

New Mexico Bureau
of
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UNCONFINED AQUIFER PUMPING TESTS

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INTRODUCTION

In the past twenty years numerous formulas have been developed for analyzing pumping tests conducted in unconfined aquifers. The pumping test method, which best fits the physical field case should always be chosen when picking the type curves to match the field time-drawdown data. The significance of several field parameters should be considered in selecting a pumping test method. Some of the parameters, which should be considered are aquifer thickness, radius of observation wells, pumping well penetration, variable pumping well penetration, vertical position of the observation well, storage coefficient, variability of specific yield, vertical anisotropy of the aquifer, horizontal anisotropy of the aquifer, heterogeneous aquifer, well storage, free surface boundary, impermeable boundaries, aquifer thinning and variable pumping rates. Pumping test methods, which consider all of these parameters, are not available. Knowing which method to use is very confusing and difficult. It is not surprising Stallman (1965) stated that "Analysis of pumping tests made in unconfined aquifers should be a fertile field for anyone slightly inclined toward pessimism". The purpose of this report is to evaluate the present status of the art of analyzing unconfined pumping tests and to give suggestions on analyzing aquifer tests. The words pumping test in this report deal with a test, which consists of a pumping well and one or more observation wells. Aquifer test will be used as synonymous to pumping test. The term

storage coefficient relates to the amount of water an aquifer yields from the expansion of water and compression of the aquifer material with reduction in head.

A review and comparison of unconfined pumping test is given first in the report. Semi-unconfined pumping tests were also reviewed, since they are closely related to the unconfined pumping test. Since vertical anisotropy of aquifers is felt to be very significant, methods used to determine the ratio of vertical to horizontal hydraulic conductivities are reviewed. Next a methodology is presented to determine the vertical anisotropy independent of the discharge of the pumping well. Four pumping tests are analyzed as examples of determining the anisotropy of aquifers by this method. Finally, the importance of pumping well penetration and observation well position is shown with some recommendations in the importance of considering certain parameters.

Figure 1 illustrates the symbols used in this report to describe the aquifer and well parameters. The pumping well penetration is described by L' and L_3' , while the piezometer's depth is described by Z' . Symbols r , D , K_v and K_h are used to represent the radius of the piezometer, thickness of the aquifer, vertical hydraulic conductivity and horizontal hydraulic conductivity respectively.

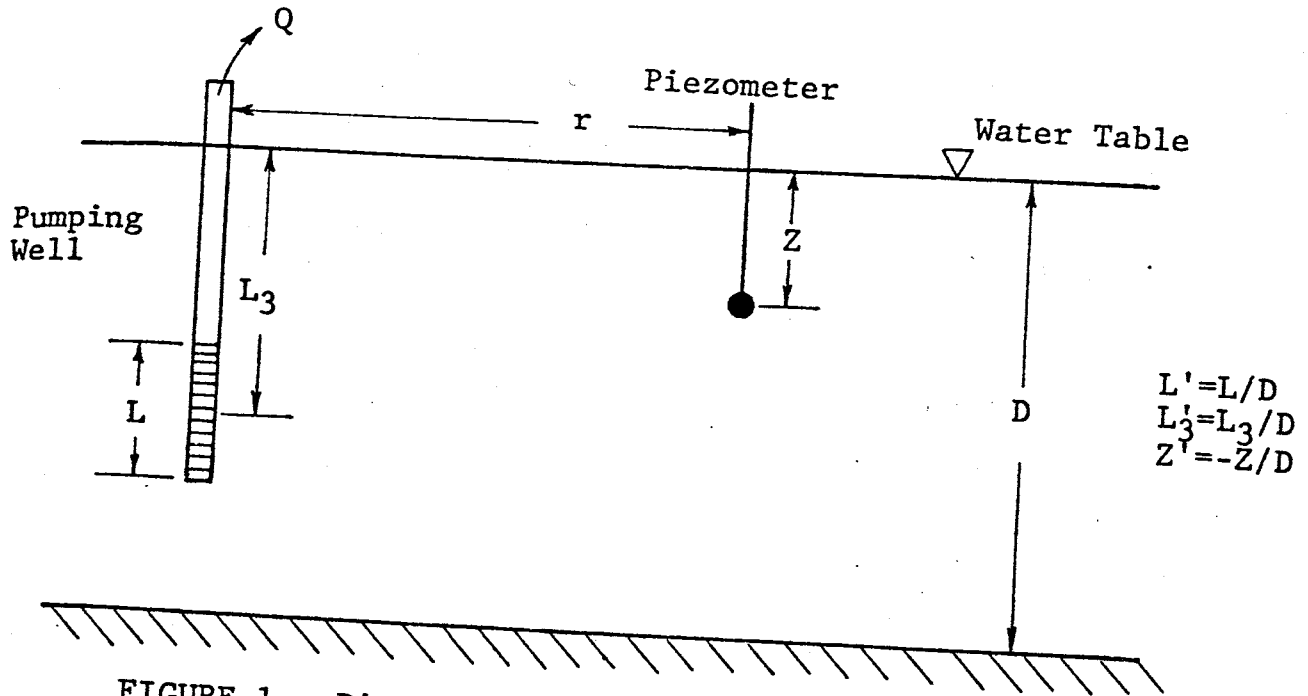


FIGURE 1. Diagram illustrating symbols used to represent aquifer and well properties.

PREVIOUS WORK

Unconfined Pumping Test Methods

The three main types of flow equations, which describe the flow to a pumping well in an unconfined aquifer are radial flow, potential flow and delayed yield. Radial flow equations consider only radial flow toward a well, that is the head along any vertical line in the aquifer is a constant. It is impossible for the head along any vertical line around pumping wells in unconfined aquifers to be a constant. Potential flow equations consider the vertical flow components as well as the radial flow components. Flow to a pumping well in an unconfined aquifer must have vertical flow component. Delayed

yield flow equations treat the specific yield as a variable of time. Table 1 summarizes the parameters considered by each pumping test method discussed for the aquifer, aquitard, pumping well, and observation well. All methods presented are for unconfined or semi-unconfined, homogeneous and infinite areal extent aquifers. None of these methods consider the seepage face problem, which occurs between the pumping well and the aquifer.

Theis (1935) introduced the radial flow equation, which correctly describes the flow to a well in a confined aquifer. The Theis equation has been used extensively to analyze unconfined pumping tests. Jacob (1944) developed an approximation for the correction of the thinning of the aquifer thickness that occurs in pumping an unconfined aquifer. Kriz (1967) solved the radial flow equation with consideration of the thinning of the aquifer. A similar solution to Kriz's with consideration of aquifer thinning was introduced by Dagan (1967b) using a perturbation analysis. Figure 2 shows a comparison between the Theis, Jacob's correction, Kriz and Dagan solutions for a dimensionless discharge, $Q/(4\pi KD^2)$ of 0.04.

where: Q = discharge in cubic feet per day
 K = hydraulic conductivity in feet per day
 D = aquifer thickness in feet

This value of dimensionless discharge represents typical field values and also represents an average effect of the thinning of the aquifer. The information in this figure was taken from Hernandez - Gobaira (1972). Since the Theis solution differs considerably from the other three solutions, the thinning of

TABLE 1. Considerations in aquifer, aquitard, pumping well, and observation well for selected unconfined and semi-unconfined pumping tests.

	AQUIFER						AQUI-TARD	PUMPING WELL				OBSERVATION WELL					
	Isotropic	Vertical Anisotropic	Finite Thickness	Infinite Thickness	Compressible Storage	Specific Yield	Delayed Yield	Compressible Storage	Specific Yield	Constant Discharge	Variable Discharge	Fully Penetrating	Partial Penetrating	Free Surface	Fully Penetrating	Point Observation	Partial Penetrating
UNCONFINED																	
Boulton (1955, 1963)	x	x								x	+				x		
Type curves Boulton (1963)										x	+						
Boulton (1954)		x	x							x	+			x			
Type curves Stallman (1961)																	
Boulton and Pontin (1971)		x	x							x							
Programmed Boulton (1971)										x		+					x
Dagan (1967)		x	x							x							
Prog. from Dagan or Hoffman										x							x
Solution given Dagan (1967)		x		x						x							x
" " " "		x		x						x							x
" " " "		x		x								*					x
" " " "		x		x								*					x
Kipp (1973)																	
Solution given Kipp (1973)	x			x													
Neuman (1972)		x	x		x	x				x					x	x	
Prog. from Neuman or Hoffman											+						
Neuman (1974)		x	x		x	x				x				x	x	x	x
Prog. from Neuman or Hoffman																	
Stallman (1963, 1965)		x	x							x							
Type curves Lohman (1972)		x	x							x							
Streltsova (1972)		x	x														
Type curves Boulton (1963)																	
Theis (1935)																	
Type curve Ferris (1963)	x		x		x					x							
SEMI-UNCONFINED																	
Boulton (1973)		x	x		x				x	x							
Type curves Boulton (1963)																	
Cooley and Case (1973)	x		x		x				x	x							
Type curves Boulton (1963)																	
Type curves not available	x		x		x				x	x							

+ Indicates pumping well approximated by line sink.
 * Indicates pumping well approximated by point sink.
 0 Indicates pumping well storage considered.

DIMENSIONLESS DRAWDOWN ($4\pi KD^2 s/O$)

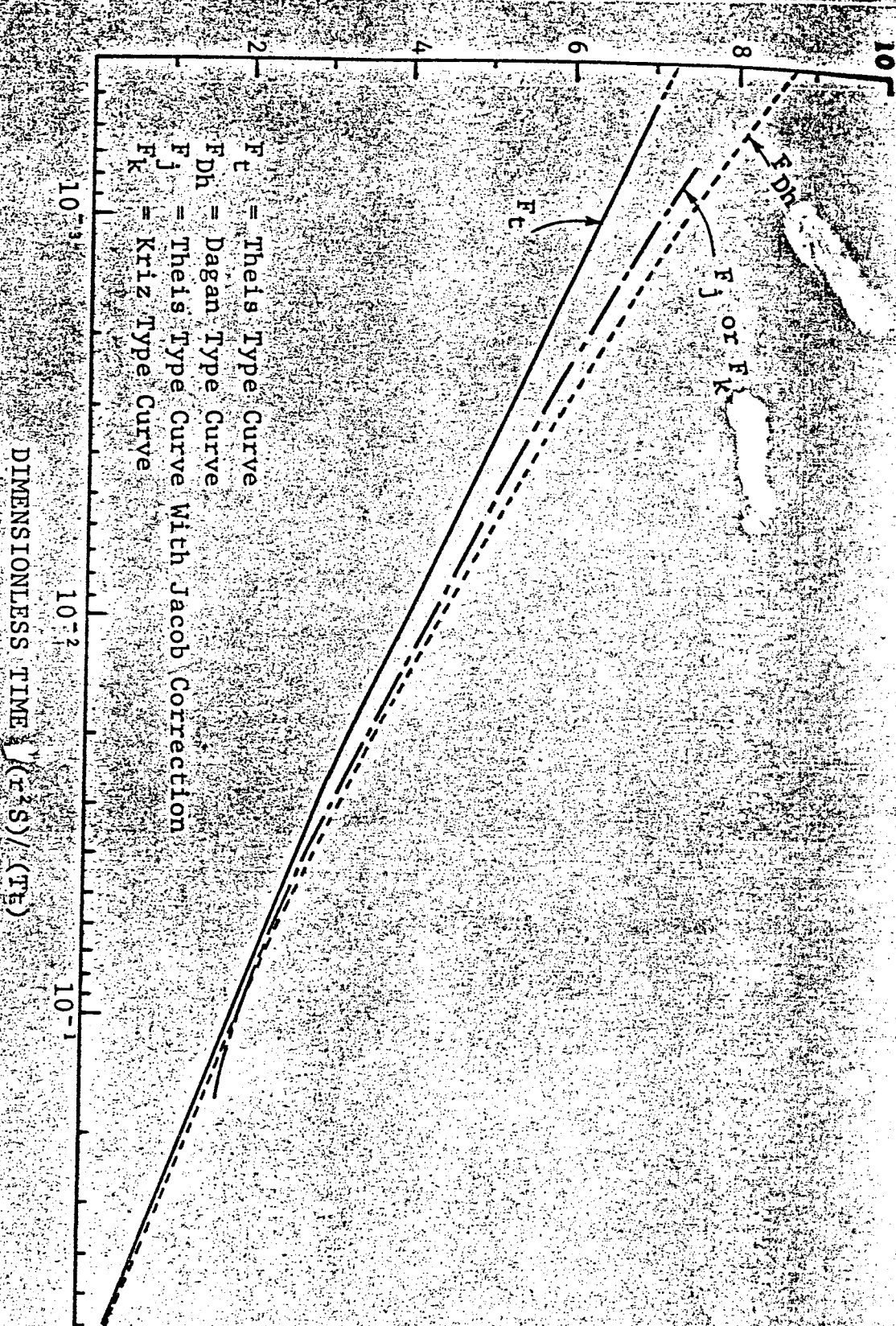


FIGURE 2. COMPARISON OF THE THEIS, DAGAN, JACOB CORRECTION AND KRIZ TYPE CURVES.

the aquifer is important in this case. Kriz's (1967) solution is felt to be more correct than Dagan's (1967b), since Dagan approximates the solution with perturbation analysis. The close agreement between Jacob's correction and Kriz's indicates that Jacob's correction is probably sufficient to handle the aquifer thinning in radial flow pumping tests.

The Jacob correction can be derived by linearizing the Dupuit flow equation with respect to $(D-s)^2$. Jacob correction is connected with the drawdown after linearization. The linearization also attaches a correction term, $D/(D-s)S_y$, to the specific yield.

where: D = aquifer thickness

s = drawdown

S_y = specific yield

Stallman (1968) presents this term and suggests that the geometric mean drawdown of the observation wells should be used for s in the correction of specific yield. Since s is a variable with time, the corrected specific yield term, $D/(D-s)S_y$, also is a variable with time. Therefore, no one value of s can be used in the correction of the specific yield. Since Jacob's correction doesn't take into account the correction of the specific yield and Kriz's solution takes into account the complete thinning problem, the correction of the specific yield is probably not very significant.

Boulton (1954) introduced the potential flow equation for flow to a well with a linear free surface boundary condition. Type curves were developed by Boulton and Stallman

(1961) for drawdown at the free surface. Stallman (1963, 1965) has introduced an electric analog solution to the potential flow with the same initial and boundary conditions as Boulton (1954). Stallman's type curves for a pumping well, which is fully penetrating or penetrates the bottom three-tenths of the aquifer were published by Lohman (1972). Dagan (1967a) developed a general analytical solution to the potential flow equation for a partially penetrating pumping well, utilizing the same initial and boundary conditions as Boulton (1954). A comparison of Dagan and Boulton (1954) for a fully penetrating pumping well and drawdown at the free surface is shown in Figure 3 for three type curves. The type curves are identical except for a small deviation as dimensionless time becomes very small. A comparison also was made between Stallman's electric analog results and Dagan's analytical solution for a partially penetrating pumping well. Deviation between the curves, shown in Figure 4, is probably due to the approximation of using discrete elements by Stallman.

Dagan (1967a) also gives three solutions for infinitely thick aquifers (Table 1). Additional considerations of these methods are variable discharge and point sink representation of the pumping well.

Consideration of the compressibility of an unconfined aquifer and the expansion of the fluid in the potential flow equation was introduced by Neuman (1972, 1974). Neuman uses the same initial and boundary condition as Boulton (1954), but adds the specific storage term to the flow equation. A

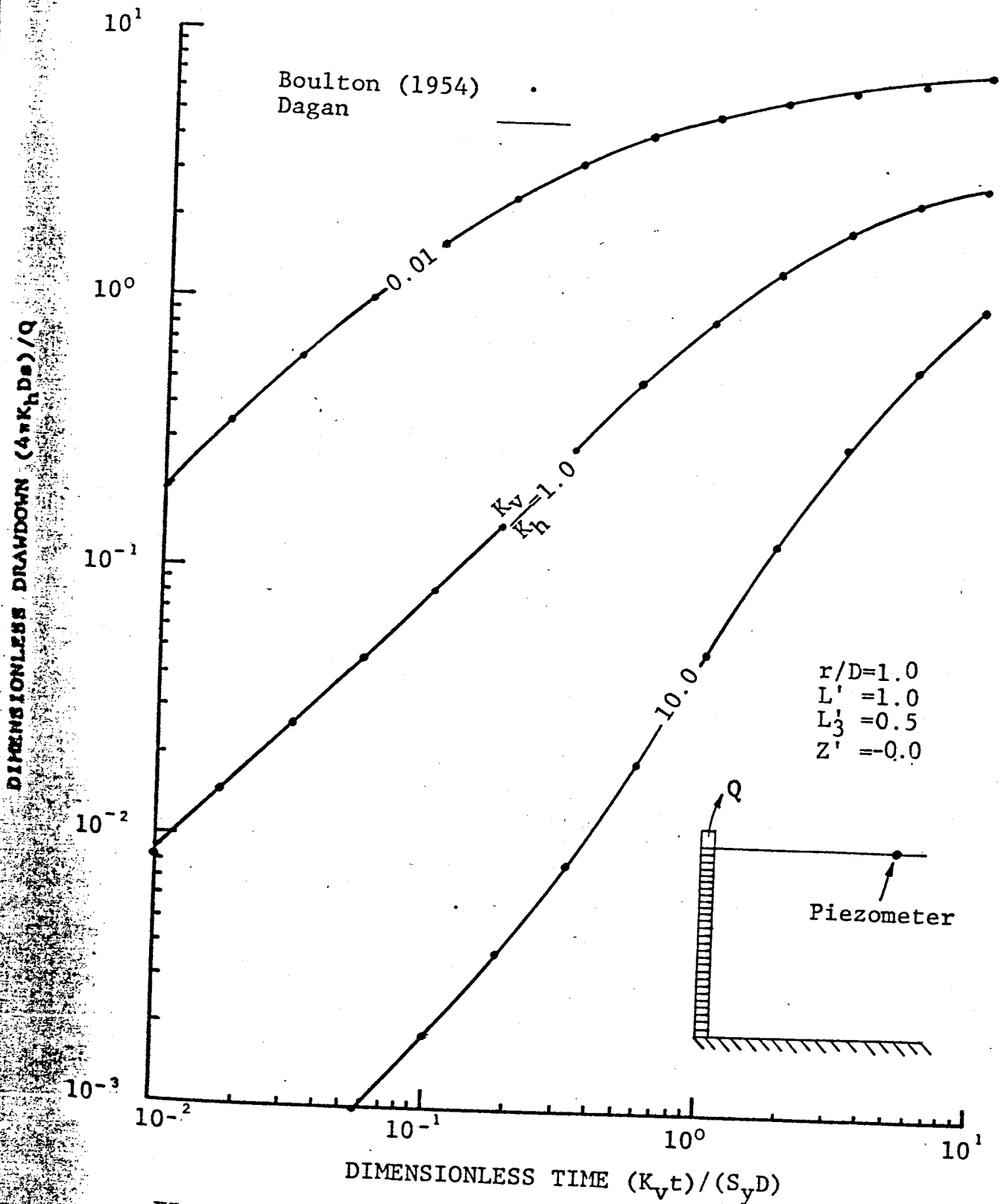


FIGURE 3. Comparison of Boulton and Dagan Type Curves.

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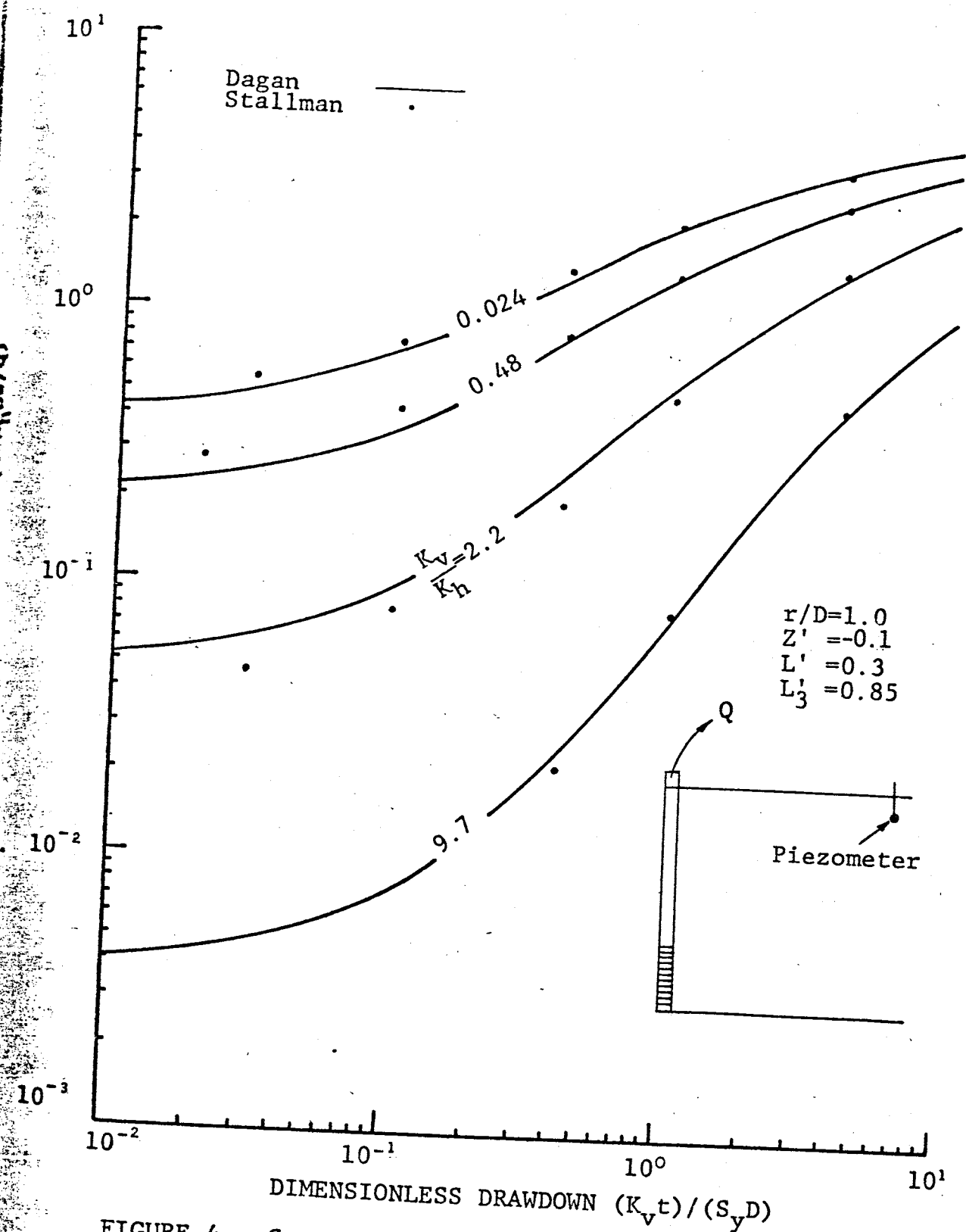


FIGURE 4. Comparison of Stallman and Dagan Type Curves.

comparison of Neuman's and Dagan's type curves shows that as time increases the two curves merge. For a S/S_y ratio of 0.1, Figure 5 shows that after dimensionless time 0.4, the type curves are equal for a K_v/K_h value of 0.1.

where: S = storage coefficient

S_y = specific yield

For smaller S/S_y ratios the two curves become identical at smaller values of dimensionless time.

