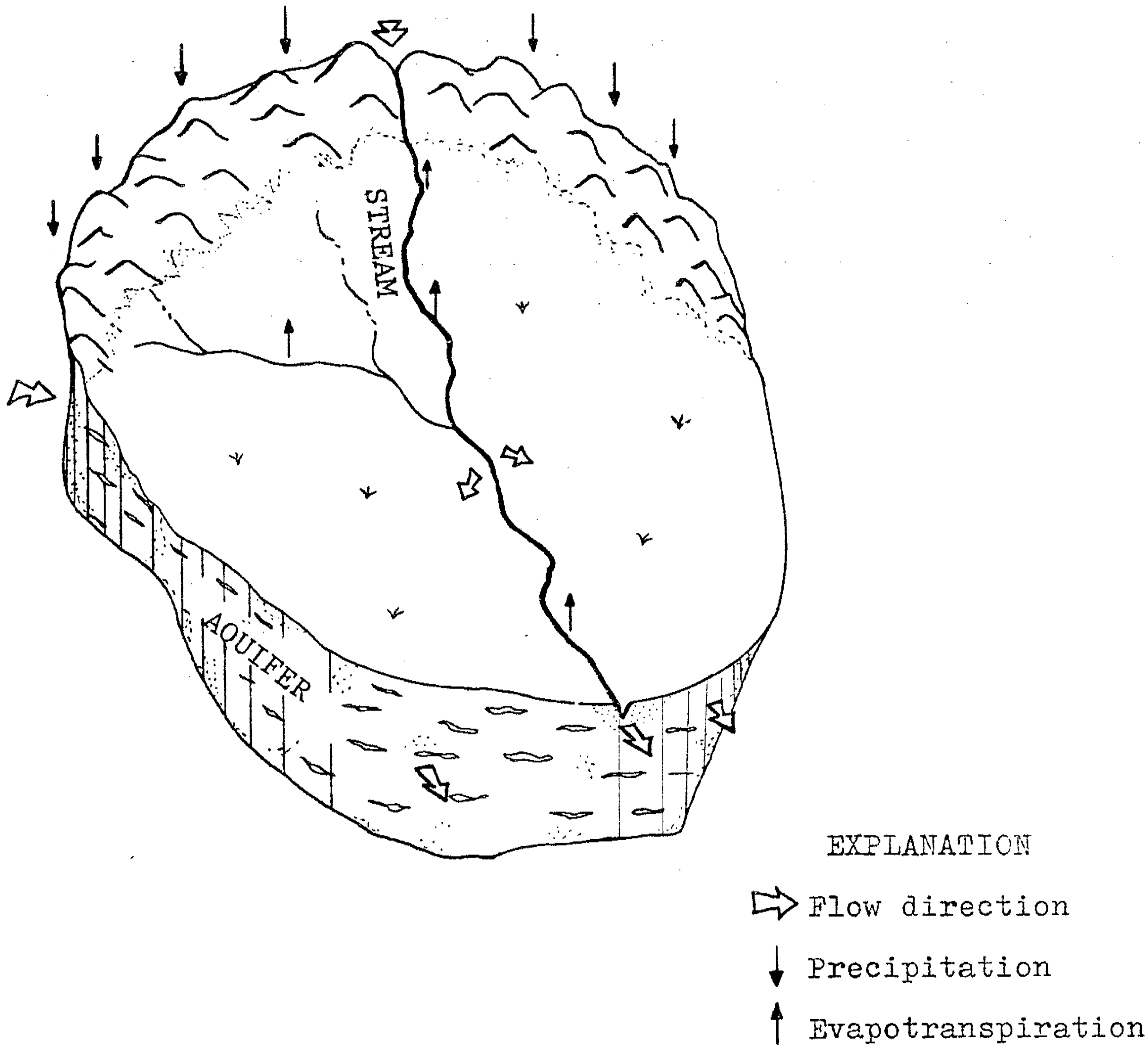


Fig. 2.1 Schematic representation of a natural stream-aquifer system.

substantial amount of water. The aquifer is unconfined or tends to be so at a regional scale. The stream can have one or several branches, is connected to the aquifer, and can be uncontrolled or controlled by a reservoir.

The system may or may not be connected to other systems. The inputs to the system are natural and artificial recharge, irrigation return flow, subsurface inflow and streamflow; the outputs from the system are pumping, subsurface outflow evapotranspiration and any downstream streamflow losses or water rights.

Development of the Lumped Model

The process that describes the behavior of the system is mass transport, governed by the law of conservation of water. The mathematical model which defines the system is an ordinary differential equation developed from the above principle and represents a water balance of the aquifer.

Amount of water that goes into the system in the interval Δt	−	Amount of water that comes out of the system in the interval Δt

= Change of amount of water stored in the aquifer in the interval Δt

or in symbols

$$(V_{in} - V_{out})/\Delta t = S\,\Delta V/\Delta t \qquad (2.1)$$

where, S is the storage coefficient (or specific yield).

A continuous representation can be found taking limits of both sides of equation 2.1

$$Q_{in} - Q_{out} = S\,dV/dt \qquad (2.2)$$

Now let

$$dV = Adh$$

and

$$y_R = (Q_{in} - Q_{out})/A$$

where y_R is the net inflow to the system, A is the area of the aquifer, and h is the mean water level in the aquifer. Substituting into equation 2.2 we obtain

$$S\,dh/dt = y_R \qquad (2.3)$$

The stream-aquifer flow may be approximated by a linear term

$$q = a\,(H - h)$$

where, $\underline{H}$ is the mean water level of the stream and $\underline{a}$ is called the subsurface outflow constant.

Introducing the above equation into (2.3) produces

$$S\, dh/dt + ah = y \qquad (2.4)$$

where, $\underline{y}$ is a fictitious net input of the system with units of L/T and based on the aquifer area $\underline{A}$, defined by

$$y = y_R + aH$$

Equation 2.4 is an ordinary differential equation which describes the aquifer and its connection to a stream in a lumped manner.

The physical model exactly represents the natural system except for the assumption of linearity in the stream-aquifer flow.

2.3 Subsurface Outflow Constant

Stream-Aquifer Interaction

The use of linear models to represent outflows is a common practice in surface hydrology (Chow, 1964, p.14.27). However, few investigators have tried to apply these simple models to depict subsurface flows.

In this section we make use of several elementary solutions for groundwater flow to determine the structure and magnitude of the subsurface outflow constant. Background information on the development of these elementary solutions can be found in texts on subsurface hydrology (see e.g., Bear, 1972). Though an actual system is nonlinear, the simplest most practical flow relationship between a stream and an aquifer, within the degree of accuracy required, is the linear one,

$$q = a(H - h) \qquad (2.5)$$

where

- q stream-aquifer flow, L/T;
- a subsurface outflow constant, 1/T;
- H mean stream stage, L;
- h mean water table in the aquifer, L.

Development of the Subsurface Outflow Constant

The subsurface outflow constant <u>a</u> is a very useful system parameter. It accounts for stream channel characteristics such as, stream bed properties; type of flow; it also subsurface outflow constant shows its relationship to the properties of the aquifer and the validity of the linear assumption in the stream-aquifer flow.

Effect of Linearity Assumption on the Stream-Aquifer Flow Computation

Let us consider a stream connected to an aquifer with natural recharge ϵ , and under steady flow conditions, as shown in Figure 2.2

Using the Dupuit approximation, the one dimensional flow equation is (Bear, 1972, p.376).

$$d/dx\,(Kh\,dh/dx) + \epsilon = 0 \qquad (2.8)$$

where K is the hydraulic conductivity of the aquifer (L/T). Integrating equation 2.8 and using the boundary condition

$$x = L, \quad dh/dx = 0$$

includes the aquifer geometry, transmissivity of the aquifer, and recharge and withdrawal distribution. This constant is related to the time that the aquifer takes to respond to an input and allows the computation of the stream-aquifer flow in a simple way.

The subsurface outflow constant, a , is defined as the stream-aquifer flow per unit aquifer area under a unit difference of mean head between the aquifer and a stream, which may be either influent or effluent.

Multiplying (2.5) by the horizontal area of the aquifer, A gives

$$Q_s = qA = aA(H - h) \tag{2.6}$$

which may be rewritten to give

$$a = (Q_s/A)/(H - h) \tag{2.7}$$

where the variables are

Q_s stream-aquifer discharge, L^3/T;

A aquifer area, L^2 .

The following mathematical procedure to find the we obtain

$$Kh \ (dh/dx) = \epsilon (L - x) \tag{2.9}$$

Integrating (2.9) with

$$x = 0, \quad h = H$$

we obtain

$$h^2 - H^2 = \epsilon x(2L - x)/K \tag{2.10}$$

Solving for <u>h</u>

$$h = H(1 + \epsilon x(2L - x)/KH^2)^{\frac{1}{2}}$$

A second order approximation of the term in parentheses is given by expanding the radical

$$h - H = \epsilon x(2L-x)/2KH - \epsilon^2 x^2 (2L-x)^2/8K^2H^3 \tag{2.11}$$

In addition,

$$\bar{h} - H = 1/L \int_0^L (h - H)dx \tag{2.12}$$

where $\bar{h}$ is the mean water table in the aquifer.
Substitute (2.11) into (2.12)

$$\bar{h} - H = \epsilon L^2(1 - \epsilon L^2/5TH)/3T \qquad (2.13)$$

where the transmissivity of the aquifer is $T = KH$.

From equation 2.11, we note that the difference between the maximum water level and the stream level, Δh , is

$$\Delta h = \epsilon L^2/2T - \epsilon^2 L^4/8T^2 H$$

or

$$\Delta h/H = (1 - \epsilon L^2/4TH)\epsilon L^2/2TH \qquad (2.14)$$

When the change in saturated thickness is small relative to the aquifer depth, H,($\Delta h/H \ll 1$) the term $\epsilon L^2/2TH$ is also small relative to unity; under these conditions it is reasonable to neglect $\epsilon L^2/5TH$ in (2.13).

Then since the flow is steady ($q = \epsilon$),

$$q = 3T(\bar{h} - H)/L^2$$

where

$$a = 3T/L^2 = \beta T/L^2 \qquad (2.15)$$

Therefore, the subsurface outflow constant <u>a</u> , depends on the transmissivity of the aquifer, <u>T</u> , a characteristic length <u>L</u> , and a dimensionless constant $\underline{\beta}$ which will be discussed subsequently.

The linear assumption in the stream-aquifer flow implies that (2.14) must be satisfied. Under steady conditions if the stream is influent or if the aquifer is subject to withdrawals as in Figure 2.3, the same analysis applies with $\epsilon < 0$ and the same subsurface outflow constant results.

Effect of Unsteady Flow on the Subsurface Outflow Constant

To examine the effect of unsteady flow on <u>a</u> , the Dupuit approximation given in linearized form (T = constant)

$$\partial (T\, \partial h/\partial x)/\partial x + \epsilon = S\, \partial h/\partial t \qquad (2.16)$$

is used with the initial condition

$$h - H = \epsilon_0 x(2L - x)/2T \qquad \text{if} \quad t < 0$$

$$\epsilon = 0 \qquad \text{if} \quad t \geqq 0$$

and boundary conditions similar to the steady state case. The solution to (2.16) is

$$h - H = \sum_{m=1,3,5,\ldots}^{\infty} A_m \exp(-m\pi\alpha/2L)^2 t \quad \mathrm{Sin}\,(m\pi x/2L) \qquad (2.17)$$

where A_m is the sine Fourier coefficient of $h - H$,

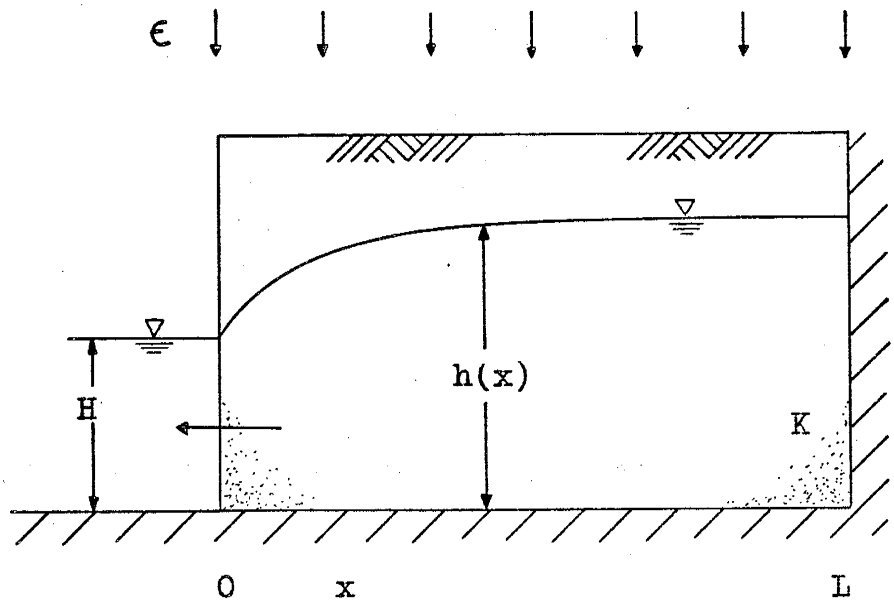


Fig. 2.2 Stream-aquifer interaction in an unconfined aquifer under stream effluent conditions

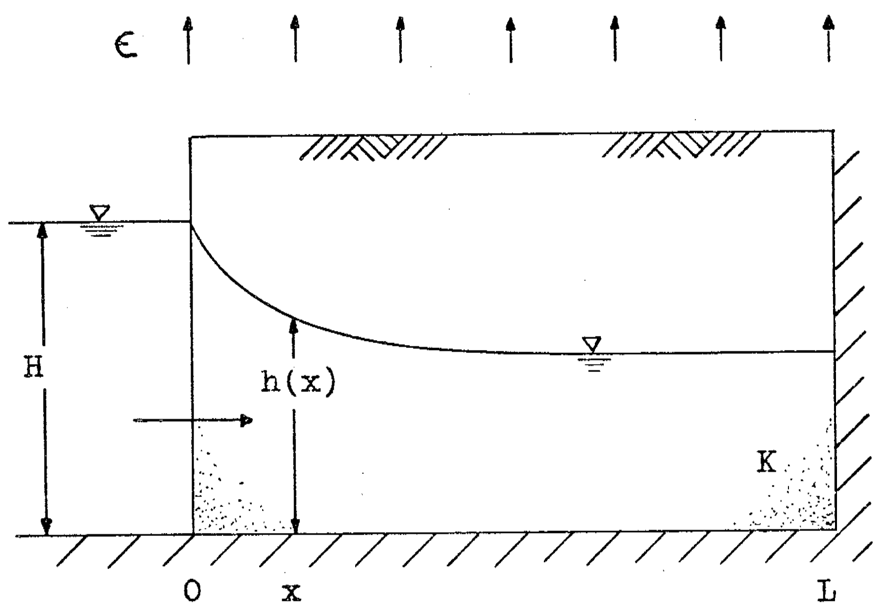


Fig. 2.3 Stream-aquifer interaction in an unconfined aquifer under stream influent conditions.

$$A_m = 8\,\epsilon_o L(1 - \cos m\pi)/Tm^3\,\pi^3$$

Using (2.17) and following a procedure similar to the steady case, an average head of the aquifer, $\underline{h}$, and the unit stream-aquifer discharge, $\underline{q}$, can be computed. Appendix A details the development of the solution of equation 2.16 and the computation of $\underline{a}$. Using (2.5) a value of the subsurface outflow constant was found for unsteady flow conditions.

$$a = T/4\,(\pi^2/L^2) = \beta\,T/L^2$$

where

$$\beta = \pi^2/4$$

The above examples of unsteady and steady flow conditions have shown us that the structure of the subsurface outflow constant remains the same except for the numerical value of the dimensionless constant, $\underline{\beta}$. Kraijenhoff Van de Leur (1958, p. B92) states that a constant ratio between the storage in the aquifer and outflow rate can be approximated in a period of depletion. His conclusion is similar to our introduction of the subsurface outflow constant.

If the water table decline is large the linearity of equation 2.5, may produce excessively large flows from stream to aquifer. In actuality, the flow approaches an asymptotic limit (see, Figure 2.4) due to two restrictions: (1) the rate of infiltration into the stream bed is limited as the water table is lowered below the stream bed and unsaturated flow conditions develop and (2) the outflow relation (equation 2.13) becomes nonlinear when the change in water level across the aquifer is of the same order as the aquifer thickness.

Therefore, the stream-aquifer flow is not only controlled by differences in head and aquifer properties but also by the streamflow and the physical characteristics of the stream bed.

Effect of Stream Clogging on the Subsurface Outflow Constant

Several situations affecting the stream-aquifer flow will be presented to give an idea of the range and type of variables that affect the dimensionless constant.

The effect of stream bed clogging on the stream-aquifer flow is shown first.

Applying Darcy's law to the aquifer flow (Figure 2.5), the head difference across the semipermeable layer, Δh, is approximated by

$$\Delta h = h_o - H \cong \epsilon Ld/HK_s \qquad (2.18)$$

where, d is the thickness of the restrictive layer and K_s its hydraulic conductivity (see Figure 2.5).

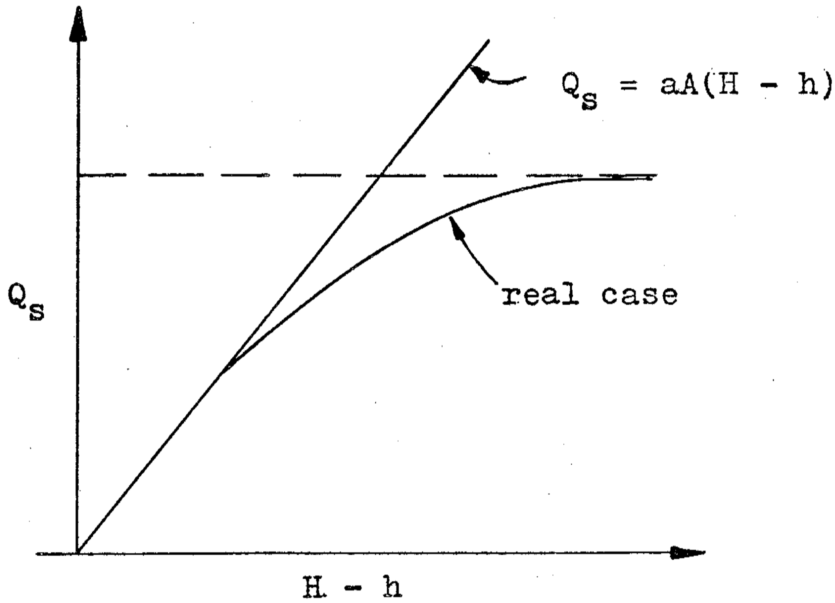


Fig. 2.4 Stream-aquifer flow as a function of the stream-aquifer head difference.

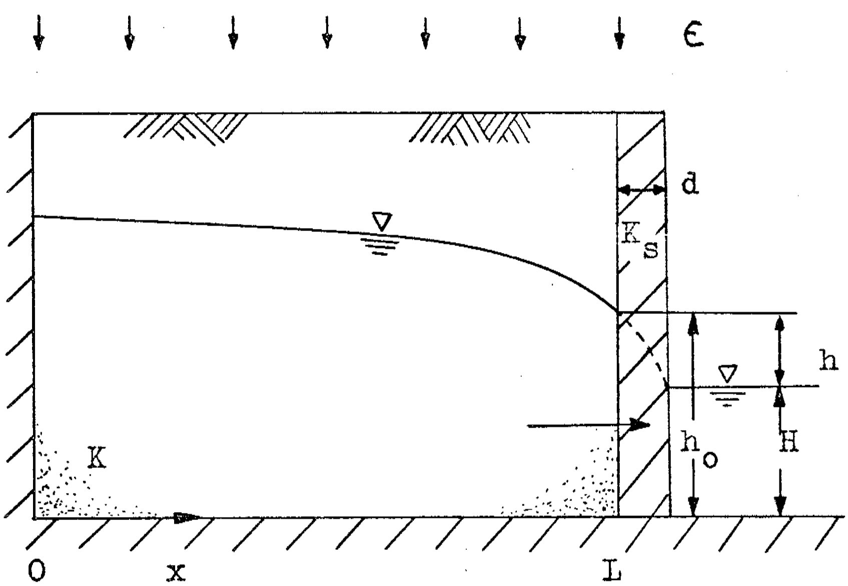


Fig. 2.5 Stream-aquifer flow, with the stream clogged by a semipermeable layer of thickness <u>d</u> and hydraulic conductivity K_s .

If steady state is considered or $q = \epsilon$, we have

$$q = a_c (h - H) \tag{2.19}$$

where a_c is the subsurface outflow constant corrected for the clogging layer. Also

$$q = a(h - h_o) \tag{2.20}$$

In Appendix B we show that

$$a_c = a/(1 + 3B^2/HL) \tag{2.21}$$

where <u>a</u>, is the subsurface outflow constant for steady state conditions, and <u>B</u> is defined as the leakage factor (Davis and De Wiest, 1967, p.225).

$$B = (T/(K_s/d))^{\frac{1}{2}}$$

As expected, a_c is smaller than <u>a</u> by a factor which depends on the square of the leakage factor. A larger <u>B</u> means smaller leakage, therefore a smaller a_c, and vice versa. Thus the stream channel characteristics can have a significant effect on the subsurface outflow constant.

<u>Effect of Converging Aquifer Flow on</u>
<u>the Subsurface Outflow Constant</u>

Two extreme cases will be considered: converging and diverging flow under steady state. The flow is radial and converges toward the system outlet, the stream, as shown in Figure 2.6.

Equation 2.16 for steady state in cylindrical coordinates is

$$1/r\ (d\ (Tr\ dh/dr)/dr) = -\epsilon \qquad (2.22)$$

where T is again taken as a constant.

The boundary conditions are

$$r = R_2 \quad , \quad dh/dr = 0 .$$

and

$$r = R_o \quad , \quad h = H$$

The solution of (2.22) is

$$T\ (h - H) = (R_o^2 - r^2)\,\epsilon/4 + (\ln(r/R_o))\,\epsilon\, R_2^2/2 \qquad (2.23)$$

The mean water level in the aquifer is

$$\bar{h} - H = 1/(R_2^2 - R_o^2)\,\pi \int_{R_o}^{R_2} (h - H)\ 2\pi r \quad dr \qquad (2.24)$$

Substituting (2.23) into (2.24), integrating and assuming $\delta = R_o/R_2 << 1$, we obtain

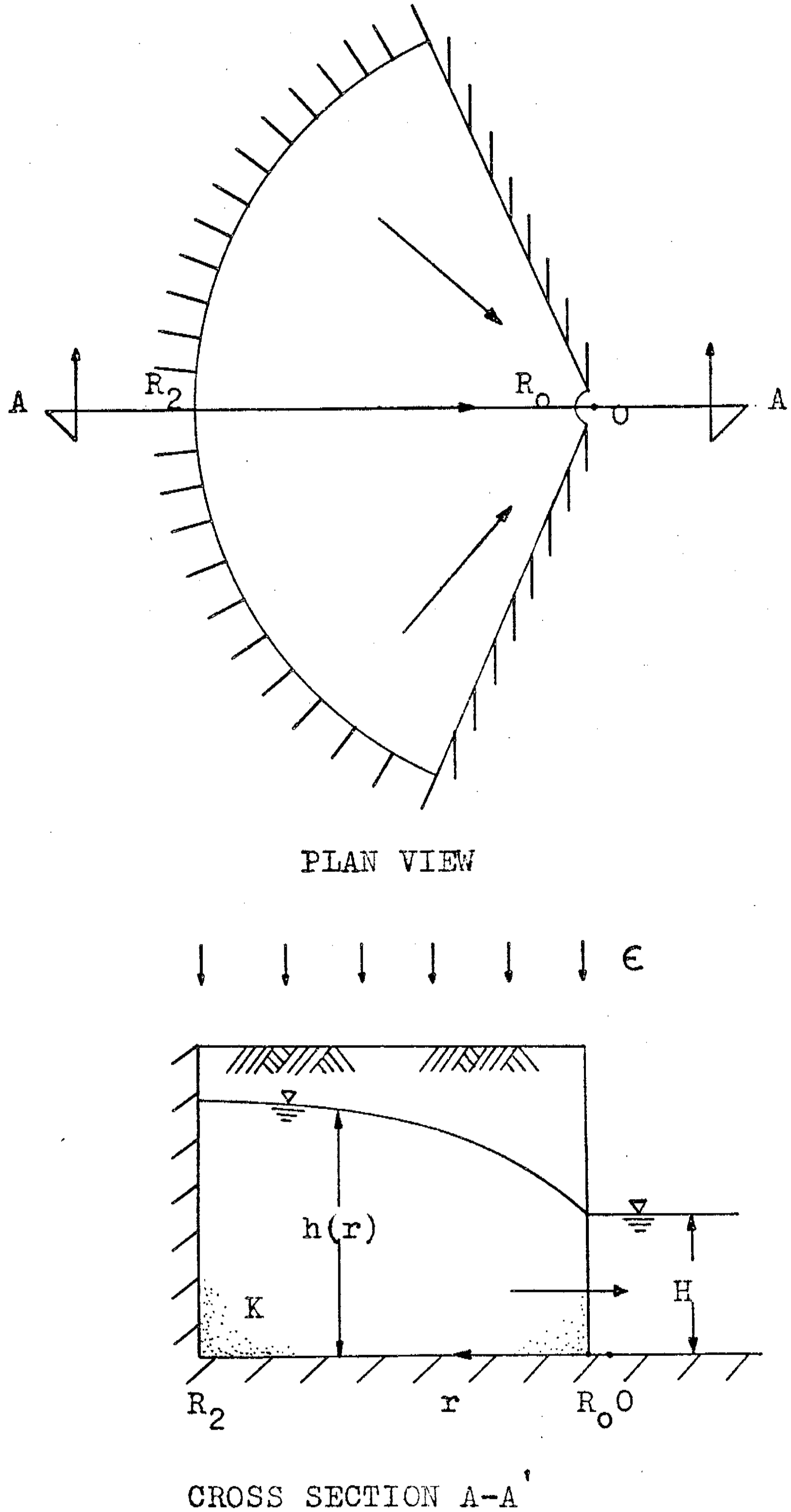


Fig. 2.6 Converging aquifer-stream type of flow.

$$\bar{h} - H = ((1 + 2\delta)\ln \delta^{-1}/2 - 3(1 + 2\delta)/8)\epsilon L^2/T \tag{2.25}$$

Using (2.5) and (2.25), we have

$$a = (1/(1 + 2\delta)\cdot(\ln \delta^{-1}/2 - 3/8))T/L^2 \tag{2.26}$$

Additional information concerning the development of the above equations is available in Appendix C .

To give an idea of the magnitude of β for this type of flow, assume that

$$\delta = R_o/R_2 = 0.1$$

and substitute into equation 2.26 to obtain

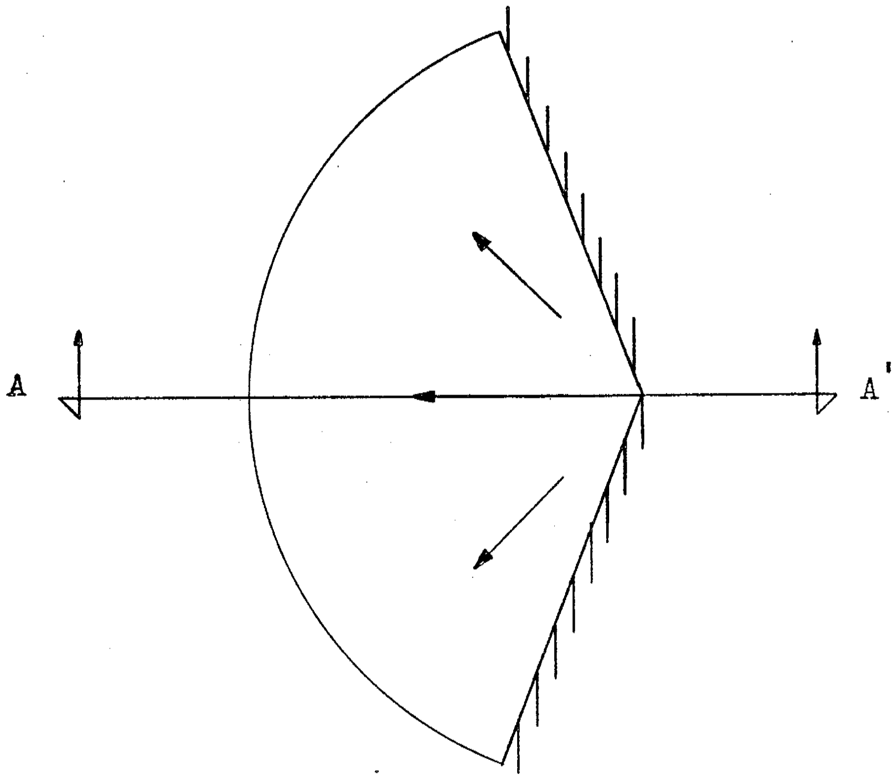
$$a = 1.07\ T/L^2$$

and hence

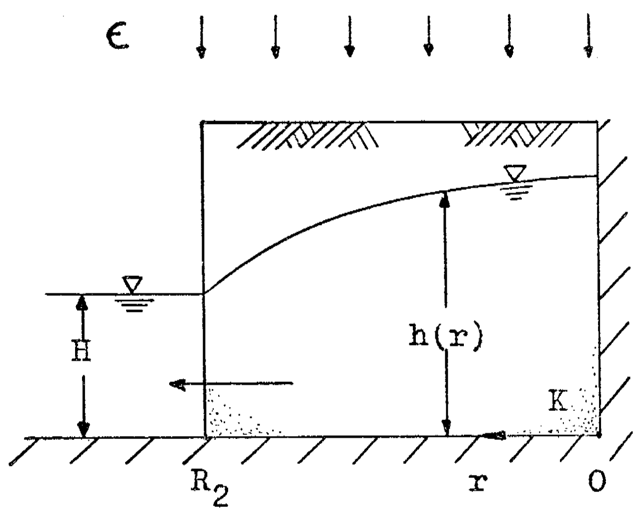
$$\beta = 1.07$$

Effect of Diverging Aquifer Flow on the Subsurface Outflow Constant

To simulate the other extreme flow case, diverging flow in an unconfined aquifer is considered. Due to the aquifer shape, the flow is forced radially to a constant head boundary as shown in Figure 2.7.



PLAN VIEW



CROSS SECTION A-A'

Fig. 2.7 Diverging aquifer-stream type of flow.

Using equation 2.22 plus the two boundary conditions

$r = 0 \quad , \quad dh/dr = 0$

$r = R_2 \quad , \quad h = H$

we obtain the solution

$$T\ (h - H) = \epsilon\ (R_2^2 - r^2)/4 \qquad (2.27)$$

Computing the average water level as

$$\bar{h} - H = \int_0^{R_2} 2\pi h\ r\ \ dr/\ \pi R_2^2 \qquad (2.28)$$

and substituting equation 2.27 into (2.28), we have

$$\bar{h} - H = \epsilon\ R_2^2/8T$$

Making use of equation 2.5

$$a = 8T/L^2 \qquad (2.29)$$

The constant <u>a</u> is identical for a semicircular or a wedge shaped aquifer with the same natural recharge $\underline{\epsilon}$.

Therefore, a reasonable range for β as a function of aquifer geometry (converging and diverging flow) is 1 to 8. This range of variation on β demonstrates the importance of a three dimensional flow on the computation of a.

Effect of Recharge Distribution on the Subsurface Outflow Constant

Consider steady flow in a homogeneous aquifer in which the recharge occurs at the upper reaches of a basin (see Figure 2.8). For the computation, the aquifer is divided into two regions. Mathematically, this problem can be posed as

$$\epsilon_1 \alpha L = \epsilon L$$

where ϵ_1 is the actual rate of recharge and ϵ is the rate of recharge applied over the entire aquifer.

The differential equation applicable to region 1 is

$$d/dx\ (T\ dh_1/dx) = -\epsilon_1 \qquad (2.30)$$

where T is a constant. The boundary conditions are

$$dh_1/dx = 0 \quad , \quad x = 0$$

$$h = h_1 \quad , \quad x = \alpha L$$

For region 2, the differential equation is

$$d/dx\ (T\ dh_2/dx) = 0 \qquad (2.31)$$

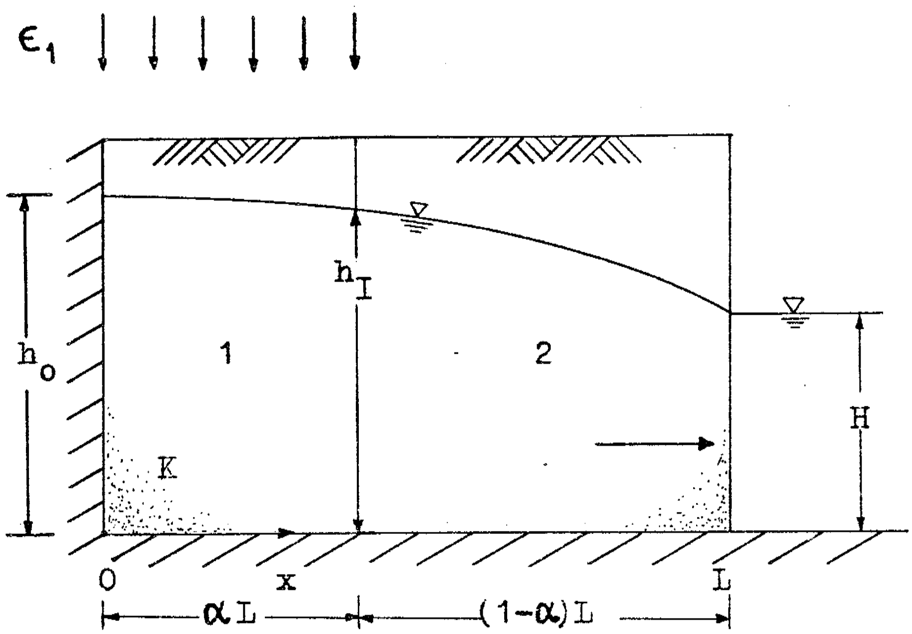


Fig. 2.8 Stream-aquifer interaction with recharge applied on a portion of the basin.

and the boundary conditions are

$$T\ dh_2/dx = T\ dh_1/dx \quad , \quad x = \alpha L$$

$$h = H \quad , \quad x = L$$

The solutions of (2.30) and (2.31) are

$$h_1 = H + (2\alpha L^2 \ (1 - \alpha) + \alpha^2 L^2 - x^2) \ \epsilon_1/2T \tag{2.32}$$

and

$$h_2 = H + \alpha \ \epsilon_1 \ L(L - x)/T \tag{2.33}$$

The mean water level is computed by

$$\bar{h} = 1/L \ (\int_0^{\alpha L} h_1 \ dx + \int_{\alpha L}^{L} h_2 \ dx) \tag{2.34}$$

Therefore,

$$\bar{h} - H = (3\alpha/2 - \alpha^3/2)\epsilon_1 L^2/3T \tag{2.35}$$

and, since $\epsilon_1 = \epsilon/\alpha = q/\alpha$,

(2.35) becomes

$$\bar{h} - H = (3/2 - \alpha^2/2) \ qL^2/3T \tag{2.36}$$

Using (2.5), we find the subsurface outflow constant

$$a = 6T/(3 - \alpha^2)L^2 \qquad (2.37)$$

With equation 2.37, we compute values for the two possible extremes which might occur in nature, and hence the range of the recharge distribution effect on the subsurface outflow constant.

If $\alpha = 1$, recharge occurs over the entire aquifer and

$$a = 3T/L^2 \qquad (2.38)$$

If $\alpha = 0$, recharge is concentrated on the impermeable boundary, and

$$a = 2T/L^2 \qquad (2.39)$$

Equation 2.38 represents, of course, the value of a already computed for the steady-state case, and equation 2.39 can easily be verified by using Darcy's law, as follows:

$$\epsilon L = T(h_o - H)/ L \quad ; \qquad h_o - H = \epsilon L^2/T$$

Since the head distribution is linear, we may write

$$\bar{h} - H = \epsilon L^2/2T$$

and with equation 2.5, we obtain

$$a = 2T/L^2$$

which verifies (2.39).

The obvious conclusion from the above analysis is that the effect of the distribution of recharge or withdrawal is not very significant, since β ranges only from 2 to 3.

The effects of different aquifer properties in individual segments of a stream-aquifer are discussed in Appendix D. It is shown there that the system can be represented by a single linear reservoir only when a and S are the same in each segment.

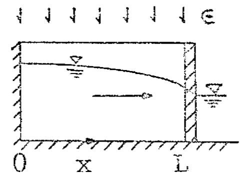
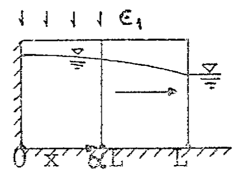
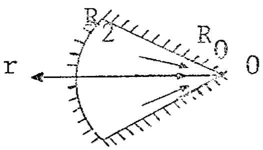
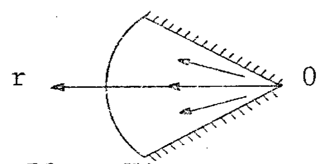
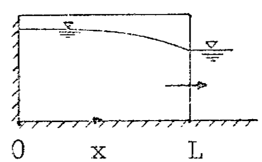
A summary of the range of a , for all studied possibilities is shown in Table 2.1. The β ranges from 1.07 to 8.

Gelhar et.al. (1974, p.94) applied spectral analysis to a Dupuit aquifer with recharge over the entire basin, and obtained values of β within the above range. Appendix E gives a mathematical justification for the use of average aquifer and stream water levels in equation 2.5.

In regions with limited field information Table 2.1 can be a helpful tool. If piezometric data exist, a more reliable selection of a can be made by the following procedure:

a) From a piezometric map, compute the mean water level.

TABLE 2.1 Type of Effects on the Subsurface Outflow Constant a, and the Respective Values of a, and β.

Type of Flow	Effect on a		Schematic Flow Picture	a	Parameters Used to Compute a	β
Steady	Clogging			$(3T/L^2)/(1+3B^2/HL)$	$B = 1.4x10\ m^2$ $H = 350\ m$ $L = 1.6x10^3\ m$ $K_s = 10^{-3}\ K$	1.71
	Recharge Distribution			$(3T/L^2)/(3/2-\alpha^2/2)$	$\alpha = 0$ $\alpha = 1$	2.0 3.0
	Aquifer Geometry	Converging	Plane View	$\frac{3T/L^2}{3(1-2\delta)(\ln\delta/2-3/8)}$	$\delta = R_0/R = 0.1$	1.07
		Diverging	Plane View	$8T/L^2$		8.0
Unsteady	Type of Flow	Falling Sinusoidal Water Table		$\pi^2 T/4L^2$		2.47

b) Apply Darcy's law to the flow passing through the stream tubes close to the stream and calculate a mean stream-aquifer flow or estimate the stream-aquifer flow from an aquifer water balance.

c) Calculate a mean stream bed elevation from a topographic map and use it to compute a mean stream elevation.

d) Apply equation 2.6 and obtain the subsurface outflow constant. Note that if the wells close to the stream are shallow an error in the stream-aquifer discharge can be introduced in the flow computation (b), due to the fact that the hydraulic gradients at the water table are greater than the gradients deeper in the aquifer.

