A STUDY OF CONTROLLING SALT WATER CONING IN AQUIFERS

by

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A DISSERTATION

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ABSTRACT

A mathematical expression was developed which would approximate the position and the shape of a sharp, upconed interface between fresh and salt water in an aquifer when the fresh water is being pumped. The computation of the interface shape was based on an empirically derived modification of Muskat's approximate formula for the upconed interface. Differing depths of well penetration and their influence on the upconed interface were investigated using this expression. Using the method of superposition this expression was then used to determine the effectiveness of two special pumping techniques, the Hydraulic Doublet and the Scavenger Well System, in controlling the upconing of the interface. The Hydraulic Doublet, a pumping drain and injection well below it, and the scavenger well system, a double pumping technique, are methods which would allow greater producing rates of the fresh water without contamination by the upconed salt water than would be possible by a pumping drain alone.

The computed results were compared with interfaces determined experimentally in a Hele-Shaw model. Close agreement was achieved for an interface which penetrated as much as 50% of the space between its initial position and the bottom of the well. Changing the producing interval of the well while keeping the pumping rate constant did not change the shape or position of the interface significantly. This also

held for varying penetrations for the hydraulic doublet and the scavenger well.

The computed interface for varying doublet injection rates also agreed quite well with the experiments. However, no reliable agreements were achieved for varying scavenger pumping rates. The computed interface also would not vary with the changing positions of the doublet or the scavenger well.

Even though the shape of the upconed interface below a well could be computed approximately, a refinement of the analytical methods used in deriving the mathematical expression for the shape of the interface is necessary before it can be applied reliably to the study of the effect of the special techniques.

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SYMBOL NOTATION

a	thickness of fresh water zone prior to pumping, L.
b	aquifer thickness; also used to designate spacing between plates of Hele-Shaw model, L.
c_i	constant, equal to $\gamma_i/K_i(\gamma_2 - \gamma_1)$, $L^{-1}T$.
C	constant
C_i	normalized constant, equal to c_i/a.
d	vertical distance between top of aquifer and the center of producing interval, L.
D	dimensionless d, equal to d/a.
f	specific discharge of special pumping technique, equal to q'/q.
F	dimensionless thickness of fresh water zone, equal to a/b.
G	Green's Function.
h	hydraulic head, L.
H	velocity potential, equal to Kh, L^2T^{-1}.
I	parameter defined by equation 23.
k	permeability of aquifer, L^2.
K	hydraulic conductivity of aquifer.
l	length of producing interval, equal to $l_2 - l_1$.
$l_{1,2}$	vertical distance from top of aquifer to top and bottom of producing interval respectively, L.
L	dimensionless l, equal to l/a.
n	porosity of aquifer.
p	fluid pressure, $ML^{-1}T^{-2}$.
q	specific discharge of pumping well, L^2T^{-1}.

Q well discharge, L^3T^{-1}.

s drawdown.

S storage coefficient, dimensionless.

$\underline{S}$ vertical distance of interface from bottom of aquifer, L.

t pumping time, T.

u,v,w, x,y,z, components of bulk fluid velocity, L^3T^{-1}.

$\underline{v}$ bulk fluid velocity, L^3,T^{-1}.

x horizontal distance from pumping well, L.

X dimensionless x, equal to x/a.

y horizontal distance perpendicular to x,z plane, L.

z vertical distance from top of aquifer, L.

Z dimensionless z, equal to z/a.

γ specific weight of fluid, $ML^{-2}T^{-2}$.

ζ vertical distance from aquifer top to interface, L.

Ƶ dimensionless ζ, equal to ζ/a.

μ dynamic viscosity, $ML^{-1}T^{-1}$.

ν kinematic viscosity, L^2T^{-1}.

Subscripts and superscripts:

i index for designating lighter or heavier water.

m index designating model parameter.

p index designating prototype parameter.

' parameter for special pumping technique.

1,2 indices designating lighter and heavier liquid.

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INTRODUCTION

Statement of the Problem

Fresh water and salt water, in a dynamic equilibrium, can exist as separated fluids in an aquifer. Being miscible liquids, they will be separated by a transition zone rather than a sharp interface. Dispersion by vertical displacement of this zone contributes to its width. However, as long as fresh water is discharged from the system some of the dispersion will be carried with it and a region of fresh water only will exist in the aquifer.

If the transition zone is small with respect to the system under consideration, it may be approximated by an interface (i.e., the fluids are effectively separated). When studying the upconing of salt water beneath a well, the extent of the transition zone most likely will be significant. In this case a line or surface of a particular salt-concentration may be thought of as an interface. Assuming then, that the transition zone may be represented by an abrupt interface, its shape and position can be determined exactly for a few two-dimensional problems and approximately for others.

In many instances, as in the case of a coastal aquifer, fresh water is pumped from aquifers which also contain salt water. Often the produced fresh water is contaminated or even made useless by the upconed salt water being drawn into the well. By proper manipulation of pumping and injection wells, this upconing of the salt water can be minimized while the flow of fresh water to the well is maximized.

Summary of Previous Investigations

The phenomenon of a transition zone or an interface in coastal aquifers and in oil reservoirs has been known and investigated for many years. Due to the nonlinearity of the boundary condition on the interface, an exact solution of this problem appears impossible except for a few steady two-dimensional cases.

Nomitsu (1927) combined the Ghyben-Herzberg relation with Darcy's law to obtain an approximate two-dimensional expression for the shape of the interface in a coastal aquifer of infinite lateral extent. This solution is valid when the vertical flow components can be neglected and was verified experimentally by Kawabata (1965). Henry (1959), using the hodograph plane, and assuming the fresh and salt water to be immiscible, derived exact solutions for the shape of the interface in a two-dimensional coastal aquifer. Rumer and Harleman (1963) experimentally confirmed Henry's solutions and studied the effect of dispersion on the interface.

Muskat (1946) treated the upconing of an initially horizontal interface towards a partially penetrating well. Meyer and Garder (1954) and Pirson (1958) derived approximate expressions for the maximum uncontaminated oil production rates from wells without drawing the underlying salt water into the well. Using various methods, Bear and Dagan (1962,1963,1964,1966a,1966b) in a series of investigations, made a comprehensive study of the interface movement and of pumping near the interface. They solved the steady two-dimensional interface problem by the use of the hodograph plane. Except for a few special cases this method results in integral solutions which are difficult, if not impossible to solve analytically. Verifying Nomitus's earlier approach, they showed that the Dupuit Approximations can be used with good results as long as the vertical flow components can be neglected. However, this often is not the case in the flow region below a drain.

The problem of upconing below a drain in an infinite two-dimensional aquifer was then investigated by Bear and Dagan (1968). Using the method of small perturbations, they obtained solutions for aquifers which are thick relative to the rises of the interface. A field test by Schmorak and Mercado (1969), supported the validity of this approach and verified the solution for transient rises of the interface up to 40% of the distance from the bottom of the drain to the initial interface position.

For relatively thin fresh water zones (as near the coast) the critical rise of the interface is correspondingly small and thus many wells become contaminated even at low pumping rates. Bardelli (1960) proposed the possibility of using an injection well below a pumping well to create a flow pattern which will prevent the salt water from reaching the pumping well while allowing larger fresh water pumping rates from it. This technique is called the Hydraulic Doublet and is shown schematically in figure 1a. Smith and Pirson (1963) showed experimentally that the doublet can be used successfully to control the coning of salt water in oil wells.

Van't Leven (1961) describes a double pumping technique to control the upconing of salt water while increasing the pumping rate of fresh water. The second well is called the Scavenger Well and is shown schematically in figure 1b. Long (1965) in a field test, has shown where a scavenger well was used successfully to prevent vertical salt water encroachment into a well producing fresh water only.

Due to the complexity of the problem, Bear and Dagan, (1966b) investigated these "special techniques" experimentally for a drain pumping from a coastal aquifer and presented their results in the form of charts. They define the efficiency of these schemes as the percentage increase of recovered fresh water in relation to the recovery without the special techniques. Based on the results of their earlier re-

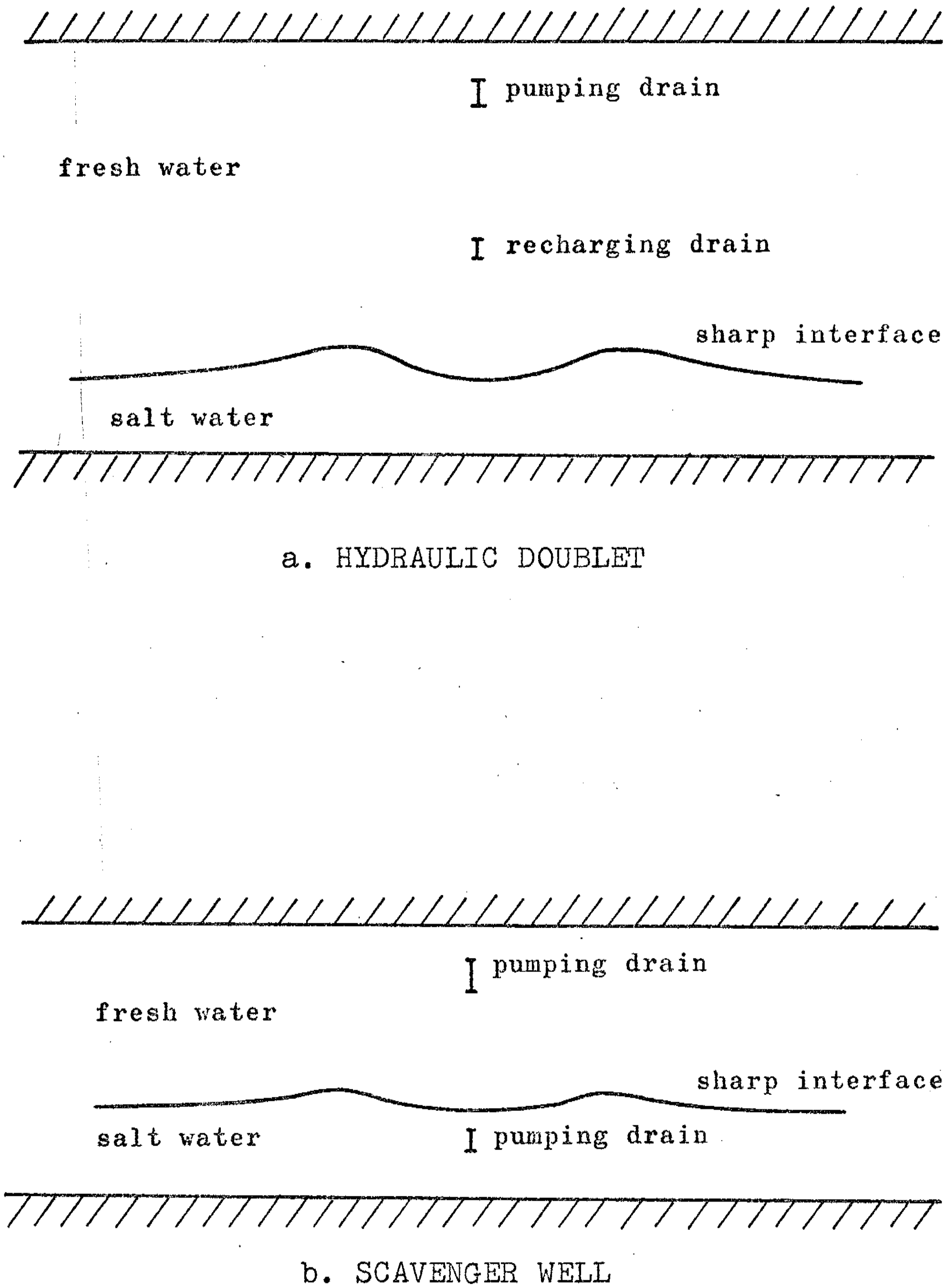


FIGURE 1: SCHEMATIC DRAWING OF EFFECT OF SPECIAL PUMPING TECHNIQUES ON UPCONED INTERFACE

search, they imposed the following constraints on their investigations:

a. The resulting rise of the interface shall be no more than 1/4 to 1/3 of the distance between the drain and the original position of the interface.

b. Continuous flushing of the transition zone to the sea must be maintained below the drain.

The two-dimensional case, shown in figure 2, was investigated with a Hele-Shaw model. An electrolytic tank was used for the three-dimensional experiments in which the spacing between wells parallel to the coast could be studied as an additional parameter. Two series of experiments were run for each case, one for the doublet and one for the scavenger well.

The following parameters (as defined in figure 2) were studied, with the thickness of the fresh water zone at the location of the drain (a) used to normalize the length parameters:

a. Distance of the drains from the toe of the interface; (x_o/a).

b. Location of the drains with respect to the original position of the interface; (d/a and d'/a).

c. Ratio of fresh water thickness at the toe and at the drains; (b/a).

d. Ratio of the flow rates; (Q'/Q).

Even though the results for the doublet were described as qualitative in nature, they did indicate that small addi-

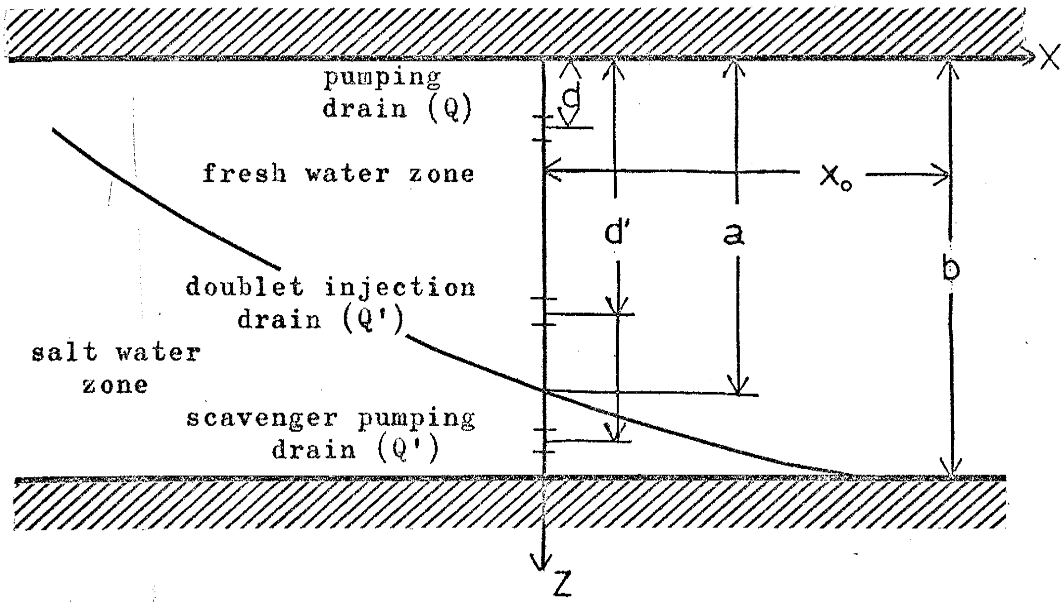


FIGURE 2: PARAMETERS FOR SPECIAL PUMPING TECHNIQUES

tional amounts of fresh water (up to 15%) may be recovered by this method. For the scavenger well the results were also qualitative. In this case, however, as much as 81 to 99% of the fresh water flowing to the sea could be recovered.