From the basic radial flow equation and the assumption that the vertical transfer of flow varies linearly with the difference of the average head and free surface head, Streltsova (1972, 1973) and Streltsova and Rushton (1973) obtain a correction for vertical flow. Streltsova shows that this approximate flow equation is the same as Boulton's (1955, 1963) delayed yield equation. Streltsova also notes that, when the discharge boundary condition is written in terms of the free surface head, the solution of the flow equation is the same as Barenblatt's et al (1960) solution for flow in fissure rocks. Streltsova's late time solution (Boulton 1963) should be very close to Dagan's, since the two solutions are for potential flow. However, since Boulton (1963) type curves are for average head and Dagan's are for point observations, graphic comparison is not possible. Streltsova's solution can be compared to Neuman, since his program has the capabilities of computing the average head. Figure 6 shows the comparison of Streltsova's early and late time type curves and Neuman's type curves for S/S_y of 0.01. The close

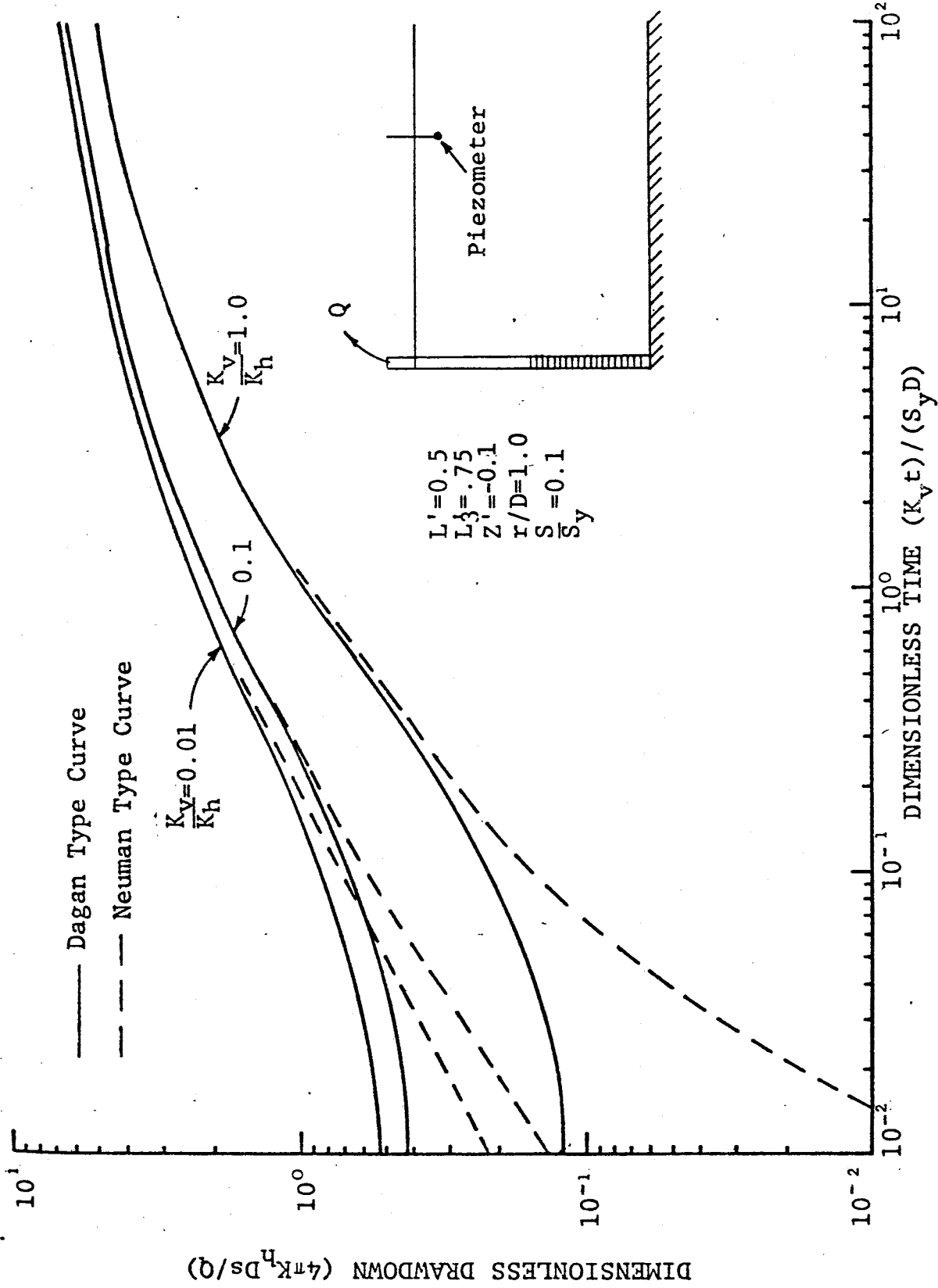


Figure 5. Comparison of Dagan and Neuman type curves.

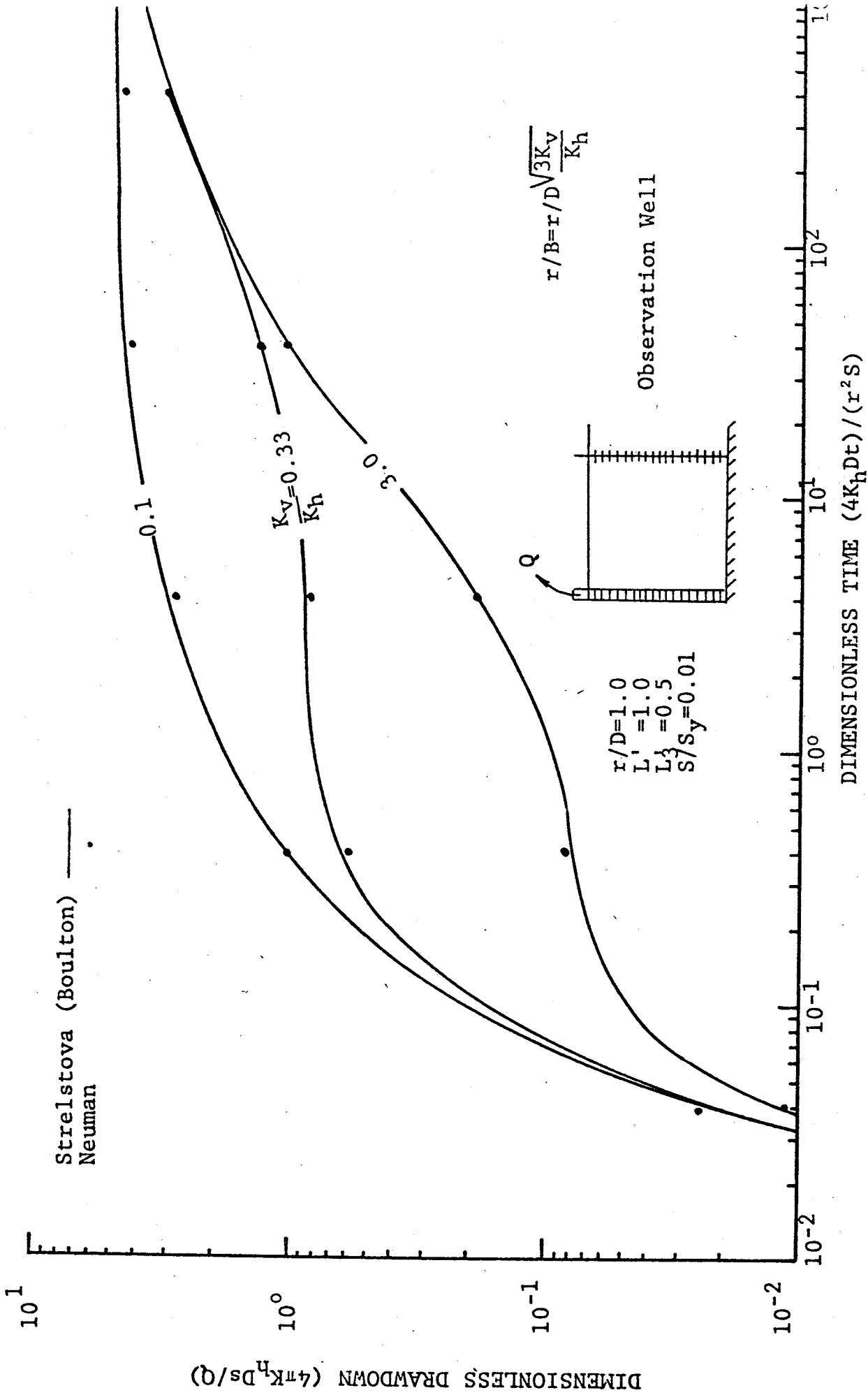


FIGURE 6. Comparison of Strelstova and Neuman Type Curves.

agreement between the two sets of curves adds validity to Streltsova's vertical transfer assumption. The relationship between the two type curves is $r/B = r/D \sqrt{3 K_v/K_h}$.

where: r = radius of observations
 B = $\sqrt{\alpha S_y/T}$
 D = aquifer thickness
 K_v = vertical hydraulic conductivity
 K_h = horizontal hydraulic conductivity
 α = $(3K_v)/(S_y D)$
 S_y = specific yield
 T = transmissivity

For comparison purposes Boulton's early time and late time type curves have been shifted one log cycle closer together, removing the flatter portion of the curves. The above relationship enables the use of Boulton's type curves for Neuman type curves for average head drawdown with a fully penetrating pumping well. To obtain Neuman type curves from those of Boulton the number of log cycles between the Theis early time curve and the Theis late time curve is $\log_{10} (S_y/S)$.

A potential flow pumping test, which considers the well storage instead of the normal infinitesimal well diameter was developed by Kipp (1973). This method is not useful for the normal pumping test, since the solution is only accurate for very small time values.

Delayed yield type flow equations were introduced by Boulton (1955) and functional type curves are given by Boulton (1963) and Prickett (1965). This is a radial flow

equation with a variable specific yield term. Boulton (1970) and Boulton and Pontin (1971) have added potential flow to the delayed yield equation.

Semi-unconfined Pumping Test Methods

Two recently developed aquifer test methods deal with semi-unconfined aquifers. This terminology is analogous to that used by Kruseman and DeRidder (1970) to describe aquifers overlain by a water table aquitard. Cooley (1971, 1972) presented a comparison between a numerical solution for a semi-unconfined aquifer and Boulton's (1963) analytical solution. Cooley and Case (1973) derived Boulton's integral for a semi-unconfined aquifer and present the analytical solutions for a compressible aquitard. Cooley and Case conclude that the unsaturated zone has very little effect on the drawdown in the aquifer. Type curves are available for a compressible aquitard for early time (Hantush, 1960) and for a noncompressible aquitard for late time (Boulton, 1963).

A solution similar to Cooley and Case's was presented by Boulton (1973) for the semi-unconfined pumping test. Boulton considered flow from the unsaturated zone in an approximate form. The partial differential equation describing the flow is the same as Boulton's (1963). Therefore, the same type curves would be used for the semi-unconfined aquifer test using either Boulton (1973) or Cooley and Case (1973).

Vertical Anisotropy Determinations

Several methods for determining the ratio of the vertical

to horizontal hydraulic conductivities have been developed. Boulton (1954), Stallman (1963), 1965), Dagan (1967a), Boulton and Pontin (1971), and Neuman (1972, 1974) all determine the hydraulic conductivity anisotropy by matching the field time-drawdown data to the best fit of a family of type curves for an observation well in an anisotropic aquifer varying K_v/K_h . Boulton and Pontin state that the anisotropic ratio can be determined only between wide limits with their method. For many aquifer tests these methods are limited, since the field data can be fitted to several different K_v/K_h type curves.

Two methods to determine anisotropy were described by Weeks (1964). One method is based on using an array of piezometers based on the finite difference form of the potential flow equation given by Stallman (1963). Weeks' results using this method were unsatisfactory. The method is of doubtful utility, since the difference in drawdown in the piezometers at distances to satisfy finite difference would be very small. The second method is based on Hantush's well equation for an anisotropic confined aquifer with partially penetrating pumping well. The analysis of the anisotropic ratio from the second method is the same as Dagan's (1967a) or Neuman's (1972, 1974). For unconfined aquifers Dagan or Neuman type curves are better suited than Weeks', since Weeks' method is based on a confined aquifer equation.

A method to determine K_v/K_h by comparing drawdown in piezometers to drawdowns from an electrolytic analog, which

simulated a steady-state confined aquifer system, was developed by Mansur and Dietrich (1965). This method is of questionable use for unconfined aquifers, since the analog is based on a steady-state confined aquifer model.

Weeks (1969) presented another method to determine anisotropy using Hantush's partially penetrating well equation for confined aquifers. Transformation of the anisotropic aquifer to an equivalent isotropic aquifer is used to determine the anisotropy. However, this method is applicable only to partially penetrating pumping wells. In addition, Weeks gives criteria for application of the method to unconfined aquifers. The application of this criteria using pumping wells, which partially penetrate the unconfined aquifer, is questionable.

METHODOLOGY

When K_v/K_h is an unknown, aquifer tests analyses consist of matching a family of type curves. When K_v/K_h is known, the family of type curves for the majority of aquifer test theories reduces to only one type curve, eliminating some judgement. The method presented here is a means of determining K_v/K_h before the type curve matching procedure is attempted. The method is based on taking the ratio of the drawdowns in two observation wells and plotting these ratios versus time on semi-log paper. The ratio of the drawdowns is then matched to a family of ratio type curves where K_v/K_h is the variable to determine the vertical anisotropy. Data

necessary to develop the ratio type curves and to determine K_v/K_h are aquifer thickness, penetration of pumping well, radius of observation wells, and depth of piezometers or penetration of observation wells. Therefore, K_v/K_h is independent of the discharge of the pumping well. The shape of the ratio curves varies depending on the relative position of the two observation wells, but tends to approach a constant as time increases and decreases. For any two observation wells the asymptotic constants are different for different K_v/K_h ratios for partially penetrating pumping wells. In some cases the early asymptotic constant can be used to estimate K_v/K_h for pumping tests. The late time asymptotic constant is not very useful in estimating K_v/K_h since the late time constant is approached slowly.

Type curves from Boulton (1954), Stallman (1963, 1965), Dagan (1967a), Boulton and Pontin (1971), or Neuman (1972, 1974) can be used in the ratio curve method. Type curves for aquifer tests which consider potential flow normally have to be constructed for each individual field situation. Therefore, it is necessary to have some convenient means of developing type curves. Dagan and Neuman methods are best suited for development of the type curves since computer programs are available for these two methods. Dagan's program (programmed by Uri Krosynski) was used in testing the ratio curve method. It is given in the Appendix with the addition of a plot subroutine.

The advantage of using the ratio curve method over the

normal type curve matching method is that the ratio curve method eliminates some normal type curves which could be matched. This will be demonstrated in the examples.

For some aquifer tests the time-drawdown data can be matched uniquely to one normal type curve. The ratio curve method is of little use with pumping tests of this type.

Steps in Analysis

1) Construct type curves for each observation well, varying K_v/K_h . To construct type curves using Dagan's program for two observation wells the following data cards are needed:

Read statements -

DAGA0820--(NCASE) Number of cases or number of type curves or number of K_v/K_h values for each observation well, (NT) the number of time values for each type curve and (NUCK) code for type of plot desired.

DAGA0830--(T) Time values used in determining the type curve.

DAGA0960--(XL3,XL) Dimensionless pumping well penetration, (R) dimensionless radius of observation well equal to $(r/D)(K_v/K_h)^{1/2}$ and (Z) dimensionless piezometer position. Do not use the following read statement if the plot subroutine is not needed.

DAGA3520--(UUM1, UUM2, VVM1, VVM2, UUU1, UUU2, VVV1, VVV2, UL, UI, VL, VI, UUL, UUI) Scaling factors for plotting graphs. A set of data cards for each observation well is needed for read statement DAGA0960 and DAGA3520. After the computation of the type curve data, the plot subroutine plots the two families of type curves on separate graphs.

2) Develop the ratio type curves by computing SD_1/SD_2 for each time value for the corresponding type curves with the same K_v/K_h value of the two observation wells.

where: SD_1 = dimensionless drawdown with most influenced observation well.

SD_2 = dimensionless drawdown with least influenced observation well.

In using Dagan's program, the SD_1/SD_2 ratio are computed in the plot subroutine; then the ratio type curves are plotted.

3) Construct time-drawdown curves for the field data from each observation well. Then compute s_1/s_2 values for the same time values to obtain ratio curve data.

where: s_1 = the drawdown values for the most influenced observation well.

s_2 = the drawdown values for the least influenced observation well.

Plot s_1/s_2 versus time on semi-log graph paper with the same vertical scale as the ratio type curve plots. The plot subroutine can be modified slightly to handle step 3 as a separate program.

4) Match the ratio curve plot of the field data from the observation wells to the family of ratio type curves to determine the K_v/K_h value. Since the abscissa of the two curves must be kept together, all possible matches can be seen from sliding one graph over the other horizontally. Note the time match point and the K_v/K_h value.

5) Match the time-drawdown plot for each observation well to its particular normal type curve with the K_v/K_h value and time match determined in step 4. From a match point determine

K_h from $SD = 4\pi K_h Ds/Q$ and determine S_y from $TD = (K_v t)/(S_y D)$.
The K_v value can then be determined from the K_v/K_h value determined in step 4 after K_h has been determined.

where: SD = dimensionless drawdown from the normal
type curve.

TD = dimensionless time from the normal type
curve.

t = time since pumping started.

PUMPING TEST EXAMPLES

Four analyses of pumping tests in vertically anisotropic unconfined aquifers are given below. The use of the ratio curve method to determine the anisotropy of the aquifers is demonstrated. The basic well data for each of the pumping tests is shown in Table 2.

Hancock Test

A pumping test in an unconfined aquifer near Hancock, Wisconsin was conducted by Weeks (1969). The wells for this test are developed in a glacial outwash. Sand points two feet long were used as observation wells during the test. In order to construct type curves, these sand points were considered to be piezometers positioned at their center. The following analysis of the Hancock data is not presented in the steps outlined in the Methodology section but presented to show the importance of knowing K_v/K_h before the normal type curve pump test analysis.

The Hancock drawdown data matches Dagan's type curves only after approximately 30 minutes of pumping. This disparity allows for a match of only the last third of the drawdown data curve with Dagan's type curve. Type curves with any K_v/K_h ratio can be equally matched with the Hancock data. This situation is typical when the early drawdown data does not match the type curve. Type curve matches are shown for wells 100 and 200 in Figures 7 and 8 for three different K_v/K_h ratios of 1.0, 0.25 and 0.01. The real time and real drawdown position of the matched field data for each K_v/K_h ratio

TABLE 2. BASIC WELL DATA FOR PUMPING TEST EXAMPLES.

NAME OF TEST	PUMPING WELL				OBSERVATION WELL				AQUIFER	
	Q	P	L'	L' ₃	tp	r	r/D	Z	Z'	D
HANCOCK	820	66-106	0.333	0.717	4765	100	0.833	11.0	-0.092	120
well 100						200	1.67	11.0	-0.092	
well 200										
MOSINEE	350	28-38	0.122	0.399	2890	20	0.244	33.4	-0.408	82
well 6						20	0.244	9.2	-0.113	
well 7										
GRAND ISLAND	540	9-37	0.281	0.231	2850	25	0.249	17.1	-0.171	100
well 1						185	1.85	4.7	-0.047	
well 16										
NEVIUS	2000	Variable			4650	20	0.538	20.6	-0.548	37.5
well 1						41	1.087	36.4	-0.969	
well 2						80	2.134	21.3	-0.567	
well 5						159	4.247	21.6	-0.576	
well 6										

Q = Well discharge.
P = Pumping well penetration.
L' and L'₃ = Dimensionless pumping well penetration.
tp = Length of pumping test.
r = Radius of observation well.
D = Aquifer thickness.
Z = Piezometer depth below top of aquifer.
Z' = Dimensionless vertical position of piezometer.

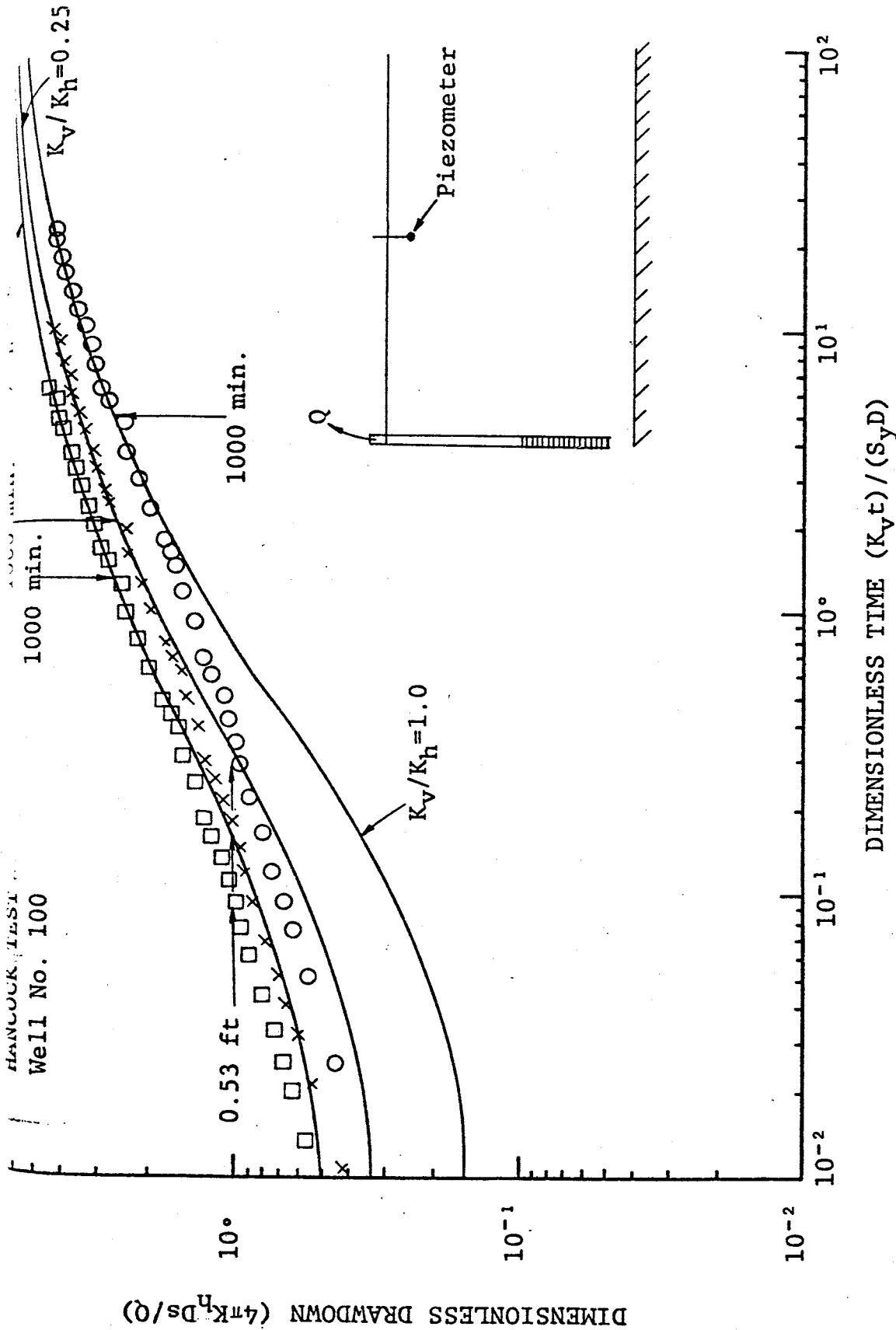


FIGURE 7. Dagan type curves and time-drawdown data for Hancock well 100.

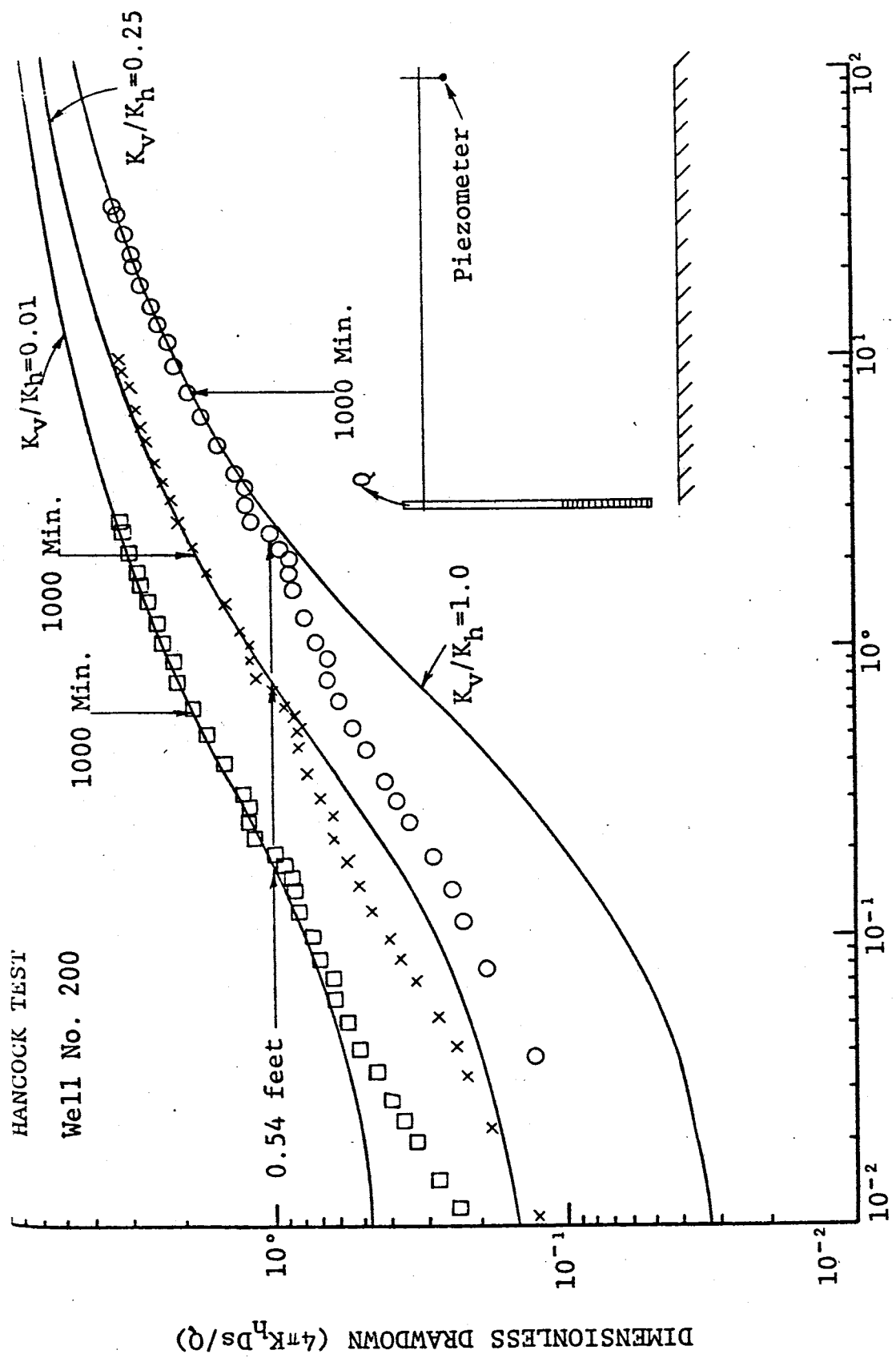


FIGURE 8. DAGAN TYPE CURVES AND TIME-DRAWDOWN DATA FOR HANCOCK WELL 200.