The previous analysis investigates the validity of the linear outflow assumption in representing the stream-aquifer interaction. It gives a rather narrow range of variation of the subsurface outflow constant and presents a simple field procedure to obtain the latter. The subsurface outflow constant groups most of the system properties and helps to define the stream-aquifer flow in a simple way. Therefore the subsurface outflow constant is a very important parameter for defining or condensing important properties of the system. Its limitations include the effect of a deep water table in the stream-aquifer flow.

2.4 Techniques Used to Solve the Lumped Model

Analytical Solution

The first technique presented for solving the lumped model is an analytical one. The solution of equation 2.4 is

$$h(t) = h_o \exp(-at/S) + 1/S \int_0^t y(\tau) \exp(-a(t-\tau)/S) d\tau \qquad (2.40)$$

where h_o is the initial condition at $t = 0$.

The first term on the right hand side represents the initial condition effect and the second term includes the effect of all past inputs $y(\tau)$, on the system. The integral is usually called the superposition or convolution integral (Miller, 1963, p.273). The mathematical development of equation 2.40 is given in Appendix F .

Finite Difference Approach

A finite difference representation of equation 2.40 can be written as

$$(h_{i+1} - h_i) S/\Delta t + (h_{i+1} + h_i) a/2 = y_i \qquad (2.41)$$

Solving equation 2.41 for h_{i+1} will give

$$h_{i+1} = (y_i \Delta t/S + (1 - a\Delta t/2S) h_i)/(1 + a\Delta t/2S) \qquad (2.42)$$

Equation 2.42 is easily programmed for an electronic computer; however, significant errors may be introduced by an improper choice of the time interval Δt , as illustrated by the following analysis.

Define the response time $t_h = S/a$ and assume, for simplicity, that y_i is zero. The exact solution of the lumped model (see, App. F) is

$$h_{i+1} = h_i \,(\exp(-\Delta t/t_h)$$

Using a Taylor series expansion, we obtain

$$h_{i+1} \cong h_i \,(1 - (\Delta t/t_h) + (\Delta t/t_h)^2/2 - (\Delta t/t_h)^3/6 + \ldots.) \quad (2.43)$$

Expanding the denominator of equation 2.42 and assuming that y_i' is zero, we obtain

$$h_{i+1} \cong h_i(1 - \Delta t/2t_h)\,(1 - (\Delta t/2t_h) + (\Delta t/2t_h)^2 - (\Delta t/2t_h)^3 + \ldots.)$$

or

$$h_{i+1} \cong h_i \,(1 - (\Delta t/t_h) + (\Delta t/t_h)^2/2 - (\Delta t/t_h)^3/4 + \ldots.) \quad (2.44)$$

The difference between equations 2.43 and 2.44 is on the order of

$$h_{exact} - h_{finite\ differences} = (\Delta t/t_h)^3/12 \qquad (2.45)$$

considering the first four terms of the expansions.

Therefore, a finite difference representation of the head $\underline{h}$, can introduce a significant error that depends on the $\Delta t/(S/a)$ ratio. Thus it is seen that the increment time of a specific problem must be selected carefully.

For instance, a ratio $\Delta t/t_h$ of approximately 0.5 produces an error of about 1%. Since the error depends on the third power (equation 2.45), $\Delta t/t_h$ ratios greater than 1 are not recommended.

Discrete Representation

A discrete representation of the exact solution of the lumped model was preferred over the finite difference approach. Equation 2.4 is rewritten

$$S\ dh/dt + a\ (h - y_i/a) = 0$$

If y_i is constant during the time interval (i, i+1) this can be written

$$S\ (d/dt)\ (h - y_i/a) + a\ (h - y_i/a) = 0$$

It has the solution

$$h - y_i/a = C \exp(-at/S)$$

where $\underline{C}$ is a constant of integration.

The initial conditions are

$$\text{at} \quad t = 0 \quad , \quad h = h_i$$

$$\text{at} \quad t = t \quad , \quad h = h_{i+1}$$

Therefore ,

$$C = h_i - y_i/a$$

which yields

$$h_{i+1} = h_i \exp(-a\Delta t/S) + (1 - \exp(-a\Delta t/S))y_i/a \tag{2.46}$$

This is equivalent to (2.40) with the input y_i a constant in a time interval Δt_i .

2.5 Response Time

The response time, t_h , of the stream-aquifer system is the time required for the water level excess over that in the stream to drop to $1/e$ times the original level when there is no net inflow to the aquifer (see Figure 2.9).

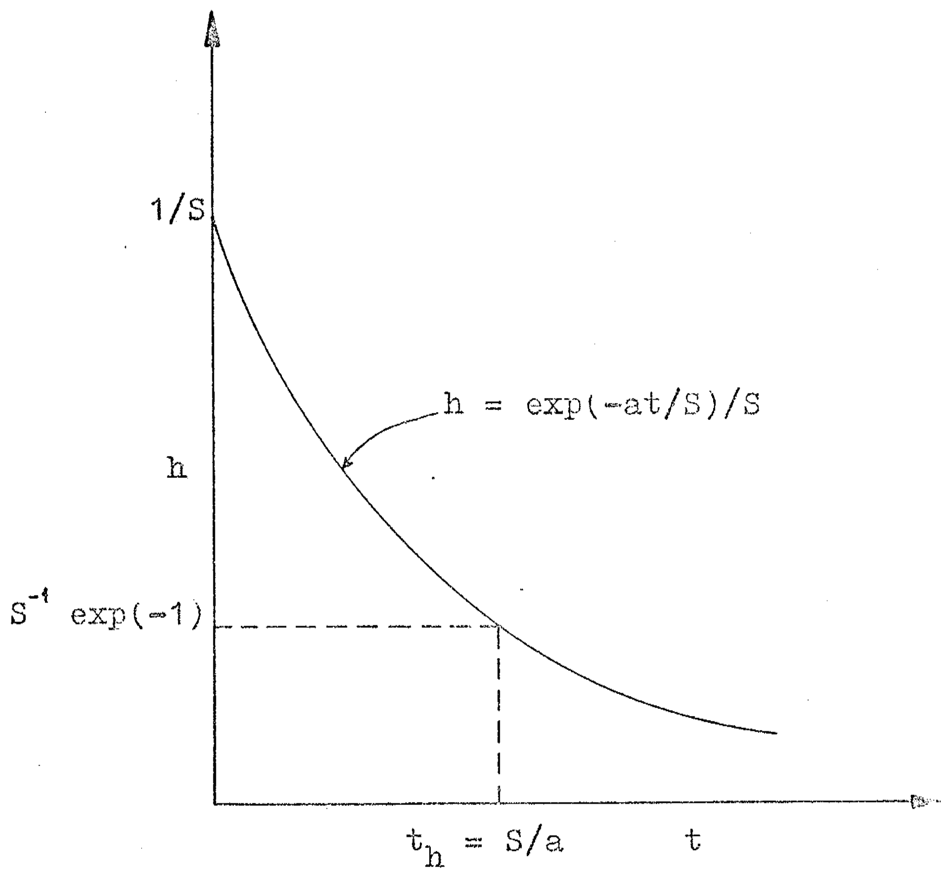


Fig. 2.9 Response time t_h , as a function of the stream-aquifer unit response.

The unit response of the system (Appendix F) is

$$h = \exp(-\ at/S)/S$$

and

$$h_{t_h} = 1/Se$$

Therefore,

$$t_h = S/a \tag{2.47}$$

where S is the storage coefficient and a is the subsurface outflow constant.

The ratio of an active volume above the aquifer storage with zero outflow, ΔV , to the stream-aquifer discharge, Q_s , is another definition of response time.

$$t_h = \Delta V/Q_s$$

or

$$t_h = AS(H - h)/Q_s$$

where $Q_s = Aa(H - h)$ is the stream-aquifer discharge. Therefore,

$$t_h = S/a$$

Note that this same definition would apply when the river is influent, but in this case the volume increment is the active aquifer deficit.

Chapman (1964) mentioned the importance of the ratio of storage to flow. He gives a table of typical values and also mentions that in arid regions a value of at least fifty years, safely let us use a steady state formula to compute the flow. In an actual situation the steady state approximation is adequate if the long-term average of groundwater flow is more important than oscillations caused by non-steady fluctuations, as noted by Kraijenhoff (1954). Hence, as the response time increases it will be more appropiate to use the steady state value of the dimensionless constant β.

2.6 Well Drawdown Correction

As noted earlier the output of the lumped model is a mean water level in the aquifer; its use is limited to specific types of problems. Costs of pumping determined by a model of this sort would be underestimated. Therefore, in order to make our physical model capable of accounting not only for average drawdowns but for local drawdowns at the wells, a correction was developed and added to the model.

Several papers exist in the field of petroleum engineering concerning the relationship of average pressures and specific pressures in a bounded reservoir. Matthews et. al.

(1954), presented a procedure using the superposition principle for finding the difference between a so-called extrapolated pressure and an average pressure for different shapes of reservoirs. Earlougher et.al. (1968) developed a simplified procedure to find the pressure distribution in a bounded square, which was used as a building block to generate flow behavior in any rectangular region. Ramey et.al. (1973) checked the results obtained by earlier authors and presented programs to solve rectangular shapes with different types of boundaries.

To develop an average drawdown correction the following assumptions are made: the flow in the aquifer is unsteady; the aquifer area influenced by a given pumping well is a square of impermeable boundaries; the aquifer is confined. Inasmuch as our model is lumped, a mean area of influence for a well was obtained by dividing the entire area of study by the number of wells. The work by Earlougher et.al. (1968) is followed closely.

First some of the variables entering in the problem are presented. A water balance of a closed square is used as the basic tool. Let

$$\bar{s} = Qt/AS$$

where $\bar{s}$ is an average drawdown, and $\underline{A}$ is the area of the square. Rearranging and multiplying the above equation by $2\pi T$, we find

$$2\pi T \bar{s}/Q = 2\pi Tt/AS$$

Noting that the left hand side of this equation is an average dimensionless pressure, $\bar{p}_D$, and the right hand side of the equation can represent a dimensionless time based on <u>A</u> , t_{DA} ; we have

$$\bar{p}_D = 2\pi t_{DA}$$

where

$$\bar{p}_D = 2\pi T \bar{s}/Q$$

and

$$t_{DA} = Tt/AS$$

A table showing the dimensionless pressure, p_D, against t_{DA} for $\sqrt{A}/r_w = 2000$ is given by Earlougher et.al. (1968). If different ratios of $\sqrt{A}/r_w$ are found a correction must be added to the p_D values given in the table; it is

$$\ln((\sqrt{A}/r)/2000) \qquad (2.48)$$

For $t_{DA} \geqq 0.2$ was found that the difference in pressure $p_D - \bar{p}_D$ was almost constant and equal to 6.29, as shown in Figure 2.10. Therefore, a simple formula is developed to represent the average drawdown at the wells and it is

$$s_w - \bar{s} = (6.29 + \ln((\sqrt{A}/r_w)/2000))Q/2\pi T \qquad (2.49)$$

where Q is an average instantaneous pumping.

The average drawdown of the aquifer $\bar{s}$ is obtained from the lumped model.

Conclusion

The use of the physical model presented in this chapter is straightforward. It was prepared with the purpose of linking it to a management model. It takes into account the stream-aquifer connection and models the aquifer under any type of input. It represents a water balance of the aquifer and the only assumption made throughout its development is a linear outflow which in most cases is satisfactory. Two useful concepts were developed: the subsurface outflow constant and the aquifer response time. The subsurface outflow constant a , condenses several properties of the system and allows for a simple stream-aquifer interaction. The aquifer response time is a ratio of an active volume above or below a basic aquifer storage with zero outflow, to the stream-aquifer flow. The analysis shows that the range of a is not large and a simple field procedure can be used to determine this parameter. Since our model is

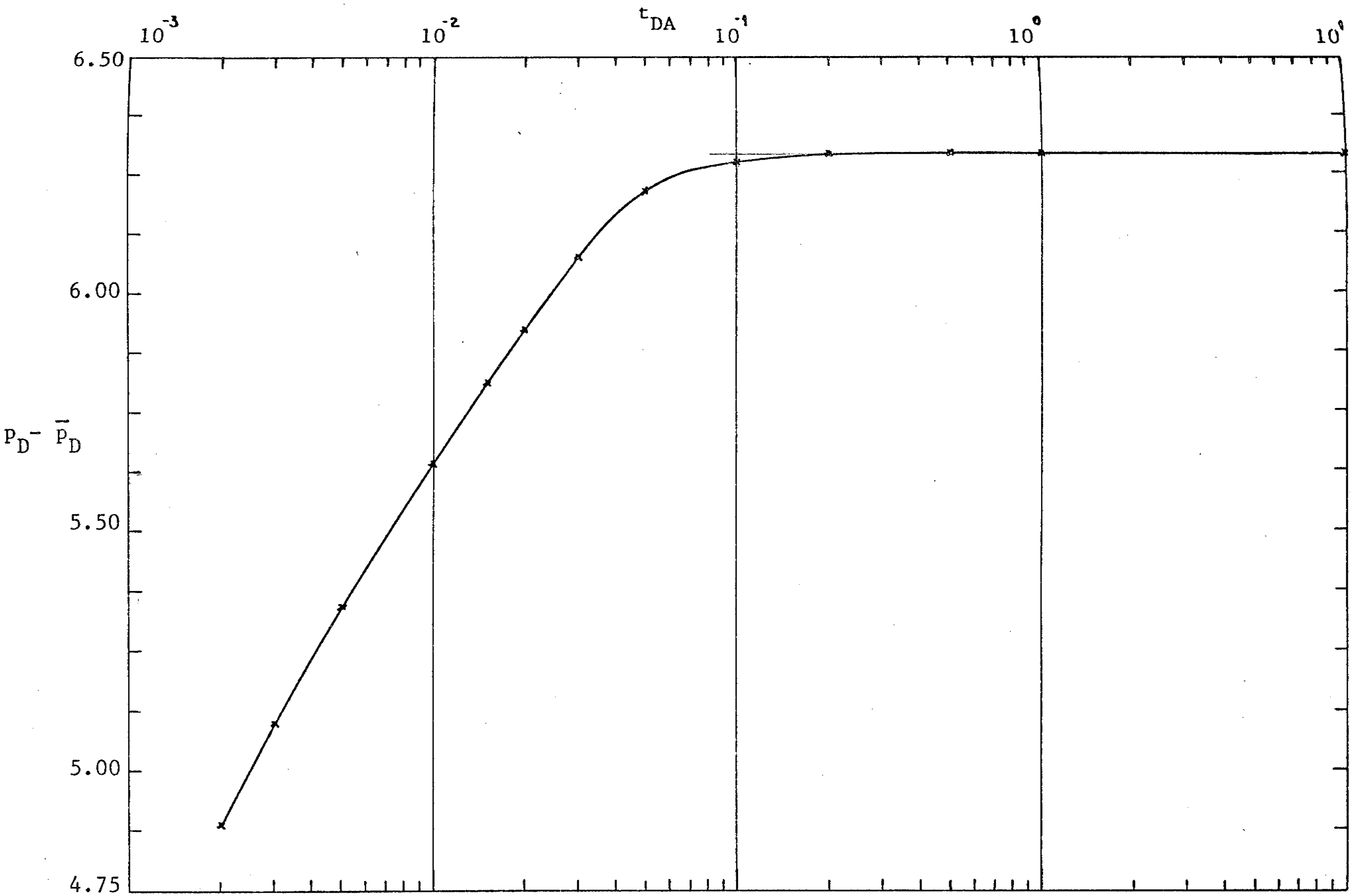


Fig. 2.10 Graph of the dimensional pressure difference $p_D - \bar{p}_D$ against the dimensionless time t_{DA} .

lumped, all the variables are worked out in terms of space averages. A drawdown correction is successfully added to the average aquifer water level, to represent the drawdown in the wells.

CHAPTER 3 MANAGEMENT MODEL DEVELOPMENT

3.1 Introduction

One objective of this study is to present a simple and logical procedure for operating a system formed by a stream connected to an aquifer under optimal economic conditions. The interaction between the economics and the physics of the system is represented by the management model, which yields the optimal policy of operation. The model may also be used for design of surface and groundwater facilities (e.g. size of a dam, well fields and main canals).

The pumping cost will be considered as variable and dependent, on the pumping volume and the total lift. Linear decision rules will be used to define the decision variables. These linear decision rules allow for more dynamics in the system operation and for a deterministic analogy of chance constraints. A suitable link will be developed between the physical and management models. This link simplifies the objective function by computing the physical model outside of the management model. As a consequence of this link, it is possible to use an iterative procedure and a standard linear programming technique to solve a nonlinear optimization problem.

3.2 Systems Analysis Approach

Terminology

Systems Analysis or Operations Research is defined as a scientific approach to problem solving for executive management (Wagner 1969, p.4). However one of the main problems encountered in the use of water resources is the difference that exists in space, time and quality between the natural water supply and the water demand. Therefore, a more specific definition of operations research is; the art or science of choosing from a number of feasible alternatives whether it be in relation to planning, design, construction or operating a water resources system. An interesting discussion on the subject is found in Hall and Dracup (1970, p.39). The analysis of the set of alternatives is carried out in an organized common-sense strategy of techniques; ranking them according to a desired criterion e.g. an optimality criterion. The set of techniques available to solve the problem of development, allocation and use of limited resources to the best advantage is called Mathematical Programming. Optimization should be understood as the problem of finding the best type of action from a set of alternatives. An optimization procedure selects an optimal policy. A set of decision variables that maximizes or minimizes a performance function subject to the system constraints is called an optimal policy. An objective function, return function, value function or criterion function is a function that establishes the criterion by which the best solution is selected.

Objectives and Limitations

The main objective of the management model is to reproduce the economics and physics of the problem and generate an optimal policy for the operation of the stream-aquifer system.

Different types of objectives of a water resource system can be thought of; social, economic and political or a combination of them. To simplify and to obtain a better understanding of the stream-aquifer management problem, the optimality criterion used in this study is based only on economic terms. This study is based on the following assumptions: (i) the management model is independent of changes in economic activities generated by decisions taken during the operation of the system, (ii) a central agency is responsible for the management of the system under non-competitive conditions, (iii) the water is used for agriculture only and (iv) no penalties are applied if the demand is not satisfied.

3.3 Mathematical Programming

Before going to the technique used to obtain the optimal policy, the main componentes of the optimization procedure will be discussed.

Objective Function

The main guiding principle used to select the objective function was the allocation of scarce water resources at the minimum possible cost. Inasmuch as operational costs need to be computed, a comparison of costs occuring at different points of time in the future was necessary. The present worth was, therefore, the economic concept used to bring out all costs to the same reference level and in order to perform this operation a nominal interest rate was used; to review these concepts in more detail an engineering economics book such as De Garno (1960) is recommended.

The objective function selected considering conjunctive use of surface and groundwater was the discounted cost, that is

$$W = \sum_{i=1}^{n} \left(C_{S_i}\; Q_{SD_i} / (1 + r/N_S)^i + C_{P_i}\; Q_{P_i} (Z - h_i^{'}) / (1 + r/N_S)^i \right) \tag{3.1}$$

where the parameters are

C_S unit cost of surface water, $\$/L^3$;

C_P unit cost of pumpage, $\$/L^4$;

Q_{SD} quantity of water diverted from the stream, L^3 ;

Q_P quantity of water pumped from the aquifer, L^3 ;

Z ground surface level, L ;

$h^{'}$ mean water level at the wells, L ;

r nominal interes rate;

n design period, T;

N_S number of seasons per year.

The first term of the right hand side of equation 3.1 represents the discounted cost of diverting water from the stream and is linear with respect to the amount of flow Q_{SD}. The second term is the discounted cost of pumping water out of the aquifer and is quadratic with respect to Q_P, since $\underline{h}'$ depends on all past inflows of the system including pumping. In other words the groundwater pumping cost is a function of the total lift and the volume pumped. Hence our problem can be classified as a nonlinear programming type, for which standard solution packages exist (Kuester, 1973, p. 105). They have certain limitations in terms of initial assumptions, preparation of data and computer storage. Dynamic Programming Techniques could also be used but they require special computer programs for each specific problem. Aron (1969, p.40) gives advantages and disadvantages of the technique.

The purpose of this study was not to test different mathematical programming procedures but to develop a simple technique able to solve nonlinear optimization problems of the type described. This is done by taking advantage of the coupling between the physical and management model.

Constraints

Three types of constraints were used for an uncontrolled stream connected to an aquifer. The first deals with the demand of water to be satisfied; the second with the surface water diversion and pumping facilities; and the last with the water requirements to be met by the stream.

The demand of water constraint is represented by

$$Q_{SD_i} + Q_{P_i} \geqq D_i \tag{3.2}$$

and says that the sum of surface water diverted from the stream Q_{SD_i}, plus the amount of water pumped out of the aquifer Q_{P_i}, should satisfy the water demand for a given period of time.

The pumping facilities constraint is

$$Q_{P_i} \leqq Q_{P_i}^{'} \tag{3.3}$$

where $Q_{P_i}^{'}$ is the maximum pumped volume allowable, at time $\underline{i}$. This constraint establishes a limit, equal to the maximum capacity of pumping, for the amount of water pumped out of the aquifer at time $\underline{i}$.

The surface water diversion constraint is defined as

$$Q_{SD_i} \leqq Q_{SD_i}^{'} \tag{3.4}$$

where $Q_{SD_i}^{'}$ is the maximum allowable diverted volume from the stream at time $\underline{i}$. The constraint says that the amount of water diverted from the stream must be less or equal to the surface facilities vailable at time $\underline{i}$.

The last constraint, called the stream requirements constraint, consists of the conservation of matter principle applied to the stream. Recall that this principle was already considered in the aquifer, and the mean water level h_i , is a result of its application. Stream and aquifer are coupled through this constraint which is very dynamic and restrictive with respect to the system operation.

Applying the conservation of matter principle to the stream under steady state conditions.

$$Q_{ST_i} - Q_{SD_i} - Q_{S_i} \geqq K1_i \tag{3.5}$$

the parameters are defined as:

Q_{ST_i} streamflow at time $\underline{i}$;
Q_{SD_i} water diverted from the stream at time $\underline{i}$;
Q_{S_i} stream-aquifer flow at time $\underline{i}$;
$K1_i$ downstream flow required at time $\underline{i}$.

Substituting equation 2.10 into 3.5, we obtain

$$Q_{ST_i} - Q_{SD_i} - Aa\ (H_i - h_i) \geqq K1_i \tag{3.6}$$

which requires that the net sum of flows through the stream must be greater than or equal to any senior right existing downstream of the study area, at the time $\underline{i}$.

Decision Variables

The linear decision rule used to define the decision variables was introduced by Charnes et.al. (1958) for an optimization problem and then applied by Revelle et.al. (1969) to a surface water management problem. Two important features of the linear decision rule are: (1) In a stochastic management problem, chance constraints can be changed to their deterministic equivalent; (2) It is highly desirable to base present decisions on a previous state of the process. A linear decision rule can be defined as $R_2 = \beta_{21} S_1 + \gamma_2$ (Charnes et.al. 1958) where β_{21}, γ_2 are the decision variables, R_2 is the unknown variable at time 2, and S_1 defines the state of the process at time 1. Several variants of this decision rule can be obtained (Charnes and Cooper 1963). Two types of decision rules are used in this study to define three decision variables. The first two decision variables use a linear decision rule such as

$$R_2 = \beta_{21} S_1$$

in which Charnes's notation is followed. The last decision variable makes use of a linear decision rule equal to that applied by Revelle et.al. (1969).

$$R_2 = S_1 + \gamma_2$$

The decision variables used in this study are:

(1) The diversion of surface water decision variable

$$\gamma_{S_i} = Q_{SD_i} / D_{i-1} \qquad (3.7)$$

(2) The groundwater decision variable

$$\gamma_{P_i} = Q_{P_i} / AS_{h_{i-1}} \qquad (3.8)$$

(3) If a dam controls the stream, a decision variable related to the dam operation is given by

$$\gamma_{B_i} = S'_{i-1} - Q_{ou_i} \qquad (3.9)$$

The decision variables are defined as follows:

γ_{S_i} ratio of the water diverted from the stream at time $\underline{i}$ to the demand at time $i-1$;

γ_{P_i} ratio of the water pumped from the aquifer at time $\underline{i}$ to the amount of water stored in the aquifer at time $i-1$;

γ_{B_i} difference between the storage of the surface reservoir $\underline{S}'$ at time $i-1$ and the volume of water released from the dam Q_{ou} at time $\underline{i}$.

Coupling of the Physical and Management Models

An important part in the development of the management model is its coupling to the physical model of the stream-aquifer system. The physical model output is in our case the mean head at the wells, $\underline{h}'$. A pumping lift can easily be found by subtracting $\underline{h}'$ from the ground surface level $\underline{Z}$.

Three alternative links can be established between the two models: (1) The physical model is located within the management model (Longenbaugh, 1970), (2) Part of the drawdown is computed outside of the management model (Maddock, 1972), and (3) The physical model is computed outside of the management model and the head is used as a link between the models. The later method was followed in the present study. Two connections are obtained: One is performed through the objective function by means of the drawdown $(Z - h_i^{'})$, which brings to the management model all the properties and past information recorded in the aquifer by the physical model output, $h_i^{'}$. The other connection is through the stream requirements constraint (equation 3.6) by means of the stream-aquifer interaction $Aa(H_i - h_i)$, where $\underline{h}$ is the mean water table in the aquifer.

Two main advantages were gained because of the use of this coupling. First, an iterative procedure using a standard linear programming package was used to solve an optimization problem having a quadratic objective function. Second, the objective function was simplified by computing the head outside of the management model since the head computation implicitly considers all past inputs of the system.

Iterative Procedure

There are procedures to linearize equations, such as

(3.1), by approximating the objective function by straight lines (separable function) and then solving the optimization problem using a linear programming package (Maass, 1966, p.501). The problem then becomes extremely cumbersome and depending on the case, sometimes it is almost impossible to solve it with the available generation of computers.

Due to the coupling between the physical and management model an iterative procedure, using a standard linear programming program, was developed to solve a nonlinear programming problem.

As noted above, equation 3.1 is quadratic in Q_p when $\underline{h}'$ is unknown and includes all past stimuli of the system. However, the same equation should be linear in Q_p if $\underline{h}'$ somehow was known. Therefore, the substitution of assumed values of $\underline{h}'$ in equation 3.1 makes the iterative procedure logical. In an initial step, the physical model computes the mean heads with assumed inputs to the system. The computed heads are then substituted into the management model; the answers are fed back into the physical model and the procedure is repeated as many times as necessary to satisfy a convergence criterion. The number of iterations required to reach an optimal solution depends on the initial estimates. Figure 3.1 shows a flow chart depicting the technique.

The iterative procedure behaved satisfactorily when the cost of water diverted from the stream and the cost of water pumped out of the aquifer were not equal. If both

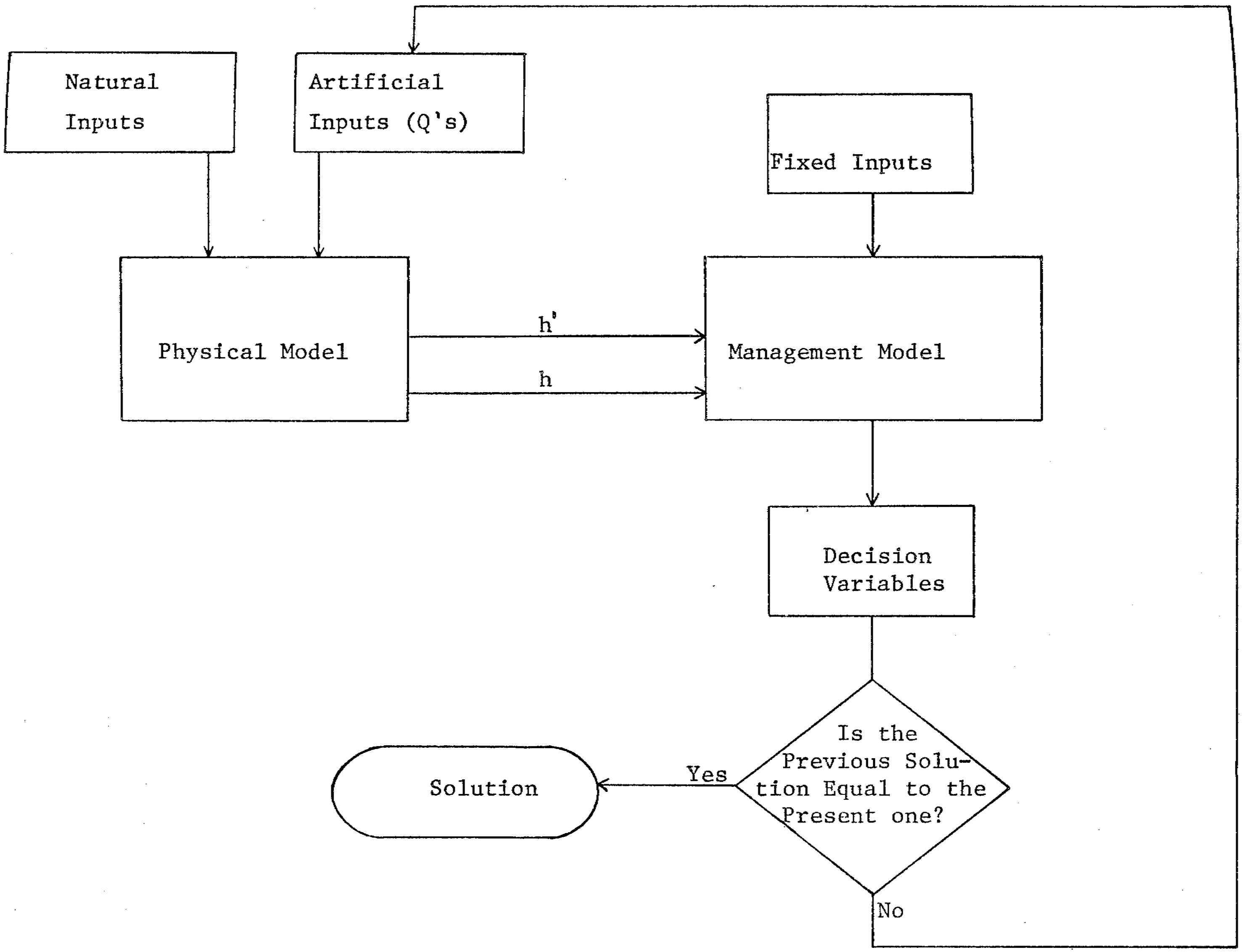


Fig. 3.1 Flow chart depicting the iterative procedure.

costs were similar, convergence problems appeared and no convergence was reached. However, this result appears reasonable since close to an optimum the model was indifferent to pumping or to use of surface water when both costs were almost the same. Furthermore, the values of the objective function for two different policies were practically same.

CHAPTER 4 STOCHASTIC REPRESENTATION OF THE PHYSICAL MODEL

4.1 Introduction

Large controversies have been raised in hydrology about the conceptual advantage of randomness over determinism (Kisiel 1969, p.23 and Yevjevich 1974). The use of stochastic approaches in groundwater hydrology has been slow in coming. In this work, a deterministic process will be presented as a special case of a stochastic or random process. The former is only concerned with the central tendency of a phenomenom and the latter also includes any unexplained variability of the studied variable. Therefore, both processes may be considered from a combined deterministic-stochastic point of view or as complements (Yevjevich, 1974, p.238).

Uncertainties or randomness in the inputs and properties of the physical model will be considered and studied. The statistics needed to represent a random stream-aquifer system are developed in this chapter. A stochastic differential equation governs the stream-aquifer system when the lumped model becomes subject to random inputs. Its solution is given in terms of ensemble averages of the aquifer head. The autocovariance function of the head as a measure of persistence, the cross correlation function as a measure of correlation between head and pumping, and the head variance as a measure of uncertainty are obtained. The subsurface outflow constant is considered to be a random variable and through a conditional probability approach an expression for the variance of the head as a function of the variance of the subsurface outflow constant is obtained.

4.2 Stochastic Differential Equation

Three different types of randomness can be related to a system; randomness in the forcing function or inputs, randomness in the coefficients or properties of the system, and a random initial condition.

The deterministic representation of the physical model was given by equation 2.4. In this section, the initial condition h_o , the storage coefficient $\underline{S}$, and the subsurface outflow constant $\underline{a}$ will be assumed to be deterministic quantities represented by their mean values. However, the total input to the system $y(t)$ will be a random variable. Equation 2.4 under these circumstances becomes a so called stochastic differential equation with a random forcing function (Syski, 1967, p.378).

$$S\, dh/dt + ah = y(t) \tag{4.1}$$

Due to the random behavior of the input $y(t)$ the filter or equation 4.1 produces a random $h(t)$. The solutions of (4.1) are of the form of equation 2.47, that is

$$h(t) = h_o \exp(-at/S) + 1/S \int_0^t y(\tau) \exp(-a(t-\tau)/S) d\tau$$

A simple way to represent the output process of the system is to take an ensemble average of all possible solutions of (4.1). Since integration and expectation commute, the solution of the stochastic differential equation 4.1 is represented by

$$E\,(h(t)) = h_o \exp(-at/S)$$

$$+ 1/S \int_o^t E\,(y(\tau))\exp(-a(t-\tau)/S)d\tau \qquad (4.2)$$

An ensemble average is defined as an average over all possible realizations at a given time. A realization is the deterministic representation obtained from measuring a stochastic process.

Equation 4.2 is valid for a stationary or non-stationary input $y(t)$. Although the system filter is time invariant, the output of the system $E(h(t))$ is a non-stationary process in the mean (see e.g., Kisiel, 1969, p.20 for definitions of stationarity).

Inputs of the System

In general, the input of the physical model $y(t)$ can be formed by the contributions of two kinds of inputs: natural inputs and man-controlled inputs. Natural inputs such as subsurface inflow, precipitation, evapotranspiration, etc. are random processes to a greater or lesser degree and with a natural persistence which can be computed from past measurements. Some random processes depend on natural phenomena, but can be controlled by man. Such processes are pumpage, water diverted from the stream, etc. Their persistence depends on the persistence of the natural inputs and of the filter characteristics.

4.3 Statistics of the Processes

Three of the most important characteristics that define a stochastic process are: its expected value or ensemble average, the autocovariance and the variance.

The Autocovariance Function of The Head

The autocovariance function of the head represents the interdependence of the stochastic process $h(t)$, at different times $\underline{r}$, $\underline{t}$ and it is the second moment about the mean values of the function $h(t)$.

$$\mathrm{Cov}\,(h(r)\,h(t)) = K_h\,(r,\,t) = E((h(r) - \mu_h(r))(h(t) - \mu_h\,(t)))$$

or

$$K_h\,(r,\,t) = E\,(h(r)\,h(t)) - \mu_h(r)\,\mu_h(t) \tag{4.3}$$

Combining equation 2.47 with (4.3), we obtain

$$K_h(r,t) = 1/S^2 \int_0^r \int_0^t K_y(u,v) \exp(-a(r-u)/S) \exp(-a(t-v)/S) du dv \quad (4.4)$$

where $K_y(u,v)$, the autocovariance of the input $y(t)$, is defined by

$$K_y(u, v) = E(y(u)\, y(v)) - \mu_y(u)\, \mu_y(v)$$

If the process $y(t)$ is a stationary process in the autocovariance, i.e., K_y is dependent on $u-v$ only, then

$$K_h(\tau) = 1/S^2 \int_0^r \int_0^t K_y(u - v) \exp(-a(r+t-u-v)/S) du dv \quad (4.5)$$

where $\tau = r - t$

The Variance of the Head

If $r = t$ in equation 4.4, we obtain the variance of, $\underline{h}$

$$\sigma_h^2(t) = K_h(t, t)$$

or

$$\sigma_h^2(t) = 1/S^2 \int_0^t \int_0^t K_y(u,v) \exp(-a(t-u)/S) \exp(-a(t-v)/S) du dv \quad (4.6)$$

In a real case the autocovariance of the input, $K_y(u, v)$ should be computed from raw data. However, if white noise is assumed feeding a system like ours, the output $\underline{h}$ is called a first order autoregressive process and has a standard autocorrelation function (Jenkins and Watts, 1968, p.162). White noise is a process which consists of uncorrelated contiguous impulses, with an autocovariance function $K_z(u) = \sigma_z^2 \delta(u)$, where $\delta(u)$ is the Dirac delta function (Jenkins and Watts, 1968, p.157).

In this study the random component $y'(t)$ of the input $y(t)$ was removed from its mean $\mu_y(t)$. The random component was assumed stationary and belonging to a first-order autoregressive process. Therefore, its autocovariance function is

$$K_y(\tau) = \exp(-|\tau|/J)\ \sigma_y^2 \qquad (4.7)$$

where, $J = -1/\ln \rho_1$ and ρ_1 is the autocorrelation function of the input for $t = 1$.

Stationary Head Variance Computed Via Spectral Analysis

Next, a procedure making use of spectral analysis (see Gelhar, 1974) will be used to obtain an asymptotic or stationary expression of the variance of the head $h(t)$.

Substituting $t_h = S/a$ into equation 4.1, we obtain

$$t_h\ dh/dt + h = y(t)/a \qquad (4.8)$$

If

$$h(t) = \mu_h(t) + f \quad \text{and} \quad y(t) = \mu_y(t) + r$$

where, f and r are stationary random components about the means, then equation 4.8 can be transformed into

$$t_h \, d\mu_h/dt + \mu_h + t_h \, df/dt + f = (\mu_y + r)/a \qquad (4.9)$$

taking ensemble averages

$$t_h \, E(d\mu_h/dt) + E(\mu_h) + t_h \, E(df/dt) + E(f)$$

$$= (E(\mu_y) + E(r))/a$$

and since $E(df/dt)$, $E(f)$ and $E(r)$ are zero.

$$t_h \, d\mu_h/dt + \mu_h = \mu_y/a \qquad (4.10)$$

by linearity, we can subtract equation 4.10 from 4.9 and get

$$t_h \, df/dt + f = r/a \qquad (4.11)$$

which is a stochastic differential equation for the random fluctuations about the mean.

Since f and r are stationary random processes, they can be represented by a Fourier-Stieljes integral in the form (Lumley and Panofsky, 1964, p.16)

$$f(t) = \int_{-\infty}^{\infty} \exp(iwt)\, dZ_f(w) \qquad (4.12)$$

and

$$r(t) = \int_{-\infty}^{\infty} \exp(iwt)\, dZ_r(w) \qquad (4.13)$$

where $\underline{w}$ is the frequency (radians/unit time). Substituting, we have

$$\int_{-\infty}^{\infty} iwt_h \exp(iwt) dZ_f(w) + \int_{-\infty}^{\infty} \exp(iwt) dZ_f(w) = 1/a \int_{-\infty}^{\infty} \exp(iwt) dZ_r(w)$$

and then

$$dZ_f(w) = dZ_r(w)/(a + iawt_h)$$

Since the random process $Z(w)$ has orthogonal increments (Lumley and Panofsky, 1964, p.16)

$$E\,(dZ_f(w_1)dZ_f^*(w_2)) = 0 \quad \text{for} \quad w_1 \neq w_2$$

$$= S_{ff}(w)dw \text{ for } w_1 = w_2 = w$$

and

$$E\,(dZ_r(w_1)\; dZ_r^*\,(w_2)) = 0 \quad \text{for} \quad w_1 \neq w_2$$

$$= S_{rr}(w)dw \quad \text{for} \quad w_1 = w_2 = w$$

where S_{ff} , S_{rr} are the spectral density functions or spectra of $\underline{f}$ and $\underline{r}$ respectively and $dZ^*(w)$ is the complex conjugate of $dZ(w)$.