These experimental results are summarized in tables and curves which can be applied to various practical problems. The number of cases which may be treated in this manner is limited to the range of the parameters investigated. Additional experiments with models of different geometrical dimensions would be necessary to extend these results to other values of the parameters. For the cases investigated, Bear and Dagan found that the analogy of the interface to a free surface, computing its shape using the Dupuit Assumptions, may be used as a good approximation.

Objectives and Method of Investigation

The objective of this investigation is to study in two dimensions the special pumping techniques and derive approximate analytical relationships involving the location of the drains, their pumping rates and the equilibrium position of the interface for these techniques. Thin aquifers will also be considered where drains of finite penetration into the aquifers will be examined. Knowledge of these relationships will be useful in optimizing the recovery of fresh water overlying salt water.

Since exact results appeared impossible at this time, Muskat's approximate method was used as an initial solution. Attempts were made to improve on this method by various schemes such as iteration on the computed interface or through the use of an "average" position of the initial interface to minimize the deviations of the computed interface. In order to verify the assumptions made, the computed interface shapes were compared with the solutions obtained by other methods and with interfaces determined experimentally in a Hele-Shaw model.

The finite penetration of the drains into an aquifer were simulated by integration along a vertical coordinate. A uniform flux over the length of the extended sink was assumed. The effect on the resulting interface by changing the penetration of the drains was then examined.

Once the interface due to a pumping drain can be approximated, the effects of the doublet or scavenger systems should be obtainable by superposition.

FORMULATION OF THE PROBLEM

The Exact Equations

Since ground water flowing through an aquifer may be considered to be an incompressible fluid flowing through a porous medium, its motion can be expressed by Darcy's Law. In vector notation this law can be written as (DeWiest, 1969)

(1) $$\underline{q} = - K \text{ grad } h$$

where $\underline{q}$ is the specific discharge vector, K is the hydraulic conductivity and h is the hydraulic head or the energy per unit weight of the liquid. The specific discharge is defined as the discharge per unit cross-sectional area of the medium and is not the true velocity of the fluid in the pore spaces. The hydraulic conducticity is dependent on the properties of the medium and the fluid passing through it.

If the aquifer is homogeneous and isotropic, K is a constant for any one fluid and Darcy's Law may be written as

(2) $$\underline{q} = \text{grad } H = n.\underline{v}$$

where H = Kh and is called the velocity potential $[L^2T^{-1}]$, n is the porosity of the medium and $\underline{v}$ is the bulk or average fluid velocity in the pore spaces. The bulk velocity may be thought of as the microscopic velocities of the fluid particles averaged over the medium and it is related to the specific discharge by the porosity of the medium. The derivation of a potential function is desirable because it makes the methods of hydrodynamics applicable to the problems of flow through porous media.

Conservation of mass for an incompressible fluid requires that

(3) $$\text{div}\ \underline{q} = 0$$

Inserting equation (2) into this expression and considering only the steady state flow results in

(4) $$\nabla^2 H = 0$$

which is Laplace's equation. In hydrodynamics the satisfaction of this equation by a potential function is associated with irrotational flow. The viscous flow of the fluid particles through the pore spaces does not meet this condition. In averaging these velocities over the porous medium, however, the rotational motions would cancel one another, resulting in the bulk velocity, which can be thought of as being irrotational. Equation (4) thus describes the steady flow of an incompressible liquid through an homogeneous and isotropic medium. The solution of this equation requires the proper boundary conditions such as the mathematical description of potential or flux along the borders of the flow domain and the location and strengths of any sources or sinks within it.

When two miscible fluids, such as fresh and salt water are present in a porous medium, the problem becomes much more complicated. Some simplifications, however, are possible under the assumptions that these liquids are immiscible and occupy different regions of the medium. To illustrate how this can be done, consider a confined aquifer of uniform thickness containing two fluids of different densities, which are separated by a sharp interface, as shown in figure 3. The velocity potential in each region is given by

$$(5) \qquad H_i = K_i h_i = (k\gamma_i/\mu_i)(\ p/\gamma_i - z) \qquad i = 1,2$$

where k is the (intrinsic) permeability of the aquifer, γ_i and μ_i are the specific weight and dynamic viscosity of the i-th fluid respectively, and p is the pressure (Bear and Dagan 1968).

The moving interface is a "material surface" which is always composed of the same fluid particles. Let the expression describing this interface be (Bear and Dagan 1968)

$$(6) \qquad \underline{S}(x,y,z,t) = z - \zeta(x,y,t) = 0$$

where ζ is the z-coordinate of the interface. The pressures along this interface are continuous or

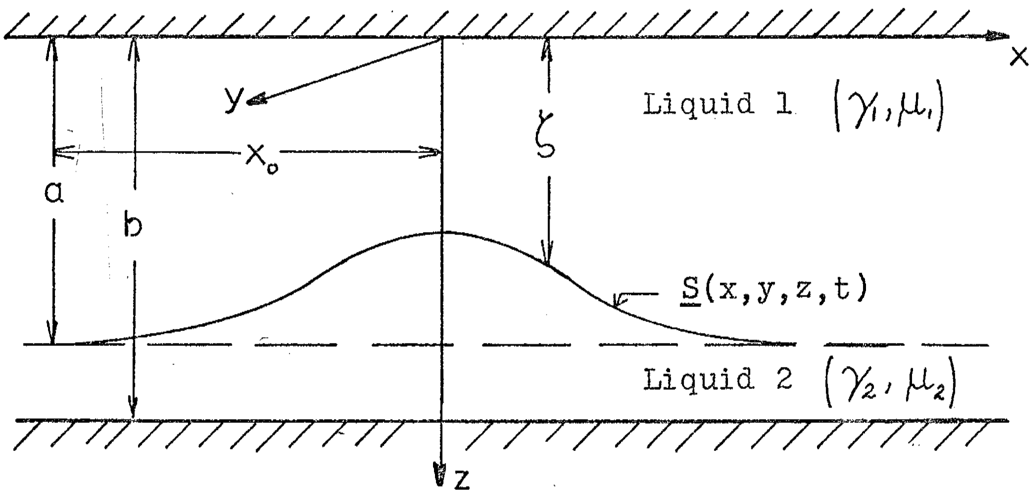


FIGURE 3: A SHARP INTERFACE BETWEEN TWO LIQUIDS IN A CONFINED AQUIFER

$$p_1 = p_2 + p_c \tag{7}$$

where p_c is the capillary pressure across the interface. The capillary pressure between fresh and sea water is small and may be neglected. Expressing the pressures on either side of the interface in terms of the hydraulic heads in each region, substituting these values into equation (7) and solving for ζ yields

$$\zeta = \gamma_1 h_1/(\gamma_2 - \gamma_1) - \gamma_2 h_2/(\gamma_2 - \gamma_1) \tag{8}$$

In terms of the velocity potentials of each region, the position of the interface may be expressed by

$$\zeta = c_1 H_1 - c_2 H_2 \tag{9}$$

where $c_1 = \gamma_1/K_1(\gamma_2 - \gamma_1)$ and $c_2 = \gamma_2/K_2(\gamma_2 - \gamma_1)$.

Since on either side of the interface each liquid always contains the same particles, the total derivatives are zero or

$$d\underline{S}/dt = 0 = \partial\underline{S}/\partial t + v_i \cdot \text{grad } \underline{S} \qquad i = 1,2 \tag{10}$$

Inserting the equation of the interface (equation 6) into this expression yields

$$\partial\zeta/\partial t + u_i\partial\zeta/\partial x + v_i\partial\zeta/\partial y - w_i = 0 \qquad i = 1,2 \tag{11}$$

where u_i, v_i and w_i are the components of the bulk velocity. In terms of the specific discharge this equation becomes

$$(12) \quad n\partial\zeta/\partial t + q_{xi}\partial\zeta/\partial x + q_{yi}\partial\zeta/\partial y - q_{zi} = 0 \qquad i = 1,2$$

Both ζ and $\underline{q}$, however, are functions of the velocity potential (equations 2 and 9). Substituting these expressions for ζ and $\underline{q}$ into equation (12), one obtains, after some rearranging,

$$(13) \begin{cases} nc_2(\partial H_2/\partial t)-nc_1(\partial H_1/\partial t)+c_1[(\partial H_1/\partial x)^2+(\partial H_1/\partial y)^2] - \\ - c_2[(\partial H_1/\partial x)(\partial H_2/\partial x)+(\partial H_1/\partial y)(\partial H_2/\partial y)]-\partial H_1/\partial z = 0 \\ \\ nc_2(\partial H_2/\partial t)-nc_1(\partial H_1/\partial t)-c_2[(\partial H_2/\partial x)^2+(\partial H_2/\partial y)^2] + \\ + c_1[(\partial H_1/\partial x)(\partial H_2/\partial x)+(\partial H_1/\partial y)(\partial H_2/\partial y)]- \partial H_2/\partial z = 0 \end{cases}$$

This expresses the condition on the interface separating the two flow regions. Due to the nonlinearity of this boundary condition, the solution to the equation of flow (equation 4) is very complicated if not impossible with present mathematical methods. Solutions to this problem may be obtained, however, by making some simplifying assumptions and approximations.

Before proceeding, it will be convenient to normalize the length parameters which appear in the determinations of the shape of the interface. This will also facilitate any practical applications which might result from this investigation. The thickness of the zone occupied by the lighter liquid prior to upconing in figure 3, is a suitable dimension for this purpose in most cases. For an assumed horizontal interface, this will also be referred to as the initial interface position. For other situations a position on the original interface may be chosen where the pumping of a well will not have an effect on the interface. In this study the dimension x_o is at a point where fresh water is recharged and the head remains constant. Since most practical applications of two liquids with different densities in an aquifer are concerned with a fresh and sea water system, the two liquids will generally be referred to as fresh and salt water. Henceforth in this study, capital letters (including Greek letters) will designate normalized parameters; for example, $X = x/a$, $Z = z/a$ and $Z = \zeta/a$. For relatively thin aquifers, especially when the salt water zone is not very thick, the total thickness of the aquifer, b, may be used to normalize the length parameters. In this case the initial interface position may be expressed as a fraction of the aquifer thickness.

Muskat Approximations

Being concerned with keeping the upconing of a salt

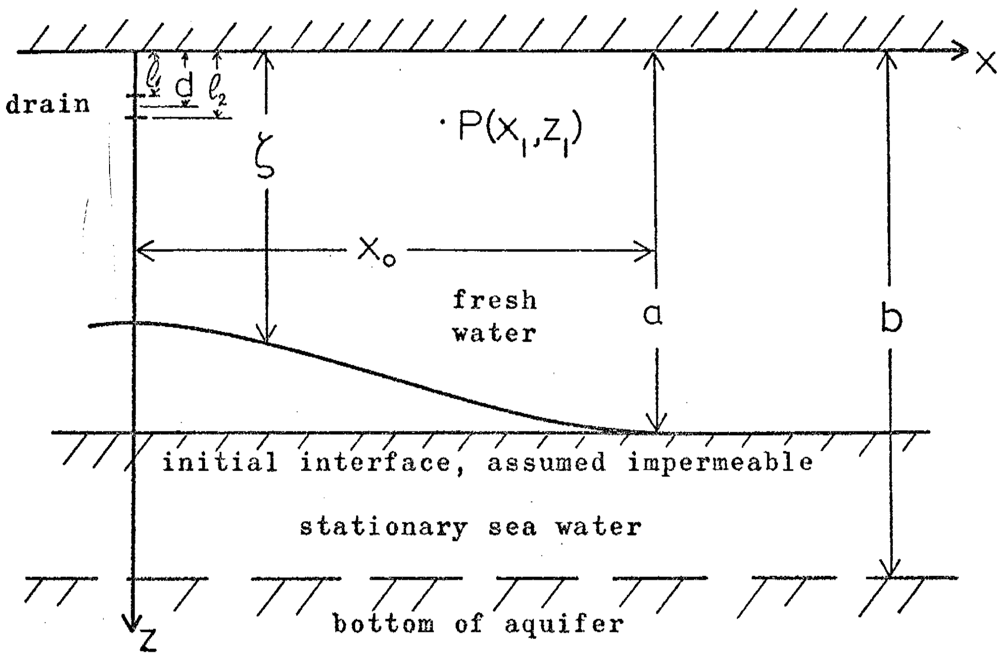


FIGURE 4: MUSKAT'S APPROXIMATION OF THE UPCONED INTERFACE

water-oil interface in an oil well to a minimum, Muskat (1946) made the following assumptions in his treatment of this problem:

a. The denser liquid remains in a static equilibrium below the upconed interface.

b. The potential distribution in the lighter liquid is not affected by the upconing of the denser liquid.

To illustrate this method, consider the two dimensional, homogeneous and isotropic aquifer shown in figure 4. A static sea water zone implies that H_2 in equation 9 is constant and thus the position of the interface will be given by

$$(14) \qquad Z = C_1H_1 - C$$

where $C_1 = c_1/a$ and C is determined from the initial hydrostatic head at the interface. Assuming that the interface was initially horizontal and that at some value of $X = X_o$ it will remain so, C may be evaluated as

$$(15) \qquad C = C_1H_{1,0} - 1$$

Substitution of C into equation 14 yields

$$(16) \qquad Z = 1 - C_1(H_{1,0} - H_1) = 1 - C_1K(h_{1,0} - h_1)$$

which relates the position of the interface to the potential difference or the drawdown, $s = h_{1,0} - h_1$, in the fresh water region.

With the assumption that the potential distribution in the fresh water zone is not affected by the upconed sea water, the initial interface will form a horizontal and impervious boundary to the flow above it. Thus the fresh water region may be thought of as a separate horizontal and confinedaquifer. This will also apply to an unconfined aquifer for steady state conditions when the water table has a very flat slope.

The investigations for this study will be made for a two-dimensional aquifer (in the x and z direction, z = + down) of uniform thickness because a Hele-Shaw model will be used to verify the assumptions made and the results obtained. The methods used, however, should also be applicable to the three-dimensional flow to wells. Smith and Pirson (1963) found that the results of the two-dimensional model techniques may be used for qualitative conclusions regarding upconing in a radial system.