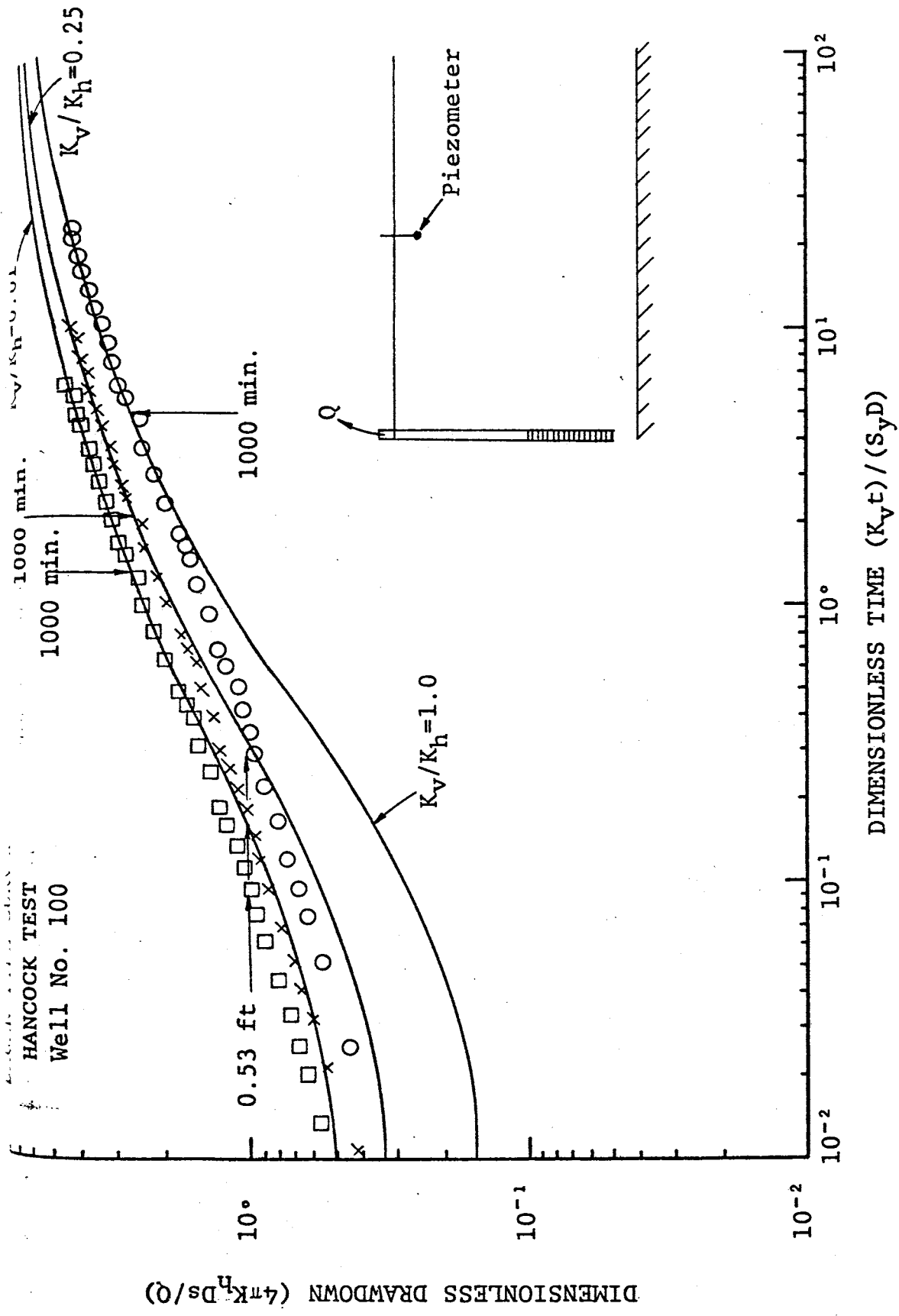


FIGURE 7. Dagan type curves and time-drawdown data for Hancock well 100.

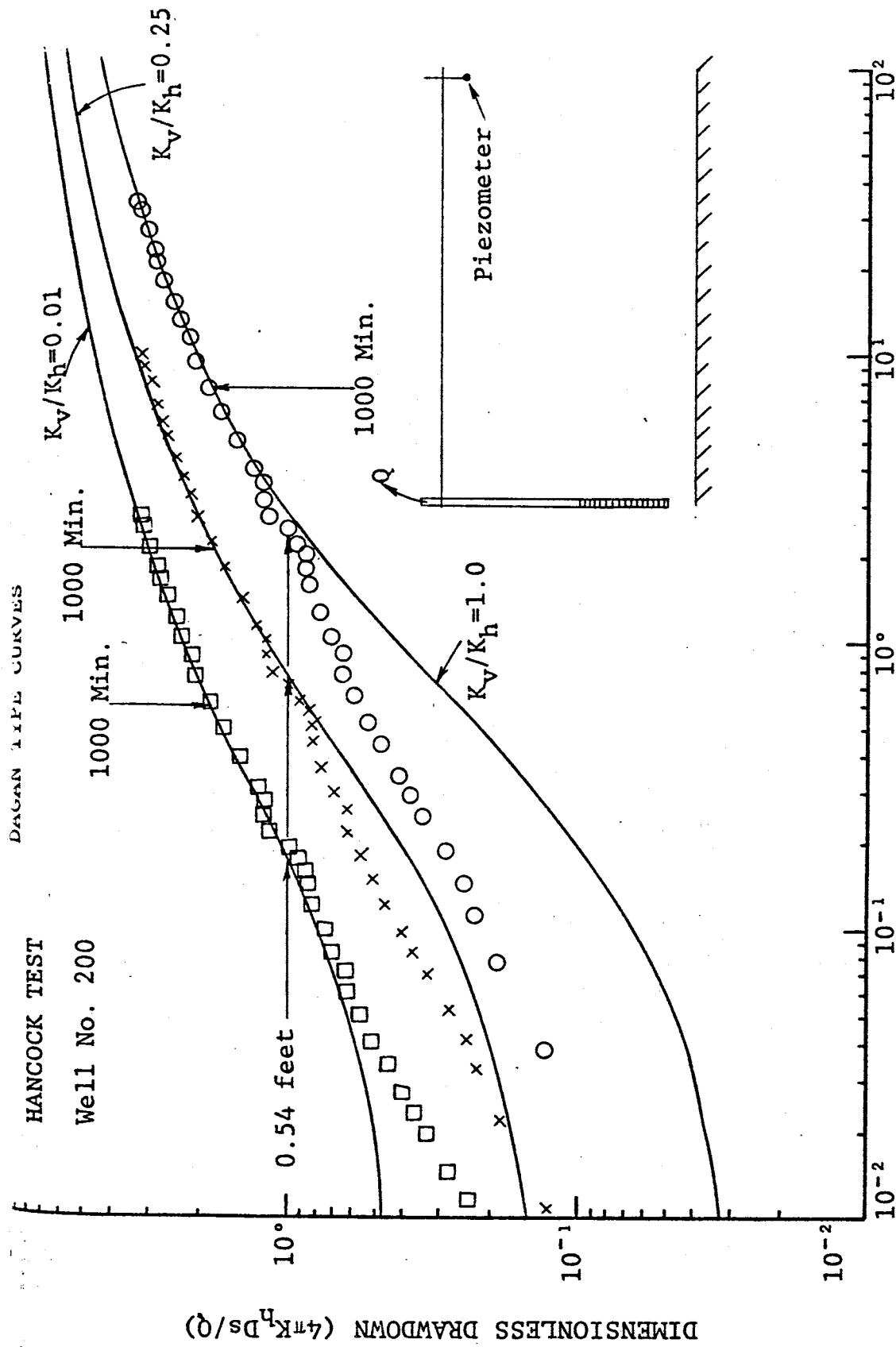


FIGURE 8. DAGAN TYPE CURVES AND TIME-DRAWDOWN DATA FOR HANCOCK WELL 200.

relative to the dimensionless time scales are indicated on the two figures. Since all three of Dagan's type curves can be matched equally well with the Hancock and similar field data, no particular value of K_v/K_h can be defined, prohibiting calculation of true K_h , K_v , and S_y values. The K_h , K_v , and S_y values for wells 100 and 200 for the three selected K_v/K_h ratios of 1.0, 0.25 and 0.01 are shown in Table 3.

TABLE 3. Pumping test results for Hancock test.

K_v/K_h	WELL 100			WELL 200		
	K_h	K_v	S_y	K_h	K_v	S_y
1.0	198	198	0.22	194	194	0.15
0.25	198	49	0.13	194	48	0.13
0.01	198	2	0.0085	194	2	0.018

A wide range in the K_v and S_y values are shown for the three different K_v/K_h ratios. These results show the importance of determining the true K_v/K_h ratio value, as K_h , K_v and S_y values will otherwise be doubtful. In Figure 7, the horizontal shift in the drawdown data for well 100 between K_v/K_h values of 1.0 and 0.01 is approximately 0.6 of a log cycle while the corresponding shift for well 200 is approximately 1.1 log cycles. The horizontal shift can be observed from the locations of the real times that are given for the drawdown data. Assuming that the correct match of the field data is within this range of K_v/K_h ratios, data from well 200 must fit within the same dimensionless time range as data from well 100 since the only variable in the dimensionless time for a particular

homogeneous aquifer is t . Therefore the type curve fit to define K_v/K_h must be within the smaller range of horizontal shift of well 100. A trial and error procedure could be used to find a match for both wells where real times are in the same position relative to the dimensionless scale, but the ratio curve method was found to expedite matters.

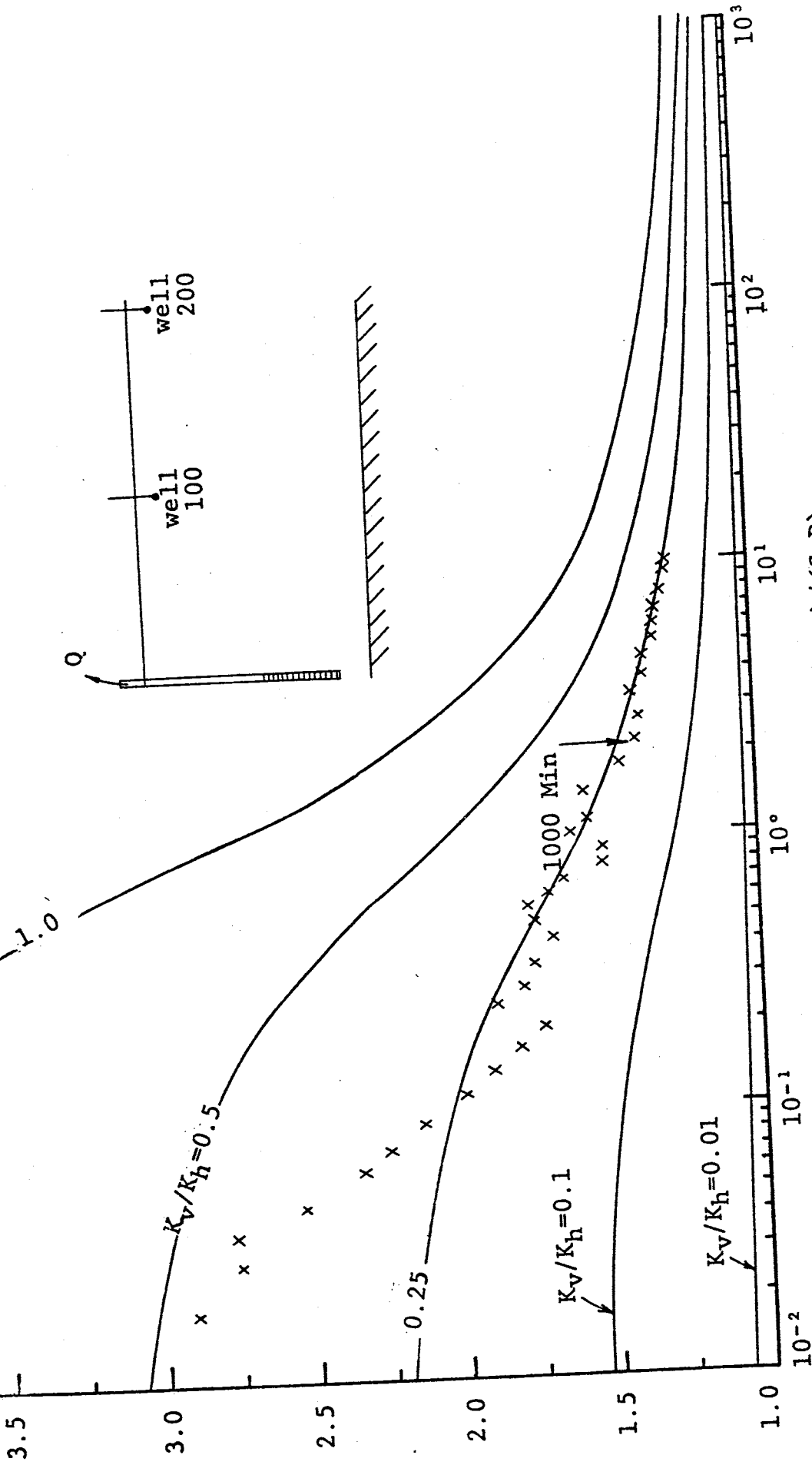
The ratio curve method was used to define the K_v/K_h ratio. Figure 9 indicates a definite match of the ratio of draw-down data from wells 100 and 200 to the ratio type curve of K_v/K_h equal 0.25. Only the last third of the ratio draw-down data can be expected to match the ratio type curve since only this portion matched the Dagan type curves. The value of K_v/K_h equal 0.25 agrees well with Weeks' (1969) results of an average K_h/K_v value of 4.4.

Mosinee Test

The Mosinee pumping test was conducted with a small partially penetrating pumping well. Wells for this test are developed near the Wisconsin River in either glacial outwash or alluvium. Sand points used in the Mosinee test were assumed to be 18 inches long and used as piezometers at their centers for type curve construction. Data from observation wells 6 and 7 were used to demonstrate the use of the ratio curve method with two observation wells at the same radius. The steps outlined in the Methodology section are followed with the Mosinee test.

Ratio type curves were constructed for K_v/K_h ratios of 1.0, 0.5, 0.2, 0.1, 0.05, 0.02, and 0.01. The match of the

HANCOCK TEST
RATIO TYPE CURVES



DIMENSIONLESS TIME $(K_v t) / (S_y D)$

FIGURE 9. Ratio type curves for Hancock wells 100 and 200.

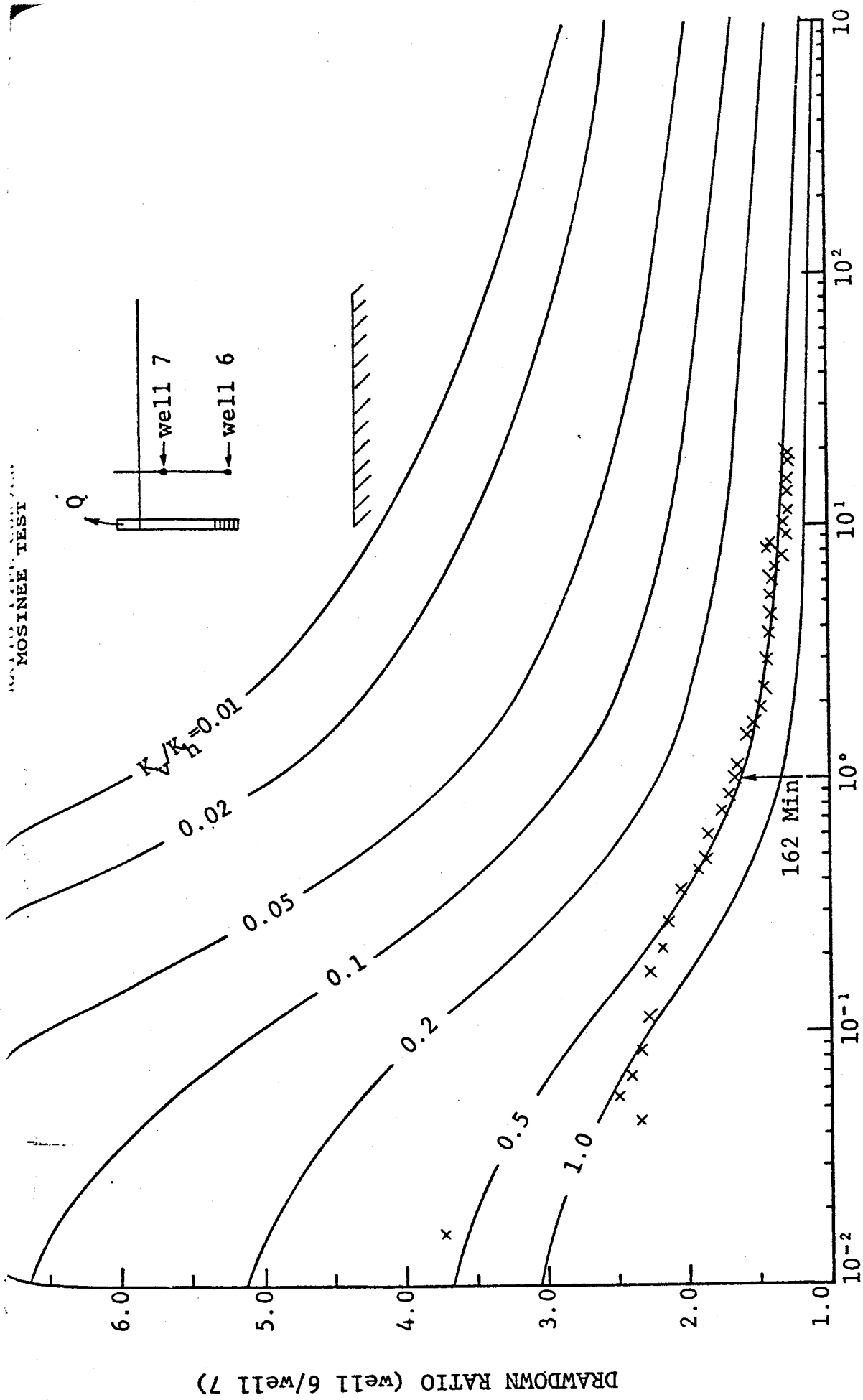
ratio of the drawdown data to the ratio type curves is shown in Figure 10. Since the ordinate of the data and type curves must match, the only two possible ratio type curves that could be matched to the ratio drawdowns were K_v/K_h values of 1.0 and 0.5. Only the last few points of the ratio drawdown data could be matched to the ratio type curve with a value of 1.0. Therefore, the K_v/K_h ratio of 0.5 is chosen as the anisotropic ratio for the Mosinee test. The time match for dimensionless time of 1.0 was 162 minutes for the ratio curve match.

The time-drawdown data for wells 6 and 7 were then matched to the Dagan type curves with a K_v/K_h value of 0.5 and the 162 minute time value. Figure 11 contains these results and shows that the match points of the two drawdown curves are the same for the two wells. As previously indicated, the match point should be the same for a homogeneous aquifer when using the $(K_v t)/(S_y D)$ dimensionless time scale. The K_h , K_v and S_y values for the Mosinee test calculated from the match in Figure 11 are, respectively, 297 feet/day, 149 feet/day and 0.20.

Weeks (1969) obtained the same K_v/K_h value for the Mosinee test using his method which utilizes Hantush's partially penetrating well equation. The close agreement between our analyses of the Mosinee and Hancock tests with Weeks' (1969) analyses gives strength to his method for long term unconfined pumping test.

Grand Island Test

Analysis of the Grand Island pumping test conducted by Wenzel (1937, 1942) is presented below. Wells for this test

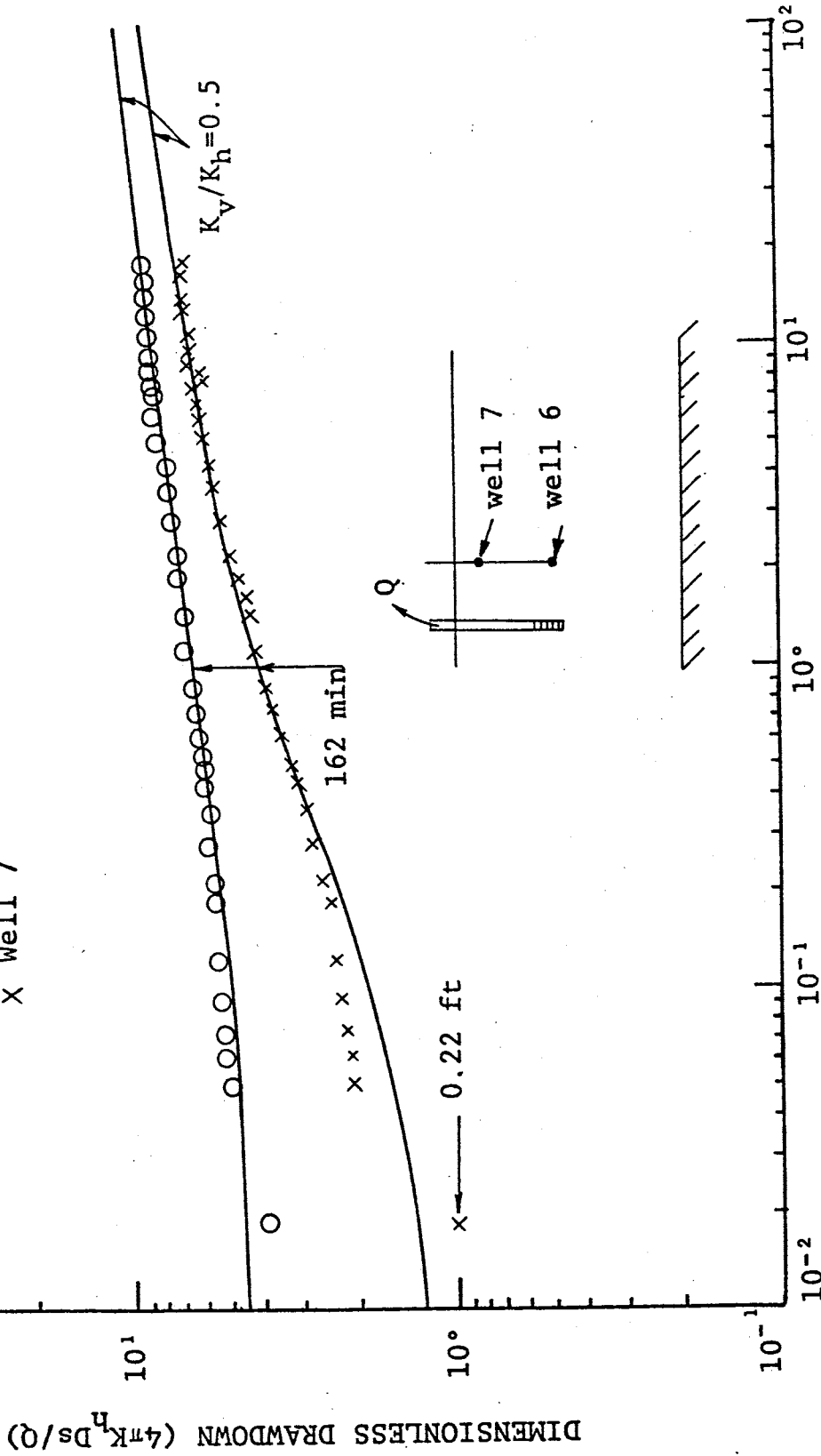


DIMENSIONLESS TIME $(K_v t) / (S_y D)$

FIGURE 10. Ratio type curves for Mosinee wells 6 and 7.

MOSINEE TEST

- Well 6
- × Well 7



DIMENSIONLESS TIME $(K_v t) / (S_y D)$

FIGURE 11. Dagan type curves and time-drawdown data for Mosinee wells 6 and 7.

are developed near the Platte River with logs indicating significant stratification. The two observation wells, 1 and 16, used in the analysis were constructed as unscreened, open-ended piezometers.

The ratio curve match defined a K_v/K_h of 0.03 for the Grand Island test as indicated in Figure 12. The ratio curve of the drawdown data for wells 1 and 16 fitted the ratio type curve fairly well. The match of the drawdown data to the normal type curves for the time value and K_v/K_h value determined in the ratio curve match is shown in Figure 13. The time-drawdown data is a poor match, while the ratio curve match is good for the same match point. The poor match of time-drawdown data makes the definition of the K_v/K_h value questionable. Aquifer properties of 376 feet per day, 11 feet per day and 0.06 were determined respectively for the horizontal and vertical hydraulic conductivity and the specific yield.

An attempt was made to explain the disagreement of the aquifer model and the field data. An assumption was made, similar to Stallman's (1963), that the aquifer was only as deep as the pumping well. The results from this assumption did not satisfactorily explain the disagreement. A heterogeneous aquifer system could possibly explain some of the discrepancy between the field data and Dagan's type curves. Two observation wells in a heterogeneous system would not have the same match point with the dimensionless time scale, $(K_v t)/(S_y D)$, therefore, different matches would yield different aquifer properties. Dagan's (1967a) analysis of 14 observation wells for the Grand Island test indicates significant heterogeneity.