Since

$$E(dZ_f(w)dZ_f^*(w)) = E(dZ_r(w)dZ_r^*(w)/((a+iaS)(a-iaS))) \tag{4.14}$$

using the previous orthogonal properties, we find

$$S_{ff}(w) = S_{rr}(w)/(a^2 + w^2\, S^2) \tag{4.15}$$

Equation 4.15 gives the relationship between the spectrum of the input and the output of the system.

The spectrum is the Fourier transform of the auto-covariance function and shows how the variance of a stochastic process is distributed with frequency. Therefore, the expression for the variance of the $\underline{f}$ process is

$$\sigma_f^2 = K_f(o) = \int_{-\infty}^{\infty} S_{ff}(w)\; dw \tag{4.16}$$

using equation 4.7, the input spectrum is

$$S_{rr}(w) = 1/2\,\pi \int_{-\infty}^{\infty} \exp(-iw\xi) K_{rr}(w)\, d\xi = T\,\sigma_r^2/(1+J^2w^2)\,\pi \tag{4.17}$$

Using (4.17), (4.15) and (4.16),

$$\sigma_f^2 = T\,\sigma_r^2/\pi \int_{-\infty}^{\infty} dw/(1 + J^2\,w^2)\ (a^2 + Sw^2)$$

and performing the integration, we obtain

$$\sigma_f^2 = \sigma_r^2\, J/a(S + Ja) \tag{4.18}$$

Equation 4.18 gives the stationary expression for the variance of the head, and was used to check values obtained from equation 4.6.

The Cross Correlation Coefficient of the Head and Pumping

An analysis similar to the previous one was carried out to find the cross correlation coefficient of the water level in the aquifer, $\underline{h}$, and the pumping discharge Q_p .

An expression relating the head $\underline{h}$ and specific pumping discharge q_p is

$$t_h\, dh/dt + h = -\, q_p/a \tag{4.19}$$

Representing $\underline{h}$ and q_p in complex form, we obtain

$$h(t) = \int_{-\infty}^{\infty} \exp(iwt)\ dZ_h(w) \qquad (4.20)$$

$$q_p(t) = \int_{-\infty}^{\infty} \exp(iwt)\ dZ_{q_p}(w) \qquad (4.21)$$

After substituting (4.20) and (4.21) into (4.19), we get

$$dZ_h(w) = -\ dZ_{q_p}(w)/(a + iwS)$$

The cross-spectral density function of q_p and h, $S_{q_p h}(w)$ can be expressed (Lumley and Panofsky, 1964, p.21) as

$$E\ (dZ_{q_p}\ (w)\ dZ_h^*(w)) = S_{q_p h}(w)\ dw$$

Therefore

$$S_{q_p h}(w) = -\ S_{q_p\ q_p}\ (a + iwS)/(a^2 + w^2\ S^2) \qquad (4.22)$$

If the random fluctuation of q_p about its mean is assumed stationary and is a first-order autoregressive process, then

$$K_{q_p} = \exp(-\ |\tau|\ /J)\ \sigma_{q_p}^2 \qquad (4.23)$$

and

$$S_{q_p h}(w) = -\ (a+iw)T\ \sigma_{q_p}^2/(a^2+w^2 S)\ (1 + J^2\ w^2)\pi \quad (4.24)$$

Since

$$K_{q_p h}(o) = 1/2\pi \int_{-\infty}^{\infty} S_{q_p h}(w)\, dw$$

then

$$K_{q_p h}(o) = -J\, \sigma_{q_p}^2 / (Ja + S) \tag{4.25}$$

Let

$$\rho_{q_p h}(o) = K_{q_p h}(o) / \sigma_{q_p} \sigma_h \tag{4.26}$$

and by analogy with (4.18) we have

$$\sigma_h = (J/a(S + Ja))^{\frac{1}{2}}\, \sigma_{q_p} \tag{4.27}$$

Now, substituting (4.27) and (4.25) into (4.26) produces

$$\rho_{Q_p h}(o) = -\,(aJ/(aJ + S))^{\frac{1}{2}} \tag{4.28}$$

where $\rho_{Q_p h}(o)$ is the cross correlation coefficient of the pumping and the head in the aquifer.

4.4 Randomness in the Subsurface Outflow Constant

Our system represents a natural phenomenon governed by chance. We do not know what the demand for water will be next year or how much rain will fall. Furthermore, we do not know how much water will be pumped nor what the water level in the aquifer will be next year. The output uncertainty will depend on the uncertainty present in the system and on the randomness and persistence of the inputs.

Much work remains to be done in the theory of stochastic differential equations. Equations (of the mixed type) with both a random forcing function and random coefficients, are difficult to solve. A conditional probability approach will be followed to obtain an expression of the uncertainty of the head as a function of a random input and of the subsurface outflow constant $\underline{a}$ in equation 4.1.

We will start our analysis by presenting an expression for the variance of a random variable $\underline{Y}$ which depends on another random variable $\underline{X}$ (Parzen, 1962, p.55).

$$\mathrm{Var}\ (Y) = E\ (\mathrm{Var}\ (Y \mid X)) + \mathrm{Var}\ (E\ (Y \mid X)) \tag{4.29}$$

The subsurface outflow constant $\underline{a}$, will be considered a random variable, leaving the storage coefficient $\underline{S}$, as a deterministic quantity. Equation 4.1 can be transformed to

$$S\ dh/dt + ah = Y_A(t) + aH(t) \tag{4.30}$$

where $y(t) = Y_A(t) + a\ H(t)$

From the linearity of equation 4.30 and from (2.47), we obtain

$$h(t) = h_o \exp(-at/S) + 1/S \int_o^t (Y_A(\tau) + aH(\tau)) \exp(-a(t-\tau)/S) d\tau$$

Taking the expected value of h(t) given <u>a</u> we get

$$E(h(t)|a) = h_o \exp(-at/S) + 1/S \int_o^t a_H(\tau) \exp(-a(t-\tau)/S) d\tau$$

$$+ 1/S \int_o^t \mu_{Y_A}(\tau) \exp(-a(t-\tau)/S) d\tau \qquad (4.31)$$

Now, making an analogy with equation 4.29, in which h(t) is a random variable that depends on another random variable, the subsurface outflow constant <u>a</u> , we have

$$\text{Var}(h(t)) = E(\text{Var}(h(t) \mid a)) + \text{Var}(E(h(t) \mid a)) \qquad (4.32)$$

In order to obtain the second part of the right hand side of equation 4.32 we use

$$\text{Var}(g(a)) = E((g(a) - E(g(a))^2)$$

for any function g(a).

Expanding g(a) in a Taylor series about E(a) and assuming a first order analysis (Cornell, 1972, p.1245), (i.e. neglecting terms beyond the first order)

$$g(a) \cong g(a)\Big]_{E(a)} + g'(a)\Big]_{E(a)} (a - E(a)) \qquad (4.33)$$

Squaring equation 4.33 and taking the expected value

(Papoulis, 1965, p.152)

$$E\left((g(a) - g(E(a)))^2\right) \cong \left(g'(a)\big]_{E(a)}\right)^2 \sigma_a^2 \quad (4.34)$$

with $g(a) = E(h(t) \mid a)$

$$\mathrm{Var}\left(E(h(t) \mid a)\right) \cong \left(\partial E(h(t) \mid a)/\partial a\big]_{E(a)}\right)^2 \sigma_a^2 \quad (4.35)$$

The first term of the right hand side of (4.32) can be obtained by taking the expected value of (4.6). Hence

$$E(\mathrm{Var}(h(t) \mid a)) = 1/S^2 \int_0^t\int_0^t E(K_y(u,v,a,)\exp(-a(t-u)/S) \cdot \exp(-a(t-v)/S))dudv \quad (4.36)$$

Expanding $K_y(u,v,a)$ about $E(a)$ and assuming a first order analysis and then taking the expected value, we obtain

$$E\left(K_y(u,v,a)\right) \cong K_y(u,v,E(a))$$

Therefore

$$\mathrm{Var}(h(t)) \cong 1/S^2 \int_0^t\int_0^t K_y(u,v,E(a)) \exp(-E(a)(t-u)/S) \cdot \exp(-E(a)(t-v)/S \, dudv + \left(\partial(E(h(t)|a)\big]_{E(a)})/\partial a\right)^2 \sigma_a^2 \quad (4.37)$$

The above expression shows the variance of the water levels due to the uncertainty in the input <u>y</u> and in the subsurface outflow constant <u>a</u> .

CHAPTER 5 STOCHASTIC REPRESENTATION OF THE MANAGEMENT MODEL

5.1 Introduction

No general procedure has yet been developed to solve the general stochastic programming problem in which some of the parameters are random. There are two bounds to the solution of a stochastic programming problem (Hadley, 1964, p.180); the lower bound can be obtained by determining the optimal value of the objective function for every possible set of parameters assuming that the random variables are known a priori, and then taking the expected value over all values of the random variables. The upper bound is obtained by replacing all the random parameters by their expected values and therefore, the variables found form a feasible but not necessarily optimal solution.

A solution of a stochastic programming problem could be obtained assuming that each model parameter can take on any one of a finite number of known values and all constraints hold for all possible combinations. However, the number of constraints becomes prohibitive if the number of possibilities is reasonably large. Also, the joint density function of the parameters must be known in this procedure.

The approach which will be followed in this work is a trade-off with respect to the solution that occurs between the two bounds. Expected values of the variables will be used in the objective function and the constraints will be of the chance constraints type or constraints that hold for most of the possible combinations but not for all. General operations research books discuss this type of constraint (e.g., Hillier, and Lieberman, 1972, p.536).

A stochastic management model will be developed to represent the management of a stream-aquifer system where economic and physical variables are governed by chance. Uncertainties for the demand of water and future availability of groundwater and surface water facilities will be considered. A stochastic management model aids in obtaining optimal operational policies and in designing water facilities.

The objective function is a function of the uncertainty in the aquifer water levels and of the cross correlation between pumping and head. It represents the discounted expected value of cost. Chance constraints (Charnes, et.al., 1958) are used to include probabilities of satisfaction of constraints. A nonstationary demand is easily reproduced by these constraints. The linear decision rule is used to define the decision variables and helps to transform the chance constraints into deterministic constraints. Finally, the computational part of the iterative procedure, used to solve the nonlinear programming problem, is discussed.

5.2 Decision Variables

The policy, control or decision variables, those deterministic variables calculated out of the optimization process, are defined as in the deterministic case (see, section 3.3) and they are:

The surface water decision variable

$$\gamma_{S_i} = Q_{SD_i} / D_{i-1}$$

The groundwater decision variable

$$\gamma_{P_i} = Q_{P_i} / ASh_{i-1}$$

The dam operation decision variable

$$\gamma_{B_i} = S'_{i-1} - Q_{ou_i}$$

The variables appearing in the decision variables definitions are random.

One advantage of the linear decision rule in defining the decision variables is that when the random events materialize in the form of preceding events (D_{i-1} , h_{i-1} and S'_{i-1}) they become known and are used in connection with the decision variables to make decisions (Q_{SD_i}, Q_{P_i}, and Q_{ou_i}) at the present. Therefore a random problem is transformed into one which deals with deterministic quantities. This feature is useful in scheduling problems; the decision policy is dynamic in the sense that current decisions depend on the previous condition of the system.

5.3 Objective Function

Due to the random nature of the variables, the objective function becomes a random variable; however, since it is meaningless to minimize a random variable, a deterministic quantity is needed. The expected value was selected because of its simplicity.

If we take the expected value of equation 3.1 (ensemble average), the objective function will represent the discounted expected value of cost.

Minimize

$$E(W) = \sum_{i=1}^{n} \mu_{C_S} \mu_{Q_{SD_i}} / (1 + r/N_S)^i$$

$$+ \sum_{i=1}^{n} \mu_{C_P} (Z \mu_{Q_{P_i}} - E(Q_{P_i} h_i))/(1 + r/N_S)^i$$

$$+ \sum_{i=1}^{n} \mu_{C_P} \psi E(Q^2_{P_i}) / (1 + r/N_S)^i \qquad (5.1)$$

where the well water level, $\underline{h}'$, is represented by $h = DC$ and the drawdown correction, DC, is given by (2.49)

$$DC = S_W - \overline{S} = \psi Q_P$$

Considering Q_P and $\underline{h}$ to be correlated, we have

$$E(Q_{P_i} h_i) = \mu_{Q_{P_i}} \mu_{h_i} + \rho_{Q_P h} \sigma_{Q_{P_i}} \sigma_{h_i} \qquad (5.2)$$

also

$$E\,(Q^2_{P_i}) = \mu^2_{Q_{P_i}} + \sigma^2_{Q_{P_i}} \tag{5.3}$$

where $\rho_{Q_p h}$ is the cross correlation coefficient of pumping and aquifer head, σ_{Q_p} is the standard deviation of pumping and σ_h is the standard deviation of aquifer head. Substituting (5.2), (5.3) and the first two decision variables into (5.1), we get

Minimize

$$E(W) = \sum_{i=1}^{n} (1/(1 + r/N_S)^i)(\mu_{C_S}\mu_{D_{i-1}}\gamma_{S_i} + \mu_{C_P} AS(Z\mu_{h_{i-1}} - \mu_{h_{i-1}}\mu_{h_i} + \mu_{DC_i}\mu_{h_{i-1}})\gamma_{P_i} + \mu_{C_P}(-\rho_{Q_P h}\sigma_{Q_{P_i}}\sigma_{h_i} + \psi\sigma^2_{Q_{P_i}})) \tag{5.4}$$

From equation 4.27, for a stationary process,

$$\sigma_h = C\,\sigma_{q_p}$$

where C is a constant which depends on aquifer properties. Above equation can be transformed into

$$\sigma_{Q_P} = C_1\sigma_h \tag{5.5}$$

where the constant $C_1 = C/A$, since $q_p = Q_P/A$ and $\sigma_{q_p} = \sigma_{Q_P}/A$. Substituting (5.5) into (5.4) we obtain

Minimize

$$E(W) = \sum_{i=1}^{n} (1/(1 + r/N_S)^{i})(\mu_{C_S}\mu_{D_{i-1}}\gamma_{S_i} + \mu_{C_P}\, AS(Z\mu_{h_{i-1}} - \mu_{h_{i-1}}\mu_{h_i} + \mu_{DC_i}\mu_{h_{i-1}})\gamma_{P_i} + \mu_{C_P}(-c_1\rho_{Q_P h} + c_1^2\psi)\,\sigma_{h_i}^2) \qquad (5.6)$$

where, the first term represents the cost of surface water diversion, the second term is that portion of the pumping cost which does not depend on the head variance, and the last term is that part of the pumping cost that depends on the head variance.

5.4 Chance Constraints

Chance constraints are not absolute but are satisfied up to specified probability levels. They were introduced by Charnes and Cooper (1959) as an approach to linear programming under uncertainty. As used in this work they include: (1) probabilities that imply degree of constraint satisfaction; (2) uncertainties of water facilities; and (3) nonstationary of the demand.

The constraint on the demand of water will be taken as an example of the procedure for representing a system restriction, by a chance constraint.

We are interested in the satisfaction of the demand constraint (see equation 3.2) under the probabilistic condition

$$P\,(D_i - (Q_{SD_i} + Q_{P_i}) \leqq 0) \geqq \lambda_1^* \qquad (5.7)$$

where λ^*_1 is a level of probability chosen to satisfy the probabilistic statement. Since the demand of water $\underline{D}$, the amount of water diverted from the stream Q_{SD}, and the aquifer pumping Q_P are random variables, (5.7) is a difficult statement to convert to certainty inequalities that can be used in the linear programming. In place of (5.7) we state

$$P(D_i \leqq \mu_{Q_{SD_i}} + \mu_{Q_{P_i}}) \geqq \lambda_1 \qquad (5.8)$$

For various values of λ_1 we give lower bounds on $P(D_i - (Q_{SD_i} + Q_{P_i}) \geqq 0)$ in Appendix G; these calculations indicate that the probability level λ^*_1 in (5.7) is slightly smaller than the value λ_1. It is shown how λ^*_1 can be estimated from a given value of λ_1.

Therefore, the cumulative distribution of the demand $F_{D_i}(d)$ at a time $\underline{i}$ (see, Figure 5.1) is defined as

$$F_{D_i}(d) = P(D_i \leqq d) \qquad (5.9)$$

where $\underline{d}$ is a variable. To insure that (5.9) is satisfied

$$d \geqq d_i(\lambda_1) \qquad (5.10)$$

where $d_i(\lambda_1)$ is the solution for x_1 of the equation $F_{D_i}(x_1) = \lambda_1$. Hence, from (5.8) and introducing the decision variables (see Section 5.2), we obtain

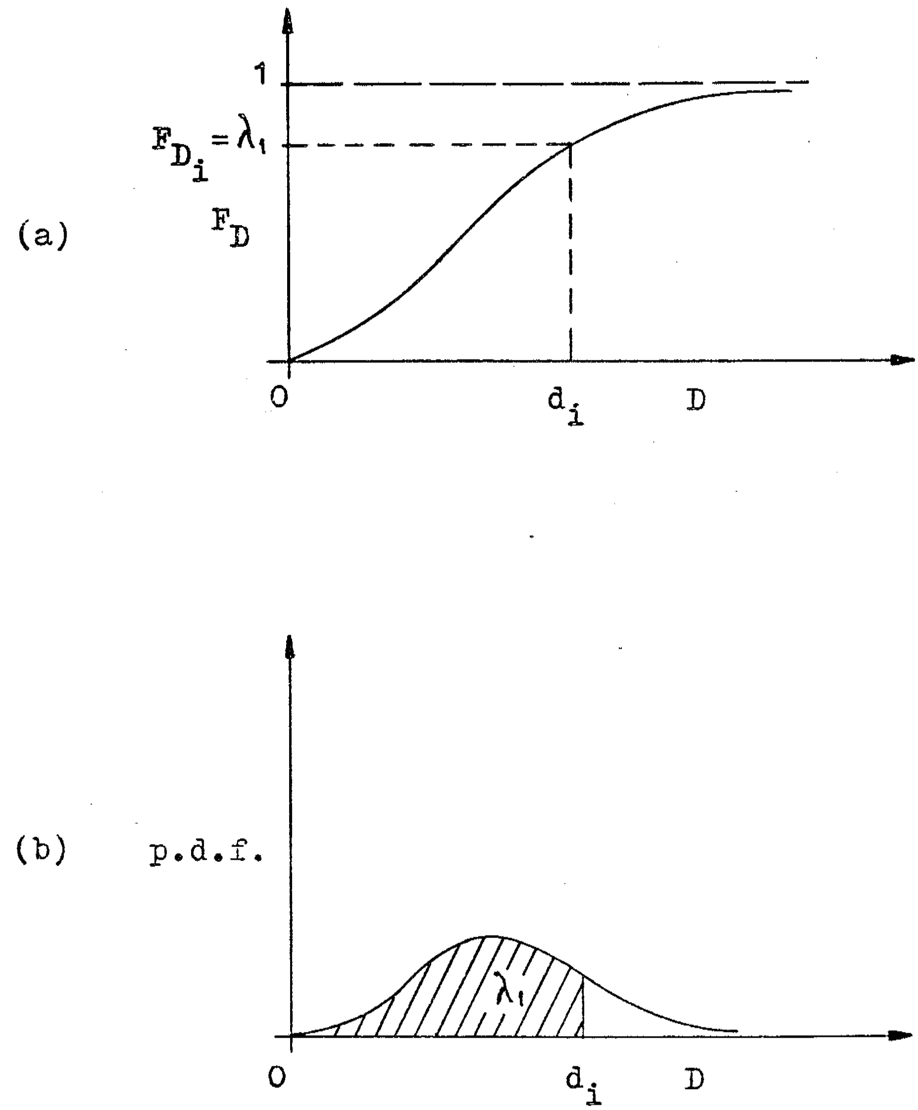


Fig. 5.1 (a) Cumulative probability distribution of the demand for a given level of probability λ, and (b) Probability density function of the demand for a given level of probability .

$$F_{D_i}(\mu_{D_{i-1}}\gamma_{S_i} + AS\mu_{h_{i-1}}\gamma_{P_i}) \geqq \lambda_1 \qquad (5.11)$$

This inequality may be called a chance representation of the demand. Equation (5.11) is equivalent to

$$\mu_{D_{i-1}}\gamma_{S_i} + AS\mu_{h_{i-1}}\gamma_{P_i} \geqq d_i\,(\lambda_1) \qquad (5.12)$$

where $d_i(\lambda_1)$ can be connected with μ_D and σ_D, to produce

$$\mu_{D_{i-1}}\gamma_{S_i} + AS\mu_{h_{i-1}}\gamma_{P_i} \geqq \mu_{D_i} + x\,\sigma_{D_i} \qquad (5.13)$$

where $\underline{x}$ depends on λ_1 and the type of probability density function of the demand which can be skewed.

Equation 5.13 is a deterministic representation of the chance constraint and can be used with a mathematical programming technique. A nonstationary demand can easily be represented by this constraint and if (5.12) is used instead of (5.13) a skew probability density function of the demand can easily be considered.

According to equation 5.13, the probability that the water demand in a given time period $\underline{i}$ be smaller than or equal to the average sum of flows Q_{SD} and Q_P must be greater than or equal to a probability level λ_1.

Another type of constraint states that the available pumping capacity at a time period $\underline{i}$ must be greater than or equal to the average amount of water pumped from the aquifer with a given probability level λ_2. In other words, there is always an uncertainty present in the available pumping capacity at a given time, due to maintenance and operational problems which shut down several of the available wells. Therefore, there is no way to know precisely how much water we shall be able to pump at a future time. The chance constraint for pumping facilities is

$$AS\mu_{h_{i-1}}\gamma_{P_i} \leq \mu_{Q'_{P_i}} + x\ \sigma_{Q'_{P_i}} \qquad (5.14)$$

where $\sigma_{Q'_{P_i}}$ represents the standard deviation of the pumping capacity and $\mu_{Q'_{P_i}}$ the expected amount of water that can be extracted from the aquifer at time period $\underline{i}$.

Equation 5.14, as well as the other constraints, can be obtained following a procedure similar to that used for the demand-of-water constraint.

The surface water diversion constraint states that the available surface water capacity at time period $\underline{i}$ must be greater than or equal to the average amount of water diverted from the stream at the same time with a probability level λ_3. The constraint can be written as

$$\mu_{D_{i-1}}\gamma_{S_i} \leq \mu_{Q'_{SD}} + x\ \sigma_{Q'_{SD}} \qquad (5.15)$$

where $\sigma_{Q'_{SD}}$ is the standard deviation of the surface water capacity and $\mu_{Q'_{SD}}$, the expected amount of water that the surface water facilities can convey at a given time $\underline{i}$.

Finally, the stream requirements constraint states that the average flow left in the stream, after all uses and interactions between the stream and the aquifer, at time period $\underline{i}$, must be greater than or equal to the downstream flow required at the same time.

$$P\,(Q_{ST_i} \geqq \mu_{Q_{SD_i}} + \mu_{Q_{S_i}} + \mu_{K1_i}) \geqq \lambda_4$$

or

$$1 - F_{Q_{ST_i}}\,(\mu_{Q_{SD_i}} + \mu_{Q_{S_i}} + \mu_{K1_i}) \geqq \lambda_4 \qquad (5.16)$$

where the variables were defined in Section 3.3 in a deterministic form. The certainty equivalent of equation 5.16 is

$$\mu_{D_{i-1}}\gamma_{S_i} + Aa(\mu_{H_i} - \mu_{h_i}) + \mu_{K1_i} \leqq d_i\,(1 - \lambda_4)$$

where $d_i(1 - \lambda_4)$ represents the $100.(1 - \lambda_4)$ percentile of the streamflow Q_{ST} .

5.5 Iterative Procedure

An iterative procedure similar to that described in Chapter 3 is used to solve the stochastic management problem.

Figure 5.2 shows a flow chart depicting the iterative procedure. The variable names are given in previous sections

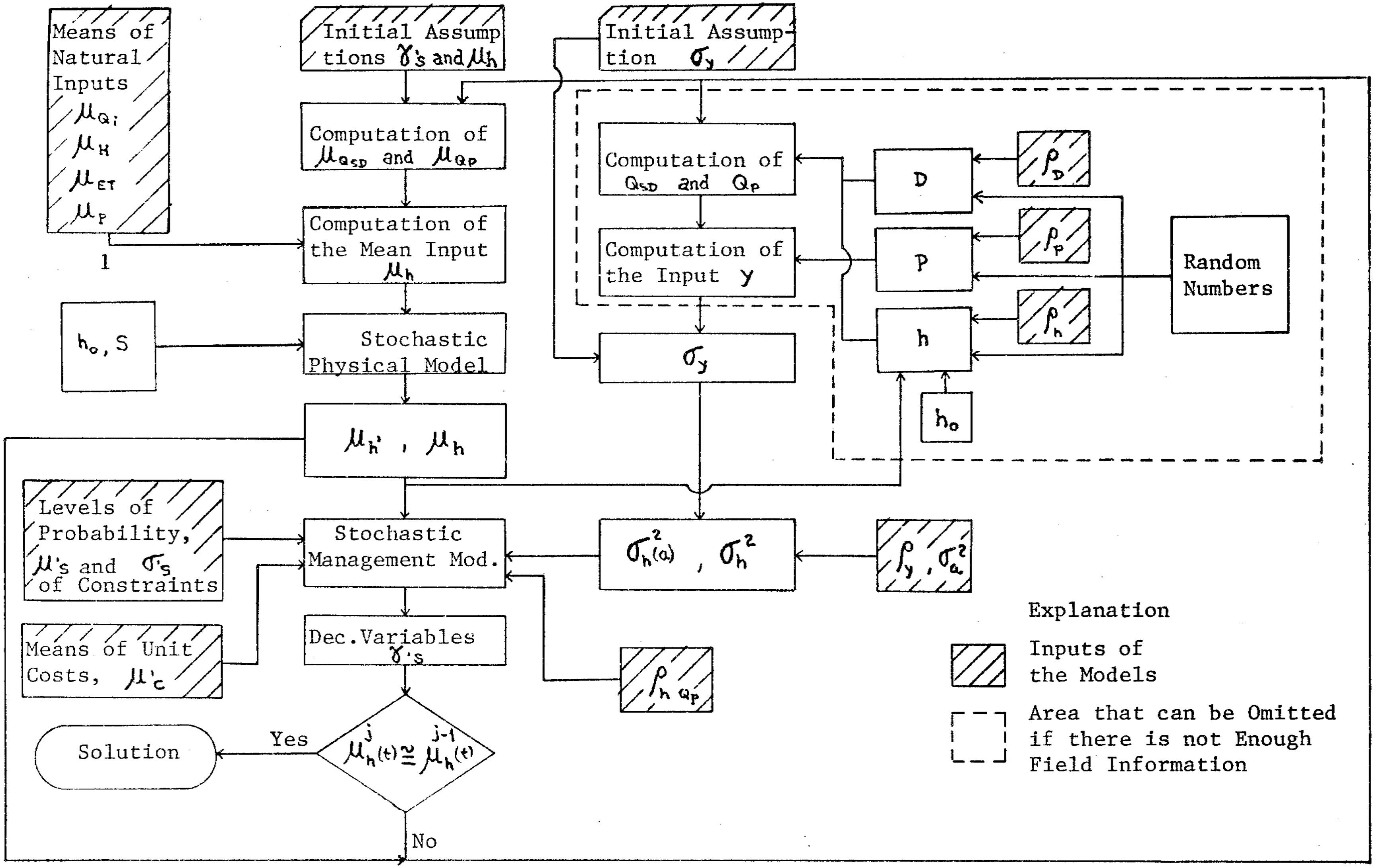


Fig. 5.2 Iterative procedure of a stochastic management problem.

or in the list of symbols. Shaded blocks show the inputs to the system and the area delimited by dashed lines represents the generation of the random input Y_i . This can be omitted if not enough field information is available. In this case, the variance of the input σ_y^2 should be assumed. Assumptions implied throughout the computation of the variance of the input are: (1) the components of the general input, Y_i , are statistically independent; (2) first order autoregressive processes are used to simulate demand of water, D_i , precipitation, P_i and the aquifer head, h_i ; (3) a normal cumulative distribution function is used to generate random numbers; (4) the variance of $\underline{y}$ is assumed constant and is estimated by $\sigma^2_y = \sum_{i=1}^{n} (y_i - \bar{y})^2/(n - 1)$; (5) precipitation is the only natural input with important randomness. Certainly most of the above assumptions could easily be relaxed; they were chosen to keep the simulation of data simpler and closer to the real case.

In summary, we have presented the stochastic management model in its simplest form. Depending on a specific problem, more terms may be added to the objective function, as well as constraints. The iterative procedure and basic concepts and equations will remain the same giving generality to the work presented.

CHAPTER 6 RESULTS AND APPLICATIONS

6.1 Introduction

To examine the reliability of the models developed herein, a comparative test was done based on a suitable case study obtained from the literature related to the subject. Furthermore, to demonstrate the viability and the versatility of these models they were applied to an actual stream-aquifer system, located in northwestern Mexico.

6.2 Comparative Study

Two features of the models were examined through comparative study: the presence of uncertainties or randomness in the stream-aquifer system and the distribution of variables in space and time. Based on a survey of related literature, a study by Maddock (1974) was chosen for the comparative work because it uniquely met the above requirements.

Maddock developed operating rules for the conjunctive use of surface water and groundwater when the demand and the natural supply (streamflow) are stochastic. He used a set of "technological functions" to condense information supplied by a distributed aquifer model and to get a link between the physical model and the stochastic management model. Also, when his water demand was represented by a Markov process (first-order autoregressive process) a demand persistence was introduced into the management model through the product of pumping and drawdown.

It was our purpose to simulate the reference problem as closely as possible. Nevertheless, a few discrepancies remain. For instance, the main source of randomness in Maddock's case was the demand, while for the model proposed here it is the mean water level in the aquifer which involves the randomness. Also in the present model, decision variables are defined differently. In Maddock's work the decision variables are defined as ratios of flow to demand at a specific time $\underline{i}$, while here the decision variables are defined as ratios of flow occurring at time $\underline{i}$ to the demand of water occurring at time $i-1$, or to the volume of water stored in the aquifer at time $i-1$. This definition is the result of the use of a linear decision rule which allows more dynamic decision variables.

The proposed management model is given by the following equations which should be compared to the development in Chapter 5 (Sections 5.2, 5.3 and 5.4). Objective function;

$$\begin{aligned} E(W) = \sum_{i=1}^{n} 1/(1+r/N_S)^i \big(& \mu_{C_S}\mu_{D_{i-1}}\gamma_{S_i} + \alpha_1\mu_{C_u}\mu_{D_{i-1}}\gamma_{u_i} \\ &+ \mu_{C_P} AS(Z\mu_{h_{i-1}} - \mu_{h_{i-1}}\mu_{h_i} + HL\mu_{h_{i-1}} + \mu_{D_{C_i}}\mu_{h_{i-1}})\gamma_{P_i} \\ &+ \mu_{C_P}(-C_1\rho_{Q_P h} + C_1^2\psi)\sigma^2_{h_i} + \alpha_2\mu_{C_R}\mu_{D_{i-1}}\gamma_{R_i}\big) \end{aligned} \qquad (6.1)$$

Constraints;

(1) Demand of water,

$$\mu_{D_{i-1}} \gamma_{S_i} + AS\mu_{h_{i-1}} \gamma_{P_i} \geq \mu_D + x\,\sigma_D \tag{6.2}$$

(2) Stream Requirements,

$$\mu_{D_{i-1}} \gamma_{S_i} - \alpha_1 \mu_{D_{i-1}} \gamma_{u_i} \leq \mu_{Q_{ST_i}} - aA\mu_H + aA\mu_{h_{i-1}} \tag{6.3}$$

(3) Water left after the demand is satisfied,

$$\gamma_{u_i} + \gamma_{R_i} = 1 \tag{6.4}$$

Decision variables;

(1) Surface water diversion,

$$\gamma_{S_i} = Q_{SD_i} \,/\, D_{i-1} \tag{6.5}$$

(2) Water returned to the stream,

$$\gamma_{u_i} = Q_{u_i} / D_{i-1} \;\; \alpha_1 \tag{6.6}$$

(3) Pumping,

$$\gamma_{P_i} = Q_{P_i} \,/\, ASh_{i-1} \tag{6.7}$$

(4) Artificial recharge,

$$\gamma_{R_i} = Q_{R_i} / D_{i-1} \alpha_2 \qquad (6.8)$$

where the parameters are

Q_{SD_i} amount of water diverted from the stream at time $\underline{i}$;

Q_{u_i} amount of water returned to the stream at time $\underline{i}$;

Q_{P_i} amount of water pumped out of the aquifer at time $\underline{i}$;

Q_{R_i} amount of water recharged to the aquifer at time $\underline{i}$.

The rest of the variables are defined in the List of Symbols. All the decision variables, except that for pumping, are in a form equivalent to that used by Maddock. Since the pumping decision variable is defined differently, the structure of the objective function is such that the variance of the head appears rather than that of the demand.

Table 6.1 summarizes the important conceptual differences between the two models. An idea of the simplicity of our physical model is demonstrated by the number of aquifer parameters needed.

$T = 0.031\ ft^2/s$

$S = 0.01$

$L = 4900\ ft$

$A = 8173$ acres

$\beta = 3$

TABLE 6.1 Significant Differences Between the Compared Models.

	Distributed Parameter Model (Maddock, 1974)	Lumped Parameter Model (Present Investigation)
Physical Model	The location of wells must be specified and the aquifer properties defined in detail. The pumping scheduling is obtained for every well.	The location of wells is not specified and the aquifer properties are average values. The pumping scheduling is obtained for the entire region.
	This model computes the drawdown at nodal points representing wells.	Since this model does not compute the drawdowns at the well locations, a drawdown correction is added in order to achieve correct pumping costs.
Management Model	A standard quadratic programming package requiring 190 k bytes of core was used.	An iterative procedure was developed to take advantage of the structure of the physical model and a simple linear programming package is used. A phase overlay computer software technique helps to reduce program storage to 144 k bytes of core. A solution requiring two iterations is solved in about 12 minutes on an IBM 360 model 44 system.
	An average demand constraint is applied to the problem.	A chance demand constraint is applied to the problem.
	"Static" decision variables.	Dynamic decision variables which use the linear decision rule.

The properties of the homogeneous aquifer, transmissivity and storage coefficient, and a map of the study area were provided by Maddock (personal communication, 1975). The characteristic length $\underline{L}$ of the system (Section 2.3) was estimated from the aquifer area and the length of the main stream channel. The location of the wells was unknown. Appendix H gives the information required by the models and the variable names. A response time t_h of one month was obtained for this system by using (2.47).

The operating rules formulated by Maddock for a coefficient of variation of 0.457 and a mean demand of water of 131 acre-ft per season are shown in Table 6.2 along with our results for the same situation. A remarkable likeness is to be noted in these results, which substantiates the competence of our stochastic management model in general and the iterative procedure in particular.

By referring to the deterministic case, we obtain the sensitivity of the discounted expected cost to changes in the demand variance. This sensitivity is expressed by Maddock as a percent error in the discounted expected cost in relation to the deterministic world assumption $P'(\sigma_D^2, 0)$, versus σ_D^2. An expression that defines $P'(\sigma_D^2, 0)$ is

$$P'(\sigma_D^2, 0) = (E(W(\sigma_D^2, 0)) - E(W(0,0))).100/E(W(0,0)) \tag{6.9}$$

where $E(W(0, 0))$ is the discounted expected cost when the true value of the demand is known (deterministic case).

TABLE 6.2 Comparison of Operating Rules of Maddock's and Proposed Models.

Season	Decision Variables									Expected Interaction Withdrawal from Stream (acre-ft)	
	Pumping			Stream Diversion		Return to the Stream		Spreading			
	$\gamma^*_{(1,n)}$	$\gamma^*_{(2,n)}$	γ^1_P	$\lambda^*_{(n)}$	γ^1_S	$\xi^*_{(n)}$	γ^1_u	$\gamma^*_{(3,n)}$	γ^1_R	$f^*_{(n)}$	Q^1_S
1	0.511	0.489	1.0	0.0	0.0	0.0	0.0	1.0	1.0	84	57
2	0.519	0.481	1.0	0.0	0.0	0.0	0.0	1.0	1.0	89	62
3	0.466	0.534	1.0	0.0	0.0	0.0	0.0	1.0	1.0	89	62
4	0.514	0.486	1.0	0.0	0.0	0.0	0.0	1.0	1.0	93	62
5	0.516	0.484	1.0	0.0	0.0	0.0	0.0	1.0	1.0	93	62
6	0.519	0.481	1.0	0.0	0.0	0.0	0.0	1.0	1.0	94	62
1	0.523	0.477	1.0	0.0	0.0	0.0	0.0	1.0	1.0	95	62
2	0.527	0.473	1.0	0.0	0.0	0.0	0.0	1.0	1.0	95	62
3	0.447	0.553	1.0	0.0	0.0	0.036	0.0	0.964	1.0	93	62
4	0.516	0.484	1.0	0.0	0.0	0.0	0.0	1.0	1.0	96	62
5	0.517	0.483	1.0	0.0	0.0	0.0	0.0	1.0	1.0	96	62
6	0.520	0.480	1.0	0.0	0.0	0.0	0.0	1.0	1.0	96	62
1	0.525	0.475	1.0	0.0	0.0	0.0	0.0	1.0	1.0	97	62
2	0.529	0.471	1.0	0.0	0.0	0.0	0.0	1.0	1.0	97	62
3	0.449	0.551	1.0	0.0	0.0	0.054	0.0	0.946	1.0	95	62
4	0.489	0.511	1.0	0.0	0.0	0.0	0.0	1.0	1.0	96	62
5	0.515	0.485	1.0	0.0	0.0	0.0	0.0	1.0	1.0	98	62
6	0.517	0.483	1.0	0.0	0.0	0.0	0.0	1.0	1.0	98	62
1	0.522	0.478	1.0	0.0	0.0	0.0	0.0	1.0	1.0	98	62
2	0.526	0.474	1.0	0.0	0.0	0.0	0.0	1.0	1.0	98	62
3	0.446	0.554	1.0	0.0	0.0	0.063	0.0	0.937	1.0	96	62
4	0.466	0.534	1.0	0.0	0.0	0.0	0.0	1.0	1.0	96	62
5	0.504	0.496	1.0	0.0	0.0	0.0	0.0	1.0	1.0	98	62
6	0.499	0.501	1.0	0.0	0.0	0.0	0.0	1.0	1.0	98	62

* Maddock's Solution

1 Present Investigation

$\gamma_P = \gamma(1,n) + \gamma(2,n)$

$\gamma(k)$ = Fraction of the demand supplied by the k th well during the n th time period

This expression for sensitivity was also utilized in the present investigation as a basis for comparison. The results of this analysis are expressed in graphical form in Figure 6.1. Maddock's result represents the point given in the text of Maddock (1974, p.9); Maddock (personal communication, 1975) has stated that his Figure 2 is incorrect. His results are only a special case of a more general stochastic representation used in the presented management model. In Maddock's problem, the expected value of the demand of water is satisfied according to any given level of probability λ, as shown in Figure 6.1 by using a chance constraint representation (Section 5.4). In addition, a non-stationary demand could easily be represented by this type of constraint (see equation 6.2).

Up to this point apparently no discrepancies exist between the management decision resulting from the two approaches. The aquifer in our case shows a decline of the mean water level which is responsible for the stream aquifer flow. However, a water balance calculation from Maddock's Table 5 implies that the mean water level of the aquifer underwent a recovery of 8.6 ft at the end of the design period instead of declining. The explanation of this phenomenon was found in the distributed nature of Maddock's model, which was able to simulate the local cone of depression of a pumping well and to induce local stream to aquifer flow even though the mean water level in the aquifer was above the stream level.