The potential distrubution in the interior of a region due to specified flux conditions on its boundaries con often be solved for by the use of Green's Functions. The Green's Function for the infinite strip with impermeable boundaries shown in figure 4, having a singularity at x_1, z_1 is given by

$$(17) \quad G(x_1,z_1,x,z) = -\tfrac{1}{2}\ln\left\{[\cosh\pi(x-x_1)/b - \cos\pi(z+z_1)/b]\cdot[\cosh\pi(x-x_1)/b - \cos\pi(z-z_1)/b]\right\}$$

This expression was obtained from Hantush and Jacob (1954) by shifting the coordinate system and letting the aquifer thickness be b instead of 2a. Letting the singularity be a drain or a line sink extending into the page, located at (0,D), then the potential difference, expressed as the drawdown, due to this drain at any point in the aquifer is given by

$$(18) \qquad s = -(q/2\pi Ka)(\tfrac{1}{2})\ln\left\{[\cosh\pi X - \cos\pi(Z+D)] \, [\cosh\pi X - \cos\pi(Z-D)]\right\}$$

Here q is the discharge per unit width of the system and it is negative for a sink and the coordinates X and Z have been normalized by the thickness of the fresh water zone a.

The drawdown between any two points in an aquifer is the difference in the two corresponding values of s. For values of X>2 the cosine terms in equation (18) become negligible with respect to the cosh terms. Using this relationship to express the potential difference in equation (16) and after some rearranging one has

$$(19) \qquad \mathcal{Z} = 1 - (C_1 q/4\pi)\left\{2\ln[\cosh\pi X_0] - \ln[\cosh\pi X - \cos\pi(Z+D)] \cdot [\cosh\pi X - \cos\,(Z-D)]\right\}$$

as the equation for the upconed interface for a given point X and Z in the aquifer. Ideally, the value of Z to be used in this equation would be the coordinate of the interface ($Z = \mathcal{Z}$), which is unknown.

As an approximation, the shape of the interface may be computed by assuming values for Z. Letting $Z = 1$ in equation (19) will be equivalent to computing the drawdowns along the initial position of the interface. In this case the interface will be given by

$$\mathcal{Z} = 1 - (C_1 q/2\pi) \left\{ \ln[\cosh \pi X_o] - \ln[\cosh \pi X + \cos \pi D] \right\} \tag{20}$$

The height of the cone below the drain ($X = 0$) is

$$\mathcal{Z} = 1 - (C_1 q/2\pi) \left\{ \ln[\cosh \pi X_o - \ln[1 + \cos \pi D] \right\} \tag{21}$$

Since it has been assumed that the upconing does not appreciably alter the drawdowns in the fresh water zone, similar to the Dupuit assumptions, equations (20) and (21) are valid only for small rises of the interface. Bear, Zaslavsky and Irmay (1968) estimate that this approach is valid only when the upconing is less than 20% of the initial distance between the horizontal interface and the sink. Because of this small range of applicability, Muskat's approximation is not suitable to compute the highest stable rise of the upconed interface without salt water entering the well. Since the objective of this study is to minimize the upconing of the interface by the use of special pumping techniques, use of these assumptions appears justified when the resultant rise of the interface does not greatly exceed 20% of the initial distance between the horizontal interface and the sink.

Improvements on Muskat's Approximation

Since the upconing salt water actually reduces the cross-sectional area of fresh water flow as the drain is approached, the interface computed by using Muskat's approximation will be too low. An improved result for the computed interface could be obtained if Z in equation (19) had a value less than one. As a first approximation one might use the value of Ƶ calculated from equation (21), as Z in equation (19) and compute a new value of Ƶ. This process may be repeated until the value of Z used is identical to the resulting value of Ƶ within a predetermined error. Since a digital computer will be used to perform the calculations to determine the interface, it will also be used for this iteration process.

Another method to improve the results of the Muskat approximation is to assume an average for the cross-sectional area of flow. This average may be thought of as an initial interface located above the actual interface. In this case the value used for Z in equation (19) to compute the position of the interface is less than one. Intuitively, the position of this fictitious initial interface would depend on the height of the resulting cone. One can therefore assume the elevation of the false interface to be a fraction of the rise of the cone as computed by Muskat's approximation. The value of this fraction will have to be determined empirically. As a first approximation one may assume this fraction to be ½ or 1/3 of the height of the computed interface. This implies that the upconed interface has a limited horizontal extent, similar to the toe of the intruded sea water in a coastal aquifer.

Finite Interval of Drain Penetration

So far, in the development of the problem, the drains have been represented by line sinks in the two-dimensional flow field. This might be a good approximation where the thickness of the fresh water zone is relatively large with respect to the length of the aquifer and the penetration of the drain is small. When this is not the case, however, the penetration of the sink into the aquifer may be significant.

The potential distribution in a region due to a system of sinks may be expressed by superposition as the sum of the potentials contributed by each sink separately. This also applies to the normal derivatives of the potentials. Thus one can visualize a drain of finite penetration as a series of line sinks (a plane in a two-dimensional flow domain). The strength of each of these line sinks may be approximated by a flux distributed uniformly over the producing interval. The potential distribution in the flow domain is obtained by integrating the line sinks over the interval $L_2 - L_1$.

For the system shown in figure 4, integration of equation (19) results in

$$(22) \qquad \Phi = (C_1 q/2\pi)\ln(\cosh\pi X_o) - (C_1 q/4\pi L)\int_{L_1}^{L_2} \ln\Big\{[\cosh\pi X - \cos\pi(Z+D)] \cdot [\cosh\pi X - \cos\pi(Z-D)]\Big\} dD$$

where $L = L_2 - L_1$ and D is now taken at the center of this interval.

A closed form for this integral was not found. However, a solution by digital computer converged rapidly. Since the form of the integral does not change significantly, it will be represented for the remainder of this study by

$$(23) \qquad I(X,Z,D,L_1,L_2) = (1/L)\int_{L_1}^{L_2} \ln\Big\{[\cosh\pi X - \cos\pi(Z+D)] \cdot [\cosh\pi X - \cos\pi(Z-D)]\Big\} dD$$

Equation (22) can be used to investigate the effect, if any, of partial penetration on the shape of the upconed interface.

SPECIAL TECHNIQUES FOR CONTROL OF UPCONING

Description of the Special Techniques

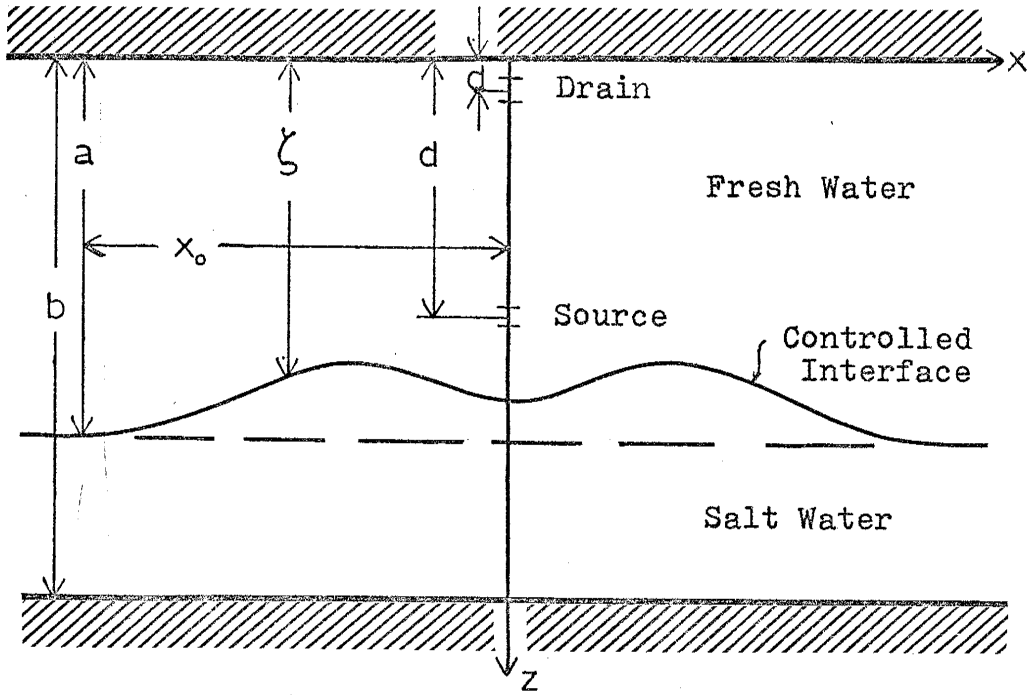
In many instances, such as in coastal aquifers, it is necessary to pump fresh water overlying salt water. Under these conditions the upconing of the salt water and the subsequent contamination of the produced fresh water can become serious. The problem then becomes one of controlling or minimizing the resulting cone through the use of special pumping techniques, if possible.

Generally these techniques fall into two types. In the first, part of the produced fresh water is injected below the pumping interval and above the original position of the interface. This method is known as the hydraulic doublet. In the second technique, salt water is produced through a second drain located below the upconed interface and is intercepted before it reaches the fresh water drain. This system of pumping is known as a scavenger system. Both of these methods have their advantages and disadvantages and are best evaluated for each particular case. A third method would be a combination of the above two. This system could be investigated by superposition of the hydraulic doublet and the scavenger well and thus it is not considered as a separate case in this report.

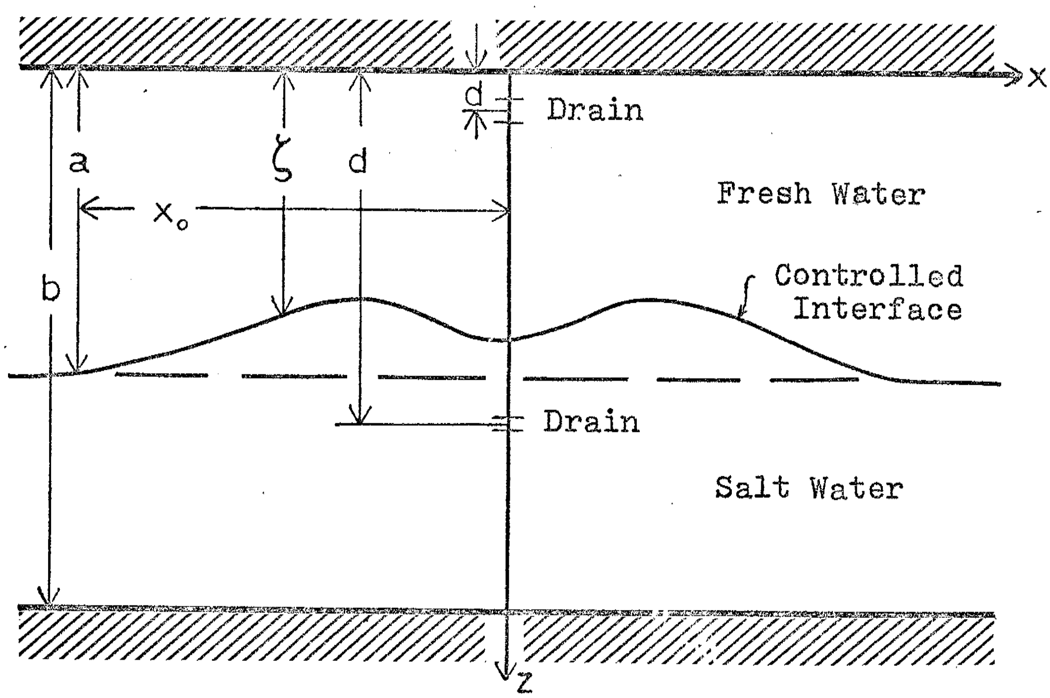
In this study the expressions developed for the special pumping techniques will consider only drains located on the z-axis. The methods, however, should be applicable to more complex systems where this is not the case. In that situation, the x-coordinate of either one of the pumping intervals (X_1 or X_1') does not drop out of the cosh terms in equation (17) and thus must be carried through in the development of the subsequent expressions for the shape of the interface.

The Hydraulic Doublet

The pumping of a partially penetrating drain causes vertical potential differences beneath the drain. Injection of fresh water at a point between the drain and the initial interface position will reduce this potential difference and thus should result in a smaller rise of the upconed interface beneath the drain. This is illustrated schematically in figure 5a. Equation (16) expresses the position of the upconed interface in terms of potential differences in the fresh water zone. The resultant potential distribution in the fresh water zone for the doublet system is obtained as the sum of the potential distribution due to the sink and the potential distribution due to the source. Superposition of these expressions yields the equation of the interface for the doublet system as



a. HYDRAULIC DOUBLET



b. SCAVENGER WELL SYSTEM

FIGURE 5: SCHEMATIC DIAGRAM OF AN INTERFACE CONTROLLED BY SPECIAL PUMPING TECHNIQUES

$$(24) \quad \mathcal{Z} = 1 - [C_1(q-q')/2\pi][\ln \cosh \pi X_o] - (C_1 q/4\pi)\ln\left\{[\cosh\pi X - \cos\pi(Z+D)][\cosh\pi X - \cos\pi(Z-D)]\right\} + (C_1 q/4\pi)\ln\left\{[\cosh\pi X - \cos\pi(Z+D')][\cosh\pi X - \cos\pi(Z-D')]\right\}$$

where the primes denote values for the source which has a positive q and which is assumed to be on the z-axis also. Letting q' be a fraction of q or

$$(25) \quad q' = f \cdot q$$

the height of the cone below the doublet (X=0) is given by

$$(26) \quad \mathcal{Z} = 1 - [C_1 q(1-f)/2\pi][\ln \cosh \pi X_o] - (C_1 q/4\pi)\left\{\ln[1-\cos\pi(Z+D)][1-\cos\pi(Z-D)] + f \cdot \ln[1-\cos\pi(Z+D')][1-\cos\pi(Z-D')]\right\}$$

Using equation (23) the equivalent expressions for finite intervals of the doublet system are

$$(27) \quad \mathcal{Z} = 1 - [C_1 q(1-f)/2\pi][\ln \cosh \pi X_o] - (C_1 q/4\pi)[I(X,Z,D,L_1,L_2)/L + f \cdot I(X,Z,D',L_1',L_2')/L']$$

and

$$(28) \quad Z = 1 - [C_1 q(1-f)/2\pi][\ln \cosh \pi X_o] - (C_1 q/4\pi)[I(0,Z,D,L_1,L_2)/L + f \cdot I(0,Z,D',L_1',L_2')/L']$$

It should be pointed out here that for Muskat's approximation, Z in equations (24) and (26) has a value of 1. Some other value, however, might be more advantageous and this value will have to be determined experimentally. When using these equations one must be careful to avoid points of singularity such as Z=D or Z=D' when X=0.

The shape of the upconed interface for various pumping rates and drain locations may be computed using equations (24) and (26). Thus one should be able to compute the optimum pumping rates for a given doublet configuration or conversely the best doublet locations may be chosen for given pumping rates.