RATIO TYPE CURVES
GRAND ISLAND TEST

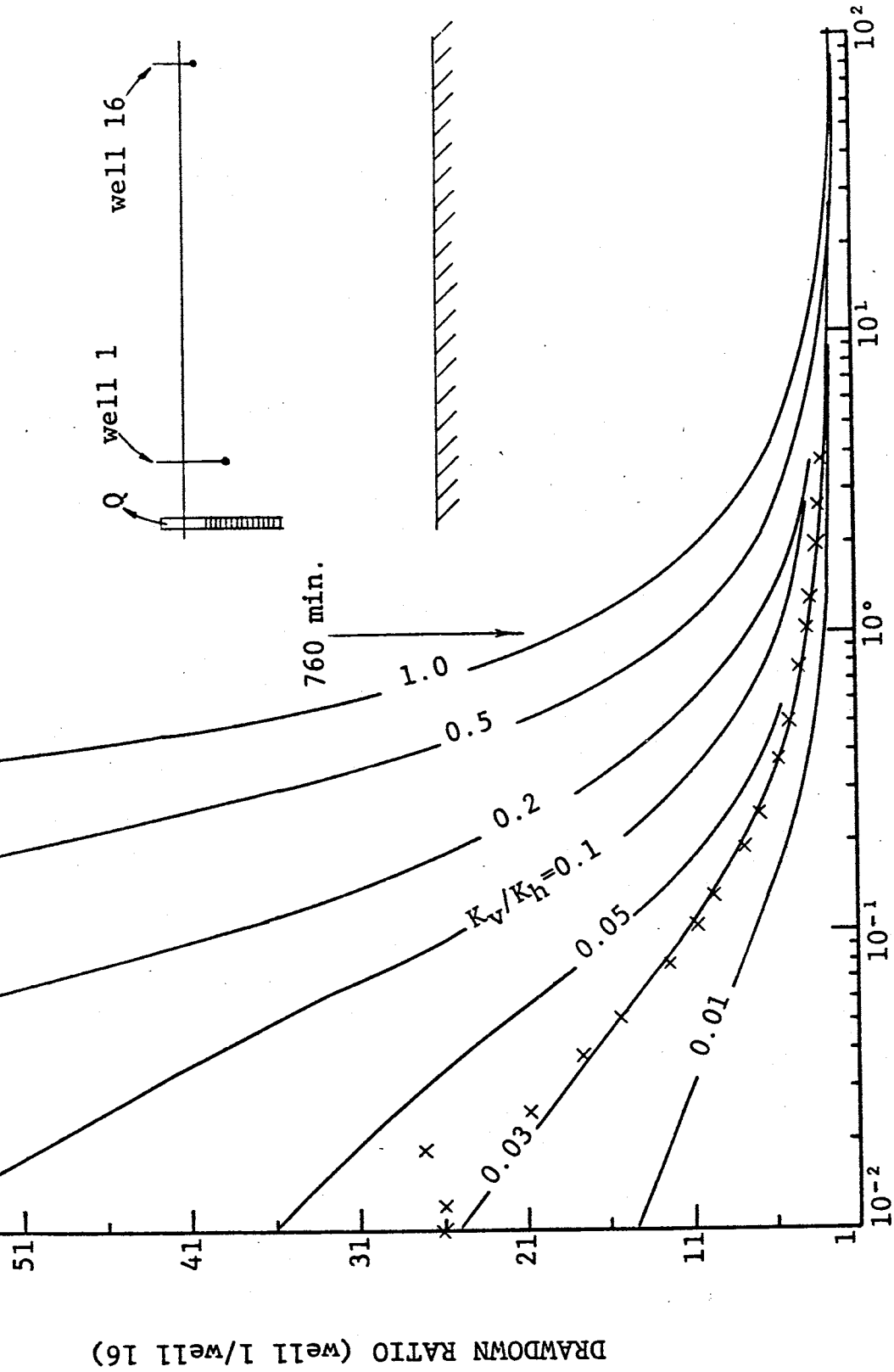
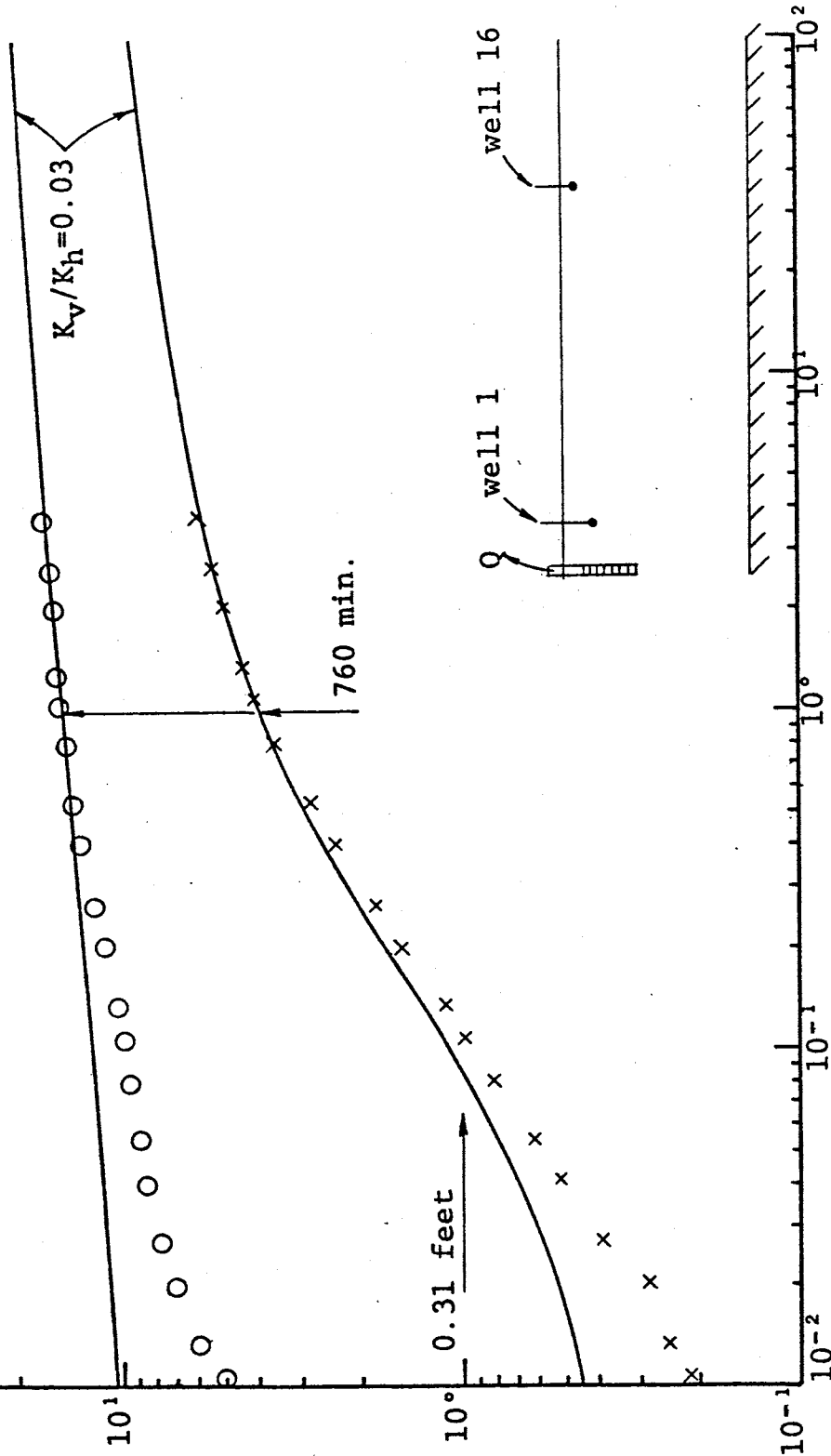


FIGURE 12. Ratio type curves for Grand Island wells 1 and 16.

Grand Island Test

- well No. 1
- x well No. 16

DIMENSIONLESS DRAWDOWN ($4\pi K_h D_s / Q$)



DIMENSIONLESS TIME ($K_v t$) / ($S_y D$)

FIGURE 13. Dagan type curves and time-drawdown data for Grand Island wells 1 and 16.

Nevius Test

The Nevius pumping test was conducted near Lamar, Colorado in the Arkansas River alluvium, which is an unconfined aquifer. Driller logs indicate that the wells used in this test have a fairly uniform material size. The test was conducted by personnel of the U. S. Geological Survey, Water Resources Division, Colorado District. The test has been analyzed by Stallman (1968) in his technique booklet on pumping test. The two foot sand points used in the test were approximated by piezometers at the sand point center for constructing Dagan's type curves.

Ratio curves for observation wells 5 and 6 of the Nevius test for values of 1.0, 0.5, 0.2, 0.1, 0.05, 0.02 and 0.01 were constructed. The initial match showed that the ratio drawdown data indicated an anisotropy of slightly greater than 0.02. The final match shown in Figure 14 results in an anisotropy of 0.025 for the Nevius test. A good approximation of K_v/K_h could have been obtained by comparing the ratio of the initial drawdown values to the early time ratio values for the ratio type curves. Aquifer properties of 928 and 23 feet per day for the respectively horizontal and vertical hydraulic conductivity were calculated from the Dagan type curve match with the time-drawdown data. The type curve matches, shown in Figure 15 for the time value determined from the ratio curve match also gives a specific yield value of 0.11 for the Nevius test.

Even though the pumping well in the Nevius test is fully perforated, a partially penetrating pumping well, which starts

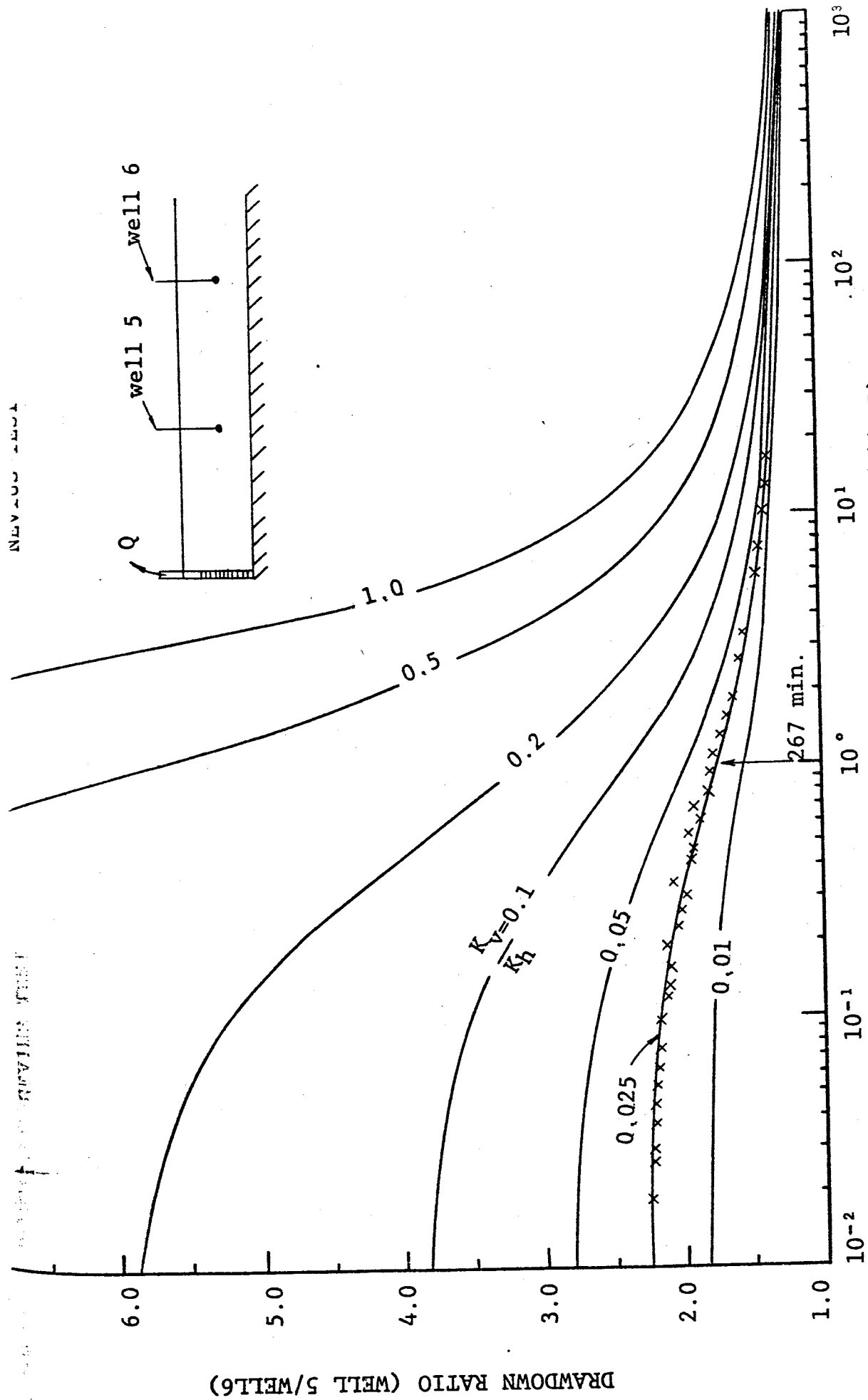
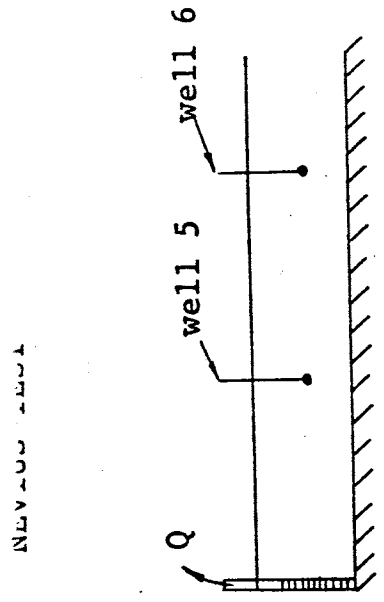


FIGURE 14. RATIO TYPE CURVES FOR NEVIUS WELLS 5 AND 6.

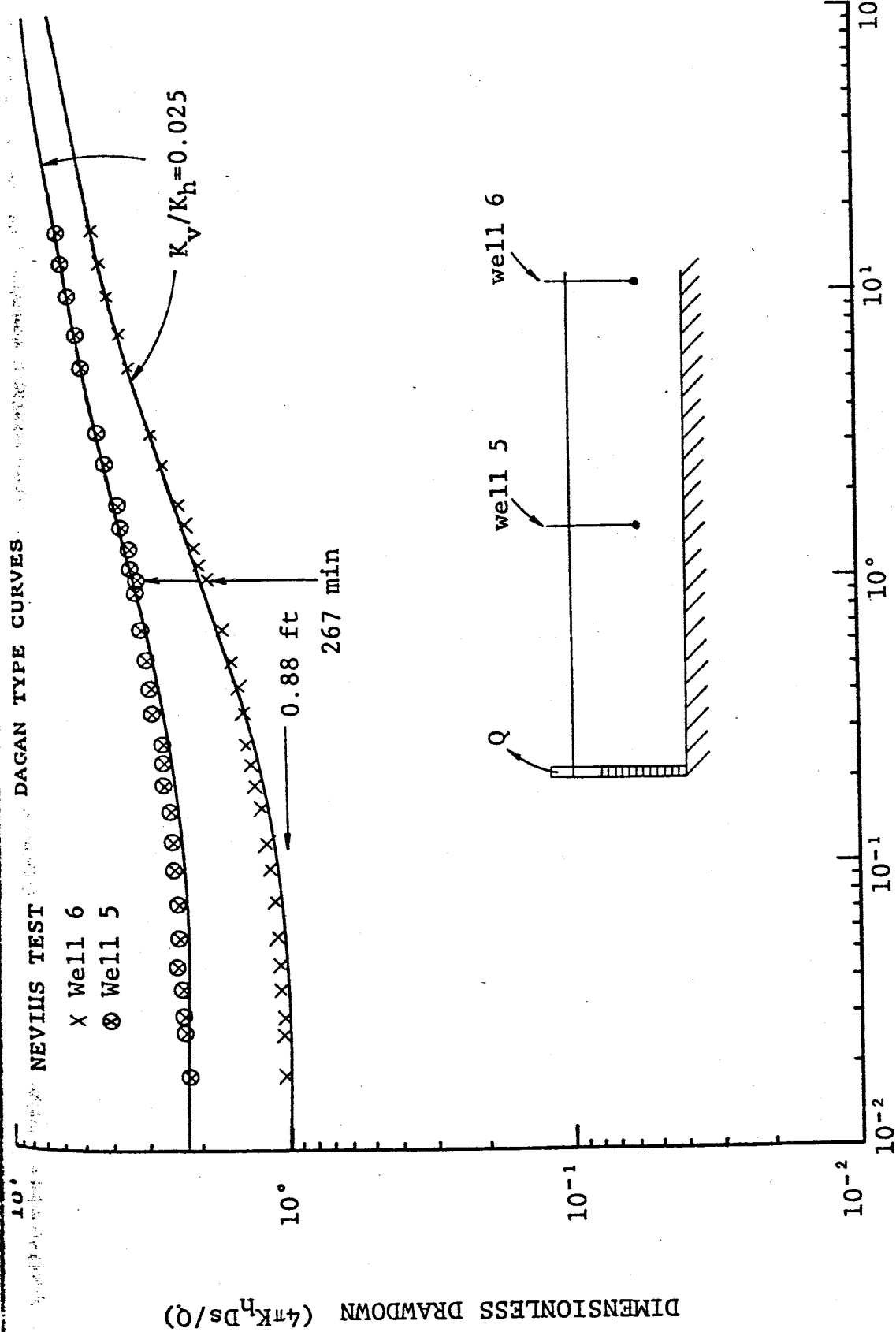
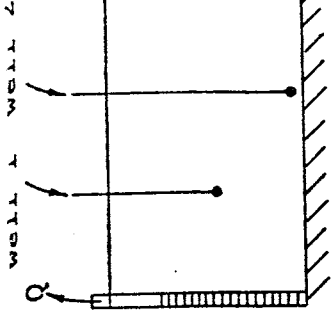


FIGURE 15. Dagan type curves and time-drawdown data for Nevius wells 5 and 6.

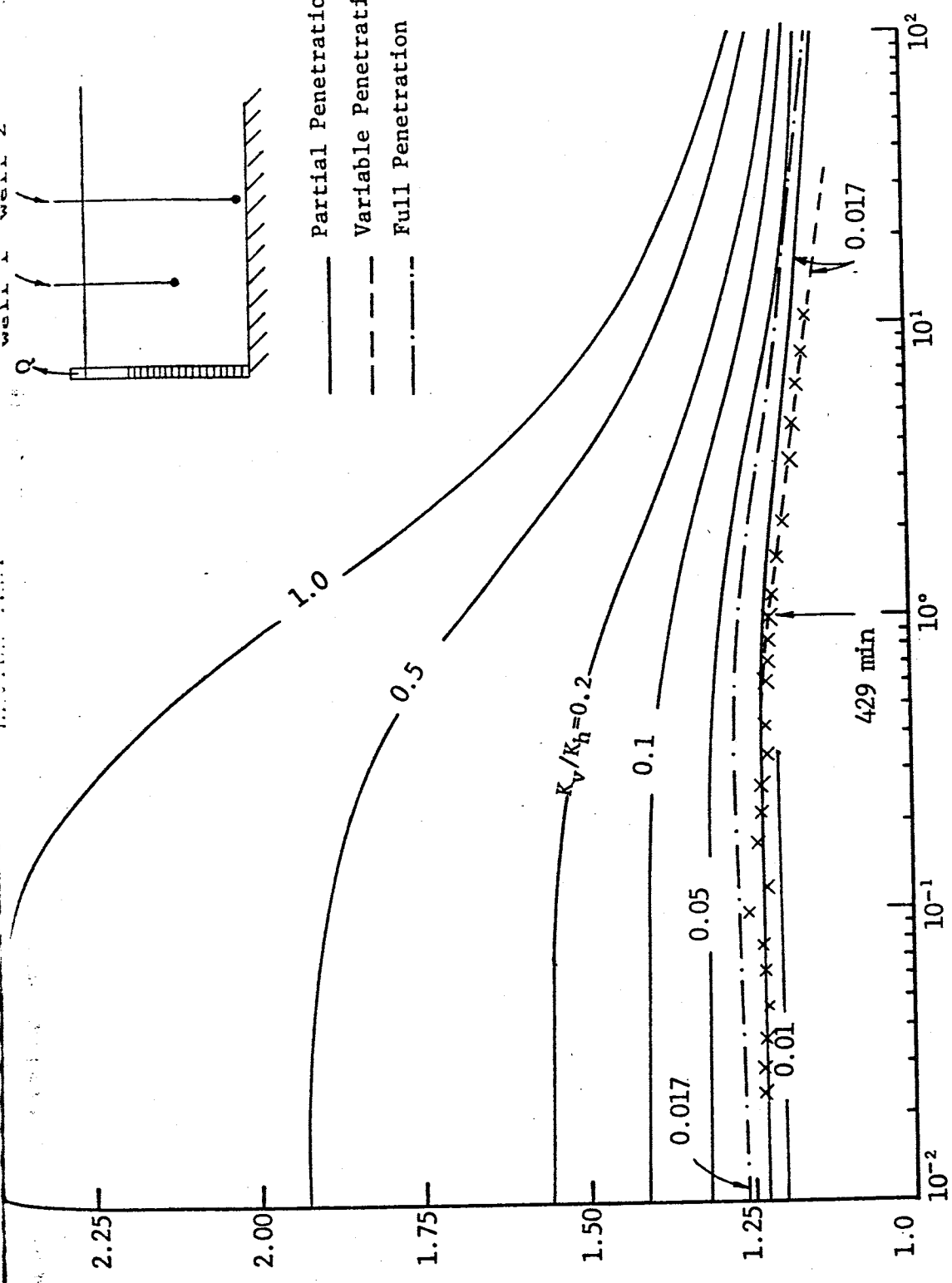
its penetration ten feet below the top of the aquifer was used to develop the Dagan type curves. The Nevius pumping well had 10 feet of drawdown after approximately 4 minutes of pumping. Therefore the pumping well in the Nevius test must react as a partially penetrating well instead of a fully penetrating well. Since unconfined aquifers will not yield water without some decline in the aquifer thickness near the pumping well, it is impossible to have a fully penetrating pumping well in unconfined aquifers.

The partial penetration of the pumping well for the construction of Dagan's type curves for observation wells 5 and 6 was approximated with a constant partial penetration. The ratio type curves for observation wells 1 and 2, shown in Figure 16, has curves developed three different ways. The solid line curves were developed for the Nevius pumping well with the penetration being below the upper ten feet of the aquifer. The dash and dot curve is for a fully penetrating pumping well. The dash curve was constructed with the penetration of the pumping well varying as the drawdown in the pumping well. This was done by first matching the ratio of the drawdowns to the solid lines to obtain a relationship between actual time and dimensionless time. After this relationship is determined, the penetration of the pumping well can be determined from the actual drawdowns in the pumping well. Each time the pumping well penetration was changed a new pumping well was started with the new pumping well penetration and a recharge well of the old penetration was started.



— Partial Penetration
 - - - Variable Penetration
 - · - · - Full Penetration

DRAWDOWN RATIO (WELL 1/WELL 2)



DIMENSIONLESS TIME $(K_v t) / (S_y D)$

FIGURE 16. Ratio type curves for Nevisius wells 1 and 2.

After the computations for all pumping well penetration were completed, the dimensionless drawdowns (well functions) were summed to form the type curve. The match of the ratios of the drawdowns for observation wells 1 and 2 to the ratio type curves (see Fig. 16) shows the importance of considering the variable penetration of the pumping well. An anisotropy of 0.017 was determined from the variable penetration ratio curve match. The match of the ratio type curves with a fully penetrating pumping well would have given a much smaller K_v/K_h than 0.017. The K_v/K_h would have been much closer to the value which Stallman (1968) determined for K_v/K_h of 0.0064 using a fully penetrating well. The Nevius pumping well had only 3.9 more feet of drawdown for 4650 minutes of pumping beyond the initial 10 feet of drawdown at 4 minutes. This small additional amount of drawdown caused a significant change in the position of the ratio type curve as time increased. Variable penetration was not used to construct type curves for observation wells 5 and 6, since they are far enough away from the pumping well not to be affected.