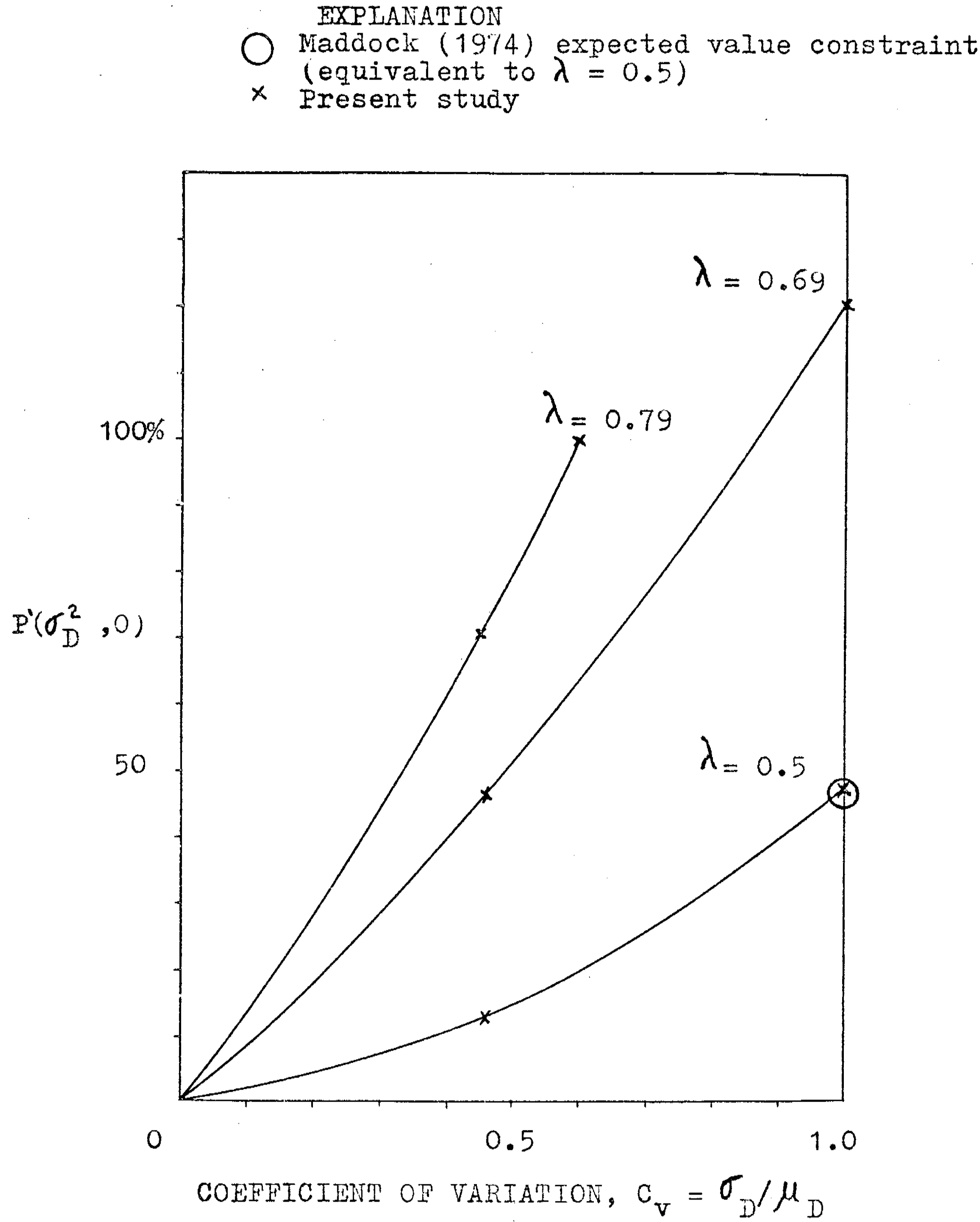


Fig. 6.1 Graph of P'(σ_D^2 , 0) versus C_v , as a function of probability level λ .

Since the cone of depression due to pumping of only two wells was the dominant effect on aquifer flow it was difficult for our lumped model to manage problems of the local flow type, even though a correction for the effect of drawdown at the wells themselves was included. As a consequence, the values of the objective function differ in magnitude. For a coefficient of variation of 0.458 and $\lambda = 0.5$, Maddock's objective function was \$3606, while in our case it was \$3976.

The advantage of our models in terms of computational effort and simple structure is illustrated by the fact that only 24 quantities, appear in the pumping term of the objective function compared to 600 of Maddock. For instance if we had 20 pumping wells, (instead of 2, as in the present case), Maddock's objective function would show 6000 different products of decision variables; while only 24 quantities would still appear in our case.

A management model such as Maddock's is very cumbersome and almost impossible to solve when the number of pumping wells is large. This is due to the large number of terms in the objective function, which is not only a consequence of the distributed representation of the problem but also of the structure of the physical model. To compute drawdown in Maddock's management model all past pumping must be included in the objective function.

A discrete representation of the convolution integral is part of our lumped parameter model solution and is used to compute all past pumping outside of the management model condensing the past information into the current aquifer head. Hence, when the physical model and the management model are brought together in the objective function many complications are avoided and an iterative procedure arises as a natural procedure for solving the nonlinear optimization problem. The iterative procedure used to solve Maddock's problem is depicted in Figure 6.2 and a listing of the computer program is supplied in Appendix I. The inputs of the models are: initial assumptions of the decision variables and the mean aquifer water levels; and the model parameters. Normally distributed random numbers were used to simulate random demand needed in the artificial recharge Q_{AR} computation. As a computational simplification, mean water levels instead of random water levels were used in the pumping computation. To compute the variance of the system input, σ_y^2, a time average was used and stationarity was assumed. The computation of the variance of the aquifer head was obtained from (4.6) and the mean water levels from (4.2). The drawdown correction was computed using (2.49).

Summary of Model Testing

The simple physical model was developed for regional studies where the effect of many wells produces a quasi-mean water level change. The resulting operating rules compare satisfactorily with Maddock's results. However under the local flow situation herein tested, in which the pumping

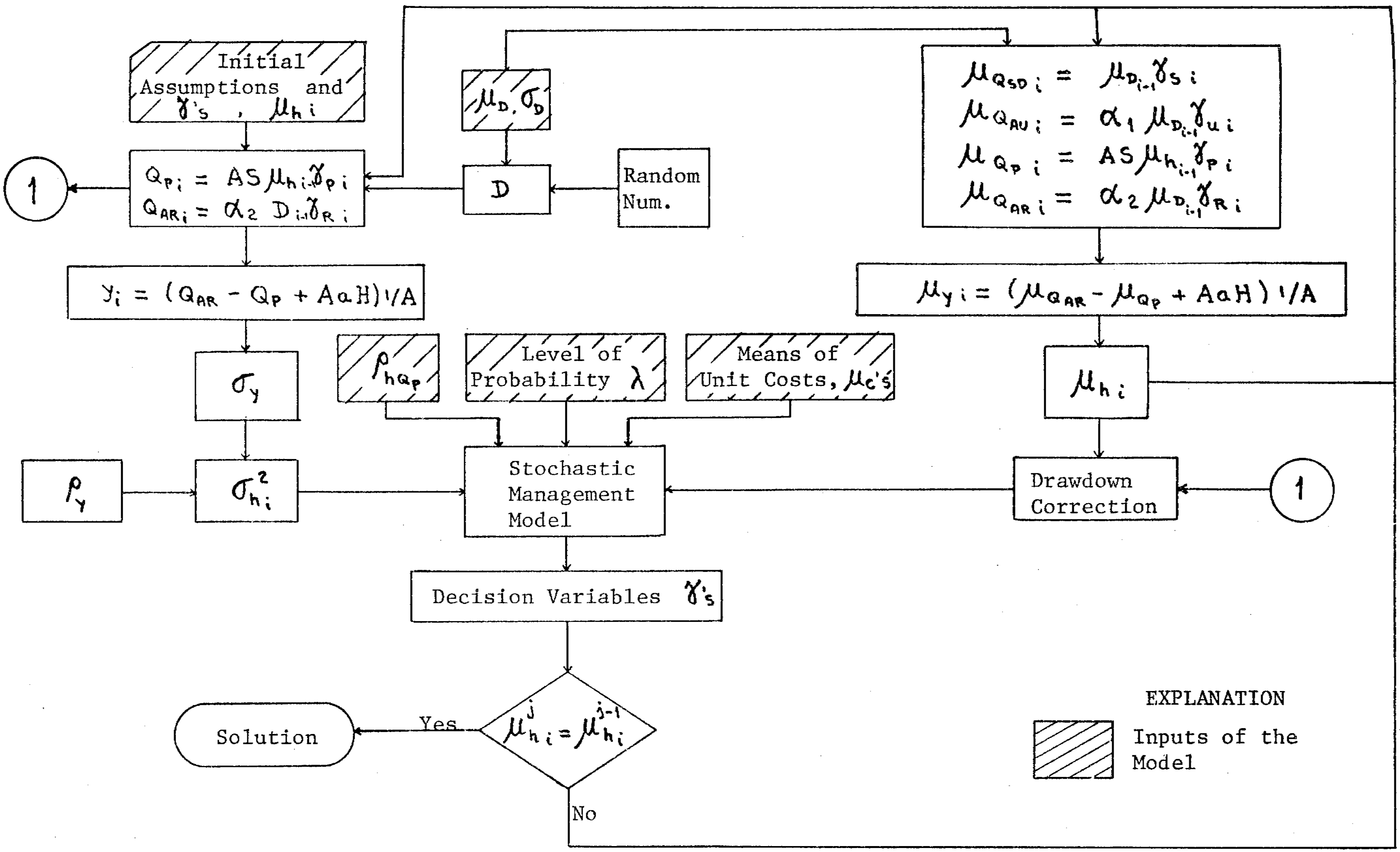


Fig. 6.2 Flow chart of the iterative procedure used to solve Maddock s problem.

effect of two wells produced cones of depression dominating the flow picture, the lumped model indicates a slightly higher value of the objective function.

The stochastic management model, with the use of a linear decision rule to define the decision variables, makes the problem more dynamic since every future decision for the system depends on a known present situation. Furthermore the use of a chance constraint representation of the demand shows that situations such as Maddock's dealing with average accomplishments (50% level of probability) are only special cases of a more general problem, in which any desired level of probability can be considered. Demands of water represented as nonstationary process, can easily be managed by a chance constraint, as used in the present work.

6.3 Application

Introduction

The objective of this part of the study was to illustrate the application of the present management scheme to a complex and realistic field problem. A system able to develop conjunctive use of surface water and groundwater and a central management agency were used to test the reliability and adaptability of the models to specific situations. A basin in northwestern Mexico was chosen as the study case. Inasmuch as not enough detailed field information was available to represent the area accurately, the results obtained from our models should not be thought of as the ultimate policies applicable to that area. Realism was important but not the most important criterion. The practical objectives of the investigation were: to formulate an optimal operational schedule for the system; and to determine the size of a projected dam for control of the stream. We especially considered uncertainties present in the natural inputs, in future demands of water, in availability of surface and groundwater facilities, and in the stream-aquifer subsurface outflow constant.

Description of the Area

The study area is crossed by the Rio Sinaloa which flows into the Gulf of Baja California. Two well fields exist, one on each side of the river. They serve mainly agriculture. The boundaries of the system are shown in Figure 6.3. Additional information on the area can be found in Appendix J.

Water Balance

To obtain the subsurface outflow constant a water balance of the studied area was carried out for the period between September of 1969 to September of 1970, this portion of the problem can be defined as the calibration part. The following equation represents the water balance of the aquifer

$$S\, dV/dt = Q_i + N_r + Q_{ret} + Q_C + Q_S - Q_P - ET \qquad (6.10)$$

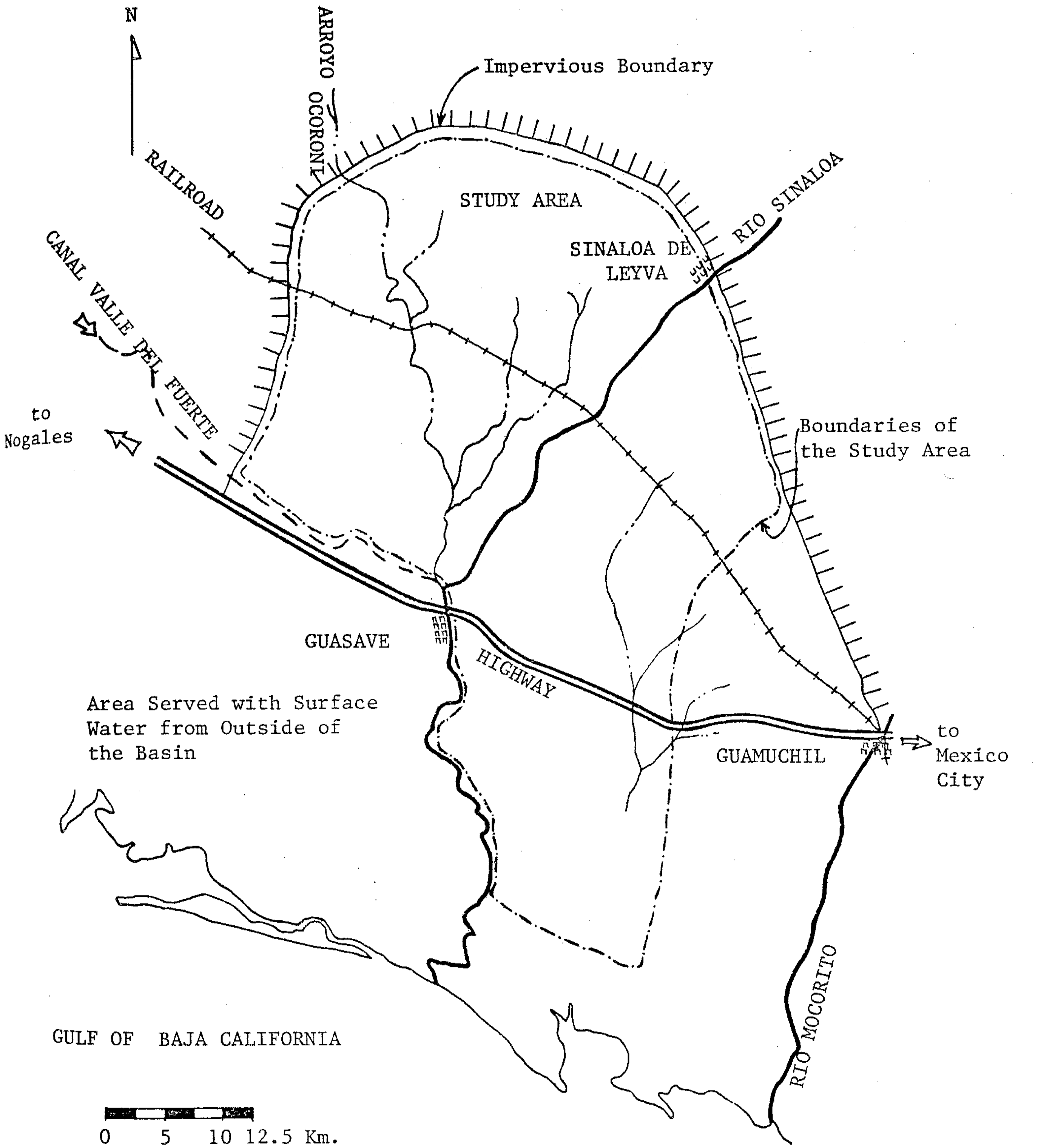


Fig. 6.3 Map of the Rio Sinaloa study area.

where

$$N_r = \alpha_1 P'$$

$$Q_{ret} = \alpha_4 \alpha_2 (Q_P + \beta_1 P + Q_{SD})$$

$$Q_S = Aa(H - h)$$

$$Q_C = \alpha_3 Q_{SD}$$

The variables are defined in Figure 6.4.

To visualize better the magnitude of the variables equation 6.10 may be expresed in terms of water depths; this is done by dividing by the aquifer area <u>A</u> .

$$S\, dh/dt = \epsilon - q_P + a(H - h) \qquad (6.11)$$

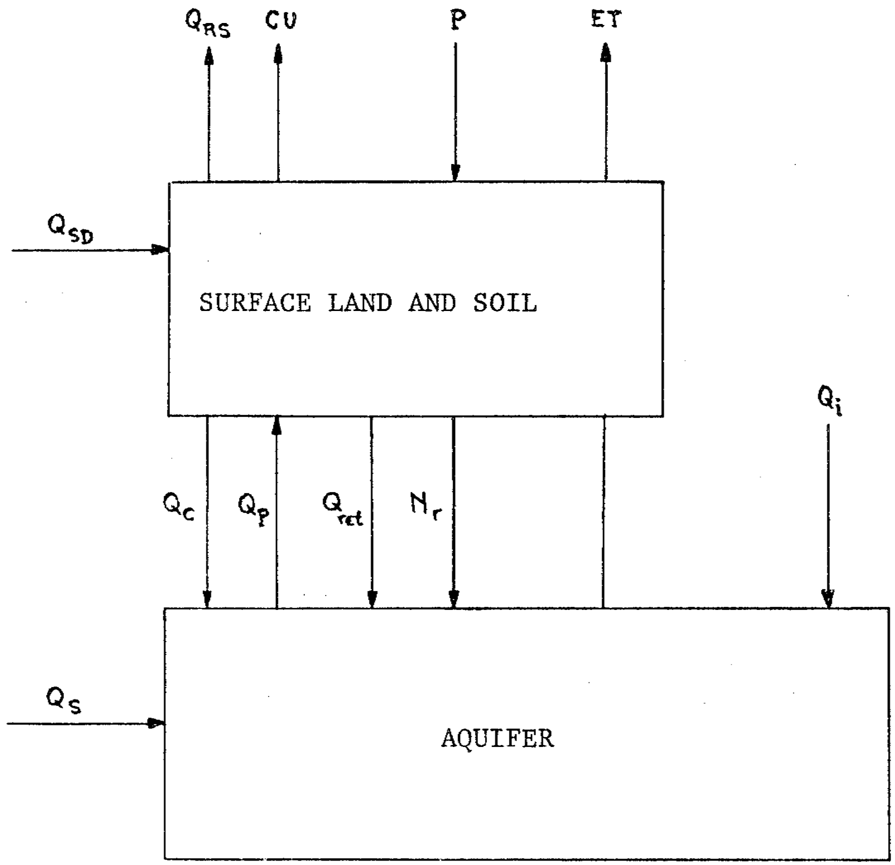
in which

$$\epsilon = q_i + \alpha_1 p' + \alpha_4 \alpha_2 (q_P + \beta_1 p + q_{SD}) + {}_3 q_{SD} - q_{ET} \qquad (6.12)$$

Information about the variables involved in (6.11) and (6.12) is presented next.

a) Irrigated Land. Irrigated land of 460 Km^2 and an aquifer of 1744 Km^2 were included in the aquifer water balance.

b) Water demand. An average consumptive use of one



EXPLANATION

Q_{SD} : Surface water applied to the field

Q_C : Conveyance loss

Q_{RS} : Surface drainage

CU : Consumptive use

P : Precipitation

ET : Evapotranspiration

Q_i : Subsurface inflow

N_r : Recharge from precipitation

Q_{ret}: Irrigation return flow

Q_P : Pumping

Q_S : Stream-aquifer flow

Fig. 6.4 Rio Sinaloa water cycle.

meter was estimated. This estimate is based on the consumptive use of the main crops and the seasonal irrigation cycles. Hence 1.7 meters of applied water was assumed reasonable.

$$\text{Demand} = 460 \times 10^6 \times 1.7 = 782 \times 10^6 \ m^3/yr$$

$$q_D = 782/1744 = 0.448 \ m/yr \qquad (6.13)$$

where q_D is the water depth required by the demand and computed for the total area of the aquifer (as opposed to the irrigated area only).

c) Subsurface Inflow. It is mainly due to the Arroyo Ocoroni (Figure 6.3) and was estimated from water table maps.

$$Q_i = 25 \times 10^6 \ m^3/yr$$

and

$$q_i = 25/1744 = 0.0143 \ m/yr \qquad (6.14)$$

where q_i is the depth of surface inflow, over the total aquifer area.

d) Pumping. Figures for pumping can be seen in Appendix J.

$$Q_P = 200 \times 10^6 \ m^3/yr$$

and

$$q_P = 200/1744 = 0.115 \ m/yr \qquad (6.15)$$

where q_P is the depth of pumped water over the total aquifer area.

e) Precipitation. An average annual precipitation of 0.45 meters was estimated for the basin; 67 percent of this precipitation falling over the irrigated land was assumed to contribute to the demand. Then

$$p = 0.45 \times 460/1744 = 0.118 \ m/yr$$

and

$$\beta_1 \ p = 0.67 \times 0.118 = 0.0793 \ m/yr \qquad (6.16)$$

where $\underline{p}$ is the depth of precipitation, referred to the aquifer area.

f) Diverted Surface Water. Since no precise information existed about surface water diverted from the stream, this quantity was obtained as follows

$$q_P + q_{SD} + \beta_1 \ p = q_D$$

where q_{SD} is the depth of diverted surface water, over the total aquifer area. Therefore,

$$q_{SD} = q_D - q_P - \beta_1 \, p$$

Substituting (6.13), (6.15) and (6.16) into above equation produces

$$q_{SD} = 0.2527 \text{ m/yr} \tag{6.17}$$

g) Irrigation Return Flow. A coefficient of infiltration of water applied to the irrigated land was computed as:

$$\alpha_2 = \frac{\text{water applied-consumptive use}}{\text{water applied}}$$

or

$$\alpha_2 = (1.7 - 1.0)/1.7 = 0.411$$

Then the depth of water infiltrated due to irrigation is

$$\alpha_2 \; q_D = 0.185 \text{ m/yr}$$

Since some of the water infiltrated returns to any available surface drainage (such as the stream) it was assumed that 55 percent of the amount of water infiltrated returned to the surface drainage system (α_4) and therefore,

$$\alpha_4 \alpha_2 q_D = 0.45 \times 0.185 = 0.0829 \text{ m/yr} \qquad (6.18)$$

h) Infiltration from Precipitation. Here, the flow of water infiltrated into the non-irrigated area is calculated assuming a coefficient of infiltration, α_1 , of 5 percent. The depth of precipitation falling on non irrigated land is

$$p' = (1744-460) \times 0.45/1744 = 0.332 \text{ m/yr}$$

and the depth of infiltrated water is

$$p' = 0.0166 \text{ m/yr} \qquad (6.19)$$

i) Conveyance Losses. A coefficient of infiltration α_3 , of 10 percent, due to the conveyance losses of canals was included in the losses as follows

$$\alpha_3 \, q_{SD} = 0.0253 \text{ m/yr} \qquad (6.20)$$

j) Evapotranspiration. This term includes losses from evapotranspiration due to phreatophytes (mainly cottonwoods along the Arroyo Ocoroni and Rio Sinaloa, see Figure 6.3). An area 130 Km long and 250 m wide was assumed to be affected by the phreatophytes. Based on an approximation of the volume density and the water consumption of cottonwoods (Robinson, 1958, p.61), a consumptive use of 1.55 meters was estimated. Therefore,

$$q_{ET} = (130 \times 0.25 \times 1.55)/1744 = 0.0289 \text{ m/yr} \quad (6.21)$$

where, q_{ET} is the depth of phreatophyte consumption referred to the aquifer area.

Making use of equation 6.12 and the previous terms, we obtain

$$\epsilon = 0.1102 \text{ m/yr} \quad (6.22)$$

The mean water level change between September 1969 and September 1970, Δh was

$$\Delta h = -0.311 \text{ m} \quad (6.23)$$

A mean water level for the entire aquifer of 19.969 meters above sea level was calculated from water table maps of the studied area.

Two points should be kept in mind when computing the mean stream stage; first, downstream of the town of Guasave

(Figure 6.3), the area to the west of the river is not considered in the analysis; second, some subsurface outflow discharges to the sea. Therefore, in the computation of the mean stream stage, the length of the stream above Guasave was included twice; the length of coast was considered and at datum level. The average stream stage elevation was then 20.6 meters.

With the values of the system variables and the water balance of (6.11) the subsurface outflow constant was computed as follows

$$- 0.311 \times 10^{-2} = 0.110 - 0.116 + 0.63a \qquad (6.24)$$

and

$$a = 4.29 \times 10^{-3} \ 1/yr \qquad (6.25)$$

The response time (see equation 2.54) can be obtained as

$$t_h = 10^{-2} / (4.29 \times 10^{-3}) = 2.33 \text{ years} \qquad (6.26)$$

A characteristic length of the aquifer can be computed once the subsurface outflow constant is known (see Section 2.3). In this case, the characteristic length is 21 km. This can easily be measured from a map of the area and is strikingly the same as that obtained using the aquifer water balance equation. This example shows how simple it is to obtain the subsurface outflow constant and to represent the system by our physical model, provided we have a clear picture of the field situation. A water balance will always be recommended as a powerful and simple tool in checking the behavior or characteristics of the physical model.

Management of the System

Plans have been made for construction of a reservoir on the Rio Sinaloa in order to control and have better use of the river. However, for best use of available water resources in the area, the conjunctive use of groundwater and surface water appears to be an advantageous alternative, and should be considered.

An aquifer, an underground reservoir built by nature and able to store, transmit and supply water, is already available. It is naturally connected to the stream, and hydraulic head differences dominate the stream-aquifer interaction. About 600 wells, approximately 120 with depth greater than 50 meters, presently extract water from the aquifer. In drought periods, the aquifer can be a reliable source of water.

Both subsystems, stream and aquifer would be operated by a central agency, charged with satisfying an estimated water demand in the "best" economic way, and with absorbing the initial costs of the irrigation and pumping facilities. No changes in the unit operational cost of the facilities, due to operation of the system, will be considered.

Optimal decisions concerning size of the dam and its operation, diversion, and operational policies for pumping will be determined. The inputs and characteristics of the system such as the subsurface outflow constant and the availability of pumping facilities are the uncertain quantities.

In a deterministic treatment, the minimization of the discounted operational cost of the system (see, e.g., equation 3.1), including the fixed cost of the dam, is represented by an objective function such as:

$$W = \sum_{i=1}^{n} 1/(1 + r/N_S)^i (C_{S_i} D_{i-1} \gamma_{S_i} + C_{R_1} \gamma_{R_1} + C_{P_i} Q_{P_i} (Z - h_i + DC_i)) \qquad (6.27)$$

See section 3.3 and the List of Symbols for description of the variables. The decision variables are:

γ_{S_i} ratio of water diverted from the stream at time i, to water demand at time $i-1$;

γ_{R_1} size of the dam in 10^6 m^3 ;

γ_{P_i} ratio of water pumped out of the aquifer at time i to amount of water stored in the aquifer at time $i-1$.

If the demand of water and inputs of the systems are random a stochastic representation of the objective function is the discounted expected value of cost. Following an analysis similar to that presented in Section 5.3 we obtain

Minimize

$$E(W) = \sum_{i=1}^{n} (1/(1 + r/N_S)^i)(\mu_{C_S}\mu_{D_{i-1}}\gamma_{S_i} + \mu_{C_R}\gamma_{R_1}$$

$$+ \mu_{C_P}\, AS(Z\mu_{h_{i-1}} - \mu_{h_{i-1}}\mu_{h_i} + \mu_{DC_i}\mu_{h_{i-1}})\gamma_{P_i}$$

$$+ \mu_{C_P}\,(- c_1\, \rho_{Q_P h} + c_1^2\, \psi)\, \sigma^2_{h_i}) \qquad (6.28)$$

The constraints of the problem are given next (see Section 5.4 for development of some of the constraints).

1) The demand constraint states that the water demand in a given period $\underline{i}$ must be smaller than or equal to the sum of the expected amount of water diverted from the stream and that pumped out of the aquifer, with a level of probability λ_1.

$$\mu_{D_{i-1}}\gamma_{S_i} + AS\mu_{h_{i-1}}\gamma_{P_i} \geqq \mu_{D_i} + x_1\ \sigma_{D_i} \qquad (6.29)$$

2) The pumping capacity constraint says that the available pumping facilities at a time $\underline{i}$, must be greater than or equal to the expected amount of water pumped out of the aquifer, with a given probability λ_2.

$$AS\,\mu_{h_{i-1}}\gamma_{P_i} \leqq \mu_{Q_P'} + x_2\ \sigma_{Q_P'} \qquad (6.30)$$

3) The diversion facilities constraint establishes that the surface water capacity at a time $\underline{i}$ must be greater than or equal to the expected amount of water diverted from the stream, with a given level of probability λ_3 .

$$\mu_{D_{i-1}} \gamma_{S_i} \leqq \mu_{Q'_{SD}} + x_3 \ \sigma_{Q'_{SD}} \qquad (6.31)$$

4) The dam freeboard constraint says that the freeboard at time $\underline{i}$ must exceed f_i with probability λ_4 .

$$\gamma_{R_1} - \gamma_{B_i} \geqq r_i \ (\lambda_4) + f_i \qquad (6.32)$$

where $r_i(\lambda_4)$ is the 100. λ_4 percentile of the streamflow Q_{ST_i} and f_i is the considered freeboard volume.

To obtain equation 6.32, a new decision variable γ_{B_i} (see Section 3.3) is introduced through a linear decision rule (Revelle et.al., 1969) which is

$$Q_{ou_i} = S'_{i-1} - \gamma_{B_i} \qquad (6.33)$$

is expressed in terms of the reservoir storage during the previous time period S'_{i-1} . The continuity equation for the reservoir is

$$S'_i = S'_{i-1} - Q_{ou_i} + Q_{ST_i} \qquad (6.34)$$

Substituting (6.33) into (6.34) produces

$$S_i' = \gamma_{B_i} + Q_{ST_i} \tag{6.35}$$

and similarly

$$S_{i-1}' = \gamma_{B_{i-1}} + Q_{ST_{i-1}} \tag{6.36}$$

Introducing (6.36) into (6.33), we obtain

$$Q_{ou_i} = \gamma_{B_{i-1}} - \gamma_{B_i} + Q_{ST_{i-1}} \tag{6.37}$$

which gives us an expression of the release from the dam Q_{ou} at time $\underline{i}$, based on the stream flow Q_{ST} at time i-1.

In a deterministic representation of the freeboard constraint

$$\gamma_{R_1} - S_i' \geqq f_i \tag{6.38}$$

Substituting equation 6.35 into 6.38, we have

$$\gamma_{R_1} - \gamma_{B_i} - f_i \geqq Q_{ST_i} \tag{6.39}$$

If the streamflow is random, then equation 6.39 can be written in terms of a probability statement, as follows:

$$P(Q_{ST_i} \leqq \gamma_{R_1} - \gamma_{B_i} - f_i) \geqq \lambda_4 \tag{6.40}$$

For mathematical programming it is better to represent it as a certainty equivalent (equation 6.32).

5) The stream requirement constraint, states that the streamflow downstream of the studied region must be greater than or equal to a given senior right K_1 with a probability λ_5.

$$w_o \gamma_{R_1} - \gamma_{B_1} - \mu_{D_1} \gamma_{S_1} \geqq aA(\mu_H - \mu_{h_1}) + K1_1 \qquad (6.41)$$

is an equation for time $i = 1$, and

$$\gamma_{B_{i-1}} - \gamma_{B_i} - \mu_{D_{i-1}} \gamma_{S_i} \geqq aA(\mu_H - \mu_{h_i}) - r_{i-1}(1-\lambda_5) + K1_i \qquad (6.42)$$

is a constraint for time $i \geqq 2$ where $w_o \gamma_{R_1}$ is the initial storage of the reservoir.

6) The dam storage constraint is defined as the storage at the dam which must be greater than a minimum storage with a probability λ_6.

$$w_m \gamma_{R_1} - \gamma_{B_i} \leqq r_i(1-\lambda_6) \qquad (6.43)$$

where $w_m \gamma_{R_1}$ is the minimum storage that the dam must have and $r_i(1-\lambda_6)$ is the $100.(1-\lambda_6)$ percentile of the stream-flow.

The deterministic constraint can be written as

$$S_i' \geqq S_{min}'$$

Then, making use of equation 6.35, we obtain

$$Q_{ST_i} \geq w_m \gamma_{R_1} - \gamma_{B_i} \tag{6.44}$$

where $S'_{min} = w_m \gamma_{R_1}$. Now, if Q_{ST_i} is random, we have

$$P(Q_{ST_i} \geq w_m \gamma_{R_1} - \gamma_{B_i}) \geq \lambda_6$$

or

$$1 - F_{Q_{ST_i}} (w_m \gamma_{R_1} - \gamma_{B_i}) \geq \lambda_6 \tag{6.45}$$

in which $F_{Q_{ST_i}}$ is the cumulative distribution of the streamflow. Equation 6.45 in its certainty representation becomes equation 6.43.

At this point, it is convenient to mention that the levels of probability λ's are selected by the designer according to available information and future estimations about the system operating policy.

A well drawdown correction DC was included in the objective function to simulate the difference between the average water level in the aquifer and the average water level at the pumping wells. Equation 2.49, previously developed, was applied to obtain the above correction. Well losses were not included in the analysis.

A mean influence area of the wells of 1.45 Km^2 was obtained from a map showing the location of the wells in the studied area. This value is based on an average distance between wells, the irrigated land area and the number of wells significantly deep (depth greater than 25 meters).

The well drawdown correction based on (2.49) is

$$s_w - \bar{s} = (Q / 2\pi T)(p_D - \bar{p}_D) \tag{6.46}$$

where Q is an average instantaneous pumping, obtained by assuming that two thirds of the number of pumping wells, WN, were pumping half of the year; it is related to the annual pumping of the area Q_P , as follows.

$$Q_P = 2/3 \times 1/2 \times WN \times Q_W \times 31.54 \tag{6.47}$$

Statistics

A gamma distribution was selected to represent the streamflow because of its non-symmetry (Fiering, 1971, p.35). The gamma cumulative distribution (Mood et.al., 1963, p.128) or incomplete gamma function is given by

$$F(x) = \int_0^x t^{\alpha} \exp(-t/\beta)dt/\alpha!\beta^{\alpha+1} \tag{6.48}$$

which can be transformed to

$$P(a, x') = F(x) = 1/\Gamma(a) \int_0^{\beta x} t_1^{a-1} \exp(-t_1)dt_1 \tag{6.49}$$

where, $\Gamma(a) = \alpha!$, $\alpha = a-1$ and $x = \beta x'$

Equation 6.49 can be related to the chi-square distribution if a is an integer (Abramowitz, et.al., 1964, p.941) as follows,

$$P(a, x') = \gamma(a, x')/\Gamma(a) = P(\chi^2/\nu) \qquad (6.50)$$

in which, $\nu = 2a$ and $\chi^2 = 2x'$ (chi-square distribution), and

$$\Gamma(a, x')/\Gamma(a) = Q(\chi^2/\nu) \qquad (6.51)$$

where

$$1 - P(\chi^2/\nu) = Q(\chi^2/\nu) \qquad (6.52)$$

A table of $Q(\chi^2/\nu)$ can be found in Abramowitz (1964, p. 978). Two parameters a and β define the shape of the gamma distribution and are related to the statistics of the streamflow as follows:

$$\mu = \beta a$$
$$\sigma^2 = \beta^2 a^2$$

The assumed values of μ and σ were $1334 \times 10^6\ m^3/yr$ and $544 \times 10^6\ m^3/yr$, respectively. Therefore,

$a = 6$

$\beta = 222.5$

$\nu = 12$

For a value of probability of $F(x) = 0.75$ we obtain $\chi^2 = 14.85$ from the tables of χ^2. Since

$$\chi^2 = 2x/\beta$$

then

$$x = r(0.75) = 1650 \times 10^6 \ m^3/yr$$

where $r(0.75)$ is the 75 th percentile of the streamflow. Repeating the procedure for $F(x) = 0.25$, we have

$$x = r(0.25) = 940 \times 10^6 \ m^3/yr$$

where $r(0.25)$ is the 25 th percentile of the streamflow.

The cross correlation function of pumping and the head of the aquifer for zero lag(computed by equation 4.28) was -0.74.

Parameters Needed by the Models

The amount of information required by the models under a stochastic formulation increases substantially compared to a deterministic formulation. The values of the parameters required for the Rio Sinaloa problem are given in Table 6.3.

A flow chart depicting the iterative procedure for solving the Rio Sinaloa management problem under random conditions is shown in Figure 6.5. The listing of the program is given in Appendix I.

Results

Figure 6.6 summarizes certain final results obtained from the application of the proposed models to the study area. These results are: the optimal operation of the system, the optimal size of the dam γ_{R_1} and the expected value of the operational cost of the system (which includes the cost of the dam). Dam, surface water diversion facilities and pumping facilities are those items optimally operated. Also shown in Figure 6.6 are: the expected water level in the aquifer, the stream-aquifer flow and the 75 th percentile of the demand.

Examination of the results shows that pumping was always at its maximum capacity throughout the entire horizon, since pumping was cheaper than diversion of water from the stream. Therefore, the amount of surface water was enough to satisfy the demand unfulfilled by pumping, and at the same time absorbed any trend or random fluctuation included in the demand.

An average well drawdown correction of about 7.29 m was found by using equation 2.49.

Because of the increasing water demand and the particular

TABLE 6.3 Parameter Values for the Rio Sinaloa Problem.