The Scavenger Well System

In the scavenger well system the vertical potential gradients due to the fresh water drain are reduced by pumping some of the sea water from below the upconed interface, as shown in figure 5b. The scavenger well may be located anywhere within the salt water region, but a higher position within the upconed zone would be more advantageous, even to the point of producing contaminated fresh water instead of sea water only. This would be especially beneficial when no contamination should reach the fresh water drain.

Since some of the salt water is produced by this pumping scheme, the assumption of a static equilibrium for the salt water is no longer valid and H_2 in equation (9) is not constant. Therefore, the potential distribution in the entire aquifer must be considered for this case and the interface between the two liquids may be thought of as a dividing streamline. In order to use Green's Functions to express the potential distribution in the aquifer, one must be able to neglect the density difference between the two liquids. Since sea water has a specific gravity of approximately 1.03, this assumption does not appear to be unreasonable.

The equation for the upconed interface is given by

$$\text{(29)} \qquad Z = F - C_1 K_1 (h_{1,0} - h_1)$$

where $F = a/b$ and the length parameters have been normalized by the aquifer thickness b. This equation is similar to equation (16) except that the computation of the interface is carried out along the initial position of the interface instead of the bottom of the aquifer. Similar to the development of the expression of the upconed interface for the hydraulic doublet, the potential distribution in the aquifer is obtained by superposition of the drawdowns due to the drain and due to the scavenger. The drawdowns for the drain are computed from equation (18) as if the aquifer contained fresh water only. Since the scavenger produces mostly salt water, its potential distributions are calculated by equation (30) as if the aquifer contained salt water only;

$$(30)\quad s = -(q'/4\pi K_2 b)\ln[\cosh\pi X - \cos\pi(Z+D)][\cosh\pi X - \cos\pi(Z-D)]$$

Substitution of the combined drawdowns into equation (29) yields

$$(31)\quad \not{z} = F - [(C_1 q + C_2 q')/2\pi]\ln(\cosh \pi X_o) - \\ -(C_1 q/4\pi)\ln\left\{[\cosh\pi X-\cos\pi(F+D)][\cosh\pi X-\cos\pi(F-D)]\right\} - \\ -(C_2 q'/4\pi)\ln\left\{[\cosh\pi X-\cos\pi(F+D')][\cosh\pi X-\cos\pi(F-D')]\right\}$$

where the primes denote the corresponding values for the scavenger drain and $C_2=\gamma_2/bK_2(\gamma_2-\gamma_1)$. The other terms have been defined previously. Letting X=0 simplifies equation (31) to

$$(32)\quad \not{z} = F - [(C_1 q + C_2 q')/2\pi]\ln(\cosh \pi X_o) - \\ -(C_1 q/4\pi)\ln\left\{[1-\cos\pi(F+ D)][1-\cos\pi(F - D)]\right\} - \\ -(C_2 q'/4\pi)\ln\left\{[1-\cos\pi(F+D')][1-\cos\pi(F-D')]\right\}$$

as the expression for the height of the upconed interface between the two drains. Using the notation of equation (23) these equations become

$$(33) \qquad Z = F - [(C_1 q + C_2 q')/2\pi]\ln(\cosh \pi X_o) - (C_1 q/4\pi L) I(X,F,D,L_1,L_2) - (C_2 q'/4\pi L') I(X,F,D',L_1',L_2')$$

and

$$(34) \qquad Z = F - [(C_1 q + C_2 q')/2\pi]\ln(\cosh \pi X_o) - (C_1 q/4\pi L) I(0,F,D,L_1,L_2) - (C_2 q'/4\pi L') I(0,F,D',L_1',L_2')$$

for drains of finite penetration into the aquifer. As with the hydraulic doublet, points of singularity must be avoided when making these computations.

For various pumping rates, as well as alternate locations of drain and scavenger, the position and shape of the resulting interface may be approximated using the equations developed here. For given circumstances, selection of the optimum conditions of discharge and drain positions should be possible, using these equations.

EXPERIMENTAL STUDIES

Model Analogy and Scaling

The analogy of the saturated flow of water through porous media with the laminar flow of viscous fluids through a capillary space is based on the similarity of the differential equations describing these flows. Hele-Shaw, in the 1890's, was the first to use two closely-spaced parallel plates to study flow patterns around variously shaped bodies. Bear (1960) presents a detailed mathematical analysis and proof of this similitude.

The velocity of saturated ground water is given by the general form of Darcy's law (equation 1).

(35) $$q = - K \text{ grad } h$$

The average velocity of an incompressible viscous liquid flowing between two parallel plates as derived from the generalized Navier-Stokes equation is (Bear, Zaslavsky and Irrmay, 1968)

(36) $$q_x = K_m \partial h/\partial x \quad \text{and} \quad q_z = K_m \partial h/\partial z$$

where $K_m = gb_m^2/12\nu$, b_m = plate spacing, g is the acceleration of gravity, ν is the kinematic viscosity $[L^2T^{-1}]$ of the liquid in the model and the subscript m denotes the model analog. K_m may be thought of as the hydraulic conductivity of the model. Comparison of these velocity equations (35 and 36) together with the similar geometry of the two systems establishes the kinematic similitude between the model and the aquifer prototype.

The two-dimensional differential equation of confined flow of groundwater is given by

$$(37) \qquad K_{xp}\partial^2 h_p/\partial x_p^2 + K_{zp}\partial^2 h_p/\partial z_p^2 = S_p \partial h_p/\partial t$$

where p is the subscript denoting the prototype aquifer and respectively K_{xp} and K_{zp} denote the hydraulic conductivity in the horizontal and vertical directions. The differential equation governing the flow in a Hele-Shaw model is (Bear, 1960)

$$(38) \qquad K_m(\partial^2 h_m/\partial x_m^2 + \partial^2 h_m/\partial z_m^2) = S_m \partial h_m/\partial t_m$$

Again similar differential equations describe model and prototype flows and thus show that the mass distribution is similar. This establishes dynamic similtude when one neglects surface tension forces.

The viscous flow model represents flow in an isotropic medium. However, by proper scale distortion an anisotropic flow domain may be simulated by the Hele-Shaw model (DeWiest

1965).

When the two homogeneous sets of equations are established for the prototype and the model analog, scaling is most readily accomplished by the method of inspectional analysis, which is described by Bear, Zavslavsky, and Irmay, (1968). This method requires that corresponding equations become proportional after substitution of model ratios. For example, the discharge in a confined aquifer of uniform thickness y as obtained from Darcy's law is

(39) $$Q_{xp} = - K_{xp} y_p b_p \partial h_p / \partial x_p$$

and

(40) $$Q_{xm} = - K_m y_m b_m \partial h_m / \partial x_m$$

for the prototype and the model respectively. By substitution of the model ratios

(41) $$x_r = x_m/x_p \qquad y_r = y_m/y_p \qquad b_r = b_m/b_p$$

$$h_r = h_m/h_p \qquad K_{xr} = K_{xm}/K_{xp} \qquad Q_{xr} = Q_{xm}/Q_{xp}$$

into equation (39) to eliminate the terms containing p, one has upon rearranging

(42) $$Q_{xm} = - (Q_{xr} x_r / K_{xr} y_r b_r h_r) K_{xm} y_m b_m \partial h_m / \partial x_m$$

For equation (42) to be identical to equation (40), the term in the brackets must equal to 1 and solving for Q_{xr} gives the ratio of the discharges in terms of other ratios.

(43) $$Q_{xr}=K_{xr}y_r b_r h_r/x_r$$

Other independent equations will yield more such conditions.

If the number of parameters involved (variables and constants) is A and the number of conditions of these parameters is B(<A), then A-B scales of the parameters may be chosen as desired, being careful to observe the imposed conditions. The remainder of these parameters can then be determined by the derived conditions. No particular aquifer was modeled in this study. Since an existing model was used for the experiments, the parameters for the computations of the upconed interface were determined from the model ratios.

Design of the Hele-Shaw Model

A schematic drawing of the Hele-Shaw model used in the present investigation is shown in figure 6. The plates of the model were made of plexiglass with the following dimensions: length 243.5 cm, height 45.7 cm and thickness 1.27 cm. These plates were held together by screws (#10-24, 3.8 cm long) on approximately 15.5 cm centers except along the bottom where they were 7.75 cm apart. Brass washers, 1.1 cm in diameter and 0.094 $\pm$ 0.002 cm thick, all being checked with a micrometer, were used as spacers separating the plates. A spacer was placed on each screw. The bottom and top of the model were sealed by several layers of Devcon Liquid Rubber applied across the open space between the plates. Even though some of the rubber entered into this space, this penetration was small (generally not more than 2 mm in isolated spots) and it did not interfere significantly with the flow pattern in the model. The top of the plates had been bevelled inward at a 45^{o} angle for a previous experiment which resulted in an effective depth of flow of 44.5 cm. The plates stood on small wooden blocks which were placed on a level steel plate. Four braces were attached to the plate to support the model in a vertical position.

The well end of the model was constructed so that fluid could be withdrawn or injected into selected intervals. These intervals were cut at the desired locations into a soft Neoprene rubber gasket (0.3 cm thick) used to seal the end of the plates. The openings in the seal connected to a groove, 0.95 x 0.64 cm, which was machined into a 46 cm x 3.9 cm x 2.5 cm plexiglass block (see fig. 6a). This groove led to the various discharge ports and could be blocked off so that no connection existed between these ports. The plexiglass block was pressed against the plates with set screws to exert pressure on the gasket between them. The discharge openings could be changed by removing the block

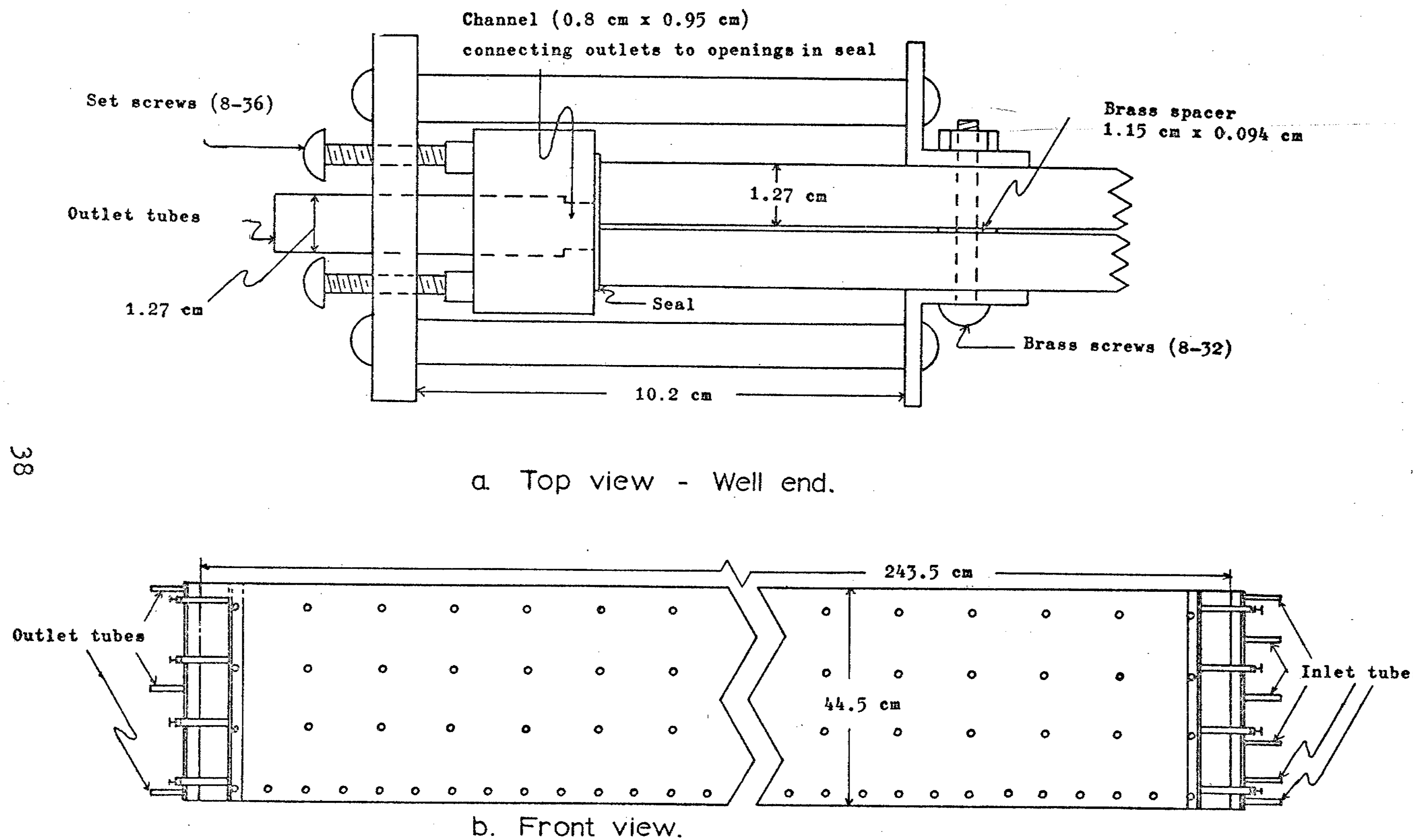


FIGURE 6: SCHEMATIC DRAWING OF THE HELE-SHAW MODEL

and cutting an altered configuration into the seal and then reassembling the block. The flow of fluids from and into the well end of the model was controlled by two positive displacement pumps (Fluid Metering, Inc. Lab Pumps) to assure constant discharge and injection rates. The discharge was measured with a 1 liter graduate cylinder and a stop watch. Figure 7 schematically shows the supporting equipment and flowpaths.

Fluids entered the model at the other end under a constant head. Construction was similar to the well and except that the groove had dimensions of 1.27 x 1.27 cm and was connected to the entire model. This groove was joined to a constant head device by 1.27 cm diameter plastic tubing connections to minimize any pressure losses. To separate the fresh and sea water, a small rubber block was placed in the groove at the desired level. The sea water was introduced below this block from a separate constant head reservoir through the remaining plastic connectors. Fresh water only could be used in the model by simply joining the salt water hoses to the fresh water reservoir.