The time value and the K_v/K_h value from the ratio curve match was used to match the time-drawdown data to Dagan's type curves for observation wells 1 and 2. The two type curves were constructed with a variably penetrating pumping well. The match shown in Figure 17 results in values of 1020 feet per day, 17 feet per day and 0.14 respectively for horizontal and vertical hydraulic conductivity and specific yield. These values are different aquifer properties determined from observation wells 5 and 6. A slightly heterogeneous system could

x well 1
 o well 2

DAGAN TYPE CURVES

DIMENSIONLESS DRAWDOWN ($4\pi K_h D_s / Q$)

$K_v / K_h = 0.017$

429 Min.

well 1 well 2

0.80 feet

DIMENSIONLESS TIME ($K_v t / (S_y D)$)

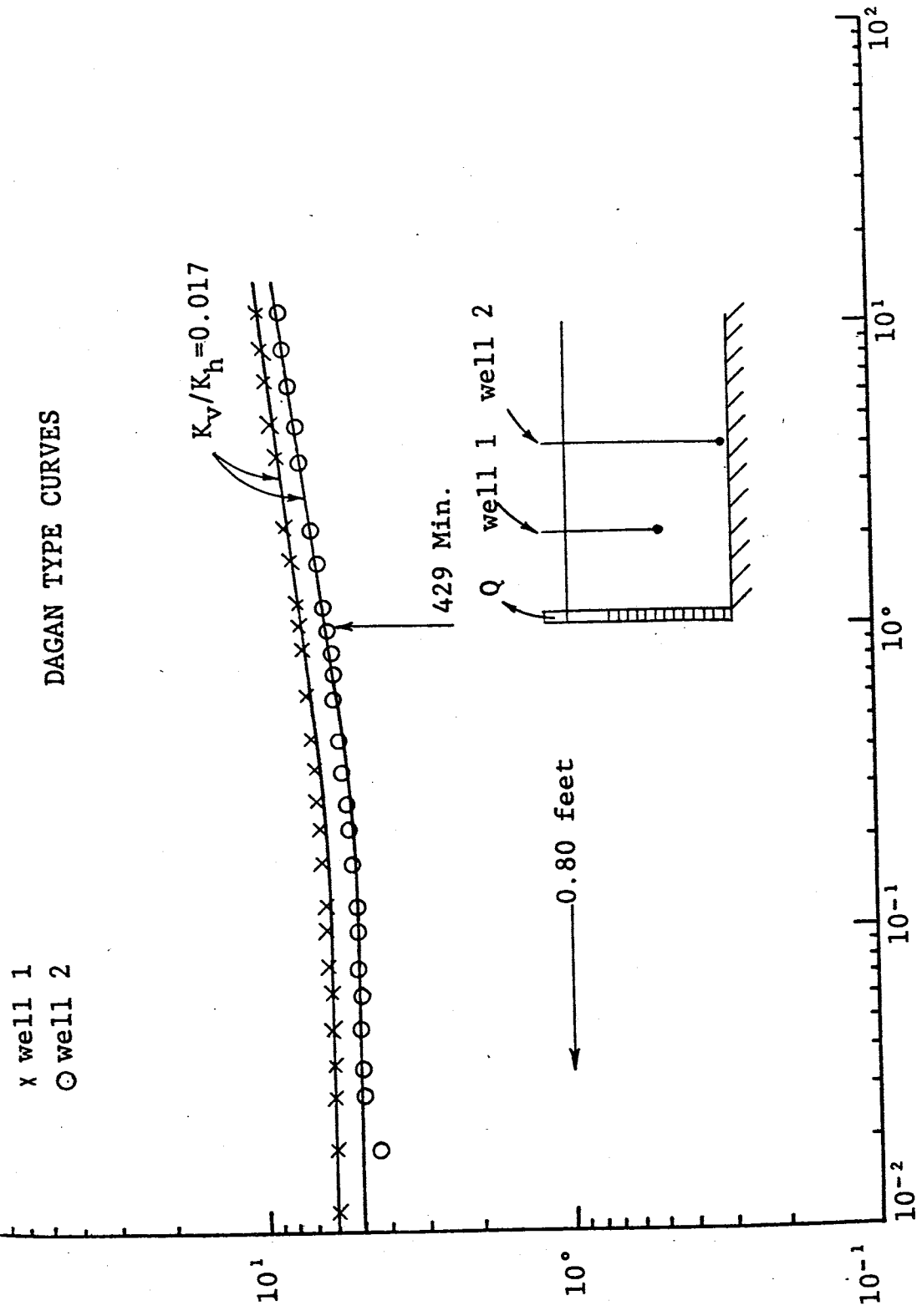


FIGURE 17. Dagan type curves and time-drawdown data for Nevius wells 1 and 2

explain the difference, since observation wells 5 and 6 are a considerable distance from observation wells 1 and 2.

The error that resulted in the Dagan type curves for observation wells 1 and 2 from treating the pumping well as fully penetrating or constant partially penetrating instead of variably penetrating is shown in Figure 18 for the Nevius test. This figure exemplifies the importance of properly representing the penetration of the pumping well in the development of the type curves. The variable penetration affect would be greater in many unconfined aquifers where more drawdown occurs in the pumping well.

A range in anisotropic ratio values of 0.017 to 0.25 has been determined for the four pumping tests analyzed. The test with the two smaller K_v/K_h ratios, Nevius and Grand Island, are from wells developed in alluvial valleys. Alluvium sediments tend to be very stratified and therefore, should normally have small K_v/K_h values. Even though the Nevius results indicate that the Arkansas alluvium is very stratified in this area, indication of this was not present from the driller's log.

RECOMMENDATIONS FOR PUMPING TEST

This section of the report was prepared as an aid to understanding certain points of interest about unconfined pumping tests. Hopefully it increases the chances for a successful pumping test. Stallman's (1968) and Kruseman and DeRidders' (1970) reports should also be consulted for pumping test preparation.

A discussion of the type of flow model to use, the required length of pumping test, suggestion for handling the

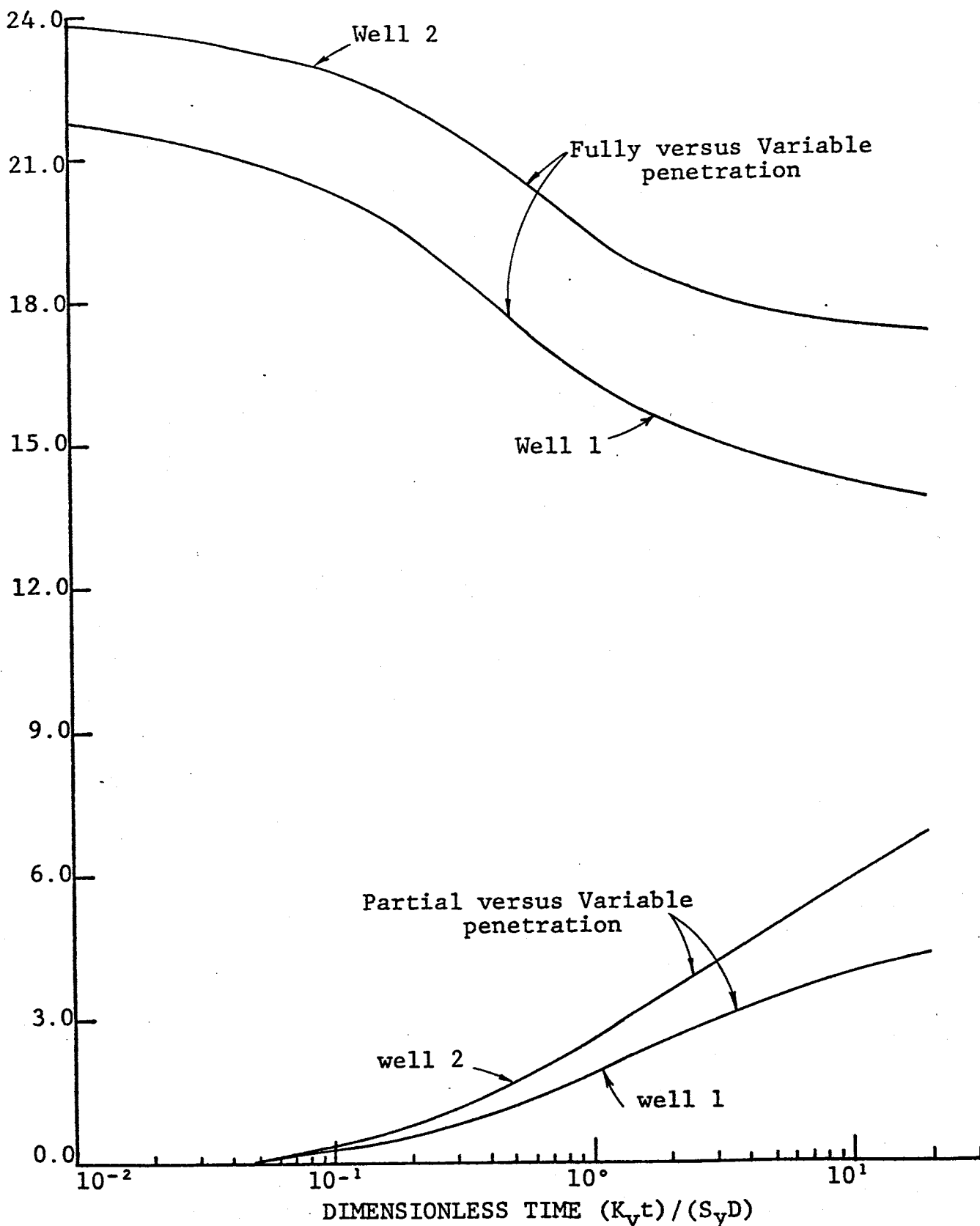


FIGURE 18. Percent error in drawdown for Nevis type curve construction with fully and partially penetrating instead of variably penetrating pumping wells.

aquifer thinning, which occurs in unconfined aquifer tests and the significance of the storage coefficient follows. Recommendations for positioning observation wells are presented next with several graphs showing percent error for incorrectly representing the position of an observation well in the type curve construction. Graphs are also shown for percent errors, which occur from representing finite length observation wells as piezometers at their center in the type curve construction. Finally, a discussion with graphs is given to demonstrate the importance of representing fully perforated pumping wells as partially or variably penetrating pumping wells.

Suggested Flow Model

Aquifer parameters are determined for many different purposes. For some purposes only transmissivity and specific yield are needed such as regional horizontal flow models and long term predictions of drawdown around a well field. The vertical conductivity is also needed for some purposes such as vertical circulation models and infiltration studies.

We recommend that a vertical flow model such as Dagan (1967a) or Neuman (1974), be used to analyze unconfined aquifer tests. Even though the vertical hydraulic conductivity is not needed for some purposes, it is needed for correct analysis of most unconfined aquifer tests. Different K_v/K_h type curves for most partially penetrating pumping wells do not merge to one type curve as time increases (see Stallman 1965). The pumping well in an unconfined aquifer must be partially or variably penetrating. The anisotropic ratio must be considered

for correct results from these pumping tests if K_v/K_h type curves do not merge.

Type curves for each observation well from the pumping test examples were compared to corresponding type curves with different K_v/K_h values with a dimensionless time scale of $(K_h Dt)/(S_y r^2)$. Type curves for observation well 6 from the Nevius test were the only curves which completely merged with each other using this dimensionless time scale. These type curves merged after approximately one-half day of pumping for observation well 6. Therefore, after one-half day of pumping observation well 6 could have been used to determine transmissivity and specific yield without considering K_v/K_h . Small errors would have occurred using observation well 5 from the Nevius pumping test to determine transmissivity and specific yield without considering K_v/K_h , while analyses using the remainder of the observation wells of all pumping tests would have introduced significant errors in the aquifer properties. The significance of considering K_v/K_h increases as the observation well's radius decreases and the aquifer's $K_v K_h$ value decreases. Factors affecting the merging of different K_v/K_h type curves include aquifer, observation well and pumping well properties. Therefore, it is very difficult to determine when and if K_v/K_h does not have to be considered to accurately evaluate aquifer properties from pumping tests. Vertical flow models are needed to correctly evaluate most unconfined aquifer tests.

Length of Pumping Test

The length of pumping test is often governed by factors other than the length of time required to define the time-drawdown curve sufficiently. The cost of running the pumping test in many cases is the controlling factor in establishing the length of pumping tests. When all other controlling factors are secondary to the definition of the time-drawdown curve, pumping tests should be run until the time-drawdown curve is satisfactorily defined. Type curves for the particular pumping test will have to be constructed before the test is conducted to be able to compare the time-drawdown data to the type curve.

The Hancock pumping test was conducted for 3.3 days, which was a minimum definition of the time-drawdown curve for this pumping test. Normally greater than 3 days of pumping would have defined the time-drawdown curve sufficiently but since the early drawdown data did not fit the type curve a greater pumping time was needed. The Mosinee and Grand Island pumping tests were run for approximately 2 days. The two day pumping test sufficiently defined the time-drawdown curve for the Mosinee test while it was not satisfactory for the Grand Island test. The poor type curve fit for the Grand Island test probably could have been improved with 2 or 3 more days of pumping. A good type curve fit was obtained with the Nevius pumping test data for a 3.2 day pumping test. This pumping test could have been stopped a day earlier without affecting the results.

Pumping tests should be conducted long enough to enable a

good match between the time-drawdown data and the type curves. A minimum of 2 days should probably be set for the length of pumping tests to prevent early misinterpretations.

Aquifer Thinning

Dagan (1967a), Neuman (1974), Boulton (1954), Streltsova (1972), and Stallman (1963) solutions do not consider the thinning of the aquifer which occurs in pumping unconfined aquifers. Stallman (page 311, 1965) suggests that Jacob's correction, which handles the thinning of the aquifer for the radial pumping test method, should be used for approximating the thinning of the aquifer in anisotropic pumping test methods. Jacob's correction is for thinning of the aquifer for the radial flow model and is not necessarily suited for the vertical flow models. Since the thinning of the aquifer in unconfined pumping tests is so important, we agree with Stallman and suggest that Jacob's correction be used on the vertical flow models such as Dagan. Jacob's correction was applied only to the Nevius pumping test, since it was not significant in the other three tests.

It can be shown that the specific yield term through linearizing the Dupuit flow equation should have a correction term on it also for the thinning of the aquifer. Stallman (1968) suggested that a geometric mean drawdown be used in the correction term, $(\frac{D}{D-s})S_y$, for the specific yield.

Where: D = aquifer thickness
s = drawdown which is approximated by geometric mean
S_y = specific yield

As stated in the Previous Work section, the Jacob's correction, which corrects only the drawdown term and not the specific yield, is very close to the true thinning equation, Kriz's equation. Therefore, we suggest that only the Jacob's correction be used to approximate the thinning affect in vertical flow models for pumping tests.

Significance of Compressibility

Neuman (1972) demonstrated with his paper that unconfined aquifers can react initially as confined aquifers and then merge with the other potential flow equations. Adding the storage coefficient to potential flow equation was a significant contribution to the understanding of drawdown in an obervation well in an unconfined aquifer. Even though the introduction of the storage coefficient is useful in understanding unconfined aquifers, its utility in analyzing field pumping tests is questionable. Neuman showed that as S/S_y decreases by a factor of ten the early time portion (confined affects) of the type curve moves one log cycle farther away from the late time portion of the type curve (see Fig. 3 Neuman 1972). This affect tends to make the early time portion impossible to be measured with normal field methods. The occurrence of the compressibility effect at very small time values is the reason why drawdowns in unconfined aquifers seem to start at a significant drawdown value.

A Neuman type curve was constructed for well 5 for the Nevius test with S/S_y and K_v/K_h values of respectively 0.1 and 0.025. The Dagan and Neuman type curves merged just before

the dimensionless time value of 0.1. For the Nevius test this is a time value of approximately 20 minutes. A more reasonable S/S_y value for an unconfined aquifer would probably be 10^{-3} . The storage coefficient would be 0.2×10^{-3} for a specific yield of 0.2. If the S/S_y for the Nevius test would be 10^{-3} , then measurements of less than 0.2 of a minute would have to be made to measure the compressibility effects. The earliest measurement for well 5 in the Nevius test was at 1 minute which did not show significant compressibility effect. Therefore, the consideration of the compressibility effect in most unconfined pumping tests is not warranted.

Observation Well Position

The position of an observation well or piezometer is important to the success of an aquifer test. Observation wells should be placed at selected positions to insure a desired result.

The radial position of the observation wells is important in defining the anisotropy. Observation wells for use in the ratio curve method should normally be positioned in the same radial direction from the pumping well to decrease the possibility of changes in aquifer properties between the two observation wells. If slight inhomogeneous conditions are present across the cone of influence of the pumping well, fallacious anisotropic ratio values are likely to result from observation wells placed on opposite sides of the pumping well. The vertical spacing between certain anisotropic type curves increases with increasing distance from the pumping well for the same vertical depth position of the observation wells. For

example, in the Hancock test the vertical distance between K_v/K_h curves of 0.01 and 1.0 for observation well 200 is greater than those for observation well 100. This larger vertical spacing between the anisotropic curves normally enables a better determination of K_v/K_h . The corresponding ratio type curves should also have a greater range of ratio values enabling a better fit. The radial position must be limited to a distance such that the drawdown is significant enough to define time-drawdown curves sufficiently. The maximum radial position with sufficient drawdown is dependent on the aquifer, pumping well and observation well properties. The pumping test with the largest r/D value analyzed in this report was slightly greater than 4 for observation well 6 from the Nevius test. The time-drawdown curve was sufficiently defined by water level measurements in this observation well. For this test an observation well with a greater distance than well 6 from the pumping well would have been satisfactory, although the maximum distance is indefinable. With all other parameters held constant, the maximum radial position of the observation well should normally be decreased with a decrease in horizontal hydraulic conductivity. Since all the parameters of the aquifer, pumping well and observation well affect the maximum desired radial position of observation wells, it is very difficult to give criteria on the radial positioning of observation wells.

A proper selection of radial position for two observation wells for use in the ratio curve method is important. If the two observation wells are at the same position vertically,

there should be significant difference between their radial positions. Successful results were obtained from two observation wells at the same depth for both the Hancock and Nevius tests. The r/D values for observation wells 100 and 200 for the Hancock test are respectively 0.833 and 1.67. The Nevius observation wells 5 and 6, which were positioned at the same depth, have r/D values of 2.134 and 4.247 respectively.

These two arrangements of two observation wells were satisfactorily spaced radially to obtain large enough differences in the ratio type curves to sufficiently define K_v/K_h accurately.

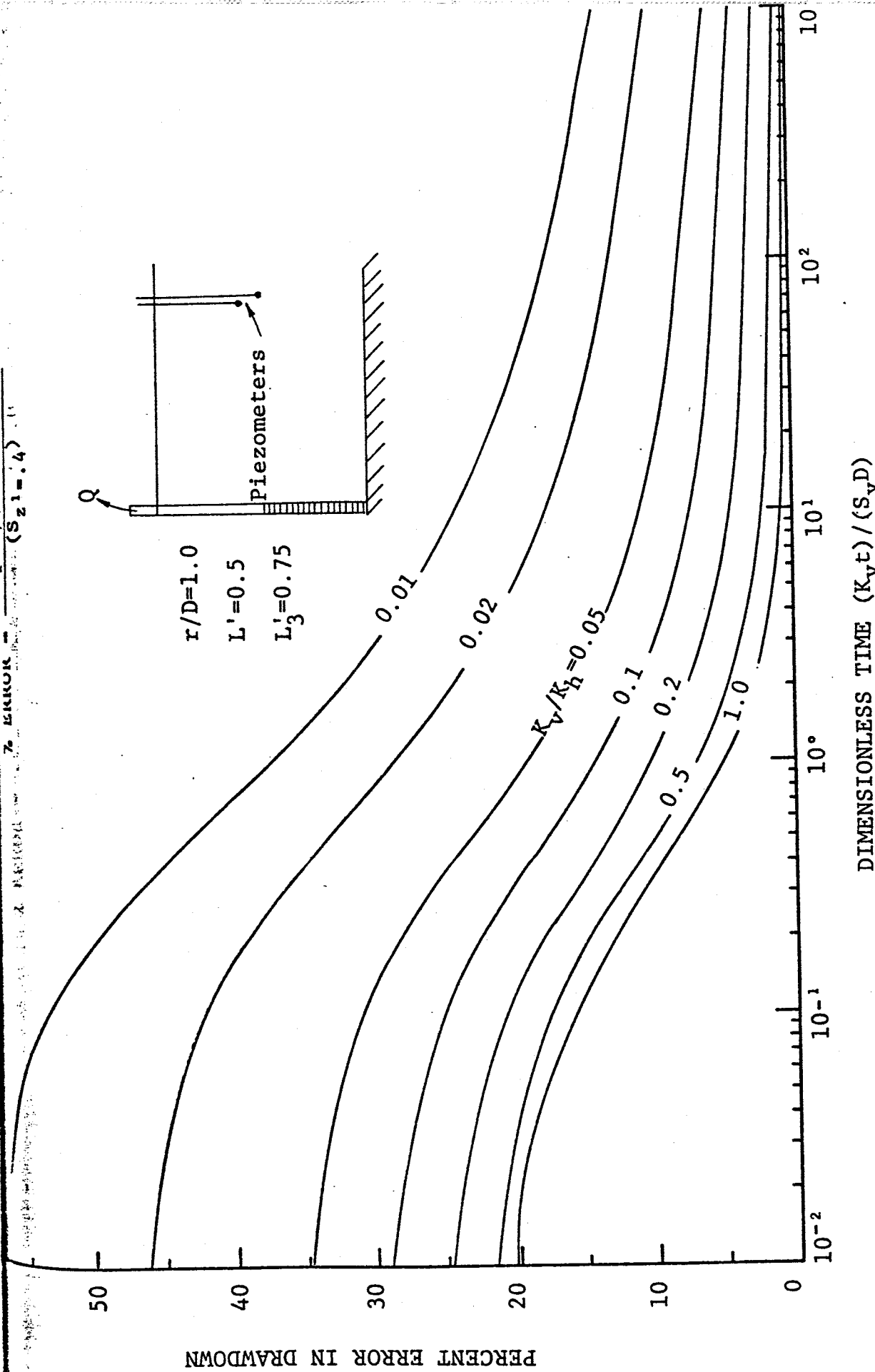
The vertical position of observation wells should also be considered. Time-drawdown curves from an observation well positioned near the free surface (water table) will have a larger vertical range in drawdown than one position deeper in the aquifer. Observation wells 6 and 7 from the Mosinee test are an example of a case in which the shallower observation well has a larger vertical range in its curves. Type curves with a large vertical range should be easier to match uniquely. Free surface drawdown curves are significantly different than curves constructed from piezometers just below the free surface. A piezometer with zero penetration is impossible to construct in the field. Observation wells which consist of screens near the free surface should be placed below the expected maximum drawdown. The effective center of the observation well would change if the drawdown drops below the top of the screen. Therefore observation wells should be placed far enough below the free surface that the drawdown will not drop below the top of the screen.

Observation wells with the same radial spacing from the pumping well should have a considerable difference in vertical spacing for satisfactory results from the ratio curve method. The Mosinee observation wells 6 and 7 are spaced the same radially but have Z' of -0.408 and -0.113 respectively for vertical positions.

The difference in vertical positioning for these two observation wells was large enough to enable a satisfactory match of the ratio curve. When positioning two observation wells with the same radius, one observation well should be near the free surface while the other should be near or below the center of the aquifer.

To test the importance of accurately representing the vertical position of piezometers in type curve construction, two different pumping well penetrations and three different vertical positions of piezometers were considered. Errors in type curve construction were computed from assumptions that vertical piezometer positions were incorrectly determined. These errors are presented in order to obtain an understanding of the importance of accurate determination of piezometer location.

The percent error in drawdown for the pumping well penetration of L' and L_3' of 0.5 and 0.75 for the three vertical positions of piezometers is shown in Figures 19, 20, and 21. The percent error in drawdown that would occur in the type curves for a vertical misplacement of ten percent above the aquifer's center is shown in Figure 19. This would be an error of 3 feet in defining the vertical position of a piezometer



DIMENSIONLESS TIME $(K_v t) / (s_y D)$

FIGURE 19. Percent error in drawdown for type curve construction of piezometer at $Z' = -0.5$ instead of the true $Z' = -0.4$.