N	Design horizon	20 years
N_S	Number of seasons per year	1
A	Aquifer area	1744 Km^2
T	Transmissivity	0.02 m^2/sec
S	Storage coefficient	0.01
β	Dimensionless constant, used to compute the subsurface outflow constant	3.0
L	Characteristic length	21 Km.
Z	Ground surface level	350 m.
H	Mean stream water level	338.63 m.
h_o	Initial water level	338 m.
WN	Number of wells	314
r_w	Average well radius	0.254 m.
A_w	Average influence area of a well	1.45 Km^2
$K1_i$	Downstream quota	0.0
$F1_i$	Dam freeboard	$200 \times 10^6\ m^3$
w_m	Minimum dam storage fraction	0.4
w_o	Initial dam storage fraction	0.6
α_1	Fraction of precipitation that actually reaches the water table	0.1845
α_2	Fraction of water applied that infiltrates	0.411
α_4	Fraction of water infiltrated that actually recharges the aquifer	0.45

TABLE 6.3 (Continued)

β_1	Fraction of precipitation that helps to satisfy the demand	0.422
ET	Expected value of evapotranspiration	50.4×10^6 m^3/yr
P	Expected value of precipitation	785×10^6 m^3/yr
Q_i	Expected value of subsurface inflow	25×10^6 m^3/yr
ρ_y	Autocorrelation function of the inputs for lag 1	0.7
$\rho_{Q_p h}$	Cross correlation function of pumping and head for lag 0	-0.74
	Probability level to satisfy the constraints	0.75
r(0.75)	75 percentile of the streamflow	1650×10^6 m^3/yr
r(0.25)	25 percentile of the streamflow	940×10^6 m^3/yr
$\mu_{Q'_{P_i}}$	Expected value of capacity of the pumping facilities	470×10^6 m^3/yr
$\mu_{Q'_{SD_i}}$	Expected value of capacity of the surface water facilities	1500×10^6 m^3/yr
σ_{D_i}	Standard deviation of the demand	200×10^6 m^3/yr
$\sigma_{Q'_{P_i}}$	Standard deviation of the pumping facilities	70×10^6 m^3/yr
$\sigma_{Q'_{SD_i}}$	Standard deviation of the surface water facilities	300×10^6 m^3/yr
σ_a	Standard deviation of the subsurface outflow constant	$0.25\bar{a}$

TABLE 6.3 (Continued)

σ_y	Standard deviation of the input	8.07×10^{-3} m/yr
r	Nominal rate of interest	5%
μ_{c_S}	Expected stream diversion costs	$6000/10^6 m^3
μ_{c_R}	Expected dam unit cost	$24444/10^6 m^3
μ_{c_P}	Expected pumping costs	$63.8/10^6 m^4
μ_{D_i}	Expected water demand in 10^6 m^3	

i	μ_{D_i}	i	μ_{D_i}	i	μ_{D_i}	i	μ_{D_i}
1	800	6	1000	11	1050	16	1050
2	850	7	1000	12	1050	17	1050
3	900	8	1050	13	1050	18	1050
4	950	9	1050	14	1050	19	1050
5	950	10	1050	15	1050	20	1050

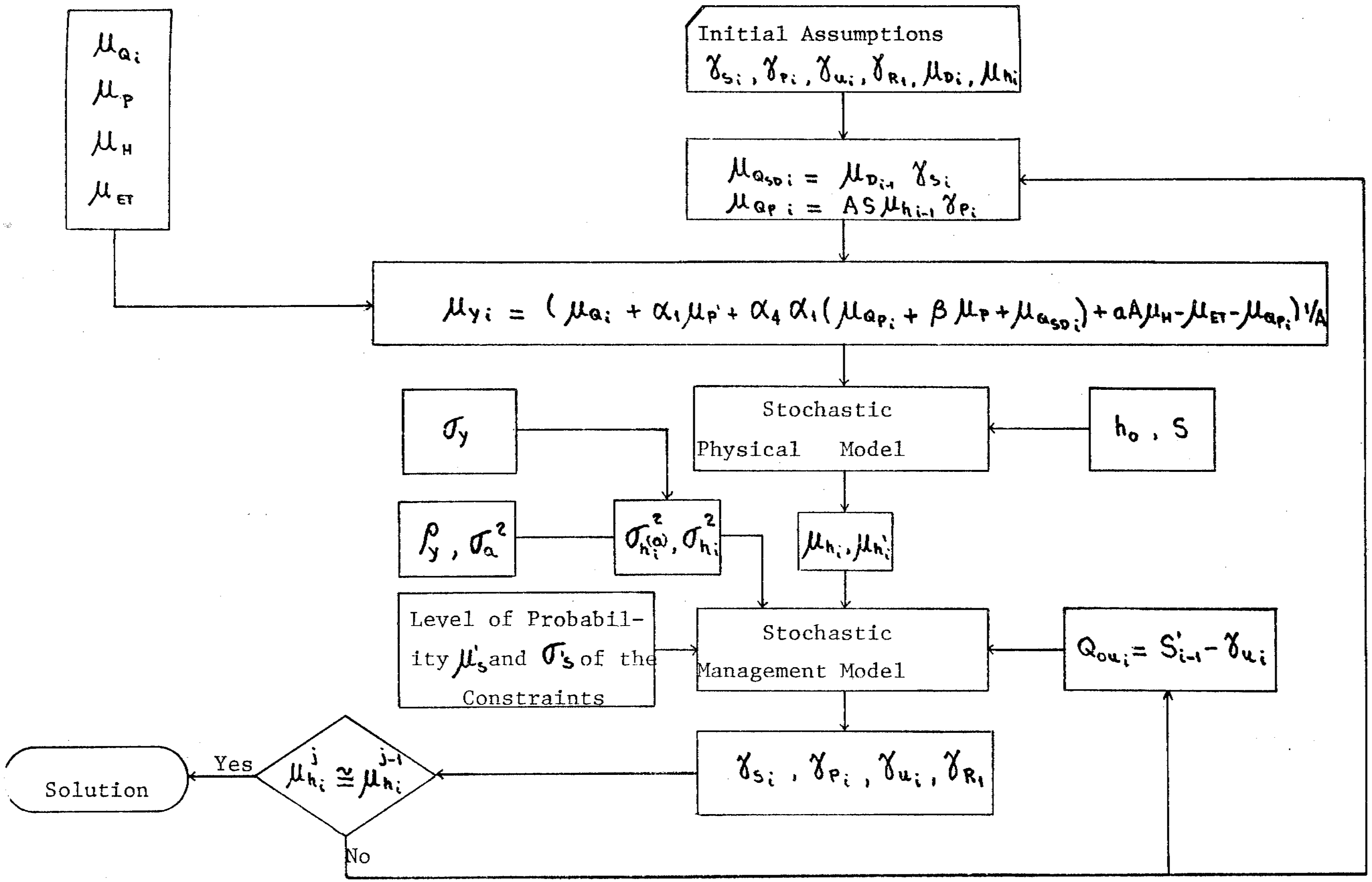


Fig. 6.5 Iterative procedure used to solve the Rio Sinaloa management problem.

Objective function	OF = \$113,224,000
Reservoir volume	γ_{R_1} = 1915 x 10^6 m^3
Std. dev. of subsurface outflow constant	$\sigma_a = 0.25\ \bar{a}$
Probability level	$\lambda = 0.75$

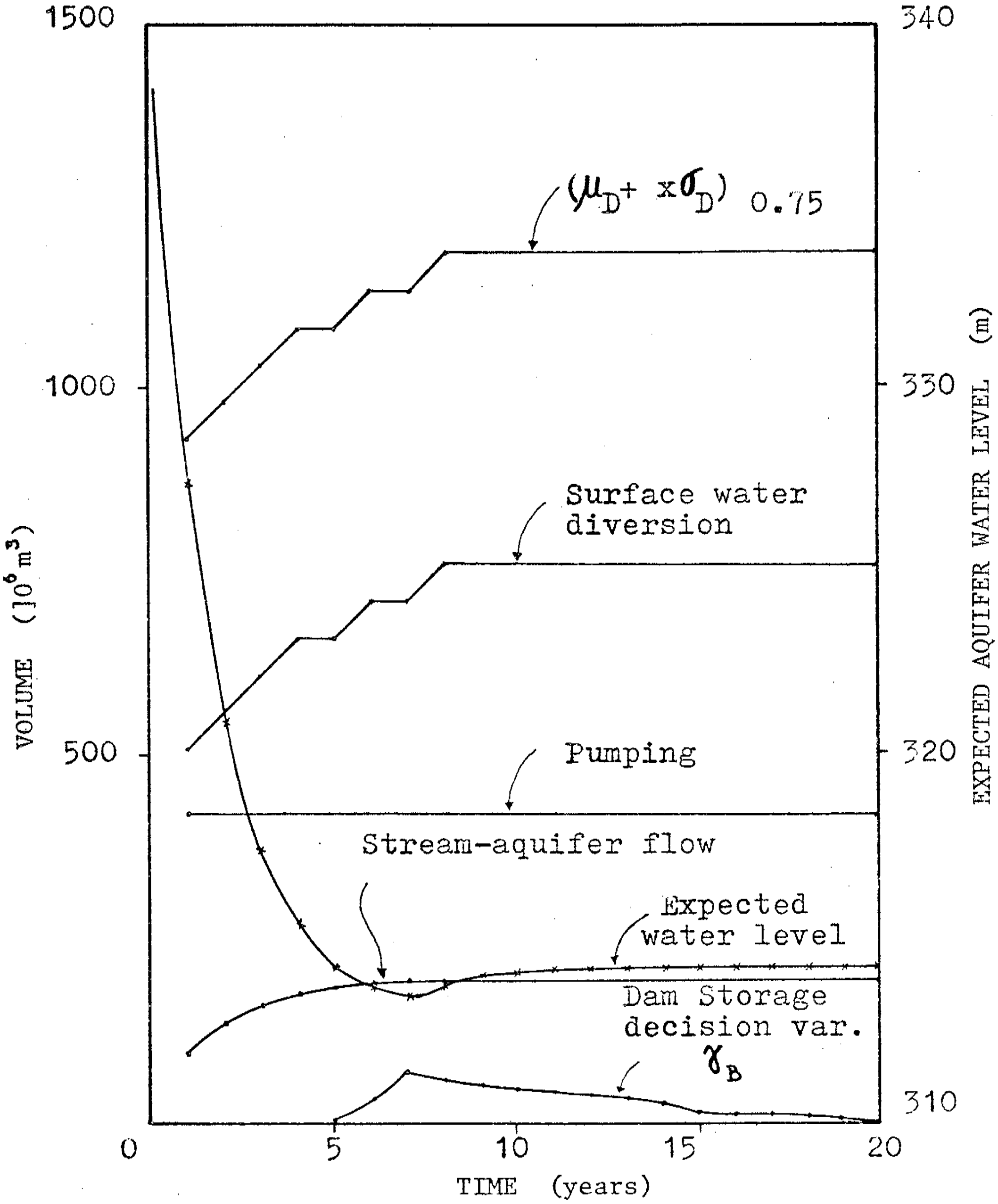


Fig. 6.6 Operational scheduling and aquifer behavior for probability level $\lambda = 0.75$.

combination of physical aquifer properties, the aquifer behaved such that the expected water levels experience a fast drop in the first five years of operation of the system. After this large initial decline of the water table, an equilibrium is rapidly reached because irrigation return flow and stream-aquifer flow increase. The amount of increase depends on the demand and on the stream-aquifer head difference.

The results of several alternatives are summarized in Table 6.4. Note that alternative B shows a 19% increase of the objective function relative to the deterministic case (alternative A), when the coefficient of variation of the demand is approximately 0.2 (see Table 6.3). This increase is comparable to that found in the Maddock problem under similar conditions (see Figure 6.1).

A graph of the variability of the water level h in the aquifer is presented in Figure 6.7. Shown is the result when the subsurface outflow constant a is a deterministic quantity ($\sigma_a = 0$) and the randomness is due to the inputs to the system. Figure 6.7 also shows the variability of h when a is treated as a random variable with a standard deviation of $0.25\bar{a}$. A sum of both variance $\sigma^2_h(\bar{a})$ and $\sigma^2_h(a)$ produces the total variance of the system. From Figure 6.7 it is seen that both cases behave in a similar manner, increasing up to a steady state value. However, $\sigma_h(a)$ has higher values than $\sigma_h(\bar{a})$ and takes almost twice the time to show its total effect. This implies that an uncertainty in a due to inadequate field information and

TABLE 6.4 Summary of the Results Obtained from Several Alternatives for the Rio Sinaloa.

	ALTERNATIVES			
	A	B	C	D
λ	deterministic	0.75	0.75	0.75
σ_a		0.0	0.25a	a
μ_{c_s} ($\$/10^6 m^3$)	6000	6000	6000	6000
μ_{c_p} ($\$/10^6 m^3$)	63.8	63.8	63.8	63.8
OF($)	95,054,000	112,993,000	113,224,000	116,703,000
γ_{R_1}($10^6 m^3$)	1534	1915	1915	1915
	ALTERNATIVES			
	E	F	G	H
λ	0.75	0.9	0.9	0.75
σ_a	0.25a	0.25a	0.25a	0.25a
μ_{c_s} ($\$/10^6 m^3$)	3000	6000	3000	3000
μ_{c_p} ($\$/10^6 m^3$)	63.8	63.8	63.8	90.0
OF($)	87,090,000	1,325,252,000	1,292,978,000	118,942,000
γ_{R_1}($10^6 m^3$)	1915	51,054	51,054	1915

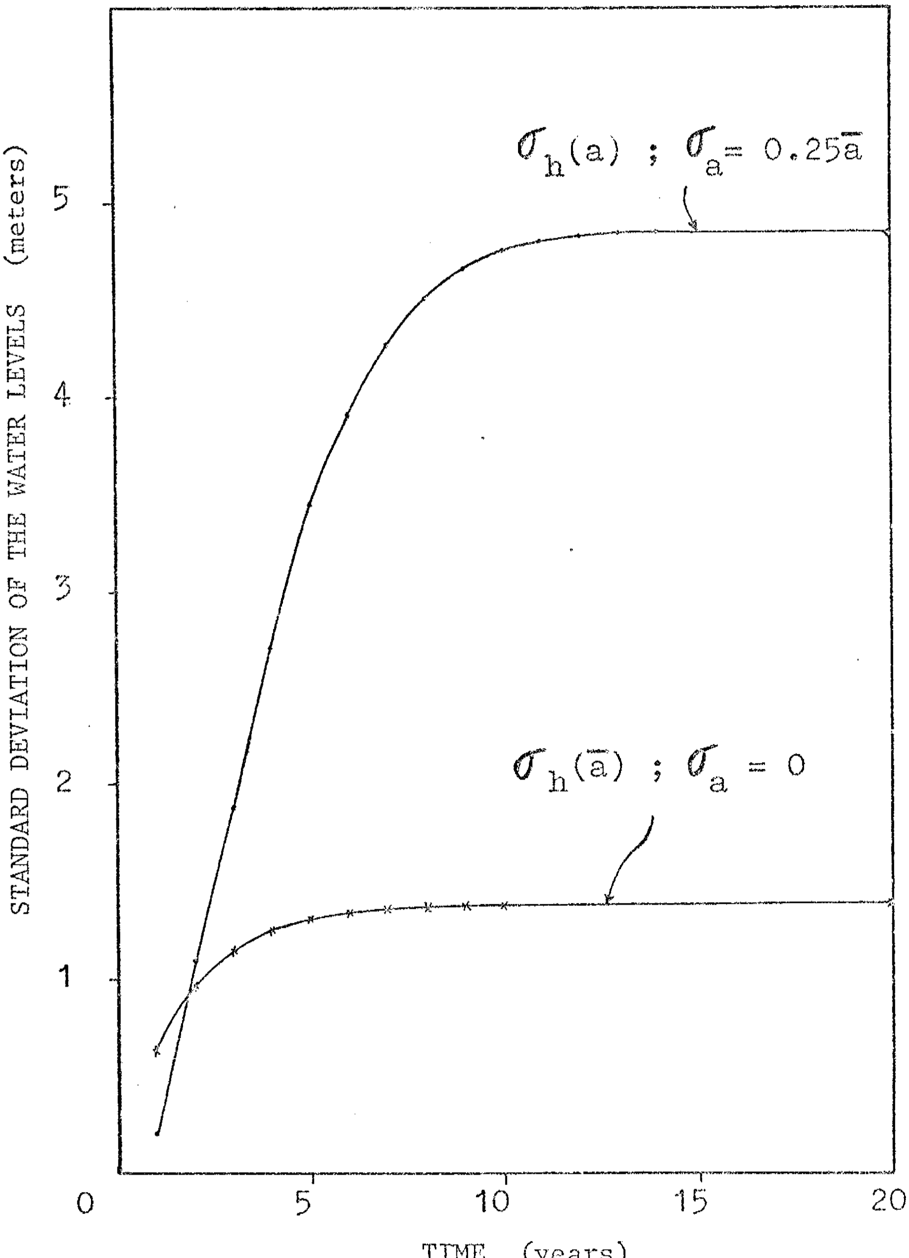


Fig. 6.7 Variability of aquifer water levels; $\sigma_h(\bar{a})$ represents uncertainty when the subsurface outflow constant is deterministic, the mean ($\bar{a} = 4.29 \times 10^{-3}$ yr^{-1}); $\sigma_h(a)$ represents uncertainty when the subsurface outflow constant is considered a random variable with $\sigma_a = 0.25\bar{a}$.

the random behavior of the system properties such as the aquifer transmissivity, increases considerably the uncertainty associated with water levels.

Sensitivity Analysis

In order to see the effect of randomness on the operation of the system, the deterministic case was run and its solution is presented as Figure 6.8. We note that both cases, deterministic and random, behave in a very similar manner; but the system under fully known conditions is cheaper to operate and the size of the dam is almost 25% smaller than in the situation where uncertainties dominate the picture.

The effect on the objective function caused by uncertainty in the subsurface outflow constant is shown in Figure 6.9. However, it should be noted that this behavior depends on the parameters of the stream-aquifer system through the coefficient part of the variance term in (6.28). In this case the effect on the objective function which is caused by uncertainty in the subsurface outflow constant is small (Figure 6.9).

The nonlinear objective function (quadratic in pumping) was solved by the proposed iterative procedure (see, Section 3.3), in which a standard linear programming package using a simplex algorithm (Kuester and Mize, 1973, p.10) was used. If reasonable initial conjectures of the value of the variables were supplied to the models, only two or three

Objective function OF = \$95,054,000
Reservoir volume $\gamma_{R_1} = 1534 \times 10^6\ m^3$

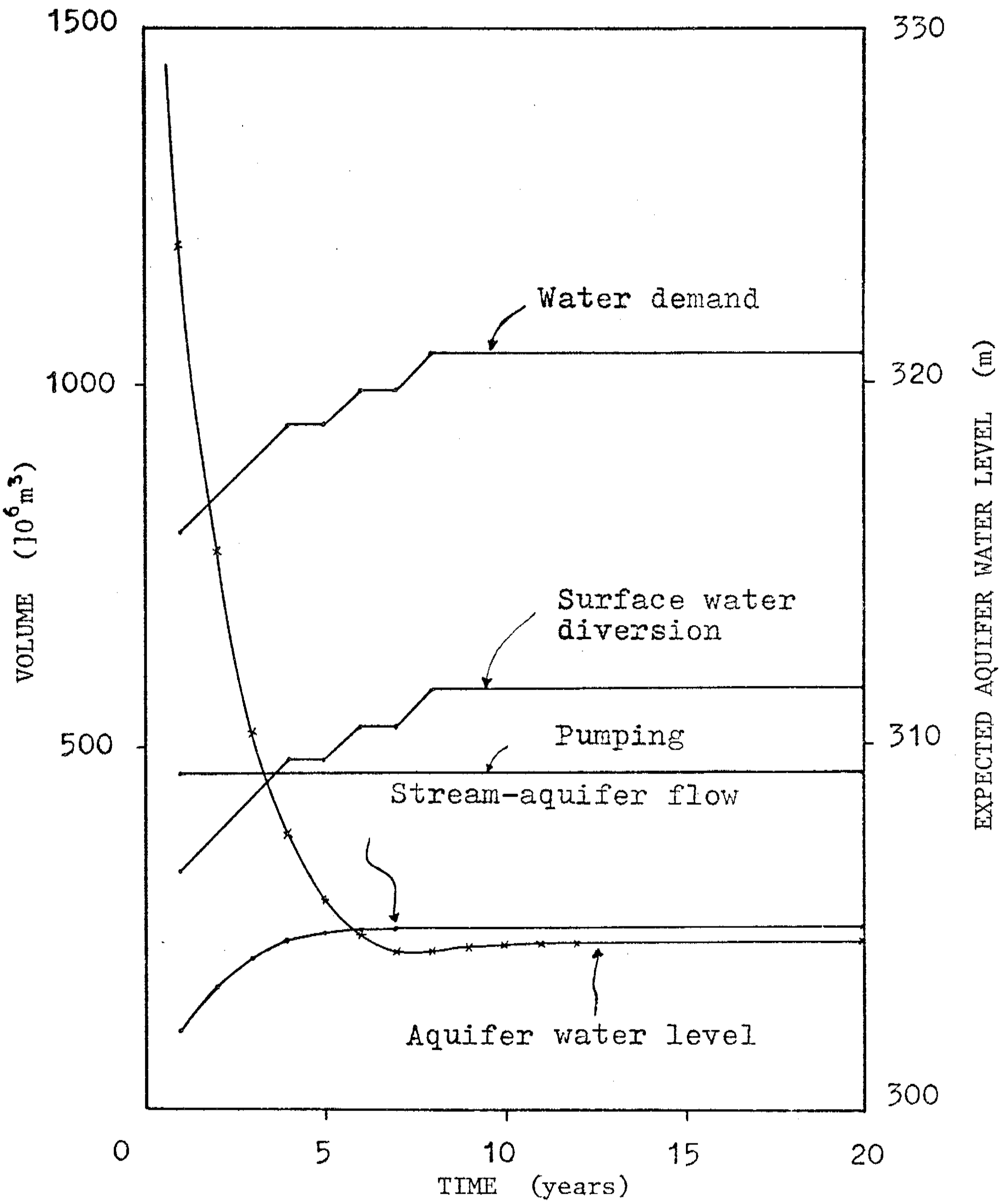


Fig. 6.8 Operational scheduling and aquifer behavior of the deterministic case.

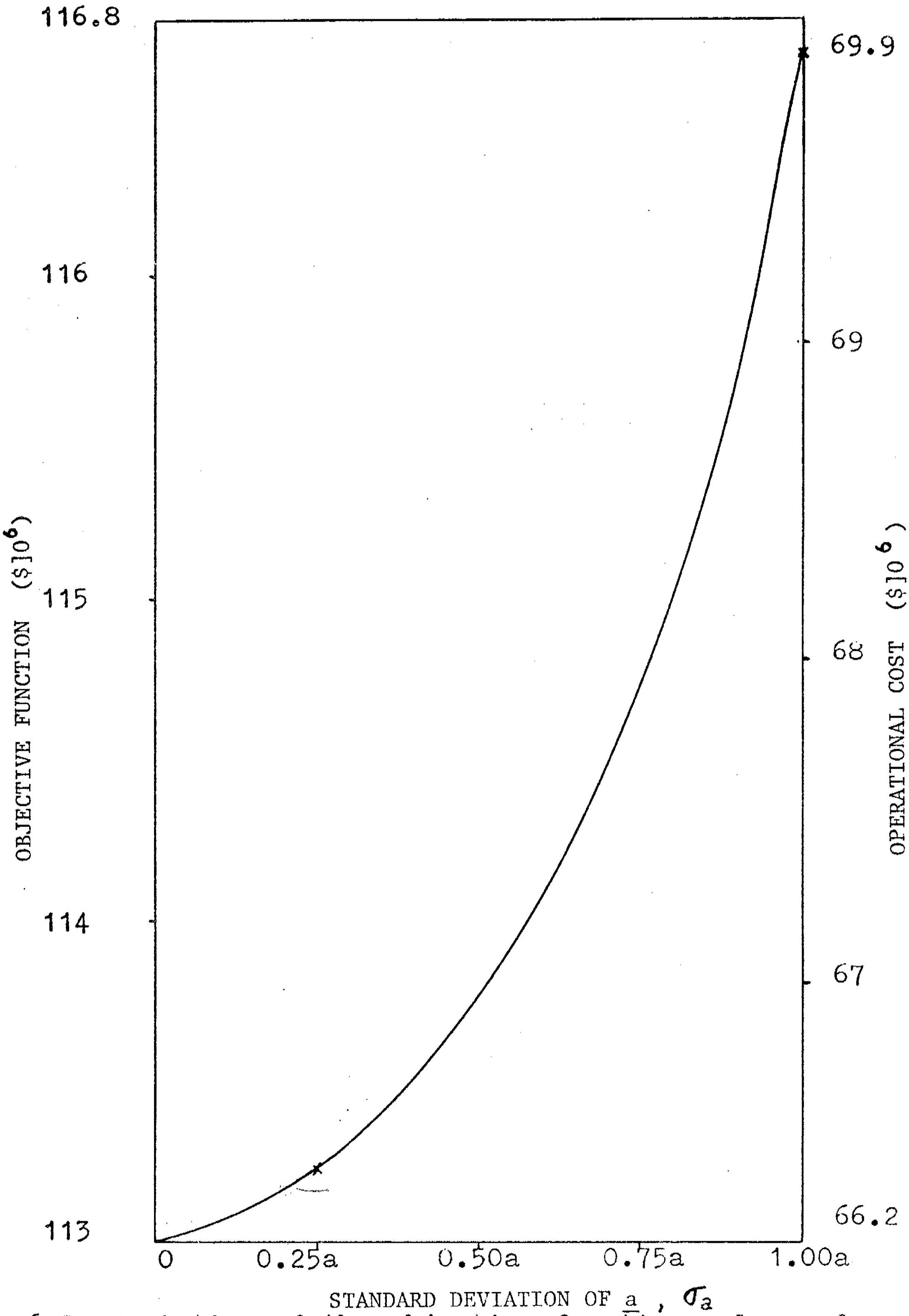


Fig. 6.9 Variation of the objective function value and operational cost of the system as a function of uncertainty in the subsurface outflow constant <u>a</u> for a level of probability $\lambda = 0.75$.

iterations in an IBM 360 model 44 were required to solve the Rio Sinaloa management problem. Less than 150 k bytes of main memory were utilized.

Convergence problems were found when the cost of diverting water from the stream Q_{SD} , and of pumping from the aquifer Q_P were approximately equal. The above problems are illustrated in Figure 6.10 and 6.11; Figure 6.10 shows the results of two iterations (subindices 1 and 2); Figure 6.11 presents the average of iterations 1 and 2 (subindex 3) as the initial assumption of the next iteration (subindex 4). Even though both objective functions are almost identical in Figure 6.11, the policies appear very different. No procedure was found to solve this convergence problem. The application of any of the two schedulings found by the models (Figure 6.11) would of course solve the management problem, since their cost is the same. The actual selection of a operational scheme would depend on factors other than those economic factors introduced in this management model.

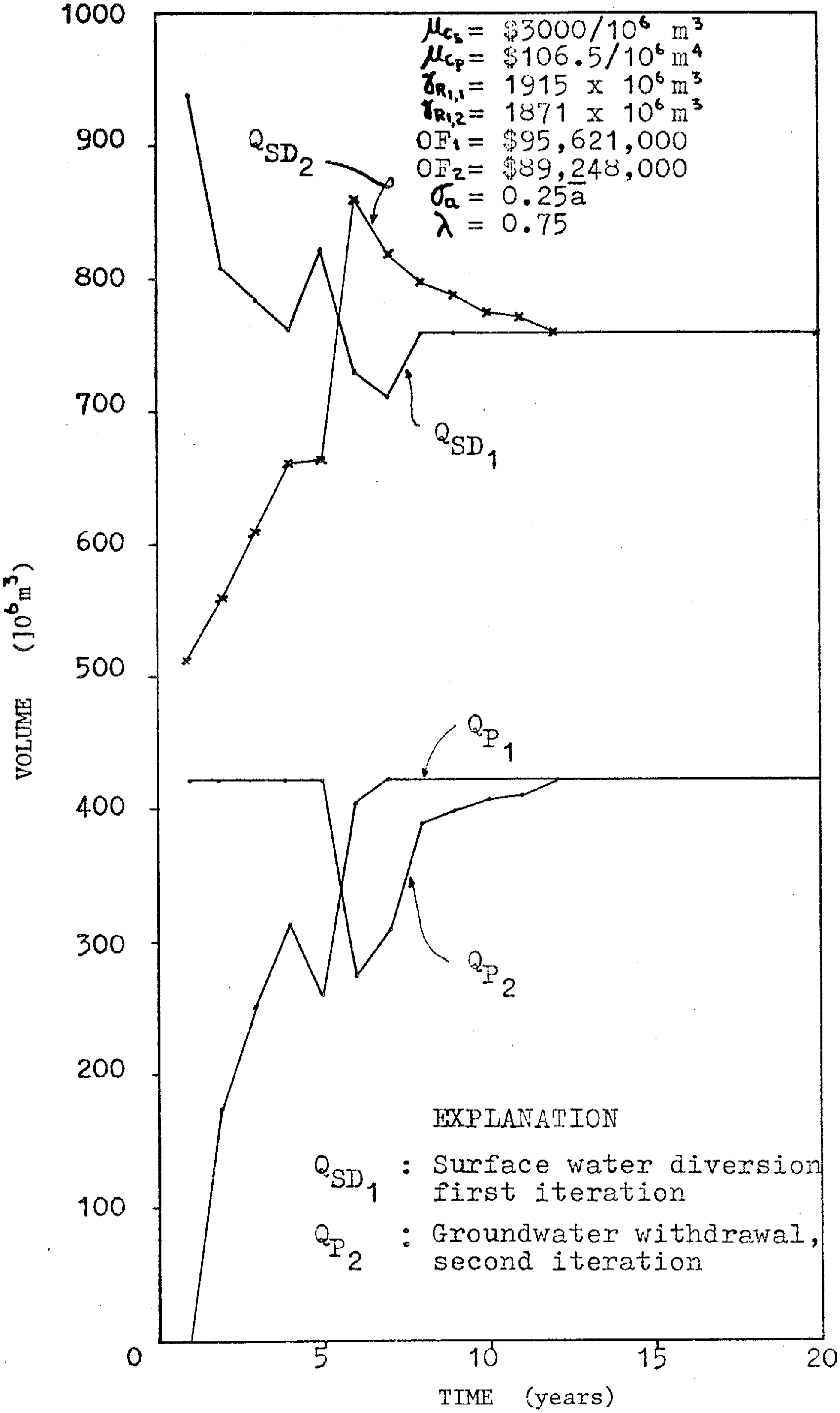


Fig. 6.10 Representation of the convergence problem when cost of surface water diversion and pumping are nearly the same.

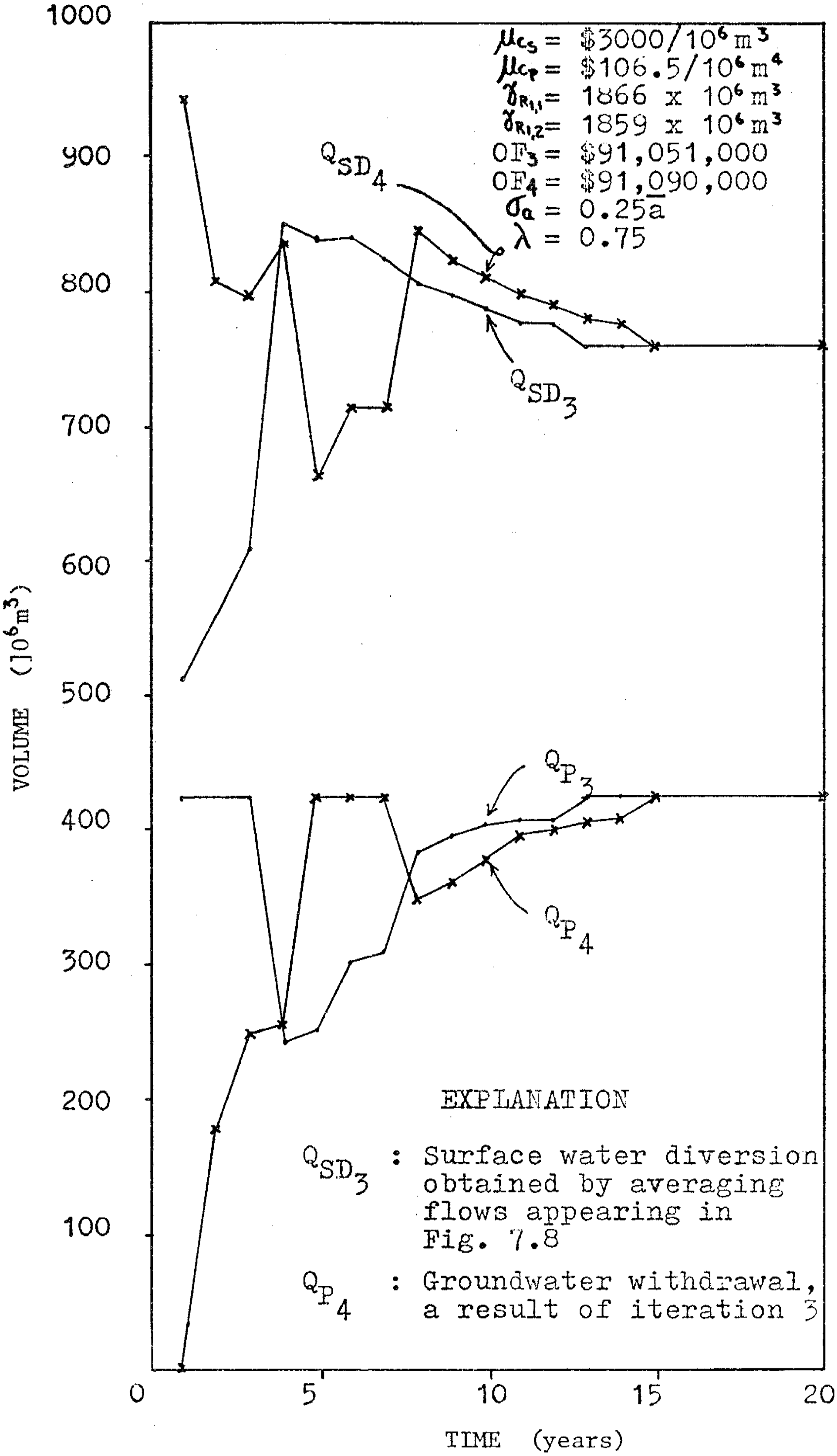


Fig. 6.11 Effect of averaging on convergence.

CHAPTER 7 SUMMARY AND CONCLUSIONS

Summary

A development of simple models to represent a stream-aquifer system and its optimal operation under situations involving uncertainties was the main concern of the study. Only physical and economic variables were considered in this work.

A lumped parameter model composed of an aquifer water balance and a linear stream-aquifer flow relationship is proposed for modeling the system. This model is not offered as a substitute for distributed models but as a simple reliable and economical alternative suitable for initial evaluation of systems with only limited field data. The stream-aquifer flow is governed by the subsurface outflow constant and a head difference between stream and aquifer. The subsurface outflow constant was found to be dependent on the average transmissivity of the aquifer, a characteristic length of the aquifer and a dimensionless constant. The effect of system properties and input characteristics on the value of the dimensionless constant was studied.

The solution of the physical or lumped parameter model is given by a convolution integral which accounts for past inputs to the system. Since the lumped model provides an average water level $\underline{h}$, an improvement was made by introducing a correction which allows the physical model to compute the head at the pumping wells. An important concept included in the analysis is the response time of the stream-aquifer system t_h ; it is a measure of the time required for the system to respond to inputs, and is related to the subsurface outflow constant and an average storage coefficient of the aquifer.

A management model was developed which minimizes the discounted operational costs of a stream-aquifer system. It is linked to the physical model by the average head of the aquifer. A simplification of the objective function was obtained by computing the average head of the aquifer outside of the management model, not including any past input to the system in the objective function. Taking advantage of this link between the models, an iterative procedure was developed for solving the quadratic optimization problem with a standard linear programming package. Linear decision rule were used to define the decision variables. By making present decisions based on previous information the operation of the system was made more dynamic.

A further step in the analysis was made by considering random natural inputs and a random demand. A stochastic differential equation with a random forcing function represented the physical model. The ensemble average, the variance, and the autocovariance of the aquifer head were computed. Using a conditional probability approach, the variance of the random subsurface outflow constant was included in the problem; a variance of the head as a function of the variance of the inputs and of the subsurface outflow constant is computed. It was found that the pumping cost depends on the interaction between the head and pumping. Hence, spectral analysis was used to find a cross correlation coefficient between these quantities. The minimization of the discounted expected value of cost is the representation of the objective function in a stochastic system. Chance constraints which allow for satisfaction of the constraints with a given probability level were used in the stochastic management model. Nonstationary demands of water are easily represented by these types of constraints.

To examine the reliability of the proposed models a comparative test was made with a study by Maddock (1974), which includes a distributed aquifer model coupled with a stochastic management model. From the comparison, similar operating rules were found and a sensitivity analysis showed that Maddock's results were a particular case of a more general problem treated by our stochastic management model. The increase in the objective function with increasing variance of the demand was in agreement with that found by Maddock. However, some differences in the objective functions were found because of the local effect on the water table of only two pumping wells. The iterative procedure as well as the link between the physical and management model proved to work satisfactorily. An advantage of our models is the small number of terms in the objective function produced by computing the mean water level of the aquifer outside of the management model.

The proposed model was applied to a basin in northwestern Mexico. The adaptability to a different situation was tested and a sensitivity analysis performed. Optimal decisions about the size and operation of a projected dam were made. The operational scheduling of water diverted from the stream and water pumped out of the aquifer were obtained to satisfy a random demand. Since the groundwater costs less than surface water, all random fluctuations in the demand of water were absorbed by the surface water. The variance of the random levels in the aquifer including a random or a deterministic subsurface outflow constant, showed similar patterns; it increased up to a steady state value. The variance of the water levels was larger when affected by a random subsurface outflow constant. Comparison of a deterministic and a stochastic case showed that for a deterministic case operational costs are less and a smaller dam is required. The study of the sensitivity of the objective function to uncertainties in the subsurface outflow constant showed that they had little effect on the discounted expected value of cost. Convergence problems in the iterative procedure were found only when the surface water cost and pumping cost were similar. In this case two different policies produced almost identical discounted cost. Otherwise, with reasonable initial assumptions, only two or three iterations where required to converge to a reasonably accurate solution.

Conclusions

The following conclusions were obtained

1. In regional studies the physical model developed in this work is very simple and reliable in its usage; it is capable of accounting for local drawdown at the wells, stream-aquifer interaction, and can easily be coupled to a management model.
2. The subsurface outflow constant is a very useful concept in modeling a stream-aquifer system. It groups parameters such as the transmissivity of the aquifer, a characteristic length of the system, and a dimensionless constant which depends on several system properties and input characteristics.
3. A simple link was made between a physical and a management model allowing us to develop an iterative procedure for solving a nonlinear optimization problem with a standard linear programming package.
4. In the stochastic management model dynamics were introduced with the use of linear decision rules to define decision variables and more generality was obtained with the use of chance constraints. A nonstationary water demand was easily represented by using chance constraints.
5. Good agreement was obtained between the results of the proposed model and a previous study by Maddock

(1974) based on a distributed aquifer model and quadratic programming. The operating rules and the sensitivity to demand uncertainty were practically the same. The value of the objective function obtained from the lumped parameter model was slightly larger; the difference is thought to be related to the interaction between the stream and the local cones of depression of the two wells.

6. The versatility of the developed model was demonstrated by applying it in the Rio Sinaloa study area in northwestern Mexico. This was a regional study involving over 400 Km^2 of land irrigated by hundreds of wells and by diversion from a stream. The optimal operational scheduling of conjunctive use of surface water and groundwater and the optimal size of a surface reservoir were obtained under random conditions.

7. The effect of uncertainty on the management of a stream-aquifer system is an important factor to be considered; under random conditions the size of the surface reservoir is larger and the cost of operation is greater than under a deterministic situation. The choice of the level of probability in the chance constraints is an important managerial decision because the expected value of the discounted cost is significantly affected by the level of probability. Uncertainty of the water demand

produced a larger increase in the operational cost than uncertainty in aquifer parameters. However, the effect of uncertainty on the aquifer water levels was found to be dependent on the system properties and input randomness.

Recommendations

1. Work remains to be done on the determination of the subsurface outflow constant when the aquifer is asymmetric with respect to the stream.
2. Different types of boundaries and shapes of well influence areas need to be used and analysed.
3. In managerial studies of conjunctive use of groundwater and surface water more emphasis should be given to the statistics of the economic variables than the statistics of the properties of the stream-aquifer system.
4. More study related to the convergence problem of the iterative procedure is needed.
5. Another approach to include a random subsurface outflow constant in the physical model, can be used: it consists of solving a stochastic differential equation with a random coefficient and a random forcing function.

APPENDIX A Computation of the Subsurface Outflow Constant for the Unsteady Case

The equation governing the flow is

$$\alpha^2 \partial^2 h/\partial x^2 + \epsilon = \partial h/\partial t \tag{A.1}$$

where $\alpha^2 = T/S$ is the hydraulic diffusivity of the aquifer, and

$$\epsilon = \epsilon_o \quad ; \quad t < 0$$

$$\epsilon = 0 \quad ; \quad t > 0$$

The initial and boundary conditions are

$$t < 0 \quad ; \quad h - H = (2L-x)\,\epsilon_o\, x/2T \tag{A.2}$$

$$x = 0 \quad ; \quad h - H = 0 \tag{A.3}$$

$$x = L \quad ; \quad \partial(h - H)/\partial x = 0 \tag{A.4}$$

The method of separation of variables is used to solve (A.1) satisfying the initial and boundary conditions. The variable $h - H$ is used throughout the analysis.