The fresh water in the model was distilled water which was recirculated. The overflows and discharges were collected in a plastic holding tank from which a small circulating pump resupplied the distilled water to the constant head device. Small amounts of Clorox were added to the tank to prevent any organic growth. The sea water was represented

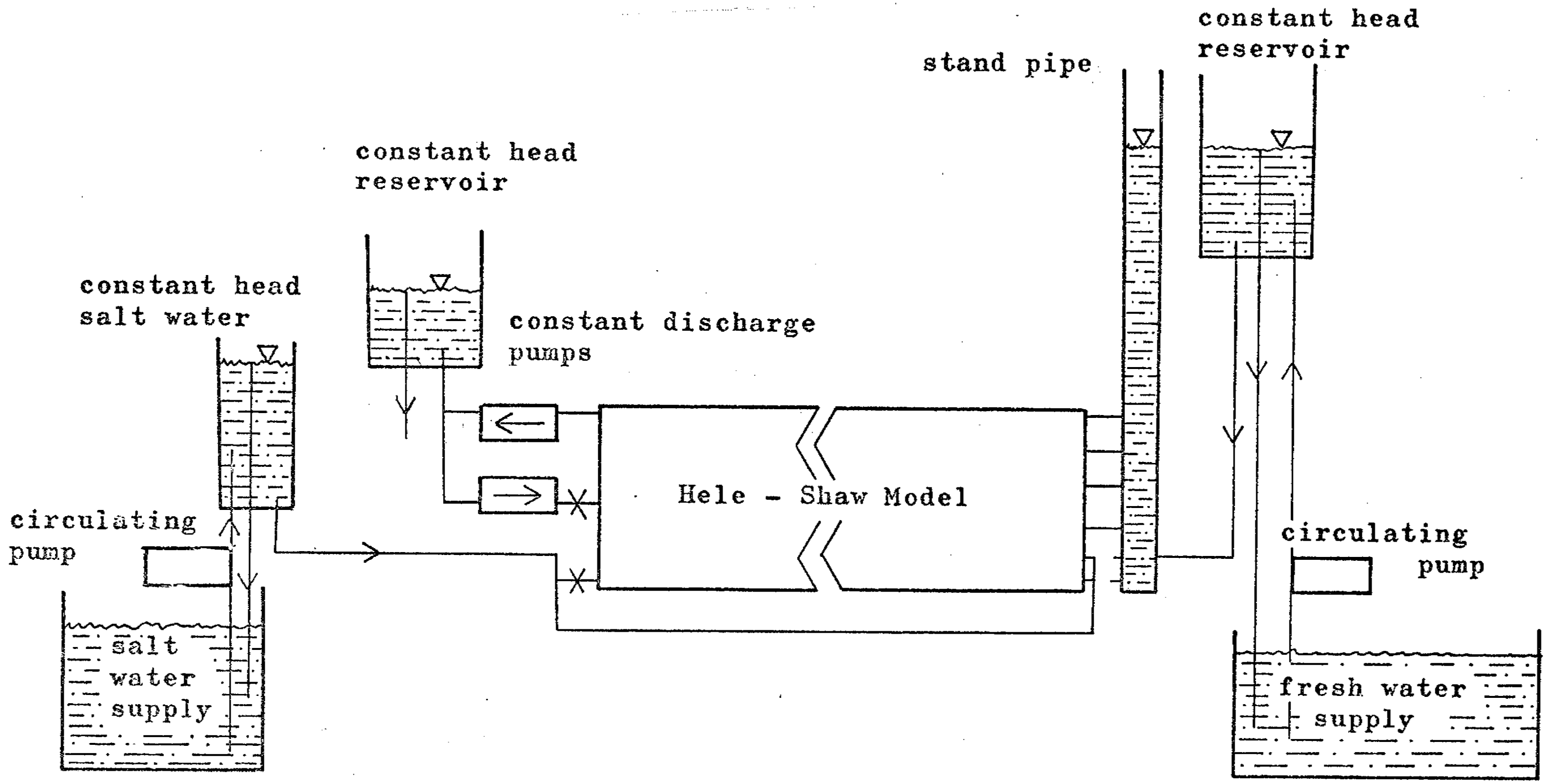


FIGURE 7: FLOW DIAGRAM AND SCHEMATIC SUPPORT EQUIPMENT FOR HELE-SHAW MODEL

by a NaCl solution to which red or blue food coloring was added for visibility. The salt water was mixed to a specific gravity of about 1.09. This heavier fluid was desirable to obtain a stable cone height in the model for the rates at which the pumps could operate.

In order to facilitate the reading of the position of the interface in the model, millimeter crossection paper was attached directly to the back of the model. Major divisions on the paper were accentuated to correspond with the coordinate axes as shown in figure 3. The interface position could be read to the nearest millimeter by making an observation perpendicular to the model.

Model Calibration

After assembling the model the spacing between the plates was found to be uneven by observing the motion of a plug of colored water. Apparently the plates had become warped very slightly during an earlier use. Careful adjustment of the tension on the spacing bolts resulted in a more uniform spacing as evidenced by a more even flow of the dyed water.

The hydraulic conductivity of the model can also be determined experimentally by computation from a problem whose solution is known, such as the phreatic flow in a thick aquifer. The equation of steady flow between two fixed heads H_o and H, separated by a distance L, is given by (Todd 1959)

(44) $$q = K_m(H_o^2 - H^2)/2L$$

The total discharge (Q) through the model is

(45) $$Q = K_m b_m(H_o^2 - H^2)/2L = (gb_m^3/24L\nu)(H_o^2 - H^2)$$

where b_m, g and ν have been defined in equation (36). This is the equation of a straight line through the origin. By plotting Q versus $(H_o^2 - H^2)$ $K_m b_m$ and consequently K_m can be determined from the slope of the straight line through the points. Several phreatic experiments were run to determine the hydraulic conductivity of the model, the results of which are shown in the appendix. The spacing between the plates was found to be 0.089 cm, resulting in a hydraulic conductivity for the model of 70 cm/sec. This value was used as the hydraulic conductivity of the model in this study.

MATHEMATICAL AND EXPERIMENTAL STUDY OF CONING PARAMETERS AND SPECIAL TECHNIQUES

Comparison of Various Approximations with Experimental Interfaces

Digital Computer Routine

The formulae derived in Chapter 2 for the computation of the interface position require that the potentials within the fresh water zone be computed at many points in the region being investigated. The points chosen would of course depend on the particular approximation used to compute the interface. Due to the number of calculations required for each interface determination, it was decided that a digital computer would greatly speed up this task. Since a digital computer would also be required for the solution of the potential distribution due to a drain with a finite penetration into the aquifer, the computational routine was developed to be used for all calculations.

The basic routine would generate X and Z grid and then would compute the potentials (or stream line values) at the grid points from equation (19). Equipotential and/or stream lines could be obtained by manually contouring the grid values obtained or by using a contour routine with the computer program. In this study the contouring seemed cumbersome and thus the interface was computed from the potential values along the initial interface positions. One of the programs actually used is included in the appendix.

Muskat's Approximation and its Modification

Using the above described computer routine, the interface position, based on Muskat's approximation, is obtained by calculating the potential differences along a line equivalent to the initial interface in the aquifer. As expected, the computed height of the cone beneath the well was always lower than the experimentally determined cone. These results have been summarized in Table I where the maximum rise of the upconed interface beneath the well only is shown for comparison.

Iteration on Muskat's approximation was a fairly simple addition to the computer routine and as such it was incorporated into the program. Initial trials showed that the fourth iteration would not significantly change the results of the third, even for cone heights exceeding 50% of the thickness of the fresh water zone. Thus the computations were carried to the third iteration only. In Table I the height of the cone beneath the drain, computed by three iterations for the same initial conditions, are compared to the observed heights and those computed by other approximations.

No improvement on Muskat's approximation was observed until the cone had reached a height of approximately 20% of the thickness of the fresh water zone. As can be seen in Table I, even at a cone height of 40%, the iteration

TABLE I

COMPARISON OF EXPERIMENTALLY DETERMINED HEIGHTS OF INTERFACE BENEATH A DRAIN (X=0) AND THE HEIGHTS AS COMPUTED BY DIFFERENT APPROXIMATIONS

OBSERVED COORDINATE Z	COORDINATE COMPUTED BY MUSKAT'S APPROXIMATION Z	COORDINATE COMPUTED BY ITERATION ON MUSKAT'S APPROXIMATION Z	COORDINATE COMPUTED BY AVERAGE INITIAL INTERFACE = 1/3 Z	COORDINATE COMPUTED BY AVERAGE INITIAL INTERFACE = $\frac{1}{2}$ Z	COORDINATE COMPUTED BY "CORRECTED" MUSKAT APPROXIMATION Z
0.901	0.922	0.922	0.896	0.882	0.896
0.849	0.883	0.883	0.844	0.824	0.844
0.793	0.843	0.843	0.791	0.764	0.792
0.746	0.808	0.807	0.744	0.711	0.745
0.694	0.770	0.768	0.693	0.653	0.695
0.640	0.729	0.731	0.644	0.596	0.646
0.584	0.694	0.696	0.595	0.544	0.602

technique would not significantly improve the computations of the interface position based on Muskat's approximation.

Average Initial Interface

Since these two attempts to compute the height of the upconed interface yielded values which were below the experimentally determined position, the interface was then computed by assuming an average initial interface position at values of ½ and 1/3 of the maximum computed cone height. For comparison with the previously obtained values, the maximum height beneath the drain for these computations are also shown in Table I. While an average initial interface of ½ of the maximum computed rise using Muskat's approximation consistently resulted in high values, the values obtained using an average initial interface of 1/3 of the computed cone height matched the experimentally determined cone height very well.

"Corrected" Muskat Approximation

The approximation based on an average initial interface of 1/3 of the computed cone height would reproduce the experimentally determined interface height beneath the drain but it would not reproduce its entire shape. This approximation would in effect shift the entire computed interface shape upward by 1/3 and it would match the experimental shape only beneath the drain. At the outer edges of the interface, Muskat's approximation and its iteration would describe the shape of the upconed interface much better. This observation led to the conclusion that the best approximate expression for the shape of the interface should result from a combination of both of the above modifications to Muskat's approximation.

Since a correction of 1/3 of the computed interface height beneath the drain added to the computed value gave satisfactory results at that point, this factor was used as a new approach for improving Muskat's approximation. Instead of basing the computation of the upconed interface position on the potential distribution along an assumed average initial interface, it could be arrived at by correcting the approximate solution along the initial interface with an empirically derived factor. The simplest factor would be a constant which is multiplied with the approximate solution. Based on previous results, this factor was assumed to be 4/3 or Muskat's approximation. Results obtained by this method are also shown in Table I in the column marked "Corrected Muskat Approximation" for comparison with the values obtained previously.

M.K. Hubbert (1953) showed that the interface between two liquids in a porous medium will be tilted when one is moving steadily while the other maintains a static equilibrium. Therefore a horizontal interface is not possible for the steady flow of fresh water overlying salt water in a confined aquifer. The effect of upconing under a drain would extend to infinity. A steady state can be established only when fresh water is recharged into the system at a constant head. The horizontal distance from the drain where this constant head is maintained is defined as x_o. In this study X_o corresponds to the normalized length of the model and varies between 5 and 7. At $X = X_o$ the original position of the interface remains fixed. It is equivalent to the coordinate of the initial horizontal base of the fresh water zone prior to pumping from the drain. This is the original interface position for Muskat's approximation.

Vertical potential gradients due to the pumping of a partially penetrating drain will cause an additional upconing of the interface from the tilted position. The horizontal distance from the drain at which this deviation begins depends on the pumping rate of the drain and on the position of the drain with respect to the original interface. Hantush (1964) showed that for all practical purposes the effects of partial penetration become negligible at $X = 1.5$ for homogeneous, isotropic aquifers. For purposes of this study this distance was determined by establishing the largest value of X at which no vertical potential differences existed for high flow rates and small drain penetrations. This value for X was found to be approximately 3 and it did not vary much for a wide range of different penetrations. Also, changes of this value between 2.5 and 4 did not have a significant influence on the shape of the computed interface.

Figure 8 is a comparison between two experimental and

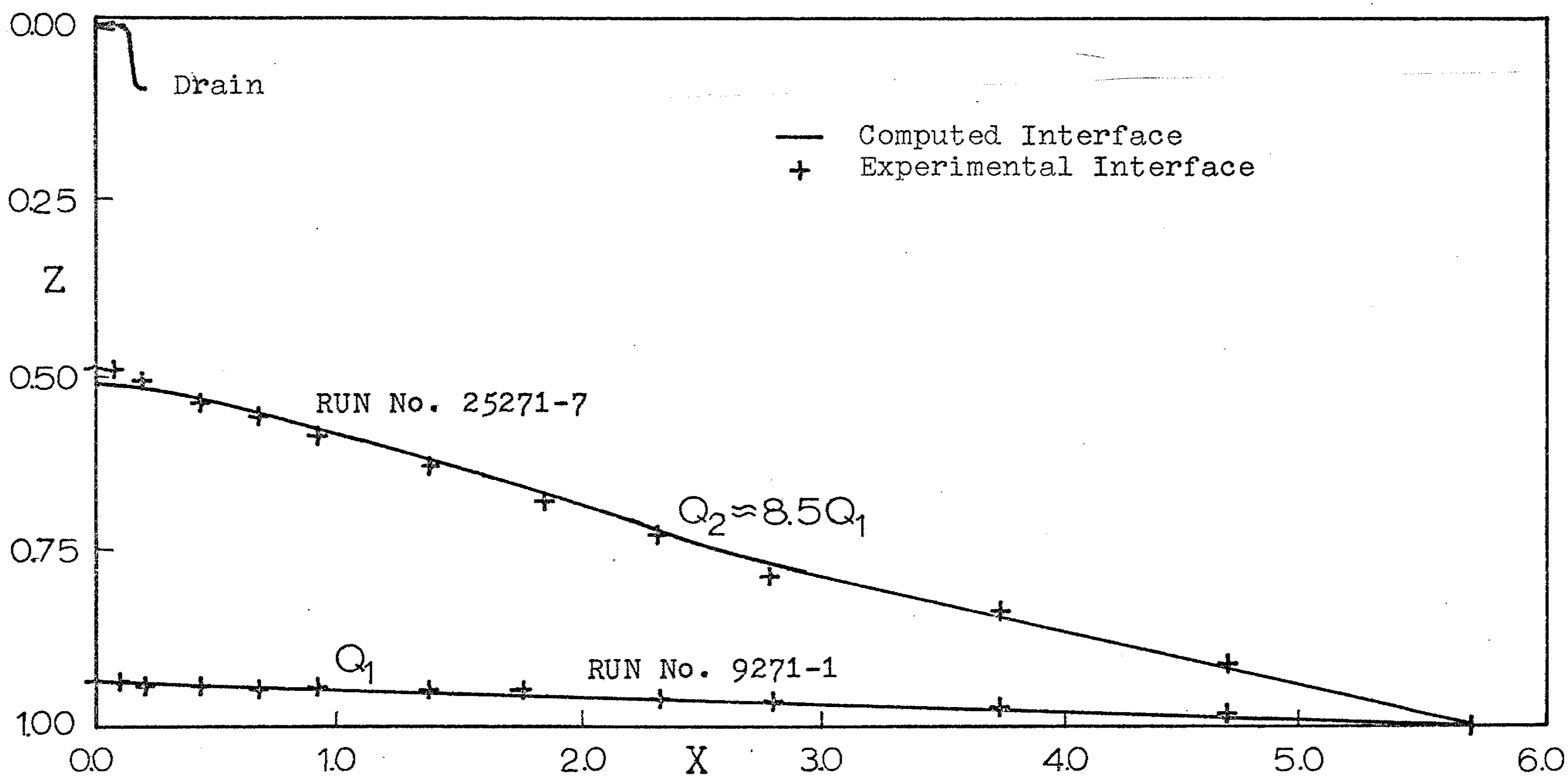


FIGURE 8: COMPARISON OF EXPERIMENTAL AND COMPUTED INTERFACE FOR DIFFERING PUMPING RATES OF THE DRAIN

computed interfaces based on Muskat's approximation corrected in the above manner. As can be seen, the correspondence between the curve and the points is good, even for a cone height of approximately 50%. Values obtained for the height of the cone beneath the drain for various flow rates are also compared with the other methods in Table I. Inspection of Figure 8 and Table I shows that the "Corrected Muskat Approximation" can be used to represent the shape of the upconed interface, facilitating the study of the effects of the special techniques upon it.