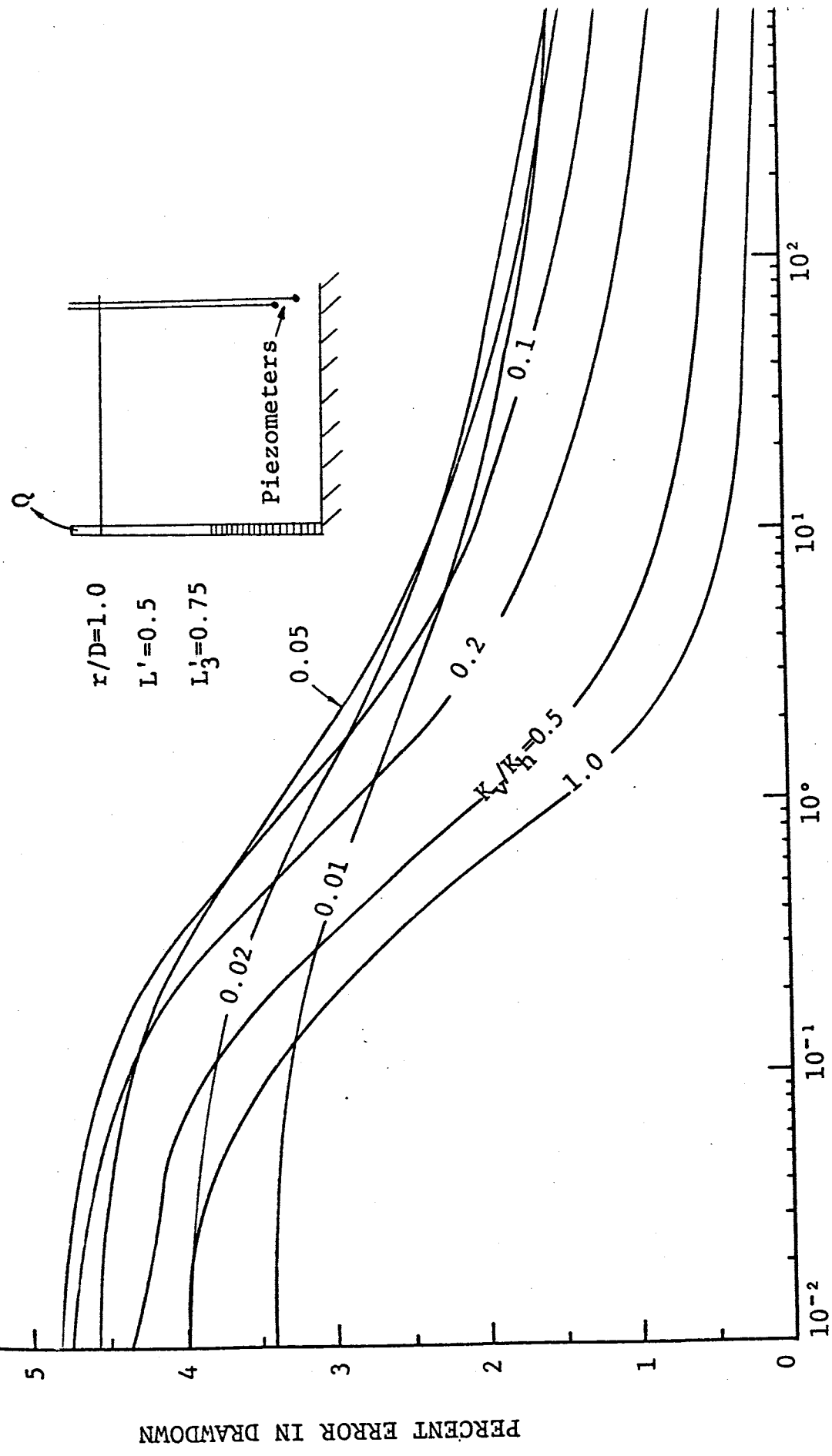
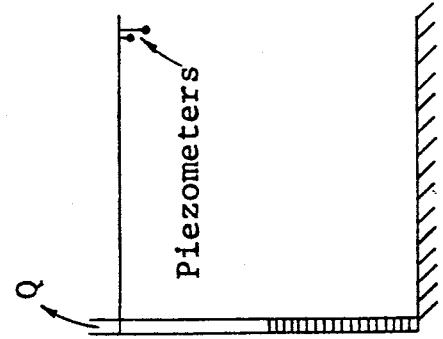


FIGURE 20. Percent error in drawdown for type curve construction of piezometer at $Z'=-0.9$ instead of the true $Z'=-0.8$.

PERCENT ERROR = $\frac{W_{m'} - W_{m'}}{W_{m'}} \times 100$ ($S_{z'} = -0.05$)

$r/D = 1.0$
 $L' = 0.5$
 $L'_3 = 0.75$

PERCENT ERROR IN DRAWDOWN



$K_v/K_h = 1.0$

$K_v/K_h = 0.01$

10⁻² 10¹ 10² 10

DIMENSIONLESS TIME ($K_v t$) / ($S_y D$)

FIGURE 21. Percent error in drawdown for type curve construction of piezometer at $Z' = -0.1$ instead of the true $Z' = -0.05$.

in an aquifer 30 feet thick. Figure 19 shows the importance of accurately knowing the vertical position of the piezometer in type curve constructions. Percent errors of nearly 60 percent occur for small dimensionless times for K_v/K_h of 0.01 and decreases to less than 15 percent for very large dimensionless times. The average error in the development of the type curve K_v/K_h of 0.01 would probably be 30 to 40 percent. Since the error varies with time, the shape of the type curve for the two different positions is different. Therefore a pumping test analyzed with the above mentioned errors would not only have errors in K_v , K_h and S_y values but the time-drawdown data would probably be matched to the wrong K_v/K_h type curve. The percent error is least for an isotropic aquifer and increases very significantly as the K_v/K_h values decreases.

Percent errors in drawdowns for the piezometer misplacement for Z' of -0.9 instead the true vertical position of -0.8 is shown in Figure 20. This figure shows that the misplacement of a piezometer is not as critical toward the bottom of the aquifer since the maximum error is less than 5 percent. The uniform increase in error with decrease in K_v/K_h does not hold true for this position but the decrease in error with increase in time does. The percent error in drawdown for type curve construction for the misplacement for Z' of -0.1 instead of the true vertical position of -0.05 is shown in Figure 21. The error in vertical position of piezometers near the free surface is very large for small dimensionless times but decreases to nearly zero as time is increased. This large dependence on time will change the shape of the constructed type curve from the true type curve considerably. Figure 21 also shows that

the percent error is not very dependant on K_v/K_h since all K_v/K_h curves between values of 0.01 and 1.0 lies between these two curves.

The three previous graphs have shown that the percent error is greatest near the free surface and decreases to insignificant amounts near the bottom of the aquifer for incorrect positioning of a piezometer vertically. Errors in type curves decreases as time increases for vertically incorrect positioning of a piezometer. The dependance on K_v/K_h in determining the percent error is greatest near the center of the aquifer and decreases in both directions.

The second pumping well penetration which was considered to evaluate the error in developing type curves with incorrect positioning of a piezometer had the following L' and L_3' values of 0.4 and 0.3. This is a pumping well which perforates the upper 10 percent. The same three vertical positions for piezometers were considered for this pumping well penetration. Figure 22 shows the percent error in drawdown for type curves that would occur if a piezometer at Z' of -0.4 was thought to be at -0.5. The pumping well penetration has changed this figure's shape from its corresponding figure (see Fig. 19) for the other pumping well penetration. The anisotropic ratio is very significant for this case as it was for the case in Figure 19. The negative errors are due to the short well penetration which stops at the same vertical level which the piezometer is thought to be.

Percent error in drawdown for type curves constructed for a piezometer at Z' of -0.8 but was thought to be at -0.9

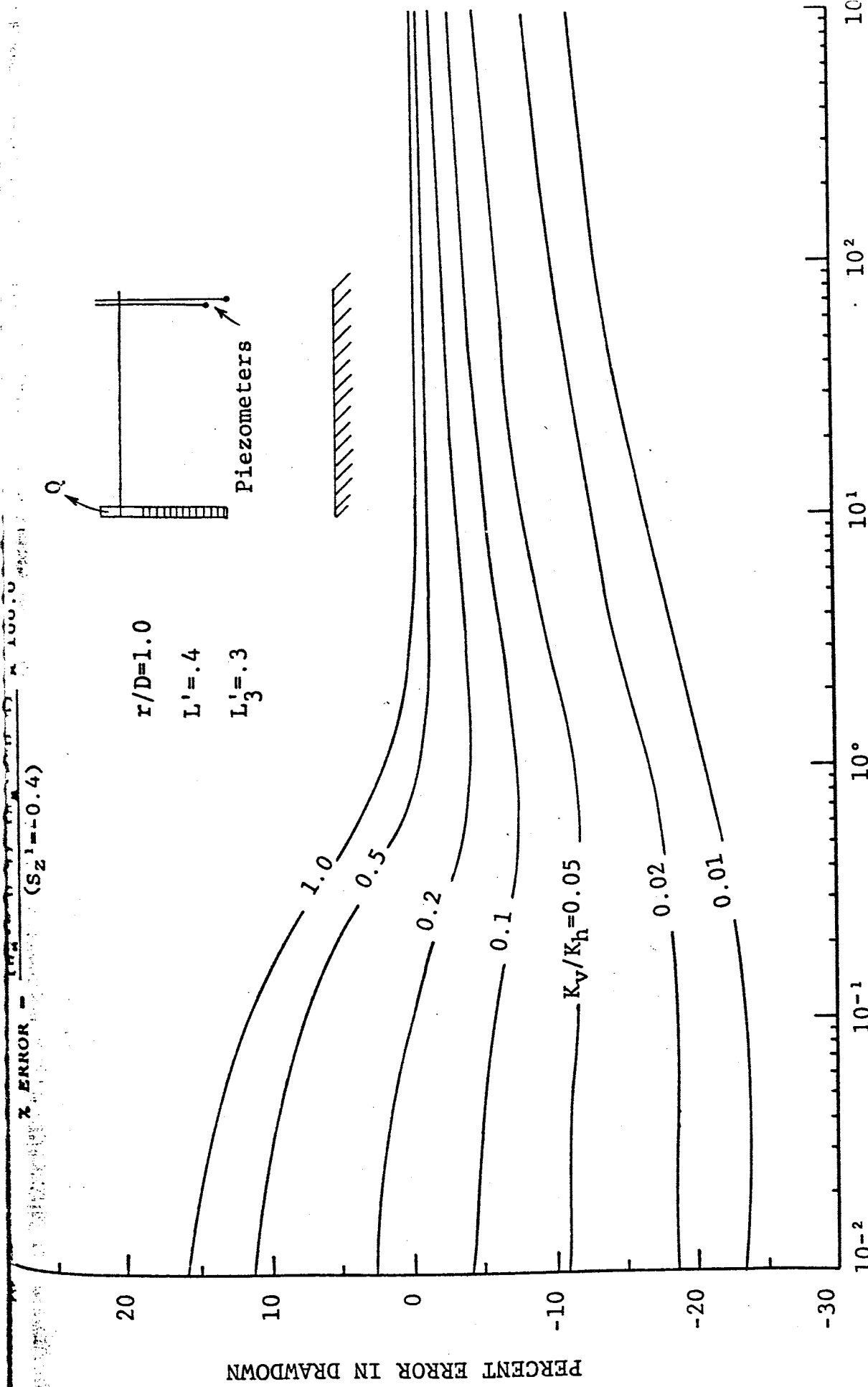


FIGURE 22. Percent error in drawdown for type curve construction of piezometer at $Z'=-0.5$ instead of the true $Z'=-0.4$.

is shown in Figure 23. The percent error curves for the lower piezometer is of the same form as the curves for the center piezometer with the percent errors decreased some. Errors for the corresponding figure (see Fig. 20) with the other pumping well penetration were smaller than the error in Figure 23. The increase in errors is due to this pumping well's penetration which causes vertical flow components to occur near the bottom of the aquifer. Figure 24 shows the percent error in drawdowns for type curves for a piezometer position for Z' of -0.05 which was thought to be at -0.1 in the type curve construction. The same shape and percent errors is shown in this figure as is shown in Figure 21 for the corresponding piezometer position with the different pumping well penetration. The percent errors are a little more dependant on anisotropy for this pumping well penetration than the first. Also K_v/K_h curves of 0.1 and 0.01 enclose the range of curves of K_v/K_h values of 0.01 to 1.0 in this figure.

The six previous figures demonstrate the importance of accurately defining the vertical position of piezometers. Careful measurements are even more important in thin aquifers. The shape and percent error of the error curves cannot generally be predicted, since they are dependant on vertical position of the piezometer and pumping well penetration. The near free surface piezometers will normally have greater percent errors.

Representing Observation Wells as Point Piezometers

Another error which could occur in type curve construction is the representation of a finite length observation well as

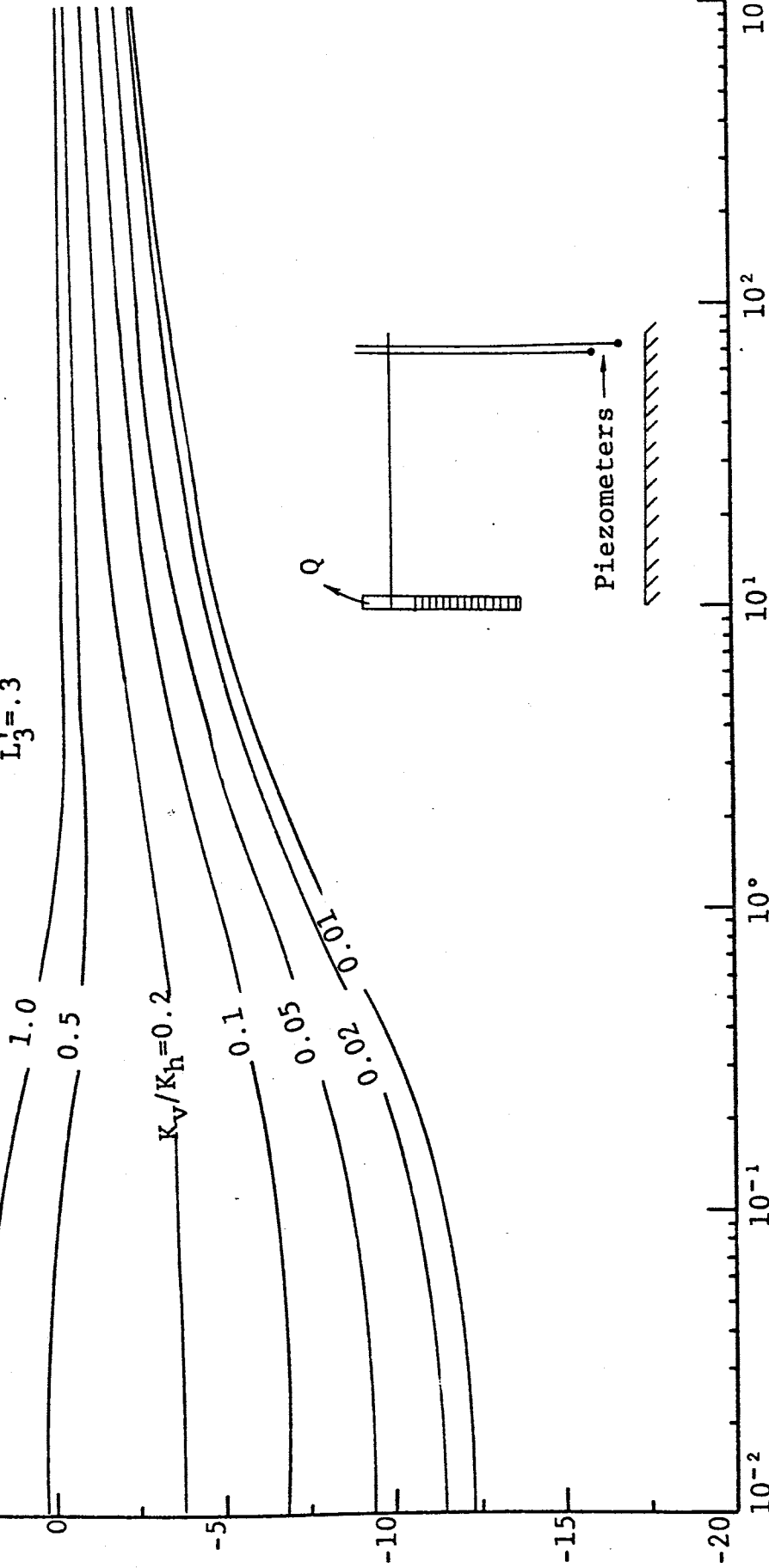
$$\% \text{ ERROR} = \frac{(S_{z1=0.9}) - (S_{z1=0.8})}{(S_{z1=0.8})} \times 100$$

$r/D=1.0$

$L'_3=0.4$

$L'_3=0.3$

PERCENT ERROR IN DRAWDOWN



DIMENSIONLESS TIME $(K_v t) / (S_y D)$

FIGURE 23. Percent error in drawdown for type curve construction of piezometer at $Z'=-0.9$ instead of the true $Z'=-0.8$.

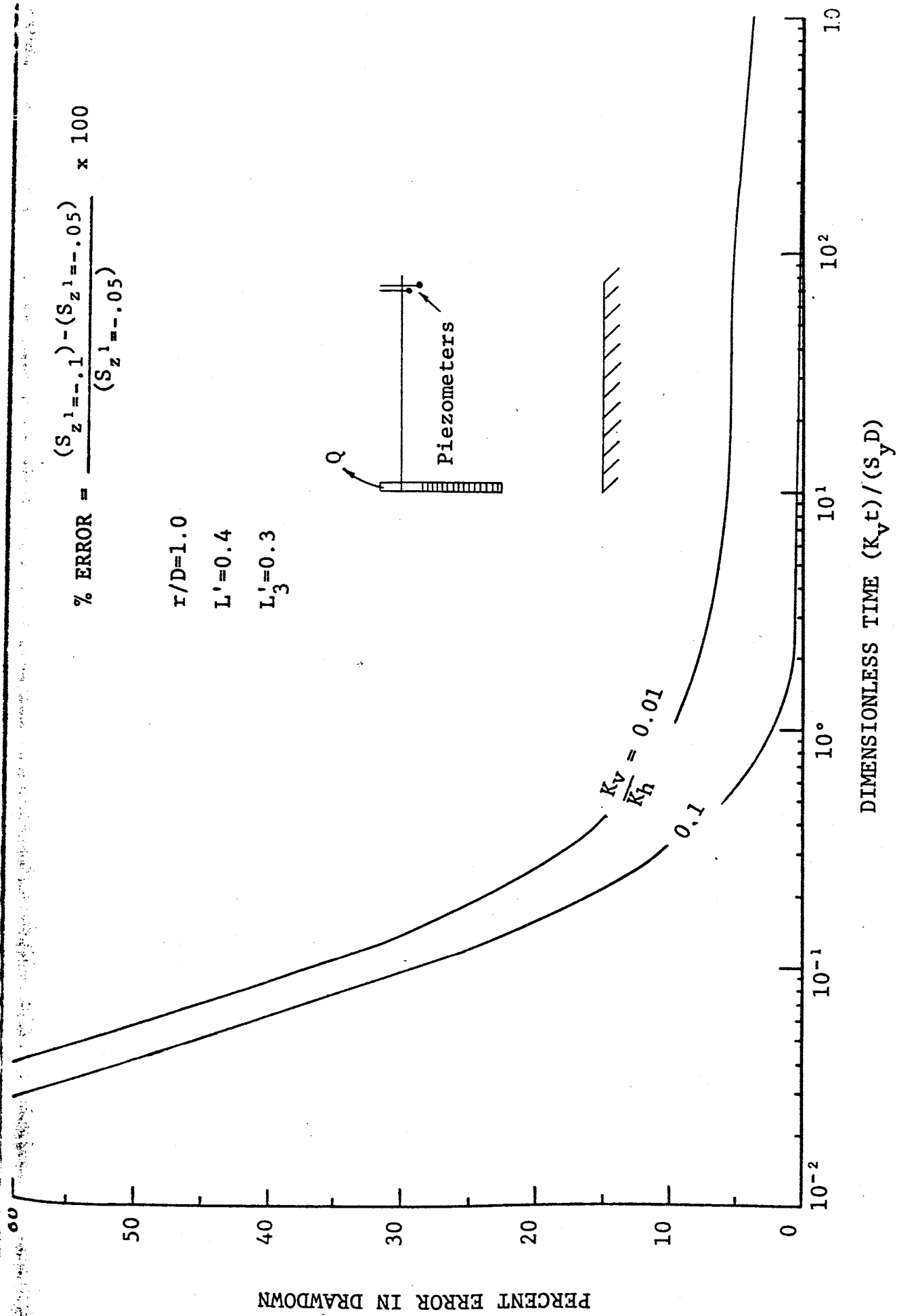
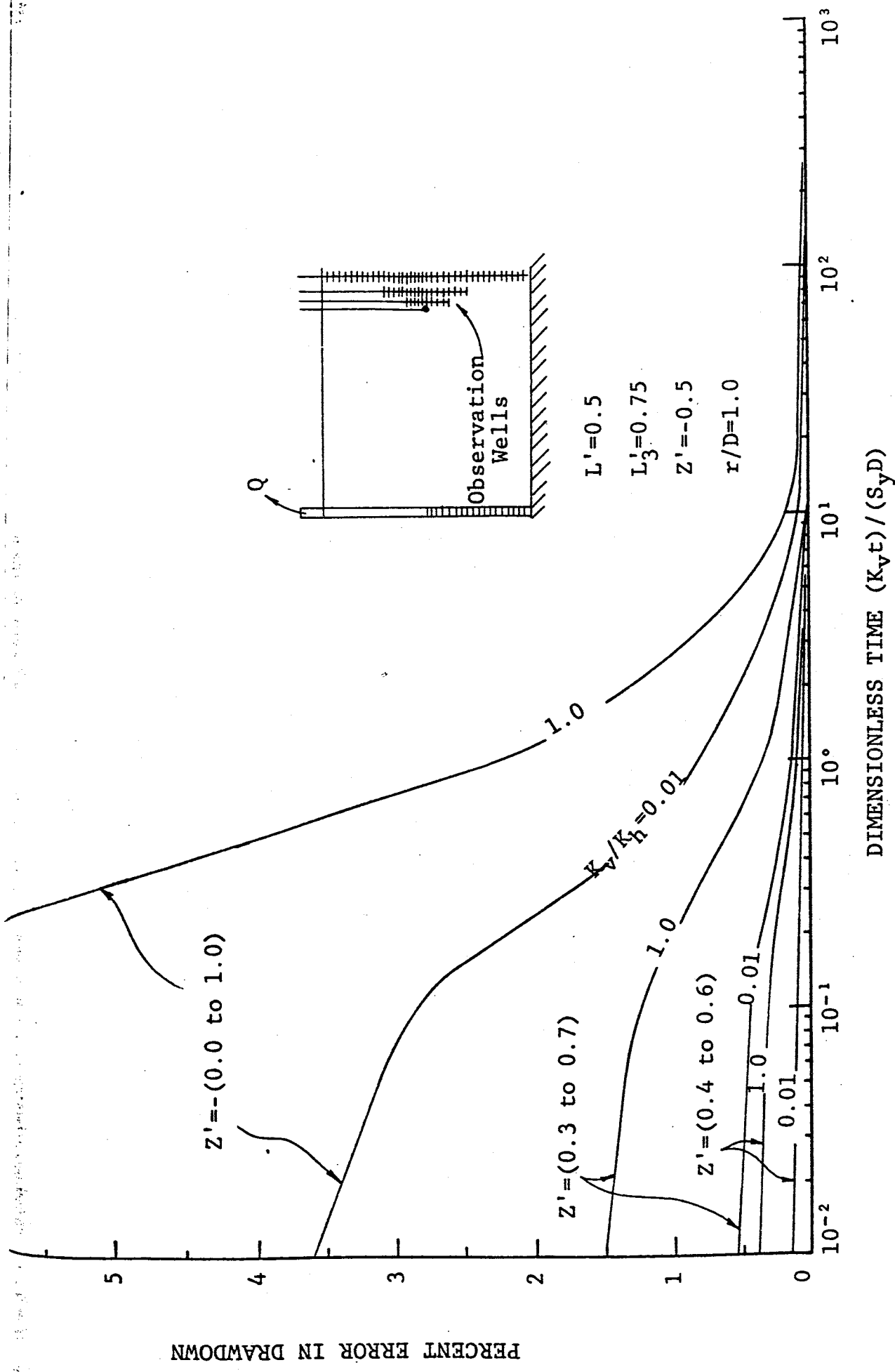


FIGURE 24. Percent error in drawdown for type curve construction of piezometer at $Z'=-.1$ instead of the true $Z'=-.05$.

a piezometer (point observation). There are unconfined pumping test methods (such as Dagan 1967a, Stallman 1963, and Bouiton and Pontin 1971) which use only solutions for point observations instead of observation wells. Therefore it is important to know how long an observation well can be and still be represented accurately as a point observation at its center. To test the error which occurs representing an observation well with a piezometer at its center, Neuman's (1974) program was used since it has the capability of determining drawdown for a finite length observation well. Three piezometer positions were used to calculate the percent error with one pumping well configuration of L' and L_3' values of 0.5 and 0.75 respectively. A value of 10^{-5} was used for S/S_y to make the type curves equal to Dagan's type curves. A r/D values of 1.0 was selected for the observation wells radial position. Figure 25 shows the percent error in drawdowns for three observation wells with their center at Z' of -0.5 which were represented in their type curve construction at their center. Three lengths of observation wells with Z' from -0.0 to -1.0, -0.3 to -0.7, and -0.4 to -0.6 are shown in the figure with two K_v/K_h values of 1.0 and 0.01. The percent error of drawdown for the fully penetrating observation well represented as a piezometer at its center is greater than 9% (which is not shown on the graph) for the isotropic aquifer and greater than 3% for the anisotropic aquifer with a K_v/K_h of 0.01 for dimensionless time of 0.01. The corresponding percent errors for the dimensionless times shown in Figure 25 for observation wells four tenths and two tenths of the aquifer thickness long are 1.5 and 0.4 for the



DIMENSIONLESS TIME $(K_v t) / (S_y D)$

FIGURE 25. Percent error in drawdown for type curve construction from representing observation wells as piezometers at their centers of $Z' = -0.5$.

isotropic case and 0.6 and 0.2 for the anisotropic case of K_v/K_h of 0.01. Therefore observation wells nearly one-half the aquifer thickness long can be represented as a piezometer at its center with maximum errors less than 2% for this case.

Figure 25 also shows that increase in dimensionless time decreases the percent error of drawdown to approximately zero. Since the percent error varies with time the error induced in the type curves would change the shape of the type curves. The increase in percent error with increase in K_v/K_h is an unexpected result. Since the r/D value being considered is one, the K_v/K_h curves are the same as $(K_v/K_h)(r^2/D^2)$ curves. Therefore the K_v/K_h curve of 0.01 can be the K_v/K_h of 1.0 with just the r/D value reduced. Figure 25 shows that the percent error increases with increase in radius to the observation well which is another unexpected result. Finally, a conclusion from Figure 25 can be drawn that an observation well with its center near the center of the aquifer can be as long as one-half the aquifer's thickness without inducing significant error into the type curves.