Let

$$h - H = X(x)T(t) \tag{A.5}$$

Substituting the above expression into (A.1) produces

$$\alpha^2 X''/X = T'/T$$

Now, let

$$\alpha^2 X''/X = -p^2 \tag{A.6}$$

and

$$T'/T = p^2 \tag{A.7}$$

Solving (A.6) and (A.7) and substituting into (A.5) we have

$$h - H = \exp(-p^2 t)\ (A \cos px/\alpha + B \sin px/\alpha) \tag{A.8}$$

where _A_ and _B_ are constants of integration, which are obtained by using (A.3) and (A.4). Now (A.8) may be transformed into

$$h - H = B \exp((-m\pi\alpha/2L)^2 t)\ \sin(m\pi x/2L) \tag{A.9}$$

In order to satisfy the initial condition (A.2)

$$h - H = \sum_{m=1,3,5,\ldots}^{\infty} A_m \exp((-m\pi\alpha/2L)^2 t)\ \sin(m\pi x/2L) \tag{A.10}$$

where

$$A_m = 1/L \int_0^{2L} f(x)\ \sin(m\pi x/L)dx$$

If

$$f(x) = (2L - x)\ \epsilon_o\ x/2$$

then

$$A_m = (1 - \cos m\pi)8\ \epsilon_o\ L/T\ m^3\ \pi^3 \tag{A.11}$$

Equations A.10 and A.11 are the solutions of (A.1) satisfying the initial and boundary condition.

The average head in the aquifer can be obtained by

$$\bar{h} = 1/2L \int_0^{2L} h\ dx \tag{A.12}$$

Substituting (A.10) into (A.12) we get

$$\bar{h} - H = \sum_{m=1}^{\infty} A_m\ \exp((-m\pi/2L)^2 Tt/S)\ (1-\cos m\pi)/m \tag{A.13}$$

since

$$q = T/L\ (\partial \bar{h}/\partial x)\Big|_{x=0} \tag{A.14}$$

we can find that

$$q = \sum_{m=1}^{\infty} A_m \exp((-m\pi/2L)^2 Tt/S)\ (Tm\pi/2L^2) \qquad (A.15)$$

Both Venetis (1969) and Kraijenhoft Van de Leur (1958) show that the terms in the series in (A.13) for $m > 1$ become very small relative to the first harmonic when time increases. Retaining only the $m = 1$ term

$$q = A_1 \exp(-\pi^2 Tt/4S)\ (T\pi/2L^2) \qquad (A.16)$$

and

$$\bar{h} - H = (A_1\ 2/\pi)\ \exp(-\pi^2 Tt/4S) \qquad (A.17)$$

Since

$$q = a\ (\bar{h} - H) \qquad (A.18)$$

then

$$a = T\pi^2/4L^2 \qquad (A.19)$$

which is the subsurface outflow constant for the unsteady case, with an initial condition equal to the steady state head solution.

APPENDIX B Computation of the Subsurface Outflow Constant for a Clogged Stream

The differential equation that governs the flow under steady flow conditions is

$$T\, d^2h/dx^2 = 0 \qquad (B.1)$$

with boundary conditions

$$x = 0 \quad ; \quad dh/dx = 0$$

$$x = L \quad ; \quad h = h_o$$

The solution of the boundary problem is

$$h = (L^2 - x^2)\,\epsilon/2T + h_o \qquad (B.2)$$

Since the flow is steady and the average water level is located at 2/3 of the maximum water level difference, because the shape of the water table is parabolic,

$$q = a\,(\overline{h} - h_o) \qquad (B.3)$$

where $a = 3T/L^2$. Analogous to (B.3), we find an expression for the flow passing through the semipermeable layer (see Fig. 2.8)

$$q = a_c (\bar{h} - H) \tag{B.4}$$

where a_c is the subsurface outflow constant which includes the clogging effect. Therefore

$$a_c = q (\bar{h} - h_o + \Delta h)^{-1} \tag{B.5}$$

where, $\Delta h = h_o - H$. Applying Darcy's law at $x = L$

$$- T \, dh/dx = K_s (h_o + H) \Delta h/2d = \epsilon L \tag{B.6}$$

If

$$(h_o + H)/2 \cong H$$

and if the flow is steady, we obtain

$$\Delta h \cong qLd/HK_s \tag{B.7}$$

Substituting (B.3) and (B.7) into (B.5), produces

$$a_c = 3T/L^2(1 + 3Td/HK_sL) \tag{B.8}$$

or

$$a_c = a/(1 + 3Td/HK_sL) \tag{B.9}$$

A further simplification gives

$$a_c = a/(1 + 3B^2/HL) \qquad (B.10)$$

where $\underline{B}$ is the leakage factor (Davis and De Wiest, 1967, p.225).

APPENDIX C Details on the Computation of the Subsurface Outflow Constant for Converging Flow

The equation governing the flow under steady conditions and as shown in Figure 2.6 is

$$1/r\ (d(Tr\ dh/dr)/dr) = -\epsilon \qquad (C.1)$$

where T is a constant. The boundaries conditions are

$$r = R_2 \quad , \quad dh/dr = 0$$

and

$$r = R_o \quad , \quad h = H$$

The solution of (C.1) is

$$T(h - H) = (R_o^2 - r^2)\epsilon/4 + (\ln (r/R_o))\,\epsilon R_2^2\ /2 \qquad (C.2)$$

The mean water level in the aquifer is

$$\bar{h} - H = 1/(R_2^2 - R_o^2)\pi \int_{R_o}^{R_2} (h - H)\ 2\pi r\ dr \qquad (C.3)$$

Let

$$dr^2 = 2r dr \qquad (C.4)$$

and

$$r \ln(r/R_o)dr = 1/4 \ln(r/R_o)^2 dr^2 \qquad (C.5)$$

Making use of (C.4) and (C.5) and substituting (C.2) into (C.3), we obtain

$$T(\bar{h} - H) = \epsilon(R_o^2/4 - (R_2^2 + R_o^2)/8$$

$$+ R_2^4 \ln(R_2^2/R_o^2)/4(R_2^2 - R_o^2) - R^2/4) \qquad (C.6)$$

If $\delta = R_o/R_2 \ll 1$, we obtain

$$(\bar{h} - H) = ((1 + 2\delta)(-\ln\delta)/2 - 3(1+2\delta)/8)\epsilon L^2/T \qquad (C.7)$$

Since

$$q = \epsilon = a(\bar{h} - H) \qquad (C.8)$$

substitute (C.7) into (C.8), to get

$$a = (1/(1 + 2\delta)((-\ln\delta)/2 - 3/8))T/L^2 \qquad (C.9)$$

APPENDIX D Effect of Different Aquifer Properties in Individual Segments of a Stream-Aquifer System

Let us start by trying to compute a subsurface outflow constant of an aquifer divided into two parts by a stream.

The equations governing the flow at each aquifer cell (see Section 2.2) are:

$$S_1 dh_1/dt + a_1 h_1 = y_1 \qquad (D.1)$$

and

$$S_2 dh_2/dt + a_2 h_2 = y_2 \qquad (D.2)$$

Multiplying both sides of (D.1) and (D.2) by the aquifer cell areas A_1 and A_2 , respectively and rearranging the equations, we obtain

$$A_1(dh_1/dt + h_1/t_{h_1}) = y_1 A_1/S_1 \qquad (D.3)$$

and

$$A_2(dh_2/dt + h_2/t_{h_2}) = y_2 A_2/S_2 \qquad (D.4)$$

where the response time of cell 1 is $t_{h_1} = S_1/a_1$ and the response time of cell 2 is $t_{h_2} = S_2/a_2$. Adding (D.3) and

(D.4) and dividing by $A = A_1 + A_2$,

$$d/dt\ (A_1h_1 + A_2h_2)/A + (A_1h_1/t_{h_1} + A_2h_2/t_{h_2})/A$$

$$= (y_1A_1/S_1 + y_2A_2/S_2)/A \qquad (D.5)$$

An equation for the entire aquifer might be

$$d\ \bar{h}/dt + \bar{h}/t_h = \bar{y} \qquad (D.6)$$

where the overbars mean weighted averages with respect to the areas. However, (D.6) is not be found unless $t_{h_1} = t_{h_2} = t_h$ and $S_1 = S_2$, i.e., $a_1 = a_2$. This last case might represent an aquifer symmetric to the stream or a combination of the transmissivity $\underline{T}$ and characteristic length $\underline{L}$, such that the T/L^2 ratio were the same (see equation 2.15). Therefore in order to represent the stream-aquifer system by only one subsurface outflow constant $\underline{a}$, the response times of each cell must be equal as well the storage coefficient $\underline{S}$. The reason for this restriction is the nonlinear relationship between the aquifer head and the response time. If the parameter $\underline{a}$ and $\underline{S}$ are significantly different it may be necessary to use more than one cell to represent the physical model.

APPENDIX E Proof that the Use of Average Heads in the Outflow Equation is Valid

To demonstrate that the use of average heads is correct in the outflow equation, we shall show that the average of any departure from the mean head in the aquifer and in the aquifer outlet is zero.

Let us consider an aquifer consisting of a sector of a circle bounded externally by a stream, as shown in Figure E.1 and with uniform recharge $\underline{\epsilon}$. For simplicity, steady flow is assumed. The equation of continuity in polar coordinates (Davis and De Wiest, 1966, p.245) is

$$(T/r)\ \partial(r\,\partial h/\partial r)/\partial r + (T/r^2)\ \partial^2 h/\partial^2\theta = -\epsilon \tag{E.1}$$

Since equation E.1 is a non homogeneous partial differential equation,

$$h = h_p + h_1\ (r, \theta) \tag{E.2}$$

where h_p is the particular integral and solution of

$$\nabla^2 h_p = -\epsilon \tag{E.3}$$

and h_1 is the complementary function and solution of

$$\nabla^2 h_1 = 0 \tag{E.4}$$

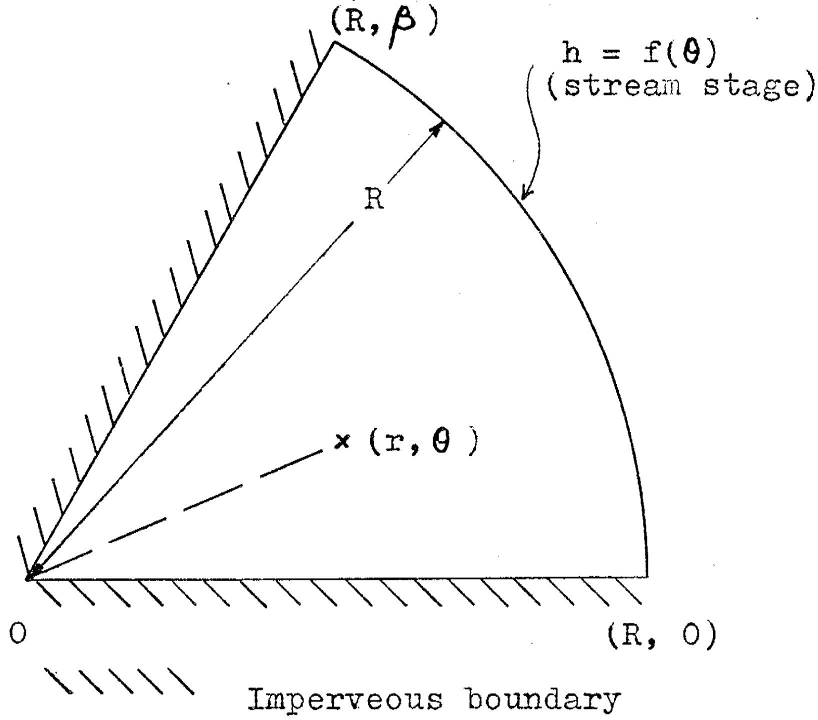


Fig. E.1 Boundaries of the aquifer.

Next the solution of (E.3) will be found. The boundary conditions are:

$$h = h_1 = f(\theta) \quad ; \quad r = R \tag{E.5}$$

$$h_p = 0 \quad ; \quad r = R \tag{E.6}$$

and

$$dh_p/dr = 0 \quad ; \quad r = 0 \tag{E.7}$$

Thus h_p refers to the average outlet head and $f(\theta)$ is the departure from the average outlet head.

Integrating (E.3) twice and using (E.6) and (E.7) we find the solution of (E.3) which is

$$h_p = (R^2 - r^2)\,\epsilon/4T \tag{E.8}$$

The solution of (E.4) will be found by using the method of separation of variables. With the boundary conditions

$$r = 0 \quad ; \quad h_1 \text{ is finite}$$

$$\theta = 0 \quad ; \quad \partial h_1/\partial\theta = 0$$

$$\theta = \beta \quad ; \quad \partial h_1/\partial\theta = 0$$

we find that

$$h_1 = a_o/2 + \sum_{m=1}^{\infty} a_m \cos(m\pi\theta/\beta)\, r^{m\pi/\beta} \qquad (E.9)$$

Equation E.9 satisfies (E.5) if

$$f(\theta) = a_o/2 + \sum_{m=1}^{\infty} a_m \cos(m\pi\theta/\beta) \qquad (E.10)$$

Since the above equation is a Fourier series,

$$a_o = 2/\beta \int_o^{\beta} f(\theta)\,d\theta = 0 \qquad (E.11)$$

because $\bar{f}(\theta) = 0$, and

$$a_m = (2/\beta R^{m\pi/\beta}) \int_o^{\beta} f(\theta) \cos m\pi\theta/\beta \; d\theta \qquad (E.12)$$

Therefore

$$h_1 = \sum_{m=1}^{\infty} a_m \cos(m\pi\theta/\beta)\, r^{m\pi/\beta} \qquad (E.13)$$

is the solution of (E.4) with the proper boundary conditions.

The outflow equation is

$$\epsilon = a(\bar{h} - \bar{H})$$

Let

$$\overline{h} = 1/A \int_0^R \int_0^\beta h(r, \theta) r \, dr \, d\theta \qquad (E.14)$$

or

$$\overline{h} = 1/A \iint h_p \, r \, dr \, d\theta + 1/A \iint h_1 r \, dr \, d\theta \qquad (E.15)$$

where $A = \beta R^2/2$. Then

$$\overline{h}_p = \epsilon/4TR^2 \int_0^R (R^2 - r^2) \, dr^2 = \epsilon R^2/8T \qquad (E.16)$$

and

$$\overline{h}_1 = 1/A \int_0^R \int_0^\beta \sum_{m=1} a_m \, r^{m\pi/\beta} \cos(m\pi\theta/\beta) \, r \, dr \, d\theta$$

But

$$\int_0^\beta \cos(m\pi\theta/\beta) \, d\theta = 0$$

Then

$$\overline{h}_1 = 0$$

This shows that the use of average heads in the outflow equation is correct for any head distribution at the outlet. Since the average of the aquifer head perturbation produced by the departure at the boundary head from its average is zero, it follows that the average boundary head can be used in the outflow expression for $\underline{H}$.

APPENDIX F Analytical Solution of the Lumped Model

Let us transform equation 2.4 which represents the lumped model to

$$dh/dt + ah/S = y/S \tag{F.1}$$

Solving the homogeneous equation, we find

$$h = h(t_o) \exp(-a(t - t_o)/S) \tag{F.2}$$

which represents a decay curve from an initial condition $h(t_o)$. From (F.2) we have

$$h(t_o) = h \exp(a(t - t_o)/S)$$

Differentiating with respect to time,

$$d/dt\ (h \exp(a(t-t_o)/S)) = 0 \tag{F.3}$$

or

$$(dh/dt + ah/S) \exp(a(t - t_o)/S) = 0$$

where $\exp(at/S)$ is the integration factor. Multiplying both sides of (F.1) by the integration factor and making use of (F.3), we obtain

$$d/dt\ (h \exp(at/S)) = y \exp(at/S)/S \tag{F.4}$$

Integrating the above equation produces

$$h(t) = h(t_o) \exp(-a(t - t_o)/S) + 1/S \int_{t_o}^{t} y(\tau) \exp(-a(t-\tau)/S)\,d\tau \tag{F.5}$$

Then if

$$t_o = 0 \quad , \quad h = h_o$$

we get

$$h(t) = h_o \exp(-at/S) + 1/S \int_{0}^{t} y(\tau) \exp(-a(t-\tau)/S)\,d\tau \tag{F.6}$$

The above integral is called a convolution integral.

A somewhat different and perhaps more illustrative procedure for finding the solution of (2.4) or (F.1) is given next.

Let us represent the rate of recharge $\underline{y}$, as a Dirac delta function or impulse function (see Figure F.1) defined as

$$\delta(t - t_o) = \begin{cases} \infty & \text{when } t = t_o \\ 0 & \text{when } t \neq t_o \end{cases}$$

and

$$\int_{-\infty}^{\infty} (t - t_o)\, dt = 1$$

Using the above definition, $y(t)$ is given by

$$y(t) = \int_{-\infty}^{\infty} (t - \tau)\, y(\tau)\, d\tau$$

Let us represent (F.1) by

$$S\, dh/dt + ah = y \qquad (F.7)$$

For a unit input in (F.7),

$$S\, dh/dt + ah = \delta(t - t_o)$$

At $t \neq t_o$

$$S\, dh/dt + ah = 0$$

whose solution is

$$h = c \exp(-at/S) \qquad (F.8)$$

When $t \cong t_o$

$$\int_{t_o-\Delta/2}^{t_o+\Delta/2} (S\ dh/dt + ah - \delta(t - t_o))\ dt = 0$$

where Δ is a small time increment. Integrating gives

$$S\ (h(t_o^+) - h(t_o^-)) + a \int_{t_o-\Delta/2}^{t_o+\Delta/2} h(t)\ dt - 1 = 0$$

and if $\Delta \rightarrow 0$

$$h(t_o^+) = 1/S \qquad (F.9)$$

Substituting (F.9) into (F.8), we get

$$h = (1/S)\ \exp(-a/S(t - t_o)) = \mu\,(t - t_o) \qquad (F.10)$$

a function called unit response or weighting function; $\mu\,(t - t_o)$ is the output of the system to a delta function input (Dooge, 1973, p.21).

If $y(t)$ changes arbitrarily as shown in Figure F.2, we can follow the next analysis,

$$\Delta \tau_k = t_k - t_{k-1}$$

where τ_k is a point between t_k and t_{k-1}.

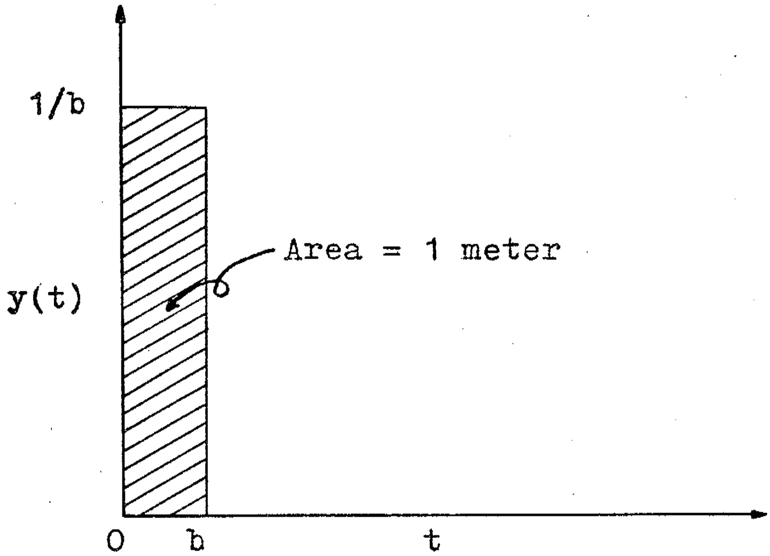


Fig. F.1 Dirac delta function representation of natural recharge $\underline{y}$.

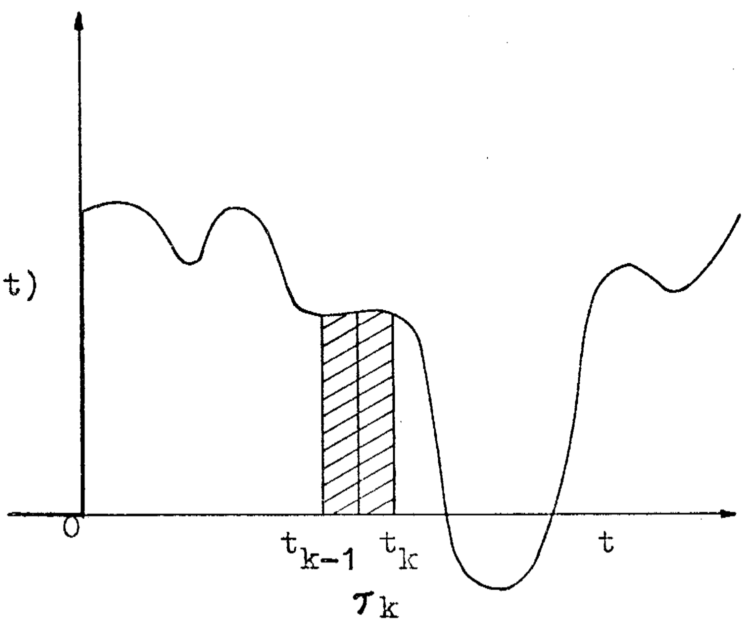


Fig. F.2 Representation of the natural recharge $\underline{y}$.

$$(y(\tau_k)\Delta\tau_k)\ \delta(t - t_k)$$

represents the shaded area in Figure F.2. The natural recharge is represented by

$$y(t) \cong \sum_{k=1}^{n} y(\tau_k)\ \delta(t - t_k)\ \Delta\tau_k$$

Since the response of the system to $\delta(t - \tau_k)$ is $\mu(t - \tau_k)$, we have

$$h(t) = 1/S \sum_{k=1}^{n} y(\tau_k)\ \mu(t - \tau_k) \qquad (F.11)$$

Substituting (F.10) into (F.11) and taking the limit when $\Delta\tau$ tends to zero, we obtain

$$h(t) = 1/S \int_{o}^{t} y(\tau)\ \exp(-a(t - \tau)/S)\, d\tau \qquad (F.12)$$

the convolution integral for a lumped model, which represents a time-invariant system.

Observe that the initial condition effect (see equation F.2) is

$$h_o(t) = h_o\ \exp(-at/S) \qquad (F.13)$$

Inasmuch as the system is linear, the superposition principle can be applied and equations F.12 and F.13 can be added together to produce the total output of the system,

$$h(t) = h_o \exp(-at/S) + 1/S \int_o^t y(\tau) \exp(-a(t-\tau)/S) d\tau \tag{F.14}$$

APPENDIX G Discussion on the Probability Level of Constraints Involving Random Quantities and Constraints Related to the Expected Values

Consider the deterministic representation of the demand chance constraint to be

$$\mu_{D_i} + x\,\sigma_D \leqq \mu_{Q_{SD_i}} + \mu_{Q_{P_i}} = \mu_{Q_{T_i}} \qquad \text{(G.1)}$$

where $\underline{x}$ corresponds to a probability level of λ_1.

To get a lower bound for

$$P\,(D_i - Q_{T_i} \leqq 0) \geqq \lambda^*_1 \qquad \text{(G.2)}$$

where λ^*_1 is the actual probability associated with the constraint stated in terms of random quantities, assume that: (1) $D_i - Q_{T_i}$ and D_i are normal random variables; (2) the correlation between Q_T and $\underline{D}$ is positive. Hence, $D_i - Q_{T_i}$ has mean $\mu_{D_i} - \mu_{Q_{T_i}}$ and variance

$\sigma^2_D + \sigma^2_{Q_T} - 2\rho\,\sigma_D\,\sigma_{Q_T}$, where

$$\sigma^2_D + \sigma^2_{Q_T} - 2\rho\,\sigma_D\,\sigma_{Q_T} \leqq \sigma^2_D + \sigma^2_{Q_T} \qquad \text{(G.3)}$$

if $\rho > 0$.

Let

$$P(D_i - Q_{T_i} \leqq 0) = P((D_i - Q_{T_i} - (\mu_{D_i} - \mu_{Q_{T_i}}))/\sigma_{D-Q_T}$$

$$\leqq (\mu_{Q_{T_i}} - \mu_{D_i})/\sigma_{D-Q_T}) \tag{G.4}$$

Now, if we satisfy (G.1), then

$$\mu_{Q_{T_i}} - \mu_{D_i} \geqq x\,\sigma_D \tag{G.5}$$

and

$$P(X \leqq a) \geqq P(X \leqq b) \tag{G.6}$$

if $a > b$. Therefore, by using (G.4) and (G.5) in (G.6), we get

$$P((D_i - Q_{T_i} - (\mu_{D_i} - \mu_{Q_{T_i}}))/\sigma_{D-Q_T} \leqq (\mu_{D_i} - \mu_{Q_{T_i}})/\sigma_{D-Q_T})$$

$$\leqq P((D_i - Q_{T_i} - (\mu_{D_i} - \mu_{Q_{T_i}}))/\sigma_{D-Q_T} \leqq x\,\sigma_D/\sigma_{D-Q_T})$$

or

$$P(D_i - Q_{T_i} \leqq 0) \geqq P((D_i - Q_{T_i} - (\mu_{D_i} - \mu_{Q_{T_i}}))/\sigma_{D-Q_T}$$

$$\leqq x\,\sigma_D / \sigma_{D-Q_T}) \tag{G.7}$$

If $\sigma_D^2 = \sigma_{Q_T}^2$ and $\rho = 0$, we have

$$\sigma_D \,/\, \sigma_{D-Q_T} = 1/\sqrt{2} = 0.707$$

where (G.7) is transformed into

$$P(D_i - Q_{T_i} \leqq 0) \geqq P(N(0,1) \leqq 0.707\, x) \qquad \text{(G.8)}$$

where $N(0,1)$ is a normal random number with mean zero and variance one. Using (G.8) we can easily find the actual probability level, λ^*_1 .

For $\lambda_1 = 0.95$, then $P(D_i - Q_{T_i} \leqq 0) \geqq 0.877 = \lambda^*_1$

and

$\lambda_1 = 0.7$, then $P(D_i - Q_{T_i} \leqq 0) \geqq 0.644 = \lambda^*_1$

As we can note the difference between the probability levels is not very large; the above procedure can be used to estimate the actual probability level λ^*_1 which is implicit in an assumed value of λ_1 .

APPENDIX H Input Data Used in the Comparative Study

N	Design horizon	4 years
N_S	Number of seasons per year	6
A	Aquifer area	8172.635 acres
T	Transmissivity	0.031 ft^2/S
S	Storage coefficient	0.01
β	Dimensionless constant	3.0
L	Characteristic length	4900 ft
Z	Ground surface level	20 ft
H	Mean stream water level	20 ft
h_o	Initial water level	20 ft
HL	Average initial lift	26.25 ft
r_w	Average well radius	1 ft
A_w	Average influence area of a well	1.78×10^8 ft^2
α_1	Fraction of developed water for return to stream	0.8
α_2	Fraction of developed water for spreading	0.5
ρ_y	Autocorrelation coefficient of the inputs	0.7
$\rho_{Q_p h}$	Cross correlation coefficient of pumping and head (from eq. 4.28)	-0.92
μ_D	Expected water demands per season	131 ac ft
σ_D	Standard deviation of water demands per season	60 ac ft

r	Nominal rate of interest	5%
μ_{c_S}	Expected stream diversion costs	\$3.4/acft
μ_{c_P}	Expected operating cost of pumping	\$0.024/acft^2
μ_{c_u}	Expected return to stream costs	\$0.105/acft
μ_{c_R}	Expected recharging costs	\$0.085/acft

$\mu_{Q_{ST}}$ Expected available streamflow

Season	$\mu_{Q_{ST}}$ ac-ft/season
1, 7, 13, 19	193
2, 8, 14, 20	115
3, 9, 15, 21	89
4,10, 16, 22	96
5,11, 17, 23	165
6,12, 18, 24	198

APPENDIX I Listing of the Computer Program

The Fortran IV program used in this work is composed of: (1) a control program which controls the number of iterations either by a mean square error test of the aquifer heads or a maximum number of iterations; (2) a Main 1 program which prepares data for linear programming; (3) a linear programming program (see, Kuester and Mize, 1973, p.10).

A phase overlay computer software technique in which Main 1 and Linear Programming share main computer core is used. This listing is for the particular case of Maddock's problem consisting of inputs, program listing, and outputs of the first iteration.

```
//SYS003 ACCESS SCRTCH                                                    00:00:00
IA86I   SYS003 SCRTCH   190    SYSRES
// EXEC COPY                                                              00:00:01

SYSIPT     423
SYS003     423
//SYS002 ACCESS SCRTCH                                                    00:00:12
//SYS003 ACCESS SDSOPT                                                    00:00:13
// LABEL 80                                                               00:00:13
IA86I   SYS002 SCRTCH   193    SYSRES
IA86I   SYS003 SDSOPT   208
// EXEC COPY                                                              00:00:13
 MADDOCK PROBLEM
ROW-ID
MATRIX
FI                        0.0
EO                        0.0
            O PROFT
            - DD1
            - DD2
            - DD3
            - DD4
            - DD5
            - DD6
            - DD7
            - DD8
            - DD9
            - DD10
            - DD11
            - DD12
            - DD13
            - DD14
            - DD15
            - DD16
            - DD17
            - DD18
            - DD19
            - DD20
            - DD21
            - DD22
            - DD23
            - DD24
            E DR1
            E DR2
            E DR3
            E DR4
            E DR5
            E DR6
            E DR7
            E DR8
            E DR9
            E DR10
            E DR11
            E DR12
            E DR13
            E DR14
            E DR15
            E DR16
            E DR17
            E DR18
            E DR19
            E DR20
            E DR21
            E DR22
            E DR23
            E DR24
            O CU1
            O CU2
            O CU3
            O CU4
            O CU5
            O CU6
            O CU7
            O CU8
            O CU9
            O CU10
            O CU11
            O CU12
            O CU13
            O CU14
            O CU15
            O CU16
            O CU17
            O CU18
            O CU19
            O CU20
            O CU21
            O CU22
            O CU23
            O CU24
         VS1    PROFT
         VS1    DD1
         VS1    DR1
         VU1    PROFT
         VU1    DR1
         VU1    CU1
         VP1    PROFT
         VP1    DD1
         VR1    PROFT
         VR1    CU1
         VS2    PROFT
         VS2    DD2
         VS2    DR2
         VU2    PROFT
         VU2    DR2
         VU2    CU2
         VP2    PROFT
         VP2    DD2
         VR2    PROFT
         VR2    CU2
         VS3    PROFT
         VS3    DD3
         VS3    DR3
         VU3    PROFT
         VU3    DR3
         VU3    CU3
```

VP3	PROFT
VP3	DD3
VR3	PROFT
VR3	CU3
VS4	PROFT
VS4	DD4
VS4	DR4
VU4	PROFT
VU4	DR4
VU4	CU4
VP4	PROFT
VP4	DD4
VR4	PROFT
VR4	CU4
VS5	PROFT
VS5	DD5
VS5	DR5
VU5	PROFT
VU5	DR5
VU5	CU5
VP5	PROFT
VP5	DD5
VR5	PROFT
VR5	CU5
VS6	PROFT
VS6	DD6
VS6	DR6
VU6	PROFT
VU6	DR6
VU6	CU6
VP6	PROFT
VP6	DD6
VR6	PROFT
VR6	CU6
VS7	PROFT
VS7	DD7
VS7	DR7
VU7	PROFT
VU7	DR7
VU7	CU7
VP7	PROFT
VP7	DD7
VR7	PROFT
VR7	CU7
VS8	PROFT
VS8	DD8
VS8	DR8
VU8	PROFT
VU8	DR8
VU8	CU8
VP8	PROFT
VP8	DD8
VR8	PROFT
VR8	CU8
VS9	PROFT
VS9	DD9
VS9	DR9
VU9	PROFT
VU9	DR9
VU9	CU9
VP9	PROFT
VP9	DD9
VR9	PROFT
VR9	CU9
VS10	PROFT
VS10	DD10
VS10	DR10
VU10	PROFT
VU10	DR10
VU10	CU10
VP10	PROFT
VP10	DD10
VR10	PROFT
VR10	CU10
VS11	PROFT
VS11	DD11
VS11	DR11
VU11	PROFT
VU11	DR11
VU11	CU11
VP11	PROFT
VP11	DD11
VR11	PROFT
VR11	CU11
VS12	PROFT
VS12	DD12
VS12	DR12
VU12	PROFT
VU12	DR12
VU12	CU12
VP12	PROFT
VP12	DD12
VR12	PROFT
VR12	CU12
VS13	PROFT
VS13	DD13
VS13	DR13
VU13	PROFT
VU13	DR13
VU13	CU13
VP13	PROFT
VP13	DD13
VR13	PROFT
VR13	CU13
VS14	PROFT
VS14	DD14
VS14	DR14
VU14	PROFT
VU14	DR14
VU14	CU14
VP14	PROFT
VP14	DD14
VR14	PROFT
VR14	CU14
VS15	PROFT

```
VS15  DD15
VS15  DR15
VU15  PROFT
VU15  DR15
VU15  CU15
VP15  PROFT
VP15  DD15
VR15  PROFT
VR15  CU15
VS16  PROFT
VS16  DD16
VS16  DR16
VU16  PROFT
VU16  DR16
VU16  CU16
VP16  PROFT
VP16  DD16
VR16  PROFT
VR16  CU16
VS17  PROFT
VS17  DD17
VS17  DR17
VU17  PROFT
VU17  DR17
VU17  CU17
VP17  PROFT
VP17  DD17
VR17  PROFT
VR17  CU17
VS18  PROFT
VS18  DD18
VS18  DR18
VU18  PROFT
VU18  DR18
VU18  CU18
VP18  PROFT
VP18  DD18
VR18  PROFT
VR18  CU18
VS19  PROFT
VS19  DD19
VS19  DR19
VU19  PROFT
VU19  DR19
VU19  CU19
VP19  PROFT
VP19  DD19
VR19  PROFT
VR19  CU19
VS20  PROFT
VS20  DD20
VS20  DR20
VU20  PROFT
VU20  DR20
VU20  CU20
VP20  PROFT
VP20  DD20
VR20  PROFT
VR20  CU20
VS21  PROFT
VS21  DD21
VS21  DR21
VU21  PROFT
VU21  DR21
VU21  CU21
VP21  PROFT
VP21  DD21
VR21  PROFT
VR21  CU21
VS22  PROFT
VS22  DD22
VS22  DR22
VU22  PROFT
VU22  DR22
VU22  CU22
VP22  PROFT
VP22  DD22
VR22  PROFT
VR22  CU22
VS23  PROFT
VS23  DD23
VS23  DR23
VU23  PROFT
VU23  DR23
VU23  CU23
VP23  PROFT
VP23  DD23
VR23  PROFT
VR23  CU23
VS24  PROFT
VS24  DD24
VS24  DR24
VU24  PROFT
VU24  DR24
VU24  CU24
VP24  PROFT
VP24  DD24
VR24  PROFT
VR24  CU24
      DD1
      DD2
      DD3
      DD4
      DD5
      DD6
      DD7
      DD8
      DD9
      DD10
      DD11
      DD12
      DD13
      DD14
      DD15
      DD16
      DD17
```

```
                DO18
                DO19
                DO20
                DO21
                DO22
                DO23
                DO24
                DR1
                DR2
                DR3
                DR4
                DR5
                DR6
                DR7
                DR8
                DR9
                DR10
                DR11
                DR12
                DR13
                DR14
                DR15
                DR16
                DR17
                DR18
                DR19
                DR20
                DR21
                DR22
                DR23
                DR24
                CU1
                CU2
                CU3
                CU4
                CU5
                CU6
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                CU10
                CU11
                CU12
                CU13
                CU14
                CU15
                CU16
                CU17
                CU18
                CU19
                CU20
                CU21
                CU22
                CU23
                CU24
3.4      0.084    39.228   -4.25825 -1.9614  51.4868  1.9614   0.0425
-2.05    0.61     0.52     -0.50    0.20     1.04     1.08
-1.08    -0.44    0.25     0.44     1.48     -0.77    -0.47
-0.13    0.0      -0.33    0.64     -0.18    -0.67    1.88
1.48     -1.55    0.92
20.      20.      20.      20.      20.      20.      20.
20.      20.      20.      20.      20.      20.      20.
20.      20.      20.      20.      20.      20.      20.
20.      20.      20.
193.     115.     89.      96.      165.     198.     193.
115.     89.      96.      165.     198.     193.     115.
89.      96.      165.     198.     193.     115.     89.
96.      165.     198.




0.08     0.08     0.08     0.08     0.08     0.08     0.08
0.08     0.08     0.08     0.08     0.08     0.08     0.08
0.08     0.08     0.08     0.08     0.08     0.08     0.08
0.08     0.08     0.08
1.0      1.0      1.0      1.0      1.0      1.0      1.0
1.0      1.0      1.0      1.0      1.0      1.0      1.0
1.0      1.0      1.0      1.0      1.0      1.0      1.0
1.0      1.0      1.0
20.0     19.6     19.5     19.5     19.5     19.5     19.5
19.5     19.5     19.5     19.5     19.5     19.5     19.5
19.5     19.5     19.5     19.5     19.5     19.5     19.5
19.5     19.5     19.5

SYSC02   423
SYS003   423
//PROG EXEC FORTRAN                                                    00:00:24

               C      ****************************************************************
               C      CONTROL PROGRAM
               C      *********************************************************************
               C      NO=MAX.NO.OF ITERATIONS
               C      ZZ=CONVERGENCE CRITERION
   0001               COMMON/SAS/Q(50),RH(50),QST(50),QSD(50),QAU(50),QP(50),QAR(50),
                     1         QSA(50),Y(50),Y1(50),RS(50),HB(50)
   0002               COMMON/SES/LGE(100),PM1(100),PM2(100),CLNM1(300),CLNM2(300),
                     1         RNM1(300),RNM2(300),RM1(100),RM2(100),
                     2         AA(20),BB(2),CC(2),DD(2),EE(1),C(8)
   0003               COMMON/SIS/VS(50),VD(50),VP(50),VR(50),CRP
   0004               COMMON/SOS/H1(50),H2(50)
               C      DATA
   0005               J=0
   0006               MA=3
   0007               N=24
   0008               NO=2
   0009               N1=10
   0010               N2=8
   0011               K1=72
   0012               K2=K1+1
```

```
      K3=N1*N
      ZZ=0.001
   12 CONTINUE
      CALL LOAD ('MAIN1   ')
      CALL MAIN1 (H1)
      J=J+1
      CALL LOAD ('LP      ')
      CALL LP (VS,VU,VP,VR)
      REWIND 3
      WRITE (MA,404) (AA(I),I=1,13)
  404 FORMAT (1X,13A4)
      WRITE (MA,405) (BB(I),I=1,2)
  405 FORMAT (2A4)
      WRITE (MA,405) (CC(I),I=1,2)
      WRITE (MA,409) ((DD(I),I=1,2),DD1)
  409 FORMAT (2A4,11X,F11.4)
      WRITE (MA,408) ((EE(I),I=1,1),EE1)
  408 FORMAT (A4,15X,F11.4)
      WRITE (MA,400) (CGL(I),RN1(I),RN2(I),I=1,K2)
  400 FORMAT (11X,A1,1X,A4,A1)
      WRITE (MA,401) (CLNM1(I),CLNM2(I),RNM1(I),RNM2(I),I=1,K3)
  401 FORMAT (7X,A4,A1,1X,A4,A1)
      WRITE (MA,402) (RM1(I),RM2(I),I=1,K1)
  402 FORMAT (13X,A4,A1)
      WRITE (MA,500) (C(K),K=1,N2)
  500 FORMAT (8F10.5)
      WRITE (MA,501) (RN(I),I=1,N)
      WRITE (MA,501) (WH(I),I=1,N)
      WRITE (MA,501) (QST(I),I=1,N)
      WRITE (MA,501) (VS(I),I=1,N)
      WRITE (MA,501) (VU(I),I=1,N)
      WRITE (MA,501) (VP(I),I=1,N)
      WRITE (MA,501) (VR(I),I=1,N)
      WRITE (MA,501) (H1(I),I=1,N)
  501 FORMAT (7F10.6)
C     MEAN SQUARE ERROR HEAD TEST
      B=0.0
      G=N-1
      DO 5 K=2,N
      B=B+(H2(K)-H1(K))*(H2(K)-H1(K))
    5 CONTINUE
      E=SQRT(B/G)
      WRITE (6,10)
   10 FORMAT (1H ,10X,73(1H-))
      WRITE (6,14) (J,E)
   14 FORMAT (1H ,14X,'ITERATION =',I3,9X,'SQRT(H.M.S.E.) =',E13.6)
      WRITE (6,40)
   40 FORMAT (1H ,10X,73(1H-)//////)
      WRITE (6,10)
      IF(E-ZZ) 20,20,11
   11 IF(J.GE.ND)GO TO 20
      REWIND 2
      REWIND 3
      GO TO 12
   20 STOP
      END
```