It should be noted that at higher pumping rates of the drain the greatest portion of the upconing is due to the tilt of the interface. This indicates that the height of the upconed interface depends on the horizontal distance between the drain and the point where the interface position remains constant. The computed interface thus consists of two portions; the tilted interface at values of X greater than 3, where it can be computed accurately, and the upconed portion at smaller values of X to which the "corrections" are applied to Muskat's approximation. An investigation of this latter upconed portion by itself was beyond the scope of this study.

As a check, the interface was computed as a free surface based on the Dupuit assumptions. This method yielded results which were better than Muskat's approximations, but not as good as the "corrected" computations used in this study. Unfortunately these free surface calculations do not correspond with the figures used in Table 1 and thus are not included there for direct comparison. They are however, included in the appendix. Since the "Corrected Muskat Approximation" agreed better with the experimental observations, no further attempt was made in this study to compute the interface based on the free surface analogy.

Range of Validity of the Computed Interface

The next step in this investigation was to determine the maximum height of the interface which could be approximated by computation. Even though stable heights of the interface greater than 50% of the thickness of the fresh water zone could be achieved experimentally, control of the interface at these heights was difficult; a very small increase in the pumping rate would cause the salt water to reach the drain. Thus it was felt that if an interface height of 50% could be described reasonably accurately, this would provide a sufficient margin of error to predict a stable interface for the special techniques.

As can be seen in Figure 8, the computational routine did approximate the interface quite well for a height of 50%. It should be noted, however, that the computed interface at this height is about 6% below the observed interface. Since small potential differences can bring about large changes in the interface position, especially when the density difference between the two liquids is small, it appears that a computed interface height of 50% is about as high as should be attempted.

Comparisons between the interface computation developed in this study and the solutions derived by Bear and Dagan for the same model parameters were not successful. Their nonsteady solutions based on the method of small perturbations could not be extrapolated to infinite time and thus were not applicable to the steady state. The steady state solutions derived by the hodograph method were developed for four specific cases which were not applicable to the model parameters of this study. A direct comparison of these methods thus was not possible.

In their investigations, however, Bear and Dagan found that the upconed interface could be approximated by treating it as a free surface using the Dupuit assumptions. In trial runs the computed position of the interface based on the free surface analogy fell between Muskat's approximation and the approximation developed in this study. A comparison of these solutions for the same model parameters is shown in Tables II and III. Inspection of these tables shows that the "Corrected Muskat Approximation" would describe the upconed interface more closely than the other methods.

Finite Interval of Penetration by Drain

Once a computational routine was established which would closely represent the interface, it was possible to investigate the effect of a finite drain penetration. An adaptive integration subroutine available on the computer was used

TABLE II

COMPARISON OF THE Z-COORDINATES AT VARIOUS VALUES AT X FOR AN OBSERVED INTERFACE WITH THOSE COMPUTED BY VARIOUS APPROXIMATIONS

	OBSERVED INTERFACE COORDINATE	COORDINATE COMPUTED BY MUSKAT'S APPROXIMATION	COORDINATE COMPUTED BY FREE SURFACE ANALOGY	COORDINATE COMPUTED BY "CORRECTED" MUSKAT APPROXIMATION
X	Z	Z	Z	Z
0.0000	0.9435	0.9561	0.9513	0.9366
0.1176	0.9482	0.9562	0.9523	0.9376
0.2353	0.9506	0.9565	0.9533	0.9389
0.4706	0.9529	0.9574	0.9554	0.9418
0.7059	0.9553	0.9588	0.9574	0.9452
0.9412	0.9576	0.9605	0.9594	0.9488
1.4118	0.9600	0.9642	0.9635	0.9564
1.8824	0.9647	0.9681	0.9675	0.9642
2.3529	0.9671	0.9720	0.9617	0.9720
2.8235	0.9759	0.9759	0.9756	0.9759
3.7647	0.9835	0.9837	0.9836	0.9837
4.7059	0.9929	0.9915	0.9915	0.9915
5.7294	1.0000	1.0000	1.0000	1.0000

TABLE III

COMPARISON OF THE Z-COORDINATES AT VARIOUS VALUES AT X FOR AN OBSERVED INTERFACE WITH THOSE COMPUTED BY VARIOUS APPROXIMATIONS

X	OBSERVED INTERFACE COORDINATES Z	COORDINATE FOR MUSKAT APPROXIMATION Z	COORDINATE COMPUTED BY FREE SURFACE ANALOGY Z	"CORRECTED" MUSKAT COORDINATE Z
0.0000	0.5435	0.7018	0.5950	0.5426
0.1176	0.5529	0.7024	0.6060	0.5499
0.2353	0.5647	0.7042	0.6169	0.5583
0.4706	0.5929	0.7019	0.6380	0.5782
0.7059	0.6165	0.7205	0.6585	0.6011
0.9412	0.6353	0.7319	0.6783	0.6257
1.4118	0.6753	0.7570	0.7164	0.6774
1.8824	0.7153	0.7832	0.7525	0.7301
2.3529	0.7553	0.8097	0.7869	0.7831
2.8235	0.8024	0.8362	0.8200	0.8362
3.7647	0.8729	0.8892	0.8823	0.8892
4.7059	0.9412	0.9423	0.9405	0.9423
5.7294	1.0000	1.0000	1.0000	1.0000

for the integration of the drain over a finite interval. Computed interfaces using model parameters were then compared with the experimentally determined interfaces. Figures 9 through 11 show the results of these comparisons.

From these graphs it is immediately obvious that increasing the interval of penetration of the drain does not change the shape of the interface significantly when the pumping rate remains constant. This should be expected however, because the flow rate of water entering each element of the drain is lower and the potential differences in the immediate vicinity of the drain are lowered. Extension of the interval of the drain thus reduces the strength of the sink in the immediate vicinity of the drain. This change in the potential differences extends no more than a few drain diameters horizontally into the aquifer. The extension of the drain apparently has only a very localized and small effect on the potential distribution on the aquifer. Thus no significant reduction of the height of the upconed interface is obtained by increasing the producing interval of the drain.

Analysis of Computed Interface

Inspection of Figure 8 shows that the "Corrected Muskat Approximation" can be used to describe the upconed interface between two fluids of different density beneath a drain in a semi-infinite aquifer up to an intruded height of 50%