The second case considered was an observation well with its center at Z' of -0.9 with a length of two tenths of the aquifer thickness. This case is shown in Figure 26 with the same pumping well penetration and radial position of the observation well as the previous case but for three K_v/K_h values of 0.01, 0.1 and 1.0. The percent errors in drawdown shown in the figure are as small as the corresponding errors were for the observation well of the same length shown in Figure 25. For large dimensionless times the K_v/K_h values curves are in the expected order of

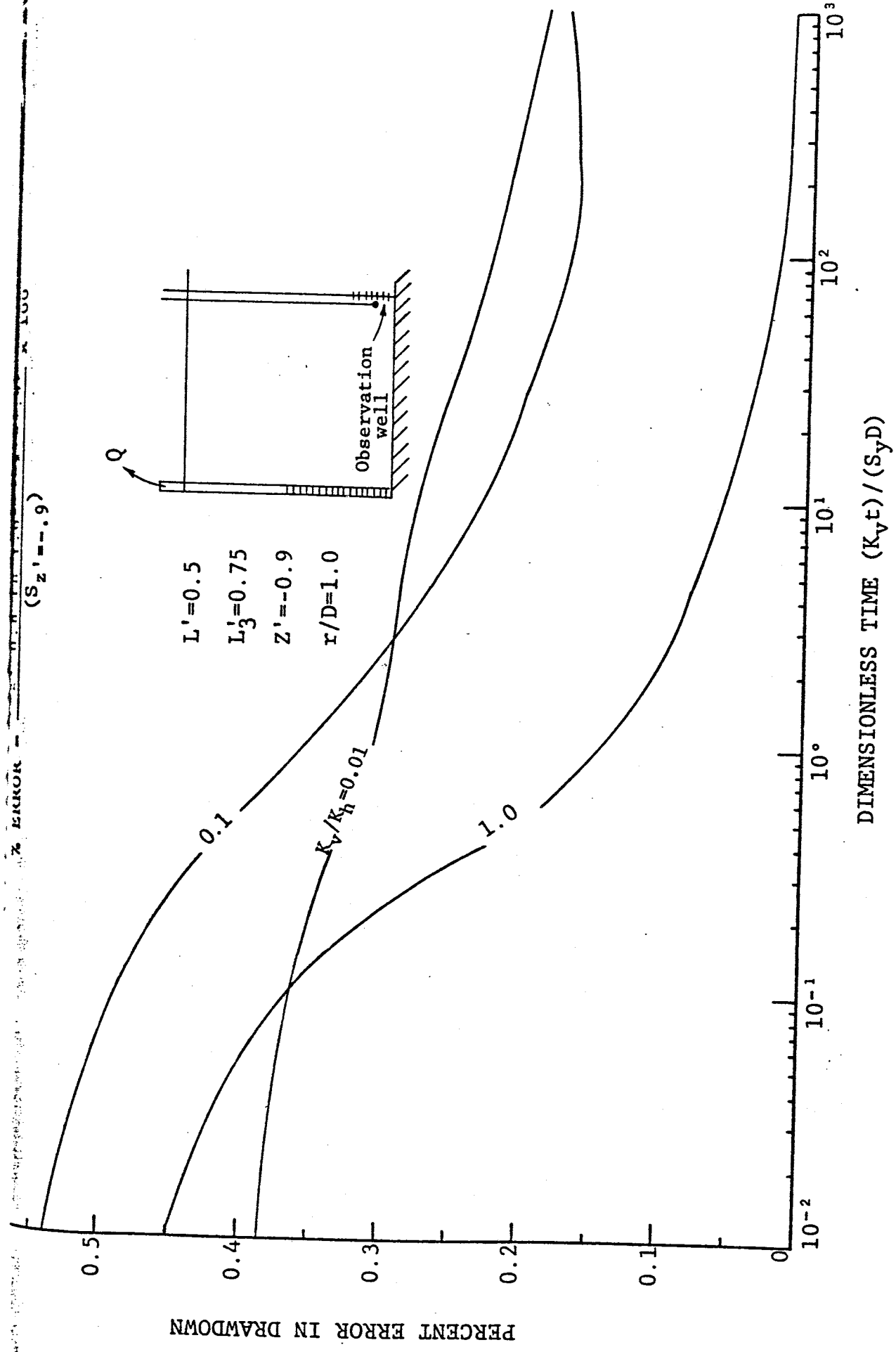


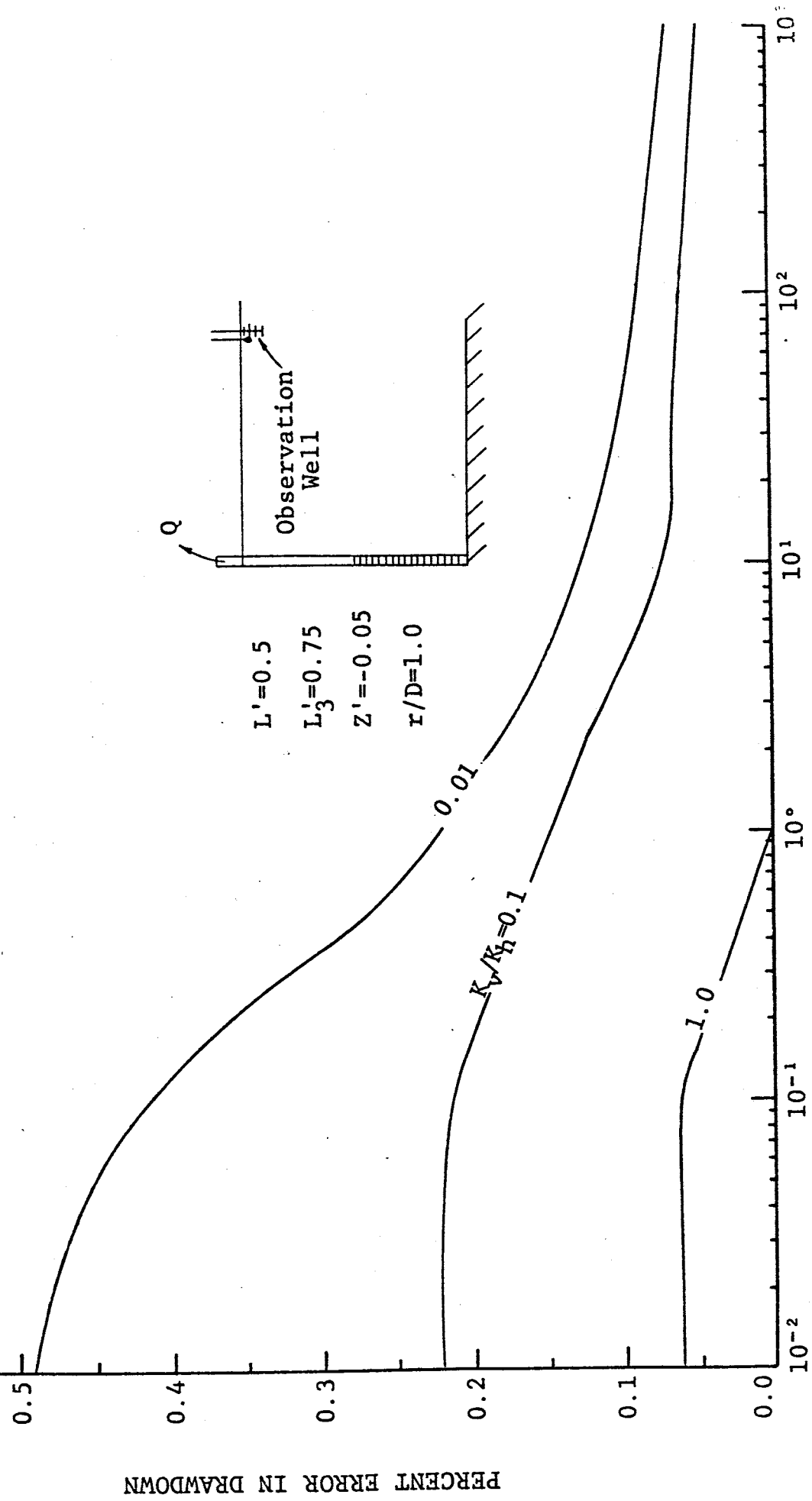
FIGURE 26. Percent error in drawdown for type curve construction from representing observation wells as piezometers at their centers of $Z' = -0.9$.

decreasing K_v/K_h for increase in error. This figure also shows that the percent error for an observation well probably will not be significant if the center of the observation well is toward the bottom of the aquifer.

The position of center of the observation well for the last case to be considered is at Z' of -0.05 with a length of one tenth the aquifer thickness. Figure 27 shows the percent error in drawdown that would occur for this observation well represented by a piezometer at its center for three K_v/K_h values of 0.01 , 0.1 and 1.0 . Since the length of the observation well is one-half the smallest length previously shown, the errors for the same length would be greater for this near free surface observation well than the other two cases. The percent errors decrease with increase in dimensionless time as the previous two cases did. The relationship between K_v/K_h and percent error is as expected. Type curves could be constructed using the center of observation well with lengths two or three tenths of the aquifer thickness without introducing significant errors into the type curves for observation wells near the free surface.

The three previous graphs have shown that using the center of the observation well to construct type curves does not induce significant errors unless the observation well is very long. Even the errors induced with fully penetrating observation wells represented as points at their centers for type curve construction could be tolerated for some cases. Observation wells with sections of screen at more than one location should be avoided if possible.

$$\% \text{ ERROR} = \frac{(M' = -0.05 \text{ or } 0.1) (S_z' = -0.05) \times 100.0}{(S_z' = -0.05)}$$



DIMENSIONLESS TIME $(K_y t) / (S_y D)$

FIGURE 27. Percent error in drawdown for type curve construction from representing observation wells as piezometers at their centers of $Z' = -0.05$.

Neuman's (1974) program computes drawdown for an observation well with one continuous screen interval.

Pumping Well Penetration

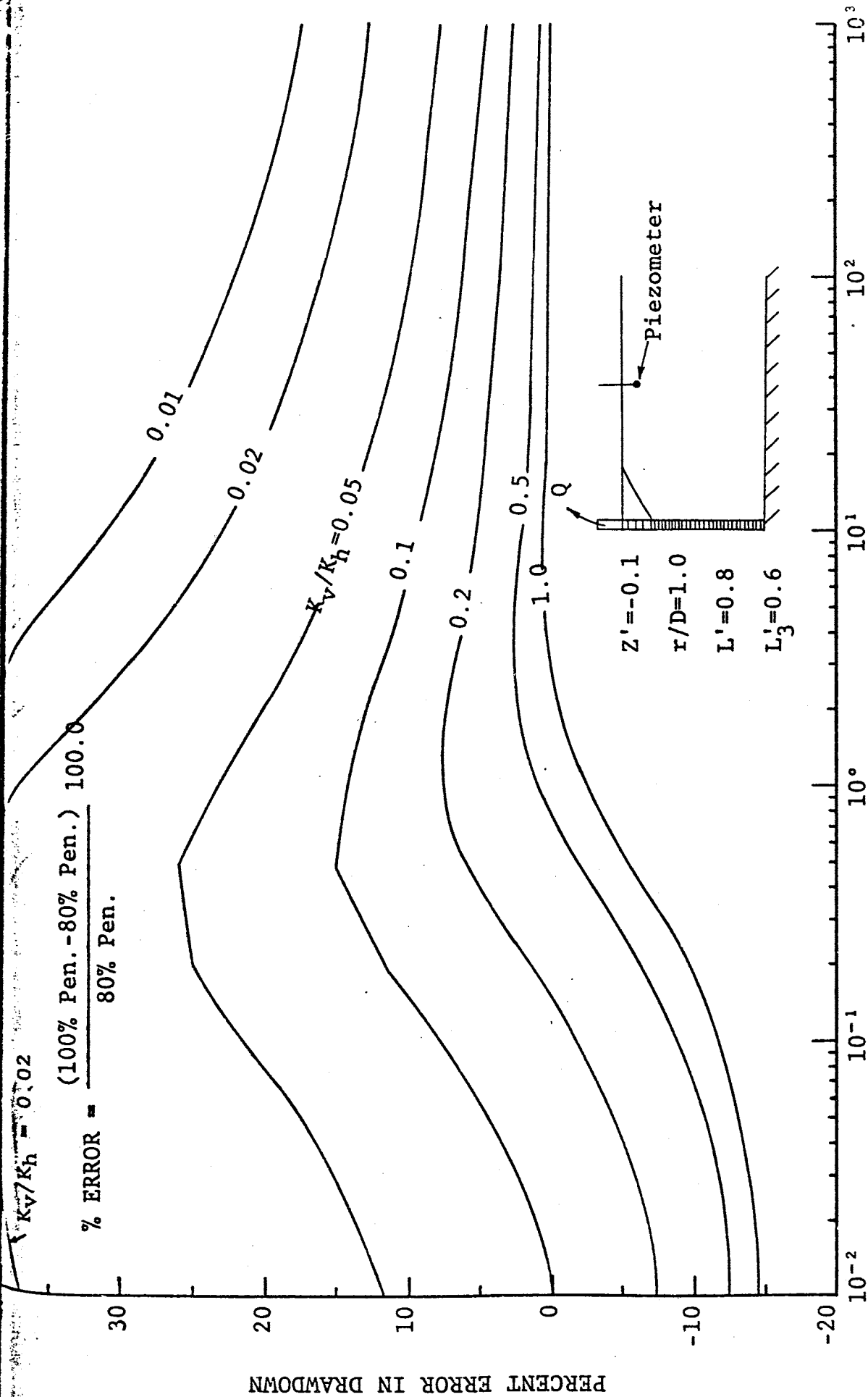
Hantush (1961) has shown the importance of partially penetrating pumping wells in confined aquifers. Partial penetration is also very important in unconfined aquifers since it adds to the vertical flow components which are normally caused by the dropping water table. Dagan (1967a) and Neuman (1974) have presented general analytical solutions to flow to a partially penetrating pumping well in unconfined aquifers.

Fully penetrating pumping wells are impossible in unconfined aquifers, since some reduction in the head of the pumping well must occur to get water to flow to the pumping well. Pumping wells which perforate the entire thickness of the aquifer thickness should not be considered as fully penetrating pumping wells in unconfined aquifers. This partial penetration effect due to drawdown in the pumping well is probably not important in very high transmissivity aquifers with low discharge pumping wells. But for most unconfined pumping tests this effect should be considered.

The majority of the drawdown in some pumping wells in unconfined aquifers occurs in the first few minutes. The partial penetration effect of fully perforated pumping wells can be approximated by a constant partially penetrating pumping well equal to the drawdown which occurs in the first few minutes. This type of partially penetrating pumping well was used for the Grand Island test and the Nevius test for observation wells 5 and 6.

To demonstrate the importance of this type partial penetration effect, the percent error in type curve construction is shown for two pumping well penetrations. Figure 28 shows the percent error in drawdown that would result in Dagan type curves if the initial drawdown in a fully perforated pumping well was 20 percent of the aquifer thickness and the pumping well was considered fully penetrating for the type curve construction. The error in the type curves is very small for an isotropic aquifer as dimensionless time increases. The error increases as the anisotropy ratio (K_v/K_h) decreases for larger dimensionless times. The error would actually be greater than that shown if some additional drawdown beyond the initial 20 percent of the aquifer thickness occurs. For the Nevius test the initial drawdown was approximately 25 percent of the aquifer thickness. Figure 29 shows the percent error in drawdown that would occur if the initial drawdown was 60 percent of the aquifer thickness and the fully perforated pumping well is treated as fully penetrating. This is an extreme case but is possible in thin alluviums which have large discharge pumping wells. This graph shows the extreme importance of considering the pumping well as a partially penetrating pumping well for this case.

The approximation of the pumping well penetration due to drawdown in the pumping well by constant partial penetration is probably not sufficient for close observation wells. The importance of considering the pumping well as a variable penetrating well was demonstrated for the Nevius pumping well for observation wells 1 and 2. This example showed that an additional



DIMENSIONLESS TIME $(K_v t) / (S_y D)$

FIGURE 28. Percent error in drawdown for type curve construction from representing a fully perforate pumping well with drawdown of 20 percent of the aquifer thickness as a fully penetrating pumping well

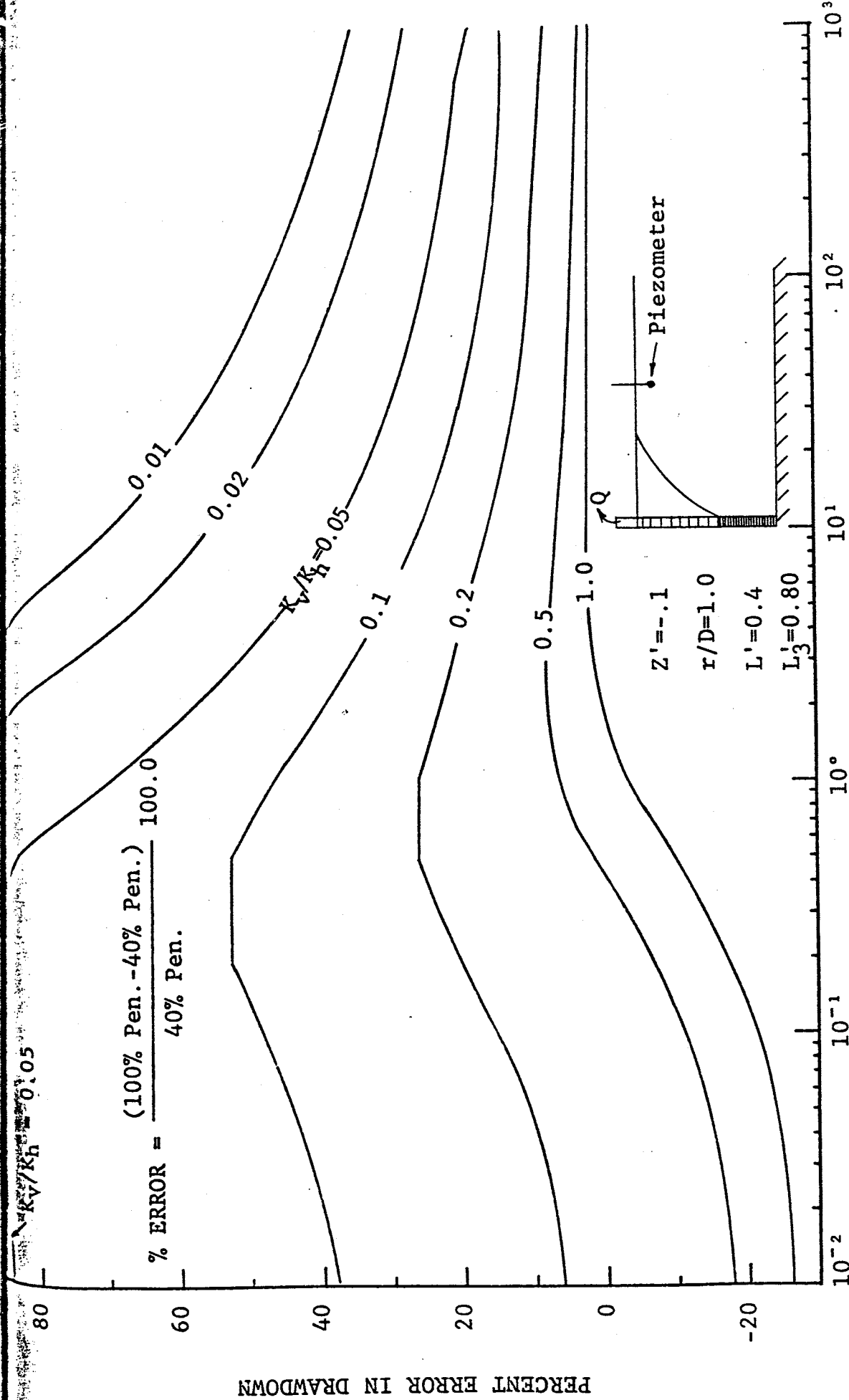


FIGURE 29. Percent error in drawdown for type curve construction from representing a fully penetrating pumping well with drawdown of 60 percent of the aquifer thickness as a fully penetrating pumping well.

3.9 feet of drawdown was significant. Observation wells 1 and 2 had r/D values of respectively 0.538 and 1.087. The variable penetration effect was important in this particular case for a dimensionless radius of greater than one. The variable penetration probably should be considered in unconfined pumping test with fully perforated pumping wells any time the dimensionless radius is less than one.

SUMMARY AND CONCLUSIONS

The numerous advances in unconfined aquifer pumping tests have also created confusion in the selection of type curves for the field hydrologist. This study is an attempt to clarify unconfined pumping test and to present suggestions of importance of certain parameters of pumping tests.

The partial penetration effect of fully perforated pumping wells in unconfined aquifers should be represented by a constant partially penetrating pumping well or a variably penetrating pumping well. Fully perforated pumping wells in unconfined aquifers should never be considered as fully penetrating pumping wells. For observation wells with small dimensionless radii (r/D values one or less) the partial penetration effect of fully perforated pumping wells probably should be treated as a variably penetrating pumping well. Variably penetrating pumping wells should be approximated as the pumping well was for the Nevis test for observation wells 1 and 2. Correct representation of a partially perforated pumping well is also important. Therefore, pumping test methods which consider partial penetration should always be used for unconfined pumping tests.

Correct representation of the position of observation wells or piezometer in type curve construction has been shown to be very important. Errors in parts of type curve construction greater than 60 percent occur for a 5 percent error in misrepresentation of the vertical position of a piezometer near the free surface. These large errors demonstrate the importance of accurately presenting the position of piezometers in type curve construction. The importance of representing the penetration of the pumping well and position of the piezometers concludes that type curves for unconfined pumping tests must be constructed for each individual test.

Small errors result from representing long observation wells as piezometers at their centers for type curve construction. Incorrect representation of the center of an observation well of any length will create large errors similar to those of misrepresenting the vertical position of a piezometer. Therefore, pumping test methods for unconfined aquifers which only give point observation drawdowns are sufficient. Neuman's (1974) program can be used for very long observation wells where errors due to representing the observation well as a piezometer at its center is of concern.

The ratio curve method, presented in the Methodology section, is a useful procedure to follow in determining K_v/K_h in analyzing pumping test. Since type curves have to be constructed for most unconfined pumping tests, the ratio curve construction takes very little additional effort. The ratio curves normally have more curvature to them than the normal type curves and are therefore easier to be uniquely matched. The vertical anisotropy value

for aquifers should be determined by using a pumping test method which considers field parameters that are thought most important plus K_v/K_h . The value of K_v/K_h should be determined by matching the family of normal type curves or by using the ratio curve method. The anisotropy ratio should be determined for unconfined aquifer test even if K_v is not needed to obtain correct transmissivity results. The importance of considering aquifers vertically anisotropic has been demonstrated and therefore a type curve including anisotropy should always be used.

The result from the Nevius pumping test analyzed in this report showed that the storage coefficient or the compressibility effects of the water and aquifer material is not significant for this pumping test in unconfined aquifers. The compressibility effects occurred at such small time values for this test that new methods to measure the drawdowns at these time values would be required. Obtaining a constant discharge from the pumping well for small time values is also a difficult problem. The storage coefficient could be significant for small times for some unconfined aquifer tests.

Boulton (1974), Dagan (1967a), Stallman (1963), Neuman (1974) and Streltsova (1972) pumping test methods are all approximately the same when considered for the same pumping test situation. Since Boulton's (1954) type curves are limited to the free surface drawdown and Stallman's type curves are limited to two pumping well configurations, these two pumping test methods have limited use since all unconfined pumping tests should consider pumping wells as partially penetrating. Therefore, we feel Dagan's and Neuman's programs should be used for developing type curves

for unconfined pumping test. Dagan's program computes type curves with significantly less computer time than Neuman's.

Since Dagan's or Neuman's method does not consider the thinning effect in unconfined aquifer test, we feel the thinning should be approximately corrected for by using Jacob's correction. The accuracy of using Jacob's correction for aquifer thinning in vertical flow models is unknown, but the importance of aquifer thinning necessitates some correction.

Development of techniques in analyzing unconfined pumping tests should be a fertile field in hydrology for many years. The problem of aquifer thinning has not been accurately handled for potential flow unconfined pumping tests. An unconfined pumping test which accurately considers pumping wells as variably penetrating should be developed. More effort should be spent to determine if the variability of specific yield has been sufficiently handled.

REFERENCES

- Barenblatt, G. I., P. Zheltov, and I. F. Kochina, 1960.
Basic concepts in the theory of seepage of homogeneous liquids in fissured rocks. J. Appl. Math. Mech. V. 24 (5).
- Boulton, N. S. 1954. The drawdown of the water-table under non-steady conditions near a pumped well in an unconfined formation. Proc. Instn. Civ. Engrs. V. 3 (3).
- Boulton, N. S. 1963. Analysis of data from non-equilibrium pumping tests allowing for delayed yield from storage. Proc. Instn. Civ. Engrs. V. 26.
- Boulton, N. S. 1955. Unsteady radial flow to a pumped well allowing for delayed yield from storage. Publication de l' Assoc. Int. d'Hydrol. Assem. Gen. Rome, No. 37.
- Boulton, N. S. 1970. Analysis of data from pumping tests in unconfined anisotropic aquifers. Jour. Hydrol. V. 10.
- Boulton, N. S. 1973. The influence of delayed drainage on data from pumping tests in unconfined aquifers. Jour. Hydrol. V. 19.
- Boulton, N. S. and J. M. A. Pontin 1971. An extended theory of delayed yield from storage applied to pumping test in unconfined anisotropic aquifers.
- Cooley, R. L. 1971. A finite difference method for unsteady flow in variably saturated porous media: Application to a single pumping well. Water Resour. Res. V. 7 (6).
- Cooley, R. L. 1972. Numerical simulation of flow in an aquifer overlain by a water table aquitard. Water Resour. Res. V. 8 (4).