TOTAL MEMORY REQUIREMENTS 000A6C BYTES

COMPILER HIGHEST SEVERITY CODE WAS 0

//MAIN1 EXEC FORTRAN 00:00:32

```
      SUBROUTINE MAIN1
C     MAIN1 PROGRAM
C     ***********************************************************************
C     COMPUTATION OF COEFF OF OBJ FUN AND CONSTRAINTS
C     ***********************************************************************
C     COMPUTATION OF COEFF OF OBJ FUN
C     ALPHA=PROBABILITY LEVEL
C     C(I)=COEFFICIENTS OF DECISION VARIABLES
C     CCH1=WATER LEVEL CORRELATION COEFF.
C     CCH2(I)=WATER LEVEL AUTOCORRELATION FUNCTION
C     CCQH=CROCC CORRELATION FUNCTION OF HEAD AND PUMPAGE
C     CLNM1=N1*N ARRAY
C     CLNM2=N1*N ARRAY
C     CRP=COST CONTRIBUTION OF RANDOM TERM
C     CS=AVERAGE DRAWDOWN AT WELLS
C     C1=CONSTANT DEPENDING OF P.D.F. KNOWLEDGE
C     K1=NO.OF CONSTRAINTS
C     K3=TOTAL NO.OF DEC.VAR.COEFFICIENTS
C     MO=DEMAND MEAN
C     N=NO.OF PERIODS
C     N1=NO.OF DEC.VAR.COEFF.BY TIME INCREMENT IN DEC.VAR.COEFF.MATRIX
C     N2=NO.OF COST COEFFICIENTS
C     R=RATE OF INTEREST
C     RNM1=N1*N ARRAY
C     RNM2=N1*N ARRAY
C     S=STORAGE COEFFICIENT
C     SDD=STANDAR DEV. OF DEMAND
C     SS=NUMBER OF SEASONS
C     SYMB=N1*N ARRAY
C     VALUE=N1*N ARRAY
C     VALUE=DEC.VAR.COEFF. MATRIX BY COLUMN
C     VALUE1=RHS. VECTOR
C     VALUF1=N ARRAY
C     VALUE2=ABS. VALUE OF VALUE
C     VALUE2=N1*N ARRAY
C     CARDS THAT SHOULD BE CHANGED TO GET THE DETERMINISTIC CASE
C     MAIN: SDD=0.0,CCH1=0.0,CCH2=0.0
C     INPUT: SDD=0.0
C     VARY: VARY1=0.0
      COMMON/SAS/D(50),WH(50),QST(50),QSU(50),QAU(50),QP(50),QAR(50),
     1           QSA(50),Y(50),YL(50),PW(50),HH(50)
      COMMON/SES/CGL(100),RN1(100),RN2(100),CLNM1(300),CLNM2(300),
     1           RNM1(300),RNM2(300),RM1(100),RM2(100),
     2           AA(20),BB(2),CC(2),DD(2),EE(1),C(5)
      COMMON/STS/VS(50),VU(50),VP(50),VR(50),CRP
      COMMON/HIS/H1(50),H2(50)
      DIMENSION H(50),SH(50),VAR(50),SD(50)
      DIMENSION C1X(50),C2X(50),CP1(50),CP2(50),CP3(50)
      DIMENSION C3X(50),CP4(50),CP5(50),CPT(50)
      DIMENSION B1(50),B2(50),B3(50),B4(50),B5(50),B6(50)
      DIMENSION VALUE(300)
      DIMENSION VALUF1(100)
```

```
      DIMENSION VALUE2(300)
      DIMENSION CCH2(50),QW(50),CS(50)
      LOGICAL*1 SYMB(300)
      LOGICAL*1 PLUS/' '/,MINUS/'-'/
      LOGICAL*1 SYMB1(300)
      LOGICAL*1 AMPER/'&'/,MENUS/'-'/
      LOGICAL*1 LGECHR(4,100)
      EQUIVALENCE (LGE,LGECHR)
      LOGICAL EQ,NE,GT,GE,LT,LE
C     DATA
      N=24
      N1=10
      N2=8
      K1=72
      K2=K1+1
      K3=N1*N
      LA=2
      READ (LA,204) (AA(I),I=1,13)
  204 FORMAT (1X,13A4)
      READ (LA,205) (BB(I),I=1,2)
  205 FORMAT (2A4)
      READ (LA,205) (CC(I),I=1,2)
      READ (LA,209) ((DD(I),I=1,2),DD1)
  209 FORMAT (2A4,11X,F11.4)
      READ (LA,208) ((EE(I),I=1,1),EE1)
  208 FORMAT (A4,15X,F11.4)
      READ (LA,200) (LGE(I),RN1(I),RN2(I),I=1,K2)
  200 FORMAT (11X,A1,1X,A4,A1)
      READ (LA,201) (CLNM1(I),CLNM2(I),RNM1(I),RNM2(I),I=1,K3)
  201 FORMAT (7X,A4,A1,1X,A4,A1)
      READ (LA,202) (RM1(I),RM2(I),I=1,K1)
  202 FORMAT (13X,A4,A1)
      READ (LA,100) (C(K),K=1,N2)
  100 FORMAT (8F10.5)
C     DATA
      AWI=1.78E3
      CCQH=-0.92
      HO=20.0
      MD=131.0
      R=0.05
      RW=1.0
      S=.01
      SDD=60.0
      SS=6.
      T=0.031
      X=0.0
      A=1.+R/SS
      UP=6.29+ALOG(SQRT(AWI)/(RW*2000.0))
      UP1=UP/(2.*3.1416*T)
      CALL INPUTY (N,AR,A1)
      CALL MEANH (H,SH,N,AR,A1)
      CALL VARH (VAR,SD,VARY,N,AR,A1)
C     FORMATION OF A NEW HEAD ARRAY,H1(I)
      H1(1)=HO
      DO 800 I=2,N
      H1(I)=H(I-1)
  800 CONTINUE
C     PRESENT VALUE OF STREAM DIV.AND RETURNED WATER COEFFICIENTS
      CRP=0.0
      DO 5 I=1,N
      B=I
      CCH2(I)=0.7**B
      C1X(I)=C(1)*MD/A**B
      C2X(I)=C(2)*MD/A**B
C     PRESENT VALUE OF PUMPING COEFFICIENTS
      QW(I)=QP(I)*43560./(2.*5.184E6)
      CS(I)=QW(I)*UP1
      CP1(I)=C(3)*H1(I)/A**B
      CP2(I)=C(4)*VAR(I)*(CCQH-0.177*177.427)/A**B
      CP3(I)=C(5)*H(I)*H1(I)/A**B
      CP4(I)=C(6)*H1(I)/A**B
      CP5(I)=C(7)*CS(I)*H1(I)/A**B
      CPT(I)=CP1(I)+CP3(I)+CP4(I)+CP5(I)
      CRP=CRP-CP2(I)
C     PRESENT VALUE OF RECHARGE COEFFICIENTS
      C3X(I)=C(8)*MD/A**B
C     CONSTRAINT COEFFICIENTS COMPUTATION
C     DEMAND CONSTRAINTS
C     IF R.H.S. IS NEGATIVE THE SIGN OF THE CONSTRAINT IS CHANGED
      B1(I)=MD
      B2(I)=AR*S*H1(I)
      B3(I)=MD+X*SDD
C     DOWNSTREAM CONSTRAINTS
      B4(I)=DST(I)-AR*A1*WH(I)+AR*A1*H(I)
      B5(I)=MD
      B6(I)=-0.8*MD
C     WRITE DOWNSTREAM CONSTRAINT BEFORE THE LAST
C     IF R.H.S. IS NEGATIVE THE TYPE OF CONSTRAINT IS CHANGED
      KA=K2-2*N
      DO 7 J=1,KA
      SYMB1(J)=LGECHR(1,J)
    7 CONTINUE
      KB=KA+N+1
      DO 8 J=KB,K2
      SYMB1(J)=LGECHR(1,J)
    8 CONTINUE
      IF (B4(I)) 101,4,4
  101 B4(I)=-B4(I)
      B5(I)=-B5(I)
      B6(I)=-B5(I)
      IF (EQ(LGE(KA+I),'&')) GO TO 3
      SYMB1(KA+I)=AMPER
      GO TO 5
    3 SYMB1(KA+I)=MENUS
      GO TO 5
    4 SYMB1(KA+I)=LGECHR(1,KA+I)
C     WATER LEFT AFTER CONSUMATIVE USE CONSTRAINTS
    5 CONTINUE
C     ****************************************************************
C     DECISION VAR COEFF MATRIX
C     ****************************************************************
C     FIRST 3 TERMS ARE VS(I)
C     SECOND 3 TERMS ARE VO(I)
C     THIRD 2 TERMS ARE VP(I)
C     FOURTH 2 TERMS ARE VR(I)
C     IF IT IS A MAX.CHANGE SIGN OF O.F.  COEFFS.
```

```
      DO 199 I=1,N
      VALUE (N1*(I-1)+1)=-C1X(I)
      VALUE (N1*(I-1)+2)=B1(I)
      VALUE (N1*(I-1)+3)=B5(I)
      VALUE (N1*(I-1)+4)=-C2X(I)
      VALUE (N1*(I-1)+5)=B6(I)
      VALUE (N1*(I-1)+6)=1.
      VALUE (N1*(I-1)+7)=-CPT(I)
      VALUE (N1*(I-1)+8)=B2(I)
      VALUE (N1*(I-1)+9)=-C3X(I)
      VALUE (N1*(I-1)+10)=1.
  199 CONTINUE
      DO 207 I=1,K3
      SYMB(I)=PLUS
      IF(VALUE(I).LT.0.0) SYMB(I)=MINUS
      VALUE2(I)=ABS(VALUE(I))
  207 CONTINUE
C     ************************************************************
C     RIGHT HAND SIDE VECTOR
C     ************************************************************
      DO 203 I=1,N
      VALUE1(I)=B3(I)
      VALUE1(N+I)=B4(I)
      VALUE1(2*N+I)=1.
  203 CONTINUE
C     STREAM-AQUIFER INTERACTION
      DO 2 K=1,N
      QSA(K)=A1*AR*(WH(K)-H(K))
    2 CONTINUE
      WRITE (6,10)
   10 FORMAT (1H ,10X,73(1H-))
      WRITE (6,11) (A1,VARY)
   11 FORMAT (1H ,12X,'I',5X,'QSD(I)',4X,'QAU(I)',4X,'QP(I)',9X,'QAR(I)'
     1,6X,'Y(I)',3X,'A1=',E13.6,2X,'VARY=',E13.6,2X,'INITIAL GUESSES')
      WRITE (6,10)
      WRITE (6,12) (J,QSD(J),QAU(J),QP(J),QAR(J),Y(J),J=1,N)
   12 FORMAT (1H ,10X,I3,1X,F10.3,1X,F10.3,1X,F10.3,1X,F10.3,1X,
     1F10.5)
      WRITE (6,40)
   40 FORMAT (1H ,10X,73(1H-)//)
      WRITE (6,10)
      WRITE (6,20) CRP
   20 FORMAT (1H ,12X,'I',8X,'HEAD MEAN',4X,'DRAW. CORR.',6X,
     1'HEAD VAR.',9X,'PUMPING RANDOM',3X,'CRP=',F10.3)
      WRITE (6,10)
      WRITE (6,30) (I,H(I),C5(I),VAR(I),CP2(I),I=1,N)
   30 FORMAT (1H ,10X,I3,5X,F10.3,5X,F10.3,5X,E15.6,5X,E15.6)
      WRITE (6,40)
      WRITE (6,10)
      WRITE (6,50)
   50 FORMAT (1H ,12X,'I',1X,'STREAM DIV COEFF=C1X',1X,
     1'RET.W.COEFF=C2X',1X,'PUMPING COEFF=CPT',2X,'A.RECH COEFF=C3X')
      WRITE (6,10)
      WRITE (6,51) (I,C1X(I),C2X(I),CPT(I),C3X(I),I=1,N)
   51 FORMAT (1H ,10X,I3,1X,E16.5,1X,E16.5,1X,E16.5,1X,E16.5)
      WRITE (6,40)
C     ********************************************************************
      WRITE (6,10)
      WRITE (6,9)
    9 FORMAT (1H ,21X,'DEMAND CONSTRAINTS')
      WRITE (6,10)
      WRITE (6,111)
  111 FORMAT (1H ,12X,'I',7X,'1=B1',7X,'2=B2',11X,'3=B3')
      WRITE (6,10)
      WRITE (6,112) (I,B1(I),B2(I),B3(I),I=1,N)
  112 FORMAT (1H ,10X,I3,2X,F10.2,2X,F10.2,4X,F10.2)
      WRITE (6,13)
   13 FORMAT (1H ,10X,73(1H-)///)
      WRITE (6,10)
      WRITE (6,19)
   19 FORMAT (1H ,22X,'DOWNSTREAM CONSTRAINTS')
      WRITE (6,10)
      WRITE (6,21)
   21 FORMAT (1H ,12X,'I',7X,'1=B5',9X,'2=B6',9X,'3=B4')
      WRITE (6,10)
      WRITE (6,22) (I,B5(I),B6(I),B4(I),I=1,N)
   22 FORMAT (1H ,10X,I3,2X,F10.2,2X,F10.2,2X,F10.2)
      WRITE (6,13)
      WRITE (6,10)
      WRITE (6,29)
   29 FORMAT (1H ,16X,'WATER LEFT AFTER C.U. CONSTRAINTS')
      WRITE (6,10)
      WRITE (6,31)
   31 FORMAT (1H ,12X,'I',7X,'1=1.',11X,'2=1.',13X,'3=1.')
      WRITE (6,13)
      WRITE (6,10)
      WRITE (6,32)
   32 FORMAT (1H ,13X,'I',9X,'QSA(I)',12X,'CCH2(I)',13X,'RN(I)',
     115X,'D(I)')
      WRITE (6,10)
      WRITE (6,39) (I,QSA(I),CCH2(I),RN(I),D(I),I=1,N)
   39 FORMAT (1H ,11X,I3,6X,E13.6,6X,E13.6,6X,E13.6,6X,E13.6)
      WRITE (6,10)
      REWIND 2
      LB=3
      WRITE (LB,60) (AA(I),I=1,13)
   60 FORMAT (1X,13A4)
      WRITE (LB,61) (BB(I),I=1,2)
   61 FORMAT (2A4)
      WRITE (LB,62) (SYMB1(I),RN1(I),RN2(I),I=1,K2)
   62 FORMAT (11X,A1,1X,A4,A1)
      WRITE (LB,61) (CC(I),I=1,2)
      WRITE (LB,63) (CLNM1(I),CLNM2(I),RNM1(I),RNM2(I),SYMB(I),
     1VALUE2(I),I=1,K3)
   63 FORMAT (7X,A4,A1,1X,A4,A1,A1,F11.4)
      WRITE (LB,207) ((DD(I),I=1,2),DD1)
      WRITE (LB,64) (RM1(I),RM2(I),VALUE1(I),I=1,K1)
   64 FORMAT (13X,A4,A1,1X,F11.4)
      WRITE (LB,208) ((EE(I),I=1,1),EE1)
      RETURN
      END

TOTAL MEMORY REQUIREMENTS 003828 BYTES

COMPILER HIGHEST SEVERITY CODE WAS  0
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```
//INPUTY EXEC FORTRAN                                                    00:00:53

          SUBROUTINE INPUTY (N,AR,A1)
C
C         ***********************************************************************
C         INPUT TO THE PHYSICAL MODEL
C         ***********************************************************************
C         A1=OUTFLOW CONSTANT IN 1/SEASON
C         AL=LEAKAGE
C         ALE=CHAR LENGTH
C         ALPHA1=RETURN FLOW COEFF
C         ALPHA2=ART RECH COEFF
C         AR=AREA
C         AT=TRANSPIRATION
C         D=DEMAND
C         HB=PREVIOUS MEAN H(I)
C         P=PRECIPITATION
C         QAR=WATER THAT ACTUALLY RECHARGES THE AQUIFER
C         QAU=WATER THAT ACTUALLY REACHES THE STREAM
C         QI=SUBSURFACE INFLOW
C         QST=STREAMFLOW
C         RN(J)=NORMAL RANDOM NUMBERS (0,1)
C         SS=NUMBER OF SEASONS PER YEAR
C         T=TRANSMISSIVITY
C         VS(J)=STREAM DIVERSION DEC VAR
C         VU(J)=RETURNED WATER DEC VAR
C         VP(J)=PUMPING DEC VAR
C         VR(J)=ART RECHARGE DEC VAR
C         WH=STREAM STAGE
C         Y(J)=EXPECTED VALUE OF INPUT (L/T)
C         Y1(J)=INPUT (L/T)
          COMMON/SAS/D(50),WH(50),QST(50),QSD(50),QAU(50),QP(50),QAR(50),
         1            QSA(50),Y(50),Y1(50),RN(50),HB(50)
          COMMON/SIS/VS(50),VU(50),VP(50),VR(50),CRP
          COMMON/SOS/H1(50),H2(50)
          ALE=4900.
          ALPHA1=0.8
          ALPHA2=0.5
          AR=8172.635
          LC=2
          MD=131.
          S=0.01
          SUD=60.0
          SS=6.
          T=0.031
        5 READ (LC,1) (RN(K),K=1,N)
          READ (LC,1) (WH(K),K=1,N)
          READ (LC,1) (QST(K),K=1,N)
          READ (LC,1) (VS(K),K=1,N)
        4 READ (LC,1) (VU(K),K=1,N)
          READ (LC,1) (VP(K),K=1,N)
        3 READ (LC,1) (VR(K),K=1,N)
          READ (LC,1) (HB(K),K=1,N)
        1 FORMAT (7F10.6)
          A1=(3.*T/ALE**2.)*86400.*360./SS
C         DISCHARGE IN ACRE-FEET/SEASON
C         DEMAND COMPUTATION
          DO 6 M=1,N
          V=SUD*RN(M)
          D(M)=MD+V
          IF (D(M).LE.0.0) D(M)=0.0
        6 CONTINUE
          DO 2 J=1,N
          QSD(J)=D(J)*VS(J)
          MQSD=MD*VS(J)
          QAU(J)=ALPHA1*D(J)*VU(J)
          MQAU=ALPHA1*MD*VU(J)
          QP(J)=AR*S*HB(J)*VP(J)
          QAR(J)=ALPHA2*D(J)*VR(J)
          MQAR=ALPHA2*MD*VR(J)
          Y1(J)=(QAR(J)-QP(J)+AR*A1*WH(J))/AR
          Y(J)=(MQAR-QP(J)+AR*A1*WH(J))/AR
        2 CONTINUE
C         FORMATION OF A NEW HEAD ARRAY,H2(I)
          DO 8 I=1,N
          H2(I)=HB(I)
        8 CONTINUE
          RETURN
          END

   TOTAL MEMORY REQUIREMENTS 00078C BYTES

 COMPILER HIGHEST SEVERITY CODE WAS  0

//MEANH EXEC FORTRAN                                                     00:01:00

          SUBROUTINE MEANH (H,SH,N,AR,A1)
C
C         ***********************************************************************
C         AVERAGE HEAD OF THE PHYSICAL MODEL
C         ***********************************************************************
C
C
C         H(I)=EXPECTED VALUE OF HEAD (L)
C         HO=INITIAL HEAD
C         S=STORAGE COEFFICIENT
          COMMON/SAS/D(50),WH(50),QST(50),QSD(50),QAU(50),QP(50),QAR(50),
         1            QSA(50),Y(50),Y1(50),RN(50),HB(50)
          DIMENSION H(50),SH(50)
C         DATA
          HO=20.0
          S=.01
          X=-A1/S
        9 X1=1.-EXP(X)
C         FIRST HEAD VALUE
        4 H(1)=HO*EXP(X)+X1*Y(1)/A1
          IF (H(1).GE.20.0) H(1)=20.
        5 SH(1)=H(1)*H(1)
C         SUCCESSIVE VALUES OF HEAD
        6 DO 1 I=2,N
          H(I)=H(I-1)*EXP(X)+X1*Y(I)/A1
          IF (H(I).LE.0.0) H(I)=0.0
          IF (H(I).GE.20.0) H(I)=20.0
          SH(I)=H(I)*H(I)
        1 CONTINUE
          RETURN
```

```
      END
 TOTAL MEMORY REQUIREMENTS 00035C BYTES
COMPILER HIGHEST SEVERITY CODE WAS  0
//VARH EXEC FORTRAN                                                    00:01:04

      SUBROUTINE VARH (VAR,SD,VARY,N,AR,A1)
C
C     ******************************************************************
C     HEAD VARIANCE OF THE PHYSICAL MODEL
C     ******************************************************************
C     CC= INPUT CORRELATION COEFF
C     S=STORAGE COEFFICIENT
C     SD=HEAD STANDARD DEVIATION
C     VAR=HEAD VARIANCE
C     VARY=INPUT VARIANCE
      COMMON/SAS/D(50),WH(50),QST(50),QSD(50),QAU(50),QP(50),QAR(50),
     1       QSA(50),Y(50),Y1(50),RN(50),HB(50)
      DIMENSION VAR(50),SD(50)
      CC=0.7
      S=0.01
C     INPUT VARIANCE
      YA=0.0
      DO 4 L=1,N
      YA=YA+Y1(L)
    4 CONTINUE
      C=N
   10 YMEAN=YA/C
      YB=0.0
      DO 5 M=1,N
    9 YB=YB+(Y1(M)-YMEAN)*(Y1(M)-YMEAN)
    5 CONTINUE
      VARY=YB/(C-1.)
    8 VARY1=VARY/S**2.
      VAR(1)=VARY1
    7 SD(1)=SQRT(VARY1)
      DO 1 I=2,N
      VARK=0.0
      DO 2 K=1,I
      VARJ=0.0
      DO 3 J=1,I
      A=K
      B=J
      E=I
      W=ABS(A-B)
C     CHECKING EXPONENT SIZE (EXP.LE.100)
      Z=2.*E-A-B
      Z1=100.*S/A1
      IF (Z.GE.Z1) Z=Z1
    6 VARJ=VARJ+VARY1*(CC**W)*EXP(-A1*Z/S)
    3 CONTINUE
      VARK=VARK+VARJ
    2 CONTINUE
      VAR(I)=VARK
      SD(I)=SQRT(VAR(I))
    1 CONTINUE
      RETURN
      END
 TOTAL MEMORY REQUIREMENTS 000540 BYTES
COMPILER HIGHEST SEVERITY CODE WAS  0
//LP EXEC FORTRAN                                                      00:01:10

      SUBROUTINE LP
      INTEGER RNM1,RNM2,CLNM1,CLNM2,BLNK,NEG,POS,SYMB,CDID,RO,MA,FI,EO,
     1IBN1(160),IBN2(160),NBN1(160),NBN2(160)
      REAL XNOP,FN,CJBAR,X,VALU,FP(160),RQ(160),B(160,160),PI(160),
     1NBP(160),XPI(160),PIVOT,LST
      COMMON/STS/VS(50),VO(50),VP(50),VR(50),CRP
      INTEGER VSINDX,VOINDX,VPINDX,VRINDX
      INTEGER CLC
      LOGICAL*1 DUMCHR
      LOGICAL*1 TITLE (40)
C
C     SET ALPHAMERIC CODES IN RO, MA, FI, EO, POS, NEG, BLNK FOR
C     COMPARING WITH LP CARD CODES
C
      RO=-640270272
      MA=-725532608
      FI=-959339344
      EO=-975814592
      POS=1346388032
      NEG=1614823488
      BLNK=1077952576
C
C     INPUT PROGRAM
C
54323 CONTINUE
        M=0
      N=1
      ISW = 0
      NROWS = 0
      NGE = 0
      NLE = 0
      NEQ = 0
      NEL = 0
      NRHS = 0
      NCOLS = 0
C
C     CLEAR MATRIX TO ZERO
C
      DO 12 I=1,160
      DO 12 J=1,160
   12 B(I,J) = 0.0
C     READ PROBLEM TITLE
      READ (2,800) TITLE
      WRITE(6,8151)
      WRITE(6,800) TITLE
      WRITE(6,801)
 8151 FORMAT(1H1)
```

```
  800 FORMAT (1X,40A1)
  801 FORMAT(//)
C
C       READ FIRST CARD - SHOULD BE ROWID
C
      READ (2,2) CDID
    2 FORMAT (A2)
      IF (CDID-RD)3,680,3
    3 WRITE(6,3333)
 3333 FORMAT (//'  ROWID CARD MISSING   '//)
 3334 STOP 1
C
C           READ AND STORE ROWID CARDS
C           INCLUDING DUMMY READ
C           FOR OBJECTIVE ROW NAME
C           GENERATE POS AND NEG SLACKS AS REQUIRED
C
  680 READ (2,681) DUMCHR
  681 FORMAT (A1)
  101 READ (2,102) CDID,LGE,RNM1,RNM2
      WRITE ( 6,102 ) CDID,LGE,RNM1,RNM2
  102 FORMAT(A2,9X,A1,1X,A4,A1)
      IF (CDID-MA)103,504,103
  504 CONTINUE
      GO TO 104
  103 M=M+1
      NROWS = NROWS+1
      IF (LGE-POS)105,106,105
  105 IF (LGE-NEG)107,108,107
  106 IBN1(M)=RNM1
      IBN2(M)=RNM2
      NLE = NLE+1
      BP(M)= 0.0
      GO TO 101
  108 IBN1(M) = RNM1
      IBN2(M) = RNM2
      NGE = NGE+1
      BP(M) = -1.0
      B(M,N) = -1.0
  401 NBN1(N) = RNM1
      NBN2(N) = RNM2
      NBP(N) = 0.0
      N = N+1
      GO TO 101
  107 IBN1(M) = RNM1
      IBN2(M) = RNM2
      NEQ = NEQ+1
      BP(M) = -2.0
      GO TO 101
C
C         READ AND STORE FIRST MATRIX ELEMENT
C
  104 READ (2,195) CDID,CLNM1,CLNM2,RNM1,RNM2,SYMB,VALUE
      WRITE ( 6,195 ) CDID,CLNM1,CLNM2,RNM1,RNM2,SYMB,VALUE
  195 FORMAT(A2,5X,A4,A1,1X,A4,A1,A1,F11.4)
      GO TO 119
  109 IF(NBN1(N) - CLNM1)111,600,111
  600 IF(NBN2(N)-CLNM2)111,601,111
  601 CONTINUE
  112 DO 113 I=1,M
      IF(IBN1(I)-RNM1)113,602,113
  602 IF(IBN2(I)-RNM2)113,603,113
  113 CONTINUE
      WRITE(6,8113)
 8113 FORMAT (//'   INCORRECT INGREDIENT CARD '//)
      STOP 2
  603 CONTINUE
  114 IF (SYMB-NEG) 116,115,116
  115 B(I,N) = -VALUE
      GO TO 117
  116 B(I,N) =  VALUE
C
C          READ AND STORE MATRIX ELEMENTS
C
  117 READ (2,195) CDID,CLNM1,CLNM2,RNM1,RNM2,SYMB,VALUE
      WRITE ( 6,195 ) CDID,CLNM1,CLNM2,RNM1,RNM2,SYMB,VALUE
      NEL = NEL+1
      GO TO 109
  111 N = N+1
      NCOLS = NCOLS+1
      IF (CDID-F1) 119,190,119
  119 NBN1(N) = CLNM1
      NBN2(N) = CLNM2
  201 IF (SYMB-NEG)202,203,202
  202 NBP(N) =  VALUE
      GO TO 117
  203 NBP(N) = -VALUE
      GO TO 117
C
C          READ AND STORE RHS ELEMENTS
C
  190 DO 191 I=1,M
  191 R0(I)=0.0
      GO TO 120
  120 READ (2,121) CDID,RNM1,RNM2,VALUE
      WRITE ( 6,121 ) CDID,RNM1,RNM2,VALUE
  121 FORMAT(A2,11X,A4,A1,1X,F11.4)
      IF (CDID-E0) 122,193,122
  122 DO 124 I=1,M
      IF(IBN1(I)-RNM1)124,610,124
  610 IF(IBN2(I)-RNM2)124,611,124
  124 CONTINUE
      WRITE(6,8124)RNM1,RNM2
 8124 FORMAT(///'  NO ROW NAME FOR   ',A4,A1///)
      STOP 3
  611 CONTINUE
  125 R0(I)=VALUE
      NRHS = NRHS+1
      GO TO 120
  193 N = N-1
      WRITE(6,551)NROWS,NCOLS,NLE,NGE,NEQ,NRHS,NEL
  551 FORMAT(' ROWS ',I2,'    COLS ',I2,'    LE ROWS ',I2,'    GE ROWS '
     1,I2,'    E ROWS ',I2,'    NONZERO RHS''S ',I2,'    NONZERO MATRIX
     1ELEMENTS   ',I5)
C
```

```
C         BLANK OUT ARTIFICIAL NAMES
C
          DO 10 I=1,M
          IF(BP(I)+1.0)19,11,10
       11 IBN1(I) = BLNK
          IBN2(I) = BLNK
          GO TO 10
       19 BP(I) = -1.0
          IBN1(I) = BLNK
          IBN2(I) = BLNK
       10 CONTINUE
C
C         ACCUMULATE COUNT OF INFEASIBILITIES
C
          NINF =0
          DO 6000 I=1,M
          IF(BP(I))6001,6000,6000
     6001 NINF = NINF+1
     6000 CONTINUE
C
C         GENERATE INDICATORS FOR MINIMIZATION OF INFEASIBILITY
C
          DO 6101 J=1,N
          XPI(J) =0.0
          DO 6101 I=1,M
          IF(BP(I))6102,6101,6101
     6102 XPI(J) = XPI(J)-B(I,J)
     6101 CONTINUE
          DO 6002 I=1,M
     6002 BP(I) = 0.0
          IPHASE = 1
C
C         MAIN ROUTINE
C
     9201 WRITE (6,9202)
     9202 FORMAT ('0 ITERATION     VAR IN      VAR OUT              OBJ FN',/)
          IT = 0
    54325 CONTINUE
C
C         CALCULATE SHADOW PRICES
C
          DO 194 J=1,N
          PI(J) = -NBP(J)
          DO 194 I=1,M
      194 PI(J) = PI(J) + BP(I)*B(I,J)
C
C         SELECT BEST NONBASIS VECTOR
C
     9101 LST = -.0000001
          KCOL = 0
          GO TO (751,552),IPHASE
      751 IF(NINF)54321,54321,552
      552 CONTINUE
          DO 9102 J=1,N
C
C         IGNORE ARTIFICIAL VARIABLES
C
          IF(NBN1(J)-BLNK+NBN2(J)-BLNK)651,9102,651
      651 CONTINUE
          GO TO (6003,6004),IPHASE
     6003 IF(XPI(J)-LST)6005,6006,6006
     6005 KCOL=J
          LST = XPI(J)
          GO TO 9102
     6004 CONTINUE
          IF(PI(J)-LST)9103,9102,9102
     9103 KCOL = J
          LST = PI(J)
     6006 CONTINUE
     9102 CONTINUE
          IF (KCOL)54321,54321,9104
C
C         DETERMINE KEYROW
C
     9104 KROW = 0
          CJBAR = LST
          LST = 1.0E20
          DO 9105 I=1,M
          IF(B(I,KCOL))9105,9105,9106
     9106 RATIO = RQ(I)/B(I,KCOL)
          IF (RATIO-LST)9107,9105,9105
     9107 LST = RATIO
          KROW=I
     9105 CONTINUE
          IF(KROW)9112,9112,9114
     9112 WRITE(6,9113) NBN1(KCOL),NBN2(KCOL)
     9113 FORMAT(' VARIABLE ',A4,A1,'  UNBOUNDED ')
          GO TO 54325
     9114 CONTINUE
C
C         TRANSFORM
C
C            DIVIDE BY PIVOT
          PIVOT = B(KROW,KCOL)
          DO 9108 J=1,N
     9108 B(KROW,J) = B(KROW,J)/PIVOT
          RQ(KROW) = RQ(KROW)/PIVOT
          DO 9109 I=1,M
          IF(I-KROW)9110,9109,9110
     9110 RQ(I) = RQ(I) - RQ(KROW)*B(I,KCOL)
          DO 9109 J=1,N
          IF(J-KCOL)9111,9109,9111
     9111 B(I,J) = B(I,J) - B(KROW,J)*B(I,KCOL)
     9109 CONTINUE
          DO 9300 I=1,M
     9300 B(I,KCOL)  = -B(I,KCOL)/PIVOT
          B(KROW,KCOL) = 1.0/PIVOT
          B(KROW,KCOL) = 1.0/PIVOT
C
C         INTERCHANGE BASIS AND NONBASIS VARIABLES
C
          RNM1 = NBN1(KCOL)
          RNM2 = NBN2(KCOL)
          NBN1(KCOL) = IBN1(KROW)
          NBN2(KCOL) = IBN2(KROW)
          IBN1(KROW) = RNM1
```

```
      IBN2(KROW) = RNM2
      LST = NBP(KCOL)
      NBP(KCOL) = BP(KROW)
      BP(KROW) = LST
      IT = IT + 1
      IF(NBN1(KCOL)-BLNK+NBN2(KCOL)-BLNK)6201,6200,6201
 6200 NINF = NINF-1
 6201 CONTINUE
C
C        COMPUTE OBJECTIVE FUNCTION
C
      FN = 0.0
      DO 9301 I=1,M
 9301 FN = FN + BP(I)*RQ(I)
      GO TO (7000,7001),IPHASE
 7000 SAVE = PI(KCOL)
      DO 7003 J=1,N
      PI(J) = PI(J) - SAVE*B(KROW,J)
      XPI(J) = XPI(J) - CJBAR*B(KROW,J)
 7003 CONTINUE
      PI(KCOL) = -SAVE/PIVOT
      XPI(KCOL) = -CJBAR/PIVOT
      GO TO 7004
 7001 CONTINUE
      DO 9302 J=1,N
 9302 PI(J) = PI(J) - CJBAR*B(KROW,J)
      PI(KCOL) = -CJBAR/PIVOT
 7004 CONTINUE
C             CHECK FOR ESSENTIAL ZERO
      DO 6111 I=1,M
      DO 6111 J=1,N
      X=B(I,J)
      IF(ABS(X)-.0000001)6112,6112,6111
 6112 B(I,J) = 0.0
 6111 CONTINUE
C
C
C         LOG ITERATION
C
      WRITE(6,9120) IT,IBN1(KROW),IBN2(KROW),NBN1(KCOL),NBN2(KCOL),FN
 9120 FORMAT(I3,7X,A4,A1,8X,A4,A1,3X,F13.3)
      GO TO 9101
C
C
54321 CONTINUE
      IF(IPHASE-1)8000,8000,54322
 8000 IPHASE = 2
      IF(NINF)8003,8003,8004
 8004 WRITE(6,8005)
 8005 FORMAT('0 SOLUTION INFEASIBLE',/)
      GO TO 54322
 8003 CONTINUE
      WRITE(6,8002)
 8002 FORMAT('0  SOLUTION FEASIBLE ',/)
      GO TO 54325
54322 CONTINUE
C
C        OUTPUT ROUTINE
C
      WRITE(6,301) IT,FN
  301 FORMAT('1','  ITERATION',I5,'  OBJ FN ',F15.3/)
      WRITE (6,21) CRP
   21 FORMAT (3X,'RANDOM PUMPING TERM',3X,F10.3)
      ZZ=FN+CRP
      WRITE (6,22) ZZ
   22 FORMAT (3X,'TOTAL OBJ FN',6X,F10.3)
      WRITE (6,302)
  302 FORMAT(3X,'BASIS VAR',17X,'AMOUNT',6X,'UNIT PROFIT',6X,'LOW
     1,6X,'HIGH',/)
      DO 3033 I=1,M
C
C        COST RANGING
C
      VALUE = 1.0E20
      LST = 1.0E20
      DO 12300 J=1,N
      IF(NBN1(J)-BLNK+NBN2(J)-BLNK)12305,12300,12305
12305 CONTINUE
      IF(B(I,J))12301,12300,12302
12302 X=PI(J)/B(I,J)
      IF(X-LST)12303,12300,12300
12303 LST=X
      GO TO 12300
12301 X=-PI(J)/B(I,J)
      IF(X-VALUE)12304,12300,12300
12304 VALUE = X
12300 CONTINUE
      LST = BP(I) - LST
      VALUE = BP(I) + VALUE
 3033 WRITE(6,304)IBN1(I),IBN2(I),RQ(I),BP(I),LST,VALUE
  304 FORMAT(7X,A4,A1,7X,F16.6,3X,F11.6,3X,F11.6,3X,F11.6)
      WRITE(6,305)
  305 FORMAT('1  VARIABLE            REDUCED COST'//)
      DO 309 J=1,N
      IF(NBN1(J)-BLNK+NBN2(J)-BLNK)311,309,311
  311 WRITE(6,310)NBN1(J),NBN2(J),PI(J)
  310 FORMAT(' ',4X,A4,A1,10X,F12.5)
  309 CONTINUE
C
C        SORT DECISION VARS AND FORM NEW ARRAYS
C
C        NP=NO.OF PERIODS
12307 NP=24
      DO 900 I=1,NP
      VS(I)=0.0
      VU(I)=0.0
      VP(I)=0.0
      VW(I)=0.0
  900 CONTINUE
      DO 901 I=1,M
      IF (LLC(IBN1(I),'VS',2)) 903,902,903
  902 CONTINUE
      VSINDX=INDX(IBN1(I),IBN2(I))
      VS(VSINDX)=RQ(I)
      GO TO 901
  903 CONTINUE
```

```
              IF (CLC(IBN1(I),'VU',2)) 913,912,913
          912 CONTINUE
              VUINDX=INDX(IBN1(I),IBN2(I))
              VU(VUINDX)=RQ(I)
              GO TO 901
          913 CONTINUE
              IF (CLC(IBN1(I),'VR',2)) 905,904,905
          904 CONTINUE
              VRINDX=INDX(IBN1(I),IBN2(I))
              VR(VRINDX)=RQ(I)
              GO TO 901
          905 CONTINUE
              IF (CLC(IBN1(I),'VP',2)) 907,906,907
          906 CONTINUE
              VPINDX=INDX(IBN1(I),IBN2(I))
              VP(VPINDX)=RQ(I)
              GO TO 901
          907 CONTINUE
          901 CONTINUE
              WRITE (6,908)
          908 FORMAT (1H ,4X,80(1H-))
              WRITE (6,909)
          909 FORMAT (1H ,5X,'I',11X,'VS(I)',13X,'VU(I)',13X,'VP(I)',
             113X,'VR(I)')
              WRITE (6,908)
              WRITE (6,910) (I,VS(I),VU(I),VP(I),VR(I),I=1,NP)
          910 FORMAT (1H ,3X,I3,2X,E16.5,2X,E16.5,2X,E16.5,2X,E16.5)
              WRITE (6,908)
      C                                                             PLUTO362
      C        RETURN TO READ NEXT PROBLEM                          PLUTO363
      C                                                             PLUTO364
              RETURN
              END

   TOTAL MEMORY REQUIREMENTS 01C704 BYTES

COMPILER HIGHEST SEVERITY CODE WAS  0

//INDX EXEC FORTRAN                                          00:01:33

              FUNCTION INDX (PART1,PART2)
              INTEGER PART1,PART2,NAME(2),CVB
              LOGICAL*1 INDNAM (8),EQ
              EQUIVALENCE (NAME(1),INDNAM(1))
              NAME(1)=PART1
              NAME(2)=PART2
              LEN=3
      C       DECIDE NO.OF DECIMALS IN VAR NAME E.G. VS14 HAS 2
              IF(EQ(INDNAM(5),' ')) LEN=2
              IF(EQ(INDNAM(4),' ')) LEN=1
      C       CVB CHANGE FROM HEX TO BINARY
              INDX=CVB(INDNAM(3),LEN)
              RETURN
              END

   TOTAL MEMORY REQUIREMENTS 000254 BYTES

COMPILER HIGHEST SEVERITY CODE WAS  0
                                                             00:01:36
// EXEC LNKEDT
LIST     PHASE COMPH,ROOT
LIST      INCLUDE PROG,L
LIST     AUTOLINK    LOAD                                    FROM  SYSREL
LIST     AUTOLINK    IBCOM#                                  FROM  SYSREL
LIST     AUTOLINK    SORT                                    FROM  SYSREL
LIST     AUTOLINK    USERUPT                                 FROM  SYSREL
LIST     AUTOLINK    FIOCS#                                  FROM  SYSREL
LIST     AUTOLINK    UNITAB#                                 FROM  SYSREL
LIST      PHASE MAIN1,*
LIST      INCLUDE MAIN1,L
LIST      INCLUDE INPUTY,L
LIST      INCLUDE MEANH,L
LIST      INCLUDE VARH,L
LIST     AUTOLINK    EQ                                      FROM  SYSREL
LIST     AUTOLINK    ALOG                                    FROM  SYSREL
LIST     AUTOLINK    FRXPR#                                  FROM  SYSREL
LIST     AUTOLINK    EXP                                     FROM  SYSREL
LIST      PHASE LP,MAIN1
LIST      INCLUDE LP,L
LIST      INCLUDE INDX,L
LIST     AUTOLINK    CLC                                     FROM  SYSREL
LIST     AUTOLINK    EQ                                      FROM  SYSREL
LIST     AUTOLINK    CVB                                     FROM  SYSREL
LIST      ENTRY

 76/029   PHASE    TRANSFER ADDR.  LOCORE   HICORE   BLOCK NO.   ESD TYPE   LABEL      LOADED
COMMON                                                           COMMON     SAS        008200
COMMON                                                           COMMON     SES        008860
COMMON                                                           COMMON     SIS        00A680
COMMON                                                           COMMON     SOS        00A9A8
ROOT      COMPH    00AB38          00AB38   00EAA7      5        CSECT      MAIN446    00AB38
                                                                 * ENTRY    MAIN44     00AB38

                                                                 CSECT      BOAOVLY    00B5A8
                                                                   ENTRY    LOAD       00B5C0
                                                                 * ENTRY    LINK       00B5B0

                                                                 CSECT      BOAIBCOM   00B6AC
                                                                   ENTRY    IBCOM#     00B6A0
                                                                   ENTRY    FIRSTIM    00C018
                                                                 * ENTRY    FISRCC#    00C979
                                                                 * ENTRY    FIXPOINT   00C3E0
                                                                 * ENTRY    MSGCC#     00C31F
                                                                 * ENTRY    ADCON#     00B75C

                                                                 CSECT      BOASSORT   00E108
                                                                   ENTRY    SORT       00E1DE

                                                                 CSECT      BOAUOPT    00E288
```

```
                                                                                ENTRY        USEROPT      00E288

                                                                                CSECT        BOAFIOCS     00E290
                                                                                  ENTRY      BUFORG#      00E784
                                                                                  ENTRY      RCBORG#      00E988
                                                                                  ENTRY      FIOCS#       00E290
                                                                               *  ENTRY      FIOCD#       00E2CA
                                                                               *  ENTRY      FIXB         00E9F8
                                                                               *  ENTRY      VARB         00EA06
76/029    PHASE       TRANSFER ADDR.   LOCORE     HICORE     BLOCK NO.      ESD TYPE     LABEL        LOADED

                                                                               *  ENTRY      VDIOCS#      00E98C

                                                                                CSECT                     00EA20
                                                                                  ENTRY      UNITAB#      00EA20

          MAIN1       00EAA8           00EAA8     0139AF        28          CSECT        MAIN1&       00EAA8
                                                                                  ENTRY      MAIN1        00EAA8

                                                                                CSECT        INPUTY&      0125D0
                                                                                  ENTRY      INPUTY       0125D0

                                                                                CSECT        MEANH&       012060
                                                                                  ENTRY      MEANH        012060

                                                                                CSECT        VARH&        0130C0
                                                                                  ENTRY      VARH         0130C0

                                                                                CSECT        EQ           013600
                                                                               *  ENTRY      GE           013628
                                                                               *  ENTRY      LE           013618
                                                                               *  ENTRY      GT           013620
                                                                               *  ENTRY      LT           013610
                                                                               *  ENTRY      NE           013608

                                                                                CSECT        BOASLOG      013558
                                                                                  ENTRY      ALOG         01357A
                                                                               *  ENTRY      ALOG10       013660

                                                                                CSECT        BOAFRXPR     013768
                                                                                  ENTRY      FRXPR#       013770

                                                                                CSECT        BOASEXP      013860
                                                                                  ENTRY      EXP          013864

          LP          00EAA8           00EAA8     02866D        57          CSECT        LP&          00EAA8
                                                                                  ENTRY      LP           00EAA8

                                                                                CSECT        INDX&        028280
                                                                                  ENTRY      INDX         028280

                                                                                CSECT        CLC          028408

                                                                                CSECT        EQ           028580
                                                                               *  ENTRY      GE           0285A8
                                                                               *  ENTRY      LE           028598
                                                                               *  ENTRY      GT           0285A0
                                                                               *  ENTRY      LT           028590
                                                                               *  ENTRY      NE           028588

                                                       3 PHASES USED  216 BLOCKS CSECT       CVB          0285D8

LINKAGE EDITOR HIGHEST SEVERITY WAS   0

//SYS002 ACCESS SCRTCH                                                                        00:01:57
//SYS003 ACCESS SCRTCH                                                                        00:01:57
1A88I   SYS002 SCRTCH    190      SYSRES
1A88I   SYS003 SCRTCH    190      SYSRES
// EXEC COMPH                                                                                 00:01:57
```