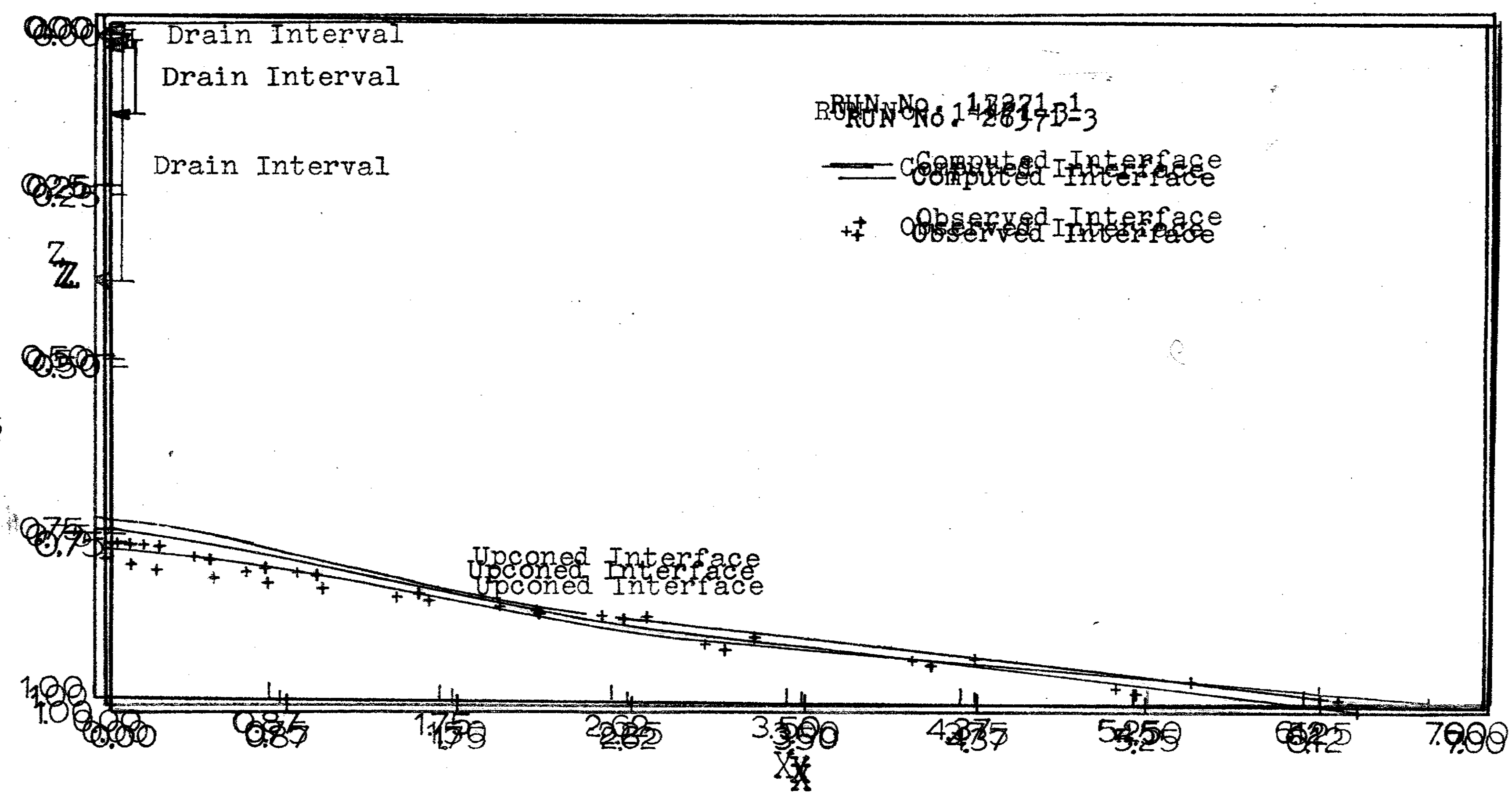


FIGURE 10: EFFECT OF FINITE DRAIN PENETRATION ON UPCONED INTERFACE

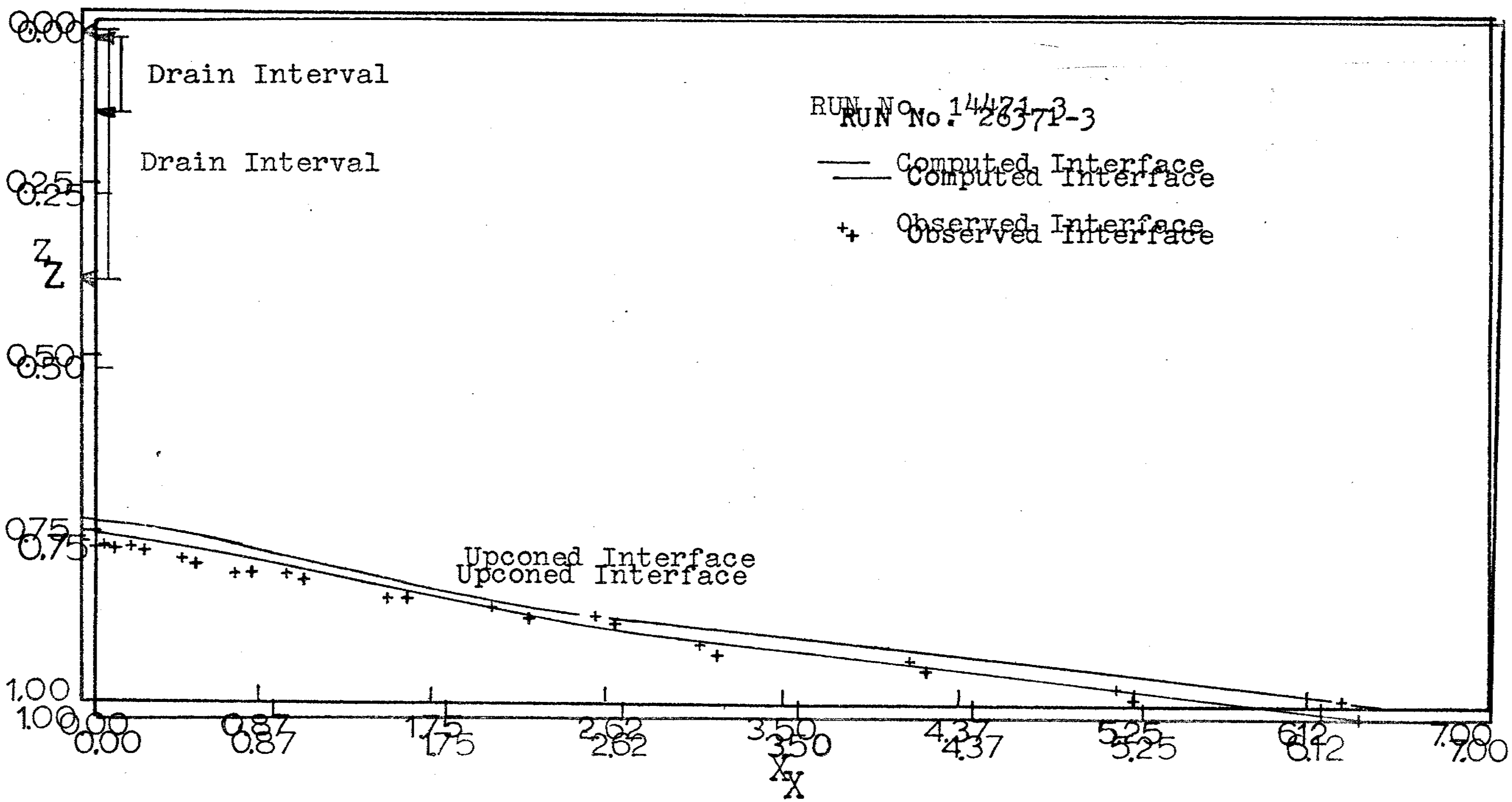


FIGURE 10: EFFECT OF FINITE DRAIN PENETRATION ON UPCONED INTERFACE

FIGURE 11: EFFECT OF FINITE DRAIN PENETRATION ON UPCONED INTERFACE

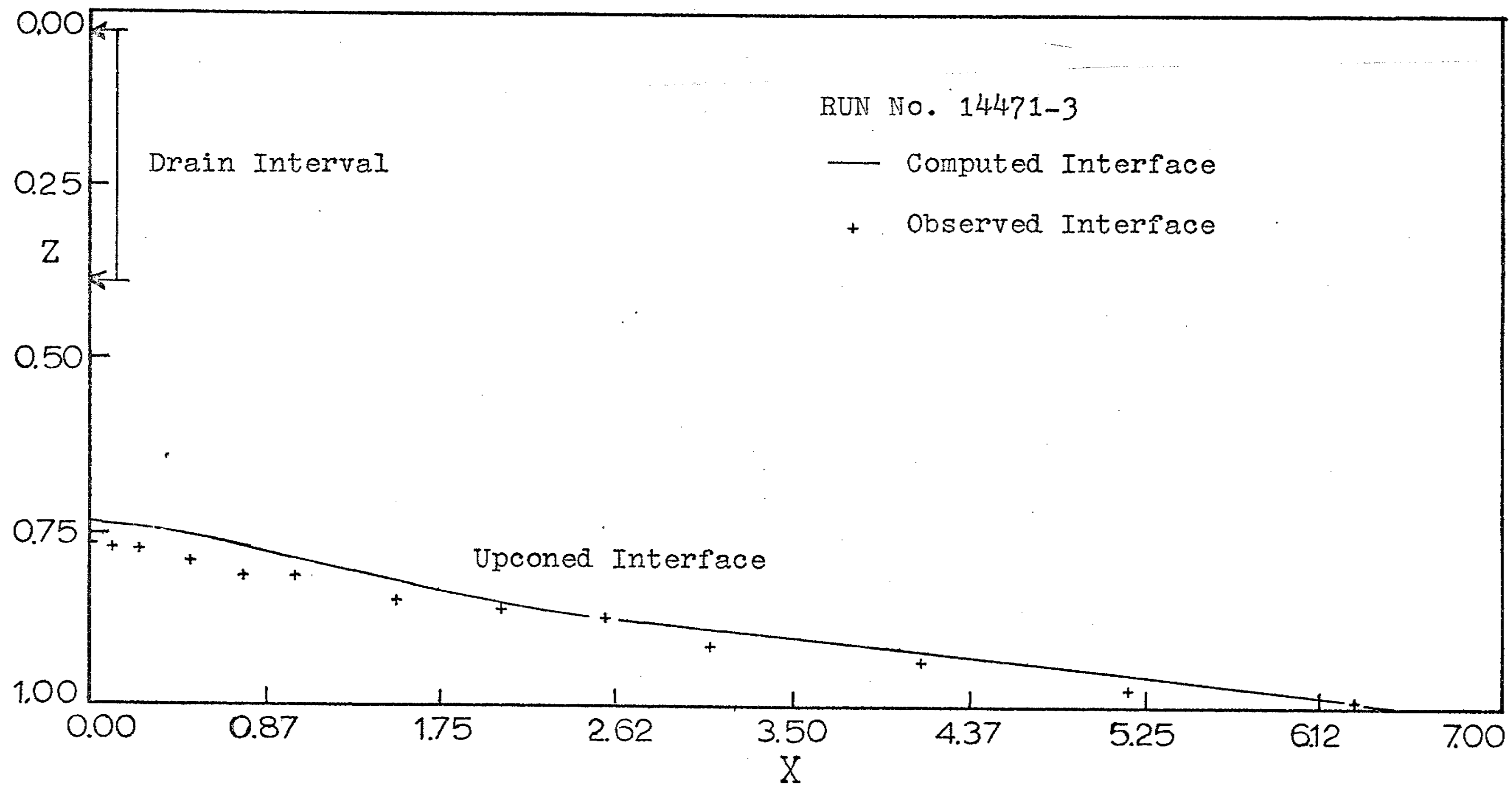


FIGURE 11: EFFECT OF FINITE DRAIN PENETRATION ON UPCONED INTERFACE

of the thickness of the fresh water. Up to this height the computer describes the interface quite well. No great change in the shape of the computed or observed interfaces was noticed with a change in the vertical length of the drain. This seems to imply that for most cases a drain can be approximated by a line sink in the two-dimensional problem, perhaps at the center of the producing interval.

In order to minimize the upconing of the interface, the drain locations were limited to the upper portions of the model. This would keep the vertical gradients in the plane beneath the drain to a minimum. It should be pointed out, however, that with small density differences in the two fluids, small changes in the vertical potential differences could have a large influence on the height of the upconed interface.

The Hydraulic Doublet

Since the computational routine would approximately describe the upconed interface between two fluids beneath a pumping drain over a relatively large range of cone heights, the next step was to investigate if this routine could be used to describe the interface position for the hydraulic doublet. Thus experiments were run to compare computed and observed interfaces. Figure 12 shows the results for two of these experiments. From this figure it

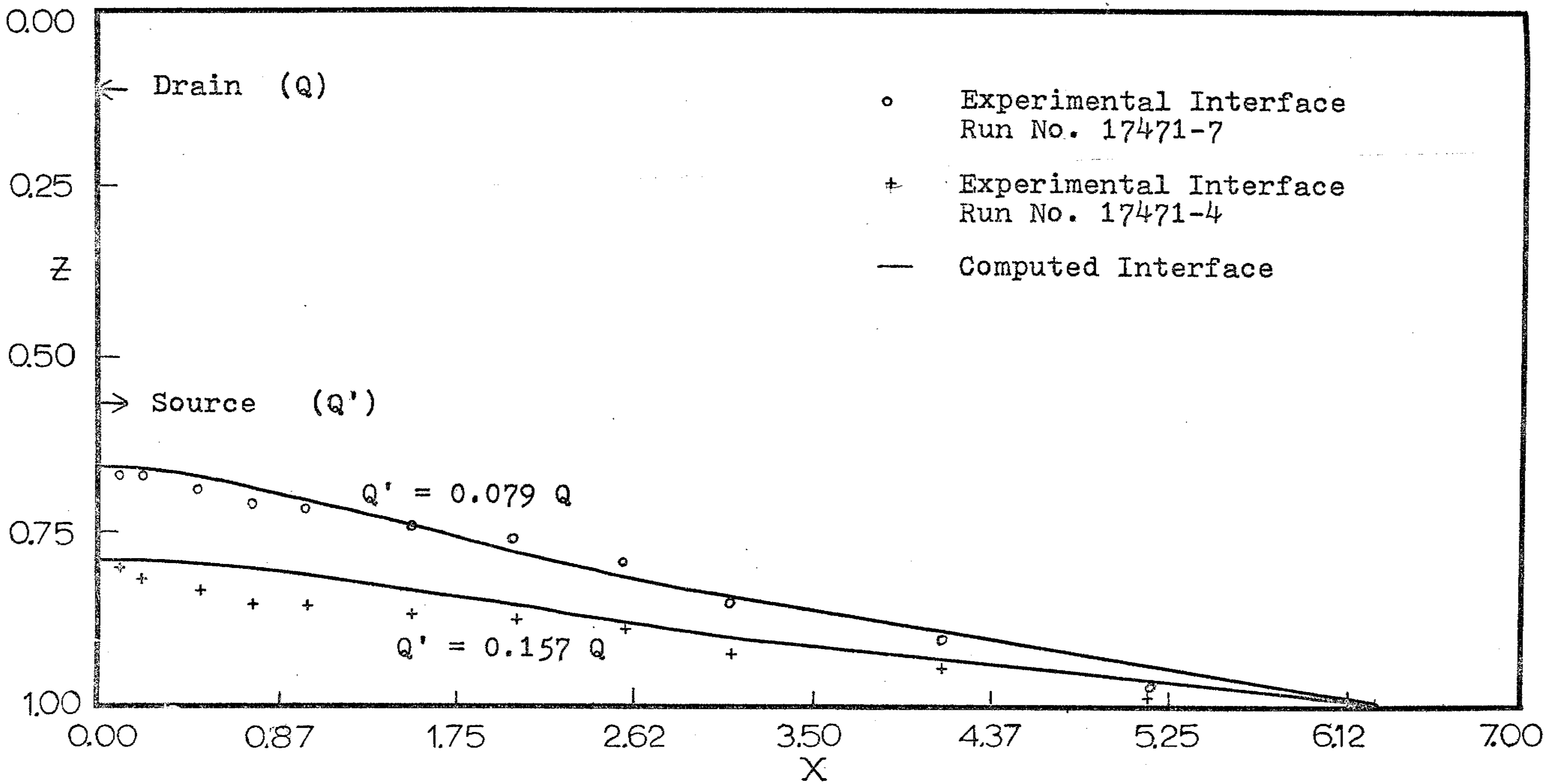


FIGURE 12: COMPARISON OF EXPERIMENTAL AND COMPUTED INTERFACES FOR THE HYDRAULIC DOUBLET SYSTEM

appears that the method can be used to compute the effects of the hydraulic doublet on the shape of the resultant interface.

Rate of Doublet Injection

The computational routine was thus used to investigate the effect of varying the rate of the injection well while its position and interval as well as the drain position and pumping rate were kept fixed. The results of these calculations are plotted in Figure 13. Examination of this figure shows that, while increasing the rate of the injection well did depress the upconed interface, it did not appreciably alter its shape. The shape of the resultant interface seemed to indicate that it corresponded to an interface determined for a drain only, pumping at the net rate.

This observation was investigated by comparing the shape and position of the interfaces of Figure 13 with those computed for a drain producing fresh water at the net pumping rates only. The values obtained for the computed interface positions were so close to those in Figure 13 that they could not be included in that plot and thus are tabulated for comparison in Table IV. This table does show that at high injection rates of the source there is a slight depression of the interface beneath the source, but this depression is very small. Thus it appears that the net producing rate of fluids has a greater influence on the shape of the upconed interface than the injection rate of the source.

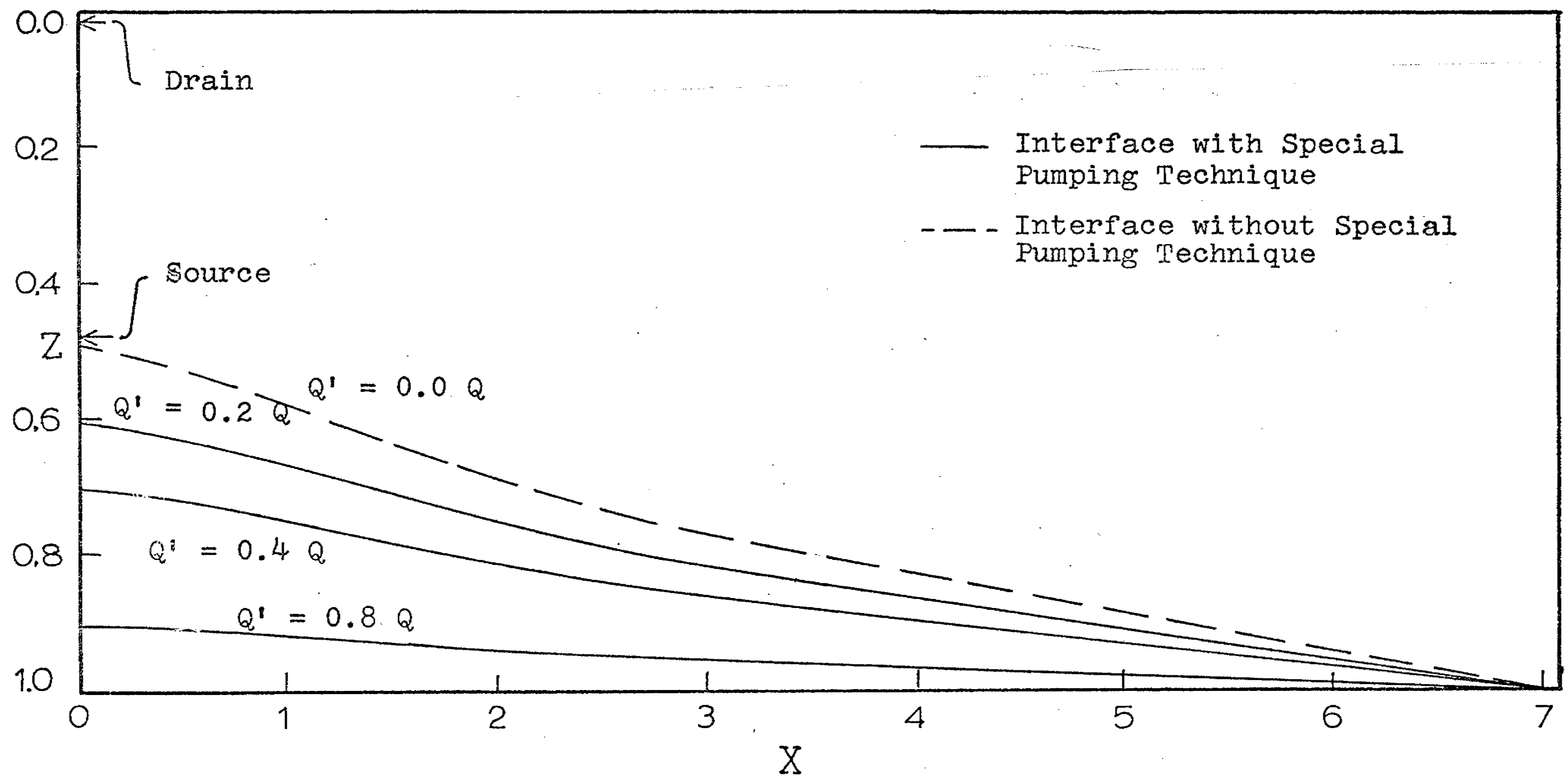


FIGURE 13: EFFECT OF VARYING INJECTION RATE ON THE SHAPE OF THE INTERFACE FOR THE HYDRAULIC DOUBLET SYSTEM

TABLE IV

COMPARISON OF INTERFACE COORDINATES FOR THE DOUBLET SYSTEM OF FIG. 1 VERSUS A DRAIN PRODUCING AT THE SAME NET PUMPING RATES

Drain location: Z = 0.0147 Coordinate of source location: Z = 4772

Injection rate:	20%*		40%		80%	
X	Z_d (1)	Z_w (2)	Z_d	Z_w	Z_d	Z_w
0.0	0.6022	0.5986	0.7062	0.6990	0.9140	0.8997
0.14	0.6077	0.6046	0.7097	0.7034	0.9137	0.9011
0.43	0.6236	0.6221	0.7196	0.7165	0.9117	0.9055
0.72	0.6446	0.6440	0.7343	0.7330	0.9135	0.9110
1.16	0.6796	0.6794	0.7599	0.7596	0.9205	0.9199
1.74	0.7259	0.7259	0.7945	0.7944	0.9316	0.9315
2.32	0.7695	0.7695	0.8271	0.8271	0.9424	0.9424
2.90	0.8098	0.8098	0.8574	0.8574	0.9525	0.9525
3.48	0.8381	0.8381	0.8786	0.8786	0.9595	0.9595
4.64	0.8906	0.8906	0.9179	0.9179	0.9726	0.9726
5.80	0.9430	0.9430	0.9572	0.9572	0.9857	0.9857
7.06	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

* The injection rate is expressed as % of the drain pumping rate.
(1) Interface coordinate for doublet system.
(2) Interface coordinate for drain only.