REFERENCES (con't)

- Cooley, R. L. and C. M. Case 1973. Effect of a water table aquitard on drawdown in an underlying pumped aquifer. Water Resour. Res. V. 9. (2).
- Dagan, G. 1967a. A method of determining the permeability and effective porosity of unconfined anisotropic aquifers. Water Resour. Res. V. 3 (4).
- Dagan, G. 1967b. Second order correction of the formula of pumping test in unconfined aquifers. Jour. of Hydr. Res. 5 No. 2.
- Ferris, J. G., D. B. Knowles, R. H. Brown, and R. W. Stallman. 1963. Theory of Aquifers tests. U. S. Geol. Survey, Water-Supply paper 1536-E.
- Hantush, M. S. 1960. Modification of the theory of leaky aquifers. J. Geophys. Res. V. 65 (11).
- Hantush, M. S. 1961. Drawdown around a partially penetrating well. J. Hyd. Div., Proc. Am. Soc. Civil Engrs., Hy4, pp. 83-98.
- Hernandez - Gobaira, M. A. 1972. Transient flow toward wells in phreatic aquifers. Unpublished Master of Science Thesis, Dept. of Civil Engr., Massachusetts Institute of Technology.
- Jacob, C. E. 1944. Notes on determining permeability of pumping tests under water - table conditions. U. S. Geol. Surv., open-file rept.
- Kipp, Jr., K. L. 1973. Unsteady flow to a partially penetrating, finite radius well in an unconfined aquifer. Water Resour. Res. V. 9 (2).

REFERENCES (con't)

- Kriz, G. J. 1967. Determination of unconfined aquifer characteristics. J. of the Hydraulic Division Proceedings of the American Society of Civil Engineers, Hy2, pp. 37-47.
- Kruseman, G. P. and N. A. DeRidder 1970. Analysis and Evaluation of Pumping Test Data. International Inst. for Land Reclamation and Improvement, Wageningen The Netherlands, Bulletin 11.
- Lohman, S. W. 1972. Ground-water hydraulics. U. S. Geol. Sur. Prof. paper 708.
- Mansur, C. I., and R. J. Dietrich. 1965. Pumping test to determine permeability ratio. Am. Soc. Civ. Engrs. Proc. 91 SM4.
- Neuman, S. P. 1972. Theory of flow in unconfined aquifers considering delayed response of the water table. Water Resour. Res. V. 8 (4).
- Neuman, S. P. 1974. Effect of partial penetration on flow in unconfined aquifers considering delayed gravity response. Water Resour. Res. V. 10 (2).
- Prickett, T. A. 1965. Type-curve solution to aquifer tests under water-table conditions. Ground Water, V. 3 (3).
- Stallman, R. W. 1961. Boulton's integral for pumping test analysis. U. S. Geol. Survey Prof. Paper 424-C.
- Stallman, R. W. 1963. Electric analog of three-dimensional flow to wells and its application to unconfined aquifers. U. S. Geol. Survey Water-Supply paper 1536-H.

REFERENCES (con't)

- Stallman, R. W. 1965. Effects of water-table conditions on water level changes near pumping wells. *Water Resour. Res.* V. 1 (2).
- Stallman, R. W. 1968. Aquifer test, design, observation, and data analysis. *U. S. Geol. Survey Techniques of Water-Resources Investigation Book 3, Chapt. B1.*
- Streltsova, T. D. 1972. Unsteady radial flow in an unconfined aquifer. *Water Resour. Res.* V. 8 (4).
- Streltsova, T. D. 1973. Flow near a pumped well in an unconfined aquifer under nonsteady conditions. *Water Resour. Res.* V. 9 (1).
- Streltsova, T. D. and K. R. Rushton 1973. Water table drawdown due to a pumped well in an unconfined aquifer. *Water Resour. Res.* V. 9 (1).
- Theis, C. V. 1935. The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using ground-water storage. *Transaction Am. Geophys. Union, 16th Annual Meeting Pt. 2.*
- Weeks, E. P. 1964. Field methods for determining vertical permeability and aquifer anisotropy. *U. S. Geol. Surv. Prof. Paper 501-D.*
- Weeks, E. P. 1969. Determining the ratio of horizontal to vertical permeability by aquifer-test analysis. *Water Resour. Res.* V. 5 (1).
- Wenzel, L. K. 1937. The Thiem method for determining permeability of water-bearing materials. *U. S. Geol. Survey Water-Supply Paper 679.*

REFERENCES (con't)

Wenzel, L. K. 1942. Methods for determining permeability of water-bearing materials. U. S. Geol. Survey Water Supply Paper 887.

APPENDIX
DAGAN'S PROGRAM

C	-----	DAGA0010
C		DAGA0020
C	PROGRAM FOR THE COMPUTATION OF TYPE CURVES IN A PUMPING TEST IN A	DAGA0030
C	HOMOGENEOUS, NON-ISOTROPIC, HORIZONTAL, PHREATIC AQUIFER OF INFINIT	DAGA0040
C	RADIAL EXTENT AND OF FINITE DEPTH 'D'.	DAGA0050
C		DAGA0060
C	THE TYPE CURVES REPRESENT THE NONDIMENSIONAL DRAWDOWN '4*3.14*S*KH	DAGA0070
C	*D/Q'	DAGA0080
C	OBTAINED IN AN OBSERVATION WELL PLACED AT A DISTANCE 'R' FROM THE	DAGA0090
C	PUMPING WELL AND OPENED AT A LEVEL 'Z' BELOW THE INITIAL HORIZON-	DAGA0100
C	TAL UNPERTURBED PHREATIC SURFACE, AS A FUNCTION OF NONDIMENSIONAL	DAGA0110
C	TIME 'KV*T/SY/N/R**2', OF OF THEIS' VARIABLE 'KH*T*D/SY/R2*12.566'	DAGA0120
C		DAGA0130
C	THE PUMPING WELL MAY BE FULLY OF PARTIALLY PENETRATING THE AQUIFER	DAGA0140
C	AND PUMPS A CONSTANT DISCHARGE 'Q'.THE NONDIMENSIONAL PENETRATION	DAGA0150
C	PARAMETERS ARE. THE LENGTH OF THE SCREEN 'L/D' AND THE DISTANCE	DAGA0160
C	FROM THE INITIAL UNPERTURBED PHREATIC SURFACE TO THE CENTRAL	DAGA0170
C	POINT OF THE SCREEN 'L3/D'.	DAGA0180
C		DAGA0190
C	THE TYPE CURVES ARE COMPUTED ACCORDING TO EQ.(42) DAGAN(1967)	DAGA0200
C		DAGA0210
C	SYMBOLS USED.	DAGA0220
C	S -DRAWDOWN OF THE WATER LEVEL IN THE OBSERVATION WELL	DAGA0230
C	KH-HORIZONTAL HYDRAULIC CONDUCTIVITY WITHIN THE AQUIFER	DAGA0240
C	KV- VERTICAL HYDRAULIC CONDUCTIVITY WITHIN THE AQUIFER	DAGA0250
C	D -INITIAL AQUIFER DEPTH	DAGA0260
C	R2- R*R	DAGA0270
C	T - TIME	DAGA0280
C	SY-EFFECTIVE POROSITY OF THE AQUIFER (SPECIFIC YIELD)	DAGA0290
C		DAGA0300
C	THE FOLLOWING DATA DECK IS REQUIRED	DAGA0310
C	CARD 1. NCASE,NT IN FORMAT 215	DAGA0320
C	NCASE IS THE NUMBER OF CASES TO BE COMPUTED (>=1)	DAGA0330
C	FOR EACH OBSERVATION WELL	DAGA0340
C	NT NUMBER OF TIME VALUES IN EACH CASE (<=20)	DAGA0350
C	NUCK IS THE TYPE OF PLOT DESIRED	DAGA0360
C	NUCK=0 PLOT ONLY DAGAN TYPE CURVES FOR EACH	DAGA0370
C	OBSERVATION WELL	DAGA0380
C	NUCK=1 PLOT DAGAN TYPE CURVES FOR EACH 2 OBSERVATION	DAGA0390
C	WELLS THEN PLOT RATIO TYPE CURVES (OBS 1/OBS 2)	DAGA0400
C	CARD 2 AND CARD 3. TT(I) (I=1,NT) IN FORMAT 10F8.0	DAGA0410
C	TT(I) ARE THE NONDIMENSIONAL TIME VALUES TO BE TAKEN.	DAGA0420
C	FOR NT =10 ONLY ONE CARD IS NEEDED.	DAGA0430
C	FOLLOWS A SET OF NCASE CARDS EACH OF WHICH CONTAINS THE FOLLOWING	DAGA0440
C	NONDIMENSIONAL PARAMETERS.	DAGA0450
C	XL3,XL,R,Z IN FORMAT 4F8.0, WHERE	DAGA0460
C	XL3 REPRESENTS L3/D (>0)	DAGA0470
C	XL REPRESENTS L/D (>0)	DAGA0480
C	R REPRESENTS SQRT(KV/KH)*R/D (>0)	DAGA0490
C	Z REPRESENTS Z/D (MUST BE NEGATIVE)	DAGA0500

(con't)
APPENDIX
DAGAN'S PROGRAM

```
C
C-----COMPUTATION OF THE INTEGRAL IS PERFORMED BY DIVIDING EACH INTERVAL
C BETWEEN TWO CONSECUTIVE ZEROS OF THE JO BESSEL FUNCTION INTO 30
C SUBINTERVALS AND APPLYING SIMPSON'S RULE.
C FINER INCREMENTS ARE USED FOR THE FIRST INTERVAL
C INTEGRATION IS INTERRUPTED WHEN THE CONTRIBUTION OF A COMPLETE
C CYCLE IS LESS THAN 1.E-5 TIMES THE INTEGRAL UP TO AND INCLUDING
C THIS CYCLE.
C-----THE WELL IS IDEAL AND HAS ZERO RADIUS, NO STORAGE, NO SEEPAGE
C FACES AND NO LOSSES. IT IS ASSUMED TO BE COMPOSED OF AN EQUAL
C STRENGTH ARRAY OF SINKS.
C-----REFERENCES.
C A METHOD OF DETERMINING THE PERMEABILITY AND EFFECTIVE POROSITY OF
C UNCONFINED ANISOTROPIC AQUIFERS, G. DAGAN, HYDRAULICS LABORATORY
C P.N., 1967. TECHNION HAIFA ISRAEL
C-----THIS PROGRAM WAS WRITTEN BY URI KROSYNSKI - JUNE 1973.
C THE ADDITION OF THE PLOT SUBROUTINE AND A FEW OTHER CHANGES
C WERE MADE BY GEORGE HOFFMAN 1974.
C-----
C-----MAIN PROGRAM
C
C COMMON/UI/P(31)/U2/T(20),NT/U3?R,R2,Z,XL2,XL3/U8/NMI
1 /U10/S(20,10)/U11/TT(20)
C DIMENSION TERM1(20),TERM2(20),ERR(20)
C READ THE DATA
C READ 100,NCASE,NT,NUCK
C READ 101,(T(I),I=1,NT)
100 FORMAT(3I5)
101 FORMAT(10F8.0)
C PARAMETERS
C XO=2.0
C YO=1.0
C PI=3.141592654
C PINV=1./PI
102 IMX=30
C
C PROCEED FOR EACH PARTICULAR CASE
C DO 7 KASE=1,NCASE
C N=500
C READ 101,XL3,XL,R,Z
C IF(XL-0.0)8,8,195
195 CONTINUE
C IF(R.LT.0.12)N=10000
C NMI=N-1
```

DAGA0510

DAGA0520

DAGA0530

DAGA0540

DAGA0550

DAGA0560

DAGA0570

DAGA0580

DAGA0590

DAGA0600

DAGA0610

DAGA0620

DAGA0630

DAGA0640

DAGA0650

DAGA0660

DAGA0670

DAGA0680

DAGA0690

DAGA0700

DAGA0710

DAGA0720

DAGA0730

DAGA0740

DAGA0750

DAGA0760

DAGA0770

DAGA0780

DAGA0790

DAGA0800

DAGA0810

DAGA0820

DAGA0830

DAGA0840

DAGA0850

DAGA0860

DAGA0870

DAGA0880

DAGA0890

DAGA0900

DAGA0910

DAGA0920

DAGA0930

DAGA0940

DAGA0950

DAGA0960

DAGA0970

DAGA0980

DAGA0990

DAGA1000

(con't)
APPENDIX
DAGAN'S PROGRAM

```
IF(Z.GE.0.0.OR.R.LE.0.0) GO TO 8
PRINT 200
200 FORMAT(/,110('-'))
PRINT 201,XL3,XL,R,Z
201 FORMAT(//,12X,'L3/D=',F12.5,5X,'L/D=',F12.5//,'SQRT(KV/KH)*R/D=',
1 F12.5,5X,'Z/D=',F12.5//,6X,'TAU',5X,'TAU/R2',5X,'12.5*KH*D*S/Q'
2/)
FACTOR=PINV/XL
R2=R*R
XL2=0.5*XL
DO 1 IT=1,NT
TT(IT)=T(IT)/R2
1 S(IT,KASE)=-0.33333333*P(2)*XL2/N
I=0
2 I=I+1
B=P(I)
DRO=(P(I+1)-B)/N
CALL CYCLE(B,DRO,TERM1)
N=30
NMI=N-1
I=I+1
B=P(I)
DRO=(P(I+1)-B)/N
CALL CYCLE(B,DRO,TERM2)
DO 3 IT=1,NT
WAVE=TERM1(IT)+TERM2(IT)
S(IT,KASE)=S(IT,KASE)+WAVE
3 ERR(IT)=ABS(WAVE/S(IT,KASE))
DO 4 IT=1,NT
IF(ERR(IT).GT.1.E-4.AND.I.LT.IMX) GO TO 2
4 CONTINUE
CALL ADTN(ADD)
DO 5 IT=1,NT
S(IT,KASE)=FACTOR*(ADD+S(IT,KASE)/R)*12.56637
5 PRINT 202,T(IT),TT(IT),S(IT,KASE)
202 FORMAT(F10.3,E12.3,E18.5/)
IF(I.LE.28) GO TO 7
DO 6 IT=1,NT
6 PRINT 203,I,ERR(IT)
203 FORMAT(17X,15,E18.5)
7 CONTINUE
C REMOVE THE FOLLOWING CARD IF A PLOT IS NOT DESIRED
CALL PLTYPE(NCASE,NUCK,XO,YO)
GO TO 102
8 CONTINUE
END
C
BLOCK DATA
COMMON/UI/P(31)
DATA P/ 0.0 , 2.40483, 5.52008, 8.65373,11.79153,
```

DAGA1010
DAGA1020
DAGA1030
DAGA1040
DAGA1050
DAGA1060
DAGA1070
DAGA1080
DAGA1090
DAGA1100
DAGA1110
DAGA1120
DAGA1130
DAGA1140
DAGA1150
DAGA1160
DAGA1170
DAGA1180
DAGA1190
DAGA1200
DAGA1210
DAGA1220
DAGA1230
DAGA1240
DAGA1250
DAGA1260
DAGA1270
DAGA1280
DAGA1290
DAGA1300
DAGA1310
DAGA1320
DAGA1330
DAGA1340
DAGA1350
DAGA1360
DAGA1370
DAGA1380
DAGA1390
DAGA1400
DAGA1410
DAGA1420
DAGA1430
DAGA1440
DAGA1450
DAGA1460
DAGA1470
DAGA1480
DAGA1490
DAGA1500

(con't)
APPENDIX
DAGAN'S PROGRAM

1 14.93092,18.07106,21.21164,24.35247,27.49348,
2 30.63461,33.77582,36.91710,40.05843,43.19979,
3 46.34119,49.48261,52.62405,55.76551,58.90698,
4 62.04847,65.18996,68.33147,71.47298,74.61450,
5 77.75603,80.89756,84.03909,87.18063,90.32217,93.46372/

END

C SUBROUTINE ADIN(ADD)
C ADD IS THE LOGARITHMIC TERM EXPRESSING THE WELL SINGULARITIES
COMMON/U3/R,R2,Z,XL2,XL3
F(X)=X+SQRT(X*X+R2)
A=XL3+XL2
B=XL3-XL2
AZ=A+Z
BZ=B+Z
ADD=F(AZ)/F(BZ)
AZ=A-Z
BZ=B-Z
ADD=ADD*F(AZ)/F(BZ)
ADD=0.25*ALOG(ADD)
RETURN
END

C SUBROUTINE CYCLE(B,DRO,TERM)
C TERM IS THE INTEGRAL OVER AN INTERVAL BETWEEN TWO SUCCESSIVE ZEROS
C OF THE BESSEL FUNCTION JO, BY SIMPSON'S RULE.

COMMON/U2/T(20),NT/U3/R,R2,Z,XL2,XL3/U4/IND/U5/POINT(20)
1 /U6/E(12,2)/U7/F(9)/U8/NM1/U9/T1,T2,T3

DIMENSION TERM(20),SEVN(20),SODD(20)

FACTOR=0.66666667*DRO

DO 1 IT=1,NT

SEVN(IT)=0.0

1 SODD(IT)=0.0

C INITIALIZE EXPONENTIALS

CALL BDR(B,DRO)

I=1

RO=B

C RUN OVER POINTS IN INTERVAL

DO 5 J=1,NM1

RO=RO+DRO

ARG=RO/R

C COMPUTE HYPERBOLIC FUNCTIONS FOR THIS VALUE OF RO AND PARTS OF

C INTEGRAND

CALL FCTN(ARG,RO)

C THE TRANSIENT TERM

CALL BRACK(ARG)

C THE VALUE OF THE INTEGRAND FOR THIS VALUE OF ARG

DO 4 IT=1,NT

IF(T2-0.1E-25)110,111,111

110 FVAL=T3*T1

DAGA1510
DAGA1520
DAGA1530
DAGA1540
DAGA1550
DAGA1560
DAGA1570
DAGA1580
DAGA1590
DAGA1600
DAGA1610
DAGA1620
DAGA1630
DAGA1640
DAGA1650
DAGA1660
DAGA1670
DAGA1680
DAGA1690
DAGA1700
DAGA1710
DAGA1720
DAGA1730
DAGA1740
DAGA1750
DAGA1760
DAGA1770
DAGA1780
DAGA1790
DAGA1800
DAGA1810
DAGA1820
DAGA1830
DAGA1840
DAGA1850
DAGA1860
DAGA1870
DAGA1880
DAGA1890
DAGA1900
DAGA1910
DAGA1920
DAGA1930
DAGA1940
DAGA1950
DAGA1960
DAGA1970
DAGA1980
DAGA1990
DAGA2000

(con't)
APPENDIX
DAGAN'S PROGRAM

	GO TO 112	DAGA2010
111	FVAL=T3*(T)-T2*POINT(IT))	DAGA2020
C	ADD ACCORDING TO SIMPSON	DAGA2030
112	CONTINUE	DAGA2040
	IF(I)2,2,3	DAGA2050
2	SODD(IT)=SODD(IT)+FVAL	DAGA2060
	GO TO 4	DAGA2070
3	SEVN(IT)=SEVN(IT)+FVAL	DAGA2080
4	CONTINUE	DAGA2090
C	ADVANCE TO NEXT ARGUMENT RO/R	DAGA2100
5	I=-I	DAGA2110
C	SIMPSON'S SUM	DAGA2120
	DO 6 IT=1,NT	DAGA2130
6	TERM(IT)=FACTOR*(SODD(IT)+2.0*SEVN(IT))	DAGA2140
	RETURN	DAGA2150
	END	DAGA2160
C		DAGA2170
	SUBROUTINE BDR(B,DRO)	DAGA2180
C	THIS SUBROUTINE INITIALIZES AND PREPARES INCREMENTS FOR THE DIF-	DAGA2190
C	FERENT EXPONENTIALS NEEDED IN ORDER TO COMPUTE THE PERTINENT	DAGA2200
C	HYPERBOLIC FUNCTIONS.	DAGA2210
C	WHEN USING A LOW VALUE OF R THE HYPERBOLIC FUNCTIONS SOON BECOME	DAGA2220
C	CLOSE TO CORRESPONDING HALF EXPONENTIALS. THE CRITERION USED IS	DAGA2230
C	ARG>60. ASSUMING THAT ARG*XL2,ARG*XL3 AND ARG*(1+Z) ARE >10.,	DAGA2240
C	THEN LABEL 2 IS PERFORMED.	DAGA2250
	COMMON/U3/R,R2,Z,XL2,XL3/U4/IND/U6/E(12,2)	DAGA2260
	TOL=60.0	DAGA2270
	D=B/R	DAGA2280
	IND=0	DAGA2290
	DO 3 I=1,2	DAGA2300
	C=DRO/R	DAGA2310
	IF(I.EQ.2) C=D	DAGA2320
	IF(D.GT.TOL) GO TO 2	DAGA2330
	E(1,I)=EXP(C)	DAGA2340
10	CONTINUE	DAGA2350
	E(3,I)=EXP(C*XL2)	DAGA2360
	E(5,I)=EXP(C*Z)	DAGA2370
11	CONTINUE	DAGA2380
	E(7,I)=EXP(C*XL3)	DAGA2390
	E(9,I)=E(1,I)*E(5,I)	DAGA2400
12	CONTINUE	DAGA2410
	E(11,I)=E(1,I)/E(7,I)	DAGA2420
	DO 1 J=2,12,2	DAGA2430
1	E(J,I)=1.0/E(J-1,I)	DAGA2440
13	CONTINUE	DAGA2450
	GO TO 3	DAGA2460
15	CONTINUE	DAGA2470
2	E(1,I)=EXP(C*(XL2+XL3-2.0))	DAGA2480
	E(2,I)=EXP(C*(XL2-XL3+Z))	DAGA2490
	E(3,I)=EXP(C*Z)	DAGA2500

(con't)
APPENDIX
DAGAN'S PROGRAM

14	CONTINUE	DAGA2510
	E(4,I)=1.0/E(3,I)	DAGA2520
	IND=1	DAGA2530
3	CONTINUE	DAGA2540
	RETURN	DAGA2550
	END	DAGA2560
C		DAGA2570
	SUBROUTINE FCIN(ARG,RO)	DAGA2580
C	THIS SUBROUTINE SHIFTS PREVIOUS EXPONENTIALS BY MEANS OF THE	DAGA2590
C	EXPONENTIAL INCREMENTS AND COMPUTES HYPERBOLIC FUNCTIONS NEEDED	DAGA2600
C	IT ALSO COMPUTES PARTS OF THE INTEGRAND	DAGA2610
C	FOR BIG VALUES OF ARG (IND=1) LABEL 3 IS PERFORMED.	DAGA2620
	COMMON/U4/IND/U6/E(12,2)/U7/F(9)/U9/T1,T2,T3	DAGA2630
	TOL=15.0	DAGA2640
	IF(IND.EQ.1) GO TO 3	DAGA2650
	DO 1 I=1,12	DAGA2660
1	E(I,2)=E(I,2)*E(I,1)	DAGA2670
	F(1)=E(1,2)-E(2,2)	DAGA2680
	F(2)=E(1,2)+E(2,2)	DAGA2690
	F(3)=E(3,2)-E(4,2)	DAGA2700
	F(4)=E(5,2)+E(6,2)	DAGA2710
	F(5)=E(7,2)+E(8,2)	DAGA2720
	F(6)=E(9,2)+E(10,2)	DAGA2730
	F(7)=E(11,2)+E(12,2)	DAGA2740
	F(8)=1.0	DAGA2750
	IF(ARG.LT.TOL) F(8)=F(1)/F(2)	DAGA2760
	F(9)=E(2,2)	DAGA2770
	DO 2 I=1,7	DAGA2780
2	F(I)=F(I)/2	DAGA2790
	T1=F(9)*F(4)*F(5)	DAGA2800
	T2=F(7)/F(2)*F(6)	DAGA2810
	T3=F(3)/F(1)/ARG*BESSEL(RO)	DAGA2820
	GO TO 5	DAGA2830
3	DO 4 I=1,4	DAGA2840
4	E(I,2)=E(I,2)*E(I,1)	DAGA2850
	F(8)=1.0	DAGA2860
	T1=0.25*E(1,2)*(E(3,2)+E(4,2))	DAGA2870
	T2=0.5*E(2,2)	DAGA2880
	T3=BESSEL(RO)/ARG	DAGA2890
5	RETURN	DAGA2900
	END	DAGA2910
C		DAGA2920
	SUBROUTINE BRACK(ARG)	DAGA2930
C	THIS SUBROUTINE COMPUTES THE TRANSIENT TERM OF THE INTEGRAND.	DAGA2940
	COMMON/U2/T(20),NT/U5/POINT(20)/U7/F(9)	DAGA2950
	TOL=150.0	DAGA2960
	DO 1 IT=1,NT	DAGA2970
	TR=ARG*T(IT)*F(8)	DAGA2980
	POINT(IT)=0.0	DAGA2990
	IF(TR.LE.TOL) POINT(IT)=EXP(-TR)	DAGA3000