AI= 0.2007981-01   VARY= 0.1301615-04

I	QSD(I)	QAU(I)	QP(I)	QAR(I)	Y(I)
1	0.0	0.0	130.762	4.000	0.39355
2	0.0	0.0	128.147	85.300	0.39347
3	0.0	0.0	127.493	61.100	0.39375
4	0.0	0.0	127.493	50.500	0.39395
5	0.0	0.0	127.493	71.500	0.39395
6	0.0	0.0	127.493	96.700	0.39395
7	0.0	0.0	127.493	97.900	0.39395
8	0.0	0.0	127.493	33.100	0.39395
9	0.0	0.0	127.493	52.300	0.39395
10	0.0	0.0	127.493	73.000	0.39395
11	0.0	0.0	127.493	78.700	0.39395
12	0.0	0.0	127.493	109.900	0.39395
13	0.0	0.0	127.493	42.400	0.39395
14	0.0	0.0	127.493	51.400	0.39395
15	0.0	0.0	127.493	61.600	0.39395
16	0.0	0.0	127.493	65.500	0.39395
17	0.0	0.0	127.493	55.600	0.39395
18	0.0	0.0	127.493	64.700	0.39395
19	0.0	0.0	127.493	60.100	0.39395
20	0.0	0.0	127.493	45.400	0.39395
21	0.0	0.0	127.493	121.900	0.39395
22	0.0	0.0	127.493	109.900	0.39395
23	0.0	0.0	127.493	19.600	0.39395
24	0.0	0.0	127.493	93.100	0.39395

CRP=   -472.673

I	HEAD MEAN	DRAW. CORR.	HEAD VAR.	PUMPING RANDOM
1	19.653	23.094	0.130161E 00	0.177681E 02
2	19.620	22.632	0.156973E 00	0.212511E 02
3	19.619	22.516	0.159756E 00	0.214491E 02
4	19.619	22.516	0.160022E 00	0.213073E 02
5	19.619	22.516	0.160047E 00	0.211345E 02
6	19.619	22.516	0.160050E 00	0.209601E 02
7	19.619	22.516	0.160050E 00	0.207869E 02
8	19.619	22.516	0.160050E 00	0.206151E 02
9	19.619	22.516	0.160050E 00	0.204448E 02
10	19.619	22.516	0.160050E 00	0.202758E 02
11	19.619	22.516	0.160050E 00	0.201082E 02
12	19.619	22.516	0.160050E 00	0.199421E 02

13	19.619	22.516	0.160050E 00	0.197775E 02
14	19.619	22.516	0.160050E 00	0.196138E 02
15	19.619	22.516	0.160050E 00	0.194517E 02
16	19.619	22.516	0.160050E 00	0.192919E 02
17	19.619	22.516	0.160050E 00	0.191315E 02
18	19.619	22.516	0.160050E 00	0.189734E 02
19	19.619	22.516	0.160050E 00	0.188166E 02
20	19.619	22.516	0.160050E 00	0.186611E 02
21	19.619	22.516	0.160050E 00	0.185069E 02
22	19.619	22.516	0.160050E 00	0.183539E 02
23	19.619	22.516	0.160050E 00	0.182025E 02
24	19.619	22.516	0.160050E 00	0.180518E 02

I	STREAM DIV COEFF=C1X	RET.W.COEFF=C2X	PUMPING COEFF=CPT	A.RECH COEFF=C3X
1	0.44172E 03	0.10913E 02	0.19332E 04	0.55215E 01
2	0.43807E 03	0.10823E 02	0.18677E 04	0.54759E 01
3	0.43445E 03	0.10733E 02	0.18448E 04	0.54306E 01
4	0.43086E 03	0.10645E 02	0.18295E 04	0.53857E 01
5	0.42730E 03	0.10557E 02	0.18144E 04	0.53412E 01
6	0.42377E 03	0.10470E 02	0.17994E 04	0.52971E 01
7	0.42026E 03	0.10383E 02	0.17845E 04	0.52533E 01
8	0.41679E 03	0.10297E 02	0.17696E 04	0.52099E 01
9	0.41335E 03	0.10212E 02	0.17551E 04	0.51668E 01
10	0.40993E 03	0.10128E 02	0.17406E 04	0.51241E 01
11	0.40654E 03	0.10044E 02	0.17262E 04	0.50818E 01
12	0.40318E 03	0.99610E 01	0.17120E 04	0.50398E 01
13	0.39985E 03	0.98786E 01	0.16976E 04	0.49981E 01
14	0.39655E 03	0.97970E 01	0.16838E 04	0.49568E 01
15	0.39327E 03	0.97160E 01	0.16699E 04	0.49159E 01
16	0.39002E 03	0.96357E 01	0.16561E 04	0.48752E 01
17	0.38679E 03	0.95561E 01	0.16424E 04	0.48349E 01
18	0.38360E 03	0.94771E 01	0.16288E 04	0.47950E 01
19	0.38043E 03	0.93988E 01	0.16154E 04	0.47554E 01
20	0.37728E 03	0.93211E 01	0.16020E 04	0.47161E 01
21	0.37417E 03	0.92441E 01	0.15888E 04	0.46771E 01
22	0.37107E 03	0.91677E 01	0.15756E 04	0.46384E 01
23	0.36801E 03	0.90919E 01	0.15626E 04	0.46001E 01
24	0.36497E 03	0.90168E 01	0.15497E 04	0.45621E 01

DEMAND CONSTRAINTS

I	1=B1	2=B2	3=B3
1	131.00	1634.53	131.00
2	131.00	1606.17	131.00
3	131.00	1603.49	131.00
4	131.00	1603.41	131.00
5	131.00	1603.40	131.00
6	131.00	1603.40	131.00
7	131.00	1603.40	131.00
8	131.00	1603.40	131.00
9	131.00	1603.40	131.00
10	131.00	1603.40	131.00
11	131.00	1603.40	131.00
12	131.00	1603.40	131.00
13	131.00	1603.40	131.00
14	131.00	1603.40	131.00
15	131.00	1603.40	131.00
16	131.00	1603.40	131.00
17	131.00	1603.40	131.00
18	131.00	1603.40	131.00
19	131.00	1603.40	131.00
20	131.00	1603.40	131.00
21	131.00	1603.40	131.00
22	131.00	1603.40	131.00
23	131.00	1603.40	131.00
24	131.00	1603.40	131.00

DOWNSTREAM CONSTRAINTS

I	1=B5	2=B6	3=B4
1	131.00	-104.80	136.06
2	131.00	-104.80	52.68
3	131.00	-104.80	26.53
4	131.00	-104.80	33.51
5	131.00	-104.80	102.50
6	131.00	-104.80	135.50
7	131.00	-104.80	130.50
8	131.00	-104.80	52.50
9	131.00	-104.80	26.50
10	131.00	-104.80	33.50
11	131.00	-104.80	102.50
12	131.00	-104.80	135.50
13	131.00	-104.80	130.50
14	131.00	-104.80	52.50
15	131.00	-104.80	26.50
16	131.00	-104.80	33.50
17	131.00	-104.80	102.50
18	131.00	-104.80	135.50
19	131.00	-104.80	130.50
20	131.00	-104.80	52.50
21	131.00	-104.80	26.50
22	131.00	-104.80	33.50
23	131.00	-104.80	102.50
24	131.00	-104.80	135.50

WATER LEFT AFTER C.U. CONSTRAINTS

I	1=1.	2=1.	3=1.

I	QSA(I)	CCH2(I)	RN(I)	D(I)
1	0.569368E 02	0.700000E 00	-0.205000E 01	0.600000E 01
2	0.623154E 02	0.490000E 00	0.610000E 00	0.167600E 03
3	0.624707E 02	0.343000E 00	0.520000E 00	0.142200E 03
4	0.624932E 02	0.240100E 00	-0.500000E 00	0.101000E 03
5	0.624957E 02	0.168070E 00	0.250000E 00	0.143000E 03
6	0.624957E 02	0.117649E 00	0.104000E 01	0.193400E 03
7	0.624957E 02	0.823544E-01	0.108000E 01	0.195800E 03
8	0.624957E 02	0.576480E-01	-0.108000E 01	0.662000E 02
9	0.624957E 02	0.403536E-01	-0.440000E 00	0.1046[illegible]E 03
10	0.624957E 02	0.282475E-01	0.250000E 00	0.146000E 03
11	0.624957E 02	0.197733E-01	0.440000E 00	0.15740[illegible]E 03
12	0.624957E 02	0.138413E-01	0.148000E 01	0.219800E 03
13	0.624957E 02	0.968891E-02	-0.770000E 00	0.848000E 02
14	0.624957E 02	0.678224E-02	-0.470000E 00	0.102800E 03
15	0.624957E 02	0.474757E-02	-0.130000E 00	0.123200E 03
16	0.624957E 02	0.332329E-02	0.0	0.131000E 03
17	0.624957E 02	0.232631E-02	-0.330000E 00	0.111200E 03
18	0.624957E 02	0.162842E-02	0.640000E 00	0.169400E 03
19	0.624957E 02	0.113989E-02	-0.160000E 00	0.120200E 03
20	0.624957E 02	0.797923E-03	-0.670000E 00	0.968000E 02
21	0.624957E 02	0.558547E-03	0.188000E 01	0.245800E 03
22	0.624957E 02	0.390982E-03	0.148000E 01	0.219800E 03
23	0.624957E 02	0.273688E-03	-0.155000E 01	0.360000E 02
24	0.624957E 02	0.191581E-03	0.920000E 00	0.182200E 03

MADDOCK PROBLEM

```
- DD1
- DD2
- DD3
- DD4
- DD5
- DD6
- DD7
- DD8
- DD9
- DD10
- DD11
- DD12
- DD13
- DD14
- DD15
- DD16
- DD17
- DD18
- DD19
- DD20
- DD21
- DD22
- DD23
- DD24
& DR1
& DR2
& DR3
& DR4
& DR5
& DR6
& DR7
& DR8
& DR9
& DR10
& DR11
& DR12
& DR13
& DR14
& DR15
& DR16
& DR17
& DR18
& DR19
& DR20
& DR21
& DR22
& DR23
& DR24
0 CU1
0 CU2
0 CU3
0 CU4
0 CU5
0 CU6
0 CU7
0 CU8
0 CU9
0 CU10
0 CU11
0 CU12
0 CU13
0 CU14
0 CU15
0 CU16
0 CU17
0 CU18
0 CU19
0 CU20
0 CU21
0 CU22
0 CU23
0 CU24
```

MA

```
VS1    PROFT-    441.7168
VS1    DD1       131.0000
VS1    DR1       131.0000
VU1    PROFT-     10.7131
VU1    DR1   -   104.8000
VU1    CU1         1.0000
VP1    PROFT-   1933.1611
VP1    DD1      1634.5259
VR1    PROFT-      5.5215
```

VR1	CU1	1.0000
VS2	PROFT-	438.0684
VS2	DD2	131.0000
VS2	DR2	131.0000
VU2	PROFT-	10.8229
VU2	DR2 -	104.8000
VU2	CU2	1.0000
VP2	PROFT-	1867.6575
VP2	DD2	1606.1717
VR2	PROFT-	5.4759
VR2	CU2	1.0000
VS3	PROFT-	434.4430
VS3	DD3	131.0000
VS3	DR3	131.0000
VU3	PROFT-	10.7334
VU3	DR3 -	104.8000
VU3	CU3	1.0000
VP3	PROFT-	1844.8347
VP3	DD3	1603.4919
VR3	PROFT-	5.4306
VR3	CU3	1.0000
VS4	PROFT-	430.8577
VS4	DD4	131.0000
VS4	DR4	131.0000
VU4	PROFT-	10.6447
VU4	DR4 -	104.8000
VU4	CU4	1.0000
VP4	PROFT-	1829.5056
VP4	DD4	1603.4148
VR4	PROFT-	5.3857
VR4	CU4	1.0000
VS5	PROFT-	427.2969
VS5	DD5	131.0000
VS5	DR5	131.0000
VU5	PROFT-	10.5568
VU5	DR5 -	104.8000
VU5	CU5	1.0000
VP5	PROFT-	1814.3735
VP5	DD5	1603.4036
VR5	PROFT-	5.3412
VR5	CU5	1.0000
VS6	PROFT-	423.7656
VS6	DD6	131.0000
VS6	DR6	131.0000
VU6	PROFT-	10.4695
VU6	DR6 -	104.8000
VU6	CU6	1.0000
VP6	PROFT-	1799.3732
VP6	DD6	1603.4021
VR6	PROFT-	5.2971
VR6	CU6	1.0000
VS7	PROFT-	420.2632
VS7	DD7	131.0000
VS7	DR7	131.0000
VU7	PROFT-	10.3830
VU7	DR7 -	104.8000
VU7	CU7	1.0000
VP7	PROFT-	1764.5056
VP7	DD7	1603.4021
VR7	PROFT-	5.2533
VR7	CU7	1.0000
VS8	PROFT-	416.7903
VS8	DD8	131.0000
VS8	DR8	131.0000
VU8	PROFT-	10.2972
VU8	DR8 -	104.8000
VU8	CU8	1.0000
VP8	PROFT-	1769.7590
VP8	DD8	1603.4021
VR8	PROFT-	5.2099
VR8	CU8	1.0000
VS9	PROFT-	413.3457
VS9	DD9	131.0000
VS9	DR9	131.0000
VU9	PROFT-	10.2121
VU9	DR9 -	104.8000
VU9	CU9	1.0000
VP9	PROFT-	1755.1331
VP9	DD9	1603.4021
VR9	PROFT-	5.1663
VR9	CU9	1.0000
VS10	PROFT-	409.9294
VS10	DD10	131.0000
VS10	DR10	131.0000
VU10	PROFT-	10.1277
VU10	DR10 -	104.8000
VU10	CU10	1.0000
VP10	PROFT-	1740.6274
VP10	DD10	1603.4021
VR10	PROFT-	5.1241
VR10	CU10	1.0000
VS11	PROFT-	406.5417
VS11	DD11	131.0000
VS11	DR11	131.0000
VU11	PROFT-	10.0440
VU11	DR11 -	104.8000
VU11	CU11	1.0000
VP11	PROFT-	1726.2422
VP11	DD11	1603.4021
VR11	PROFT-	5.0318
VR11	CU11	1.0000
VS12	PROFT-	403.1819
VS12	DD12	131.0000
VS12	DR12	131.0000
VU12	PROFT-	9.9610
VU12	DR12 -	104.8000
VU12	CU12	1.0000
VP12	PROFT-	1711.9763
VP12	DD12	1603.4021
VR12	PROFT-	5.0398
VR12	CU12	1.0000
VS13	PROFT-	399.3499
VS13	DD13	131.0000

VS13	DR13	131.0000
VU13	PROFT-	9.8786
VU13	DR13 -	104.8000
VU13	CU13	1.0000
VP13	PROFT-	1697.8279
VP13	DD13	1603.4021
VR13	PROFT-	4.9981
VR13	CU13	1.0000
VS14	PROFT-	396.5457
VS14	DD14	131.0000
VS14	DR14	131.0000
VU14	PROFT-	9.7970
VU14	DR14 -	104.8000
VU14	CU14	1.0000
VP14	PROFT-	1683.7971
VP14	DD14	1603.4021
VR14	PROFT-	4.9568
VR14	CU14	1.0000
VS15	PROFT-	393.2683
VS15	DD15	131.0000
VS15	DR15	131.0000
VU15	PROFT-	9.7160
VU15	DR15 -	104.8000
VU15	CU15	1.0000
VP15	PROFT-	1669.8811
VP15	DD15	1603.4021
VR15	PROFT-	4.9159
VR15	CU15	1.0000
VS16	PROFT-	390.0183
VS16	DD16	131.0000
VS16	DR16	131.0000
VU16	PROFT-	9.6357
VU16	DR16 -	104.8000
VU16	CU16	1.0000
VP16	PROFT-	1656.0808
VP16	DD16	1603.4021
VR16	PROFT-	4.8752
VR16	CU16	1.0000
VS17	PROFT-	386.7949
VS17	DD17	131.0000
VS17	DR17	131.0000
VU17	PROFT-	9.5561
VU17	DR17 -	104.8000
VU17	CU17	1.0000
VP17	PROFT-	1642.3943
VP17	DD17	1603.4021
VR17	PROFT-	4.8349
VR17	CU17	1.0000
VS18	PROFT-	383.5984
VS18	DD18	131.0000
VS18	DR18	131.0000
VU18	PROFT-	9.4771
VU18	DR18 -	104.8000
VU18	CU18	1.0000
VP18	PROFT-	1628.8206
VP18	DD18	1603.4021
VR18	PROFT-	4.7950
VR18	CU18	1.0000
VS19	PROFT-	380.4282
VS19	DD19	131.0000
VS19	DR19	131.0000
VU19	PROFT-	9.3988
VU19	DR19 -	104.8000
VU19	CU19	1.0000
VP19	PROFT-	1615.3678
VP19	DD19	1603.4021
VR19	PROFT-	4.7554
VR19	CU19	1.0000
VS20	PROFT-	377.2842
VS20	DD20	131.0000
VS20	DR20	131.0000
VU20	PROFT-	9.3211
VU20	DR20 -	104.8000
VU20	CU20	1.0000
VP20	PROFT-	1602.0100
VP20	DD20	1603.4021
VR20	PROFT-	4.7161
VR20	CU20	1.0000
VS21	PROFT-	374.1663
VS21	DD21	131.0000
VS21	DR21	131.0000
VU21	PROFT-	9.2441
VU21	DR21 -	104.8000
VU21	CU21	1.0000
VP21	PROFT-	1588.7710
VP21	DD21	1603.4021
VR21	PROFT-	4.6771
VR21	CU21	1.0000
VS22	PROFT-	371.0740
VS22	DD22	131.0000
VS22	DR22	131.0000
VU22	PROFT-	9.1677
VU22	DR22 -	104.8000
VU22	CU22	1.0000
VP22	PROFT-	1575.6404
VP22	DD22	1603.4021
VR22	PROFT-	4.6384
VR22	CU22	1.0000
VS23	PROFT-	368.0073
VS23	DD23	131.0000
VS23	DR23	131.0000
VU23	PROFT-	9.0919
VU23	DR23 -	104.8000
VU23	CU23	1.0000
VP23	PROFT-	1562.6189
VP23	DD23	1603.4021
VR23	PROFT-	4.6001
VR23	CU23	1.0000
VS24	PROFT-	364.9658
VS24	DD24	131.0000
VS24	DR24	131.0000
VU24	PROFT-	9.0168
VU24	DR24 -	104.8000
VU24	CU24	1.0000
VP24	PROFT-	1549.7046
VP24	DD24	1603.4021

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      VR24   PROFT-      4.5621
      VR24   CU24        1.0000
F1                          0.0
             DD1       131.0000
             DD2       131.0000
             DD3       131.0000
             DD4       131.0000
             DD5       131.0000
             DD6       131.0000
             DD7       131.0000
             DD8       131.0000
             DD9       131.0000
             DD10      131.0000
             DD11      131.0000
             DD12      131.0000
             DD13      131.0000
             DD14      131.0000
             DD15      131.0000
             DD16      131.0000
             DD17      131.0000
             DD18      131.0000
             DD19      131.0000
             DD20      131.0000
             DD21      131.0000
             DD22      131.0000
             DD23      131.0000
             DD24      131.0000
             DR1       136.0632
             DR2        52.6846
             DR3        26.5293
             DR4        33.5066
             DR5       102.5042
             DR6       135.5042
             DR7       130.5042
             DR8        52.5042
             DR9        26.5042
             DR10       33.5042
             DR11      102.5042
             DR12      135.5042
             DR13      130.5042
             DR14       52.5042
             DR15       26.5042
             DR16       33.5042
             DR17      102.5042
             DR18      135.5042
             DR19      130.5042
             DR20       52.5042
             DR21       26.5042
             DR22       33.5042
             DR23      102.5042
             DR24      135.5042
             CU1         1.0000
             CU2         1.0000
             CU3         1.0000
             CU4         1.0000
             CU5         1.0000
             CU6         1.0000
             CU7         1.0000
             CU8         1.0000
             CU9         1.0000
             CU10        1.0000
             CU11        1.0000
             CU12        1.0000
             CU13        1.0000
             CU14        1.0000
             CU15        1.0000
             CU16        1.0000
             CU17        1.0000
             CU18        1.0000
             CU19        1.0000
             CU20        1.0000
             CU21        1.0000
             CU22        1.0000
             CU23        1.0000
             CU24        1.0000
ED                          0.0
ROWS 72,   COLS 96,   LE ROWS 24,   GE ROWS 24,   E ROWS 24,   NONZERO RHS'S 72,   NONZERO MATRIX ELEMENTS

 ITERATION     VAR IN       VAR OUT          OBJ FN

         1     VP1                         -154.934
         2     VP2                         -307.261
         3     VP3                         -457.978
         4     VP4                         -607.449
         5     VP5                         -755.686
         6     VP6                         -902.697
         7     VP7                        -1048.493
         8     VP8                        -1193.034
         9     VP9                        -1336.431
        10     VP10                       -1478.692
        11     VP11                       -1619.728
        12     VP12                       -1759.598
        13     VP13                       -1898.313
        14     VP14                       -2035.881
        15     VP15                       -2172.312
        16     VP16                       -2307.616
        17     VP17                       -2441.802
        18     VP18                       -2574.878
        19     VP19                       -2706.855
        20     VP20                       -2837.741
        21     VP21                       -2967.545
        22     VP22                       -3096.277
        23     VP23                       -3223.945
        24     VP24                       -3350.557
        25     VU1                        -3361.470
        26     VU2                        -3372.293
        27     VU3                        -3383.026
        28     VU4                        -3393.671
        29     VU5                        -3404.228
        30     VU6                        -3414.697
        31     VU7                        -3425.030
        32     VU8                        -3435.377
        33     VU9                        -3445.589
        34     VU10                       -3455.717
        35     VU11                       -3465.760
        36     VU12                       -3475.721
```

37	VU13		-3485.600
38	VU14		-3495.397
39	VU15		-3505.113
40	VU16		-3514.748
41	VU17		-3524.304
42	VU18		-3533.781
43	VU19		-3543.180
44	VU20		-3552.501
45	VU21		-3561.745
46	VU22		-3570.912
47	VU23		-3580.004
48	VU24		-3589.021

SOLUTION FEASIBLE

49	VR1	VU1	-3583.629
50	VR2	VU2	-3578.282
51	VR3	VU3	-3572.979
52	VR4	VU4	-3567.720
53	VR5	VU5	-3562.504
54	VR6	VU6	-3557.332
55	VR7	VU7	-3552.202
56	VR8	VU8	-3547.115
57	VR9	VU9	-3542.070
58	VR10	VU10	-3537.066
59	VR11	VU11	-3532.104
60	VR12	VU12	-3527.183
61	VR13	VU13	-3522.302
62	VR14	VU14	-3517.462
63	VR15	VU15	-3512.662
64	VR16	VU16	-3507.902
65	VR17	VU17	-3503.180
66	VR18	VU18	-3498.498
67	VR19	VU19	-3493.855
68	VR20	VU20	-3489.250
69	VR21	VU21	-3484.683
70	VR22	VU22	-3480.154
71	VR23	VU23	-3475.662
72	VR24	VU24	-3471.208

ITERATION 72 OBJ FN -3471.208

RANDOM PUMPING TERM -472.873
TOTAL OBJ FN -3944.080

BASIS VAR	AMOUNT	UNIT PROFIT	LOW	HIGH
VP1	0.080146	***********	***********	-0.003732
VP2	0.081560	***********	***********	-0.003456
VP3	0.081697	***********	***********	-0.000977
VP4	0.081701	***********	***********	-0.000244
VP5	0.081701	***********	***********	-0.000977
VP6	0.081701	***********	***********	-0.001709
VP7	0.081701	***********	***********	-0.001221
VP8	0.081701	***********	***********	-0.001709
VP9	0.081701	***********	***********	-0.000244
VP10	0.081701	***********	***********	-0.001465
VP11	0.081701	***********	***********	-0.000488
VP12	0.081701	***********	***********	-0.001465
VP13	0.081701	***********	***********	-0.000488
VP14	0.081701	***********	***********	-0.000977
VP15	0.081701	***********	***********	-0.001465
VP16	0.081701	***********	***********	-0.001465
VP17	0.081701	***********	***********	-0.003732
VP18	0.081701	***********	***********	-0.000977
VP19	0.081701	***********	***********	-0.000488
VP20	0.081701	***********	***********	-0.000244
VP21	0.081701	***********	***********	-0.000244
VP22	0.081701	***********	***********	-0.000244
VP23	0.081701	***********	***********	-0.000244
VP24	0.081701	***********	***********	-0.000244
DR1	136.063202	0.0	-2.189194	0.051447
DR2	52.634601	0.0	-2.181231	0.051021
DR3	26.529297	0.0	-2.165635	0.050599
DR4	33.506607	0.0	-2.147933	0.050181
DR5	102.504196	0.0	-2.130231	0.049767
DR6	135.504196	0.0	-2.112627	0.049355
DR7	130.504196	0.0	-2.095166	0.048948
DR8	52.504196	0.0	-2.077652	0.048543
DR9	26.504196	0.0	-2.060678	0.048142
DR10	33.504196	0.0	-2.043649	0.047744
DR11	102.504196	0.0	-2.026760	0.047349
DR12	135.504196	0.0	-2.010010	0.046956
DR13	130.504196	0.0	-1.993379	0.046570
DR14	52.504196	0.0	-1.976926	0.046185
DR15	26.504196	0.0	-1.960587	0.045802
DR16	33.504196	0.0	-1.944385	0.045425
DR17	102.504196	0.0	-1.928315	0.045050
DR18	135.504196	0.0	-1.912379	0.044677
DR19	130.504196	0.0	-1.896574	0.044307
DR20	52.504196	0.0	-1.880900	0.043941
DR21	26.504196	0.0	-1.865356	0.043578
DR22	33.504196	0.0	-1.849940	0.043219
DR23	102.504196	0.0	-1.834652	0.042861
DR24	135.504196	0.0	-1.819489	0.042507
VR1	1.000000	-5.521500	-10.913100	***********
VR2	1.000000	-5.475900	-10.822900	***********
VR3	1.000000	-5.430600	-10.733400	***********
VR4	1.000000	-5.385700	-10.644700	***********
VR5	1.000000	-5.341200	-10.556800	***********
VR6	1.000000	-5.297100	-10.469500	***********
VR7	1.000000	-5.253300	-10.383000	***********
VR8	1.000000	-5.209900	-10.297200	***********
VR9	1.000000	-5.166800	-10.212100	***********
VR10	1.000000	-5.124100	-10.127700	***********
VR11	1.000000	-5.081800	-10.044000	***********
VR12	1.000000	-5.039800	-9.961000	***********
VR13	1.000000	-4.998100	-9.878600	***********
VR14	1.000000	-4.956800	-9.797000	***********
VR15	1.000000	-4.915900	-9.716000	***********
VR16	1.000000	-4.875200	-9.635700	***********
VR17	1.000000	-4.834900	-9.556100	***********
VR18	1.000000	-4.795000	-9.477100	***********
VR19	1.000000	-4.755400	-9.398800	***********
VR20	1.000000	-4.716100	-9.321100	***********
VR21	1.000000	-4.677100	-9.244100	***********
VR22	1.000000	-4.638400	-9.167700	***********
VR23	1.000000	-4.600100	-9.091900	***********
VR24	1.000000	-4.562100	-9.016800	***********

VARIABLE	REDUCED COST
DD1	1.18270
DD2	1.16280
DD3	1.15051
DD4	1.14101
DD5	1.13158
DD6	1.12222
DD7	1.11295
DD8	1.10375
DD9	1.09463
DD10	1.08558
DD11	1.07661
DD12	1.06771
DD13	1.05889
DD14	1.05014
DD15	1.04146
DD16	1.03285
DD17	1.02432
DD18	1.01585
DD19	1.00746
DD20	0.99913
DD21	0.99087
DD22	0.98269
DD23	0.97456
DD24	0.96651
VS1	286.78442
VU1	5.39160
VS2	285.74121
VU2	5.34700
VS3	283.73096
VU3	5.30280
VS4	281.38574
VU4	5.25900
VS5	279.06030
VU5	5.21560
VS6	276.75415
VU6	5.17240
VS7	274.46680
VU7	5.12970
VS8	272.19873
VU8	5.08730
VS9	269.94897
VU9	5.04530
VS10	267.71802
VU10	5.00360
VS11	265.50562
VU11	4.96220
VS12	263.31128
VU12	4.92120
VS13	261.13525
VU13	4.88050
VS14	258.97729
VU14	4.84020
VS15	256.83691
VU15	4.80010
VS16	254.71449
VU16	4.76050
VS17	252.60931
VU17	4.72120
VS18	250.52177
VU18	4.68210
VS19	248.45128
VU19	4.64340
VS20	246.39801
VU20	4.60500
VS21	244.36174
VU21	4.56700
VS22	242.34224
VU22	4.52930
VS23	240.33946
VU23	4.49180
VS24	238.35307
VU24	4.45470

I	VS(I)	VU(I)	VP(I)	VR(I)
1	0.0	0.0	0.80146E-01	0.10000E 01
2	0.0	0.0	0.81560E-01	0.10000E 01
3	0.0	0.0	0.81697E-01	0.10000E 01
4	0.0	0.0	0.81701E-01	0.10000E 01
5	0.0	0.0	0.81701E-01	0.10000E 01
6	0.0	0.0	0.81701E-01	0.10000E 01
7	0.0	0.0	0.81701E-01	0.10000E 01
8	0.0	0.0	0.81701E-01	0.10000E 01
9	0.0	0.0	0.81701E-01	0.10000E 01
10	0.0	0.0	0.81701E-01	0.10000E 01
11	0.0	0.0	0.81701E-01	0.10000E 01
12	0.0	0.0	0.81701E-01	0.10000E 01
13	0.0	0.0	0.81701E-01	0.10000E 01
14	0.0	0.0	0.81701E-01	0.10000E 01
15	0.0	0.0	0.81701E-01	0.10000E 01
16	0.0	0.0	0.81701E-01	0.10000E 01
17	0.0	0.0	0.81701E-01	0.10000E 01
18	0.0	0.0	0.81701E-01	0.10000E 01
19	0.0	0.0	0.81701E-01	0.10000E 01
20	0.0	0.0	0.81701E-01	0.10000E 01
21	0.0	0.0	0.81701E-01	0.10000E 01
22	0.0	0.0	0.81701E-01	0.10000E 01
23	0.0	0.0	0.81701E-01	0.10000E 01
24	0.0	0.0	0.81701E-01	0.10000E 01

ITERATION = 1   SQRT(M.S.E.) = 0.117131E 00

APPENDIX J Basic Information on Rio Sinaloa Study Area

Population of the valley in which the study area is located:	150,000 inhabitants
Area of the irrigated land:	460 Km^2
Area of the aquifer:	1744 Km^2
Main aquifer:	Quaternary alluvium
Irrigation cycles and average consumptive use:	Spring (46.8 cm), summer (77.7 cm) and winter (48 cm)
Main crops:	Cotton, wheat, bastard saffron, sorghum, corn, soybean and cantaloupe.
Climate:	Semiarid
Rainy season:	July to October and sometimes is extended to January
Average precipitation:	450 mm.
Monthly mean temperature:	29°C maximum and 17°C minimum
Annual mean evaporation:	About 2300 mm
Topography of the study area:	The area to the west of the river slopes to the southwest and descend from 45 m above sea level to 20 m above sea level. The area to the east of the river slopes to the south-

	west and descends from 65 m above sea level to sea level.
Pumping figures:	East side of the river, 300 wells extracting $149 x 10^6 m^3/yr$ (in 1971); west side of the river 240 wells extracting $50 x 10^6 m^3/yr$
Well depth distribution:	About 68 wells with a depth greater than 50 m are located at the east side of the river; about 22 wells with a depth greater than 50 m are located at the west side of the river
Average aquifer transmissivity:	$0.02 m^2/s$
Average aquifer storage coefficient:	0.01

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