Position of Injection Well with Respect to Interface

The effect of changing the vertical position of the source beneath the drain was investigated when all other parameters were kept constant. The computational routine was used for this because of the difficulty in keeping all the other experimental parameters constant while changing the position of the source. Figures 14, 15, and 16 are representative examples of these computations. Again it can be seen that changing the position of the source does not appreciably alter the shape of the upconed interface. Only very small changes directly beneath the source can be detected.

It should be noted that the interface is not depressed below the position of the source even when this source is located below the computed interface. Model experiments of this situation did not show a stable, depressed interface either. The fresh water would just bubble through the salt water without establishing a stable cone shape. This condition would occur only when the injection rate of the source is small compared to the pumping rate of the drain. A stable depression of the cone would result when the injection rate was large enough to depress the interface below the source position. Thus again it appears that the net pumping rate has a greater influence on the computed interface shape than the position of the source.

Even though the computed interface does not accurately describe the shape of the upconed interface, it does appear

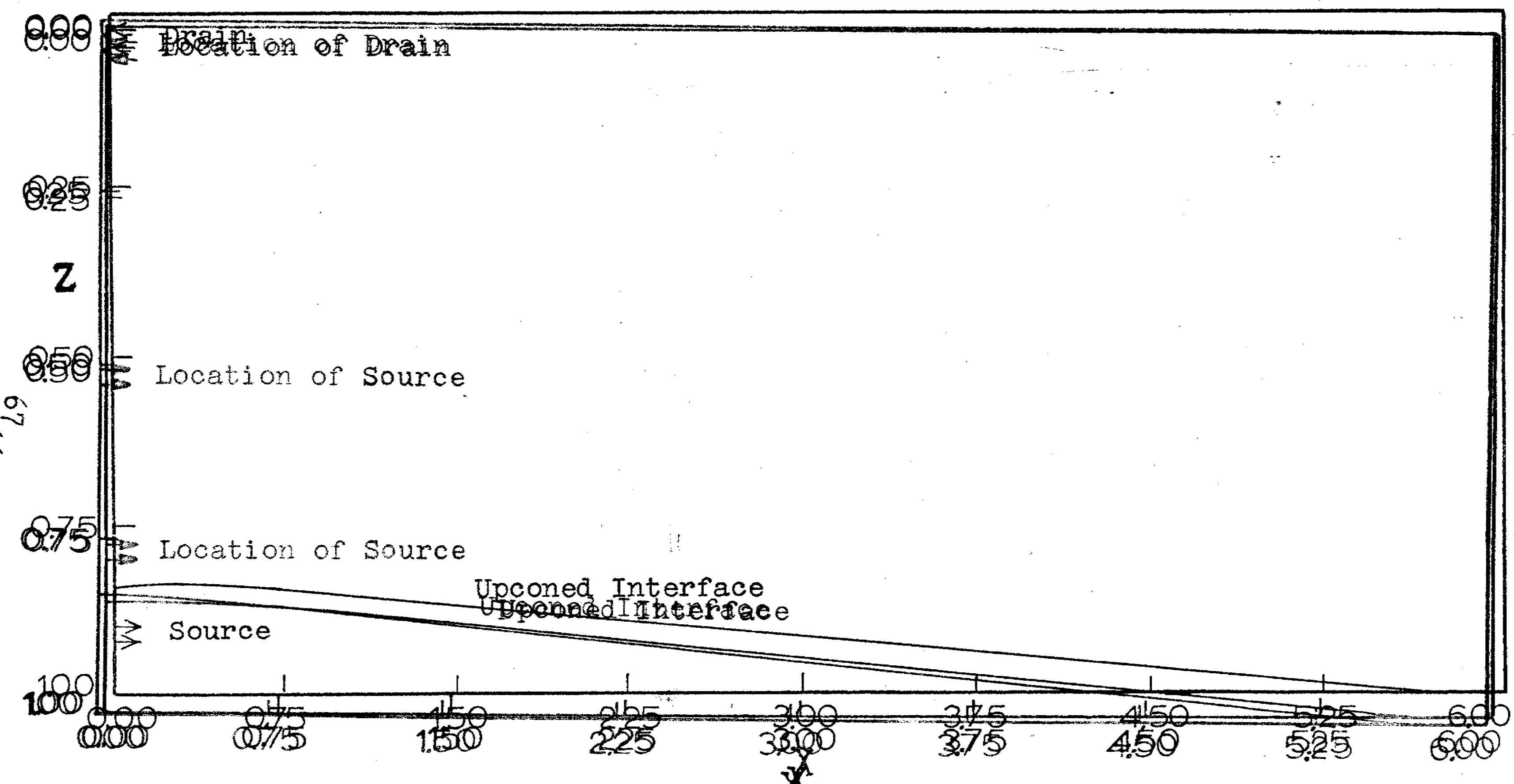


FIGURE 16: EFFECT OF VARYING LOCATION OF SOURCE ON THE SHAPE OF THE INTERFACE FOR THE HYDRAULIC DOUBLET SYSTEM

FIGURE 15: EFFECT OF VARYING LOCATION OF SOURCE ON THE SHAPE OF THE INTERFACE FOR THE HYDRAULIC DOUBLET SYSTEM

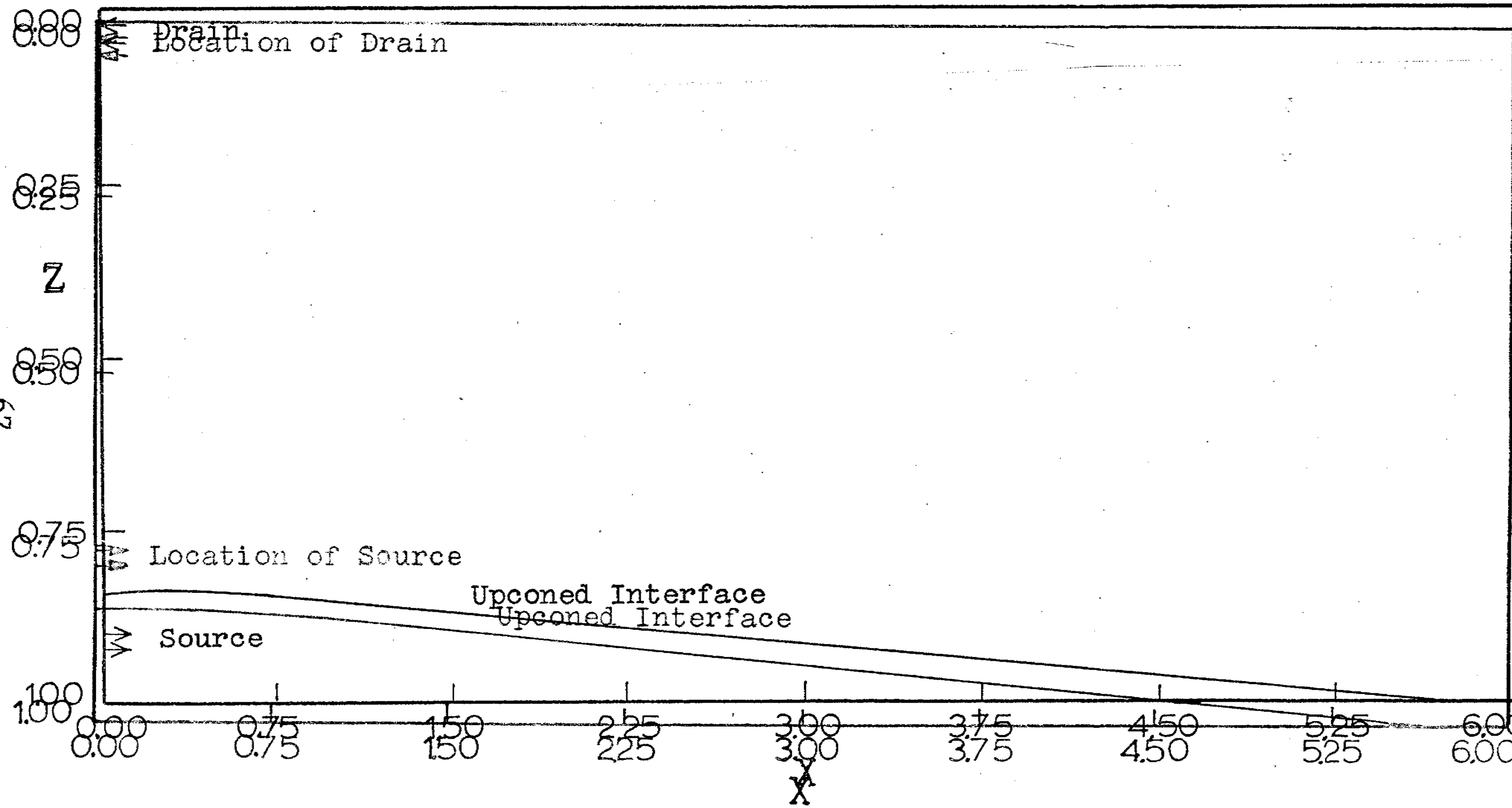


FIGURE 16: EFFECT OF VARYING LOCATION OF SOURCE ON THE SHAPE OF THE INTERFACE FOR THE HYDRAULIC DOUBLET SYSTEM

FIGURE 15: EFFECT OF VARYING LOCATION OF SOURCE ON THE SHAPE OF THE INTERFACE FOR THE HYDRAULIC DOUBLET SYSTEM

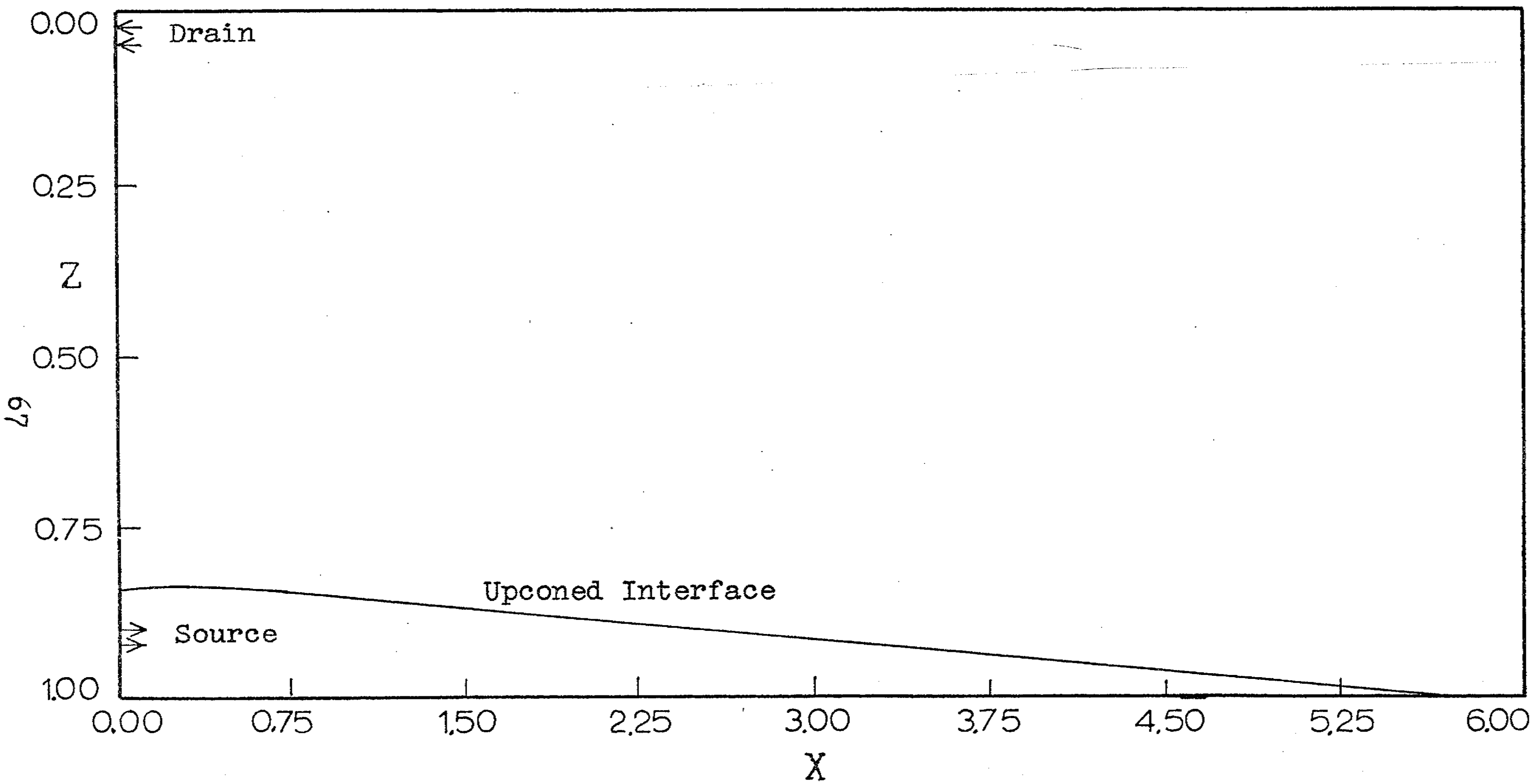


FIGURE 16: EFFECT OF VARYING LOCATION OF SOURCE ON THE SHAPE OF THE INTERFACE FOR THE HYDRAULIC DOUBLET SYSTEM

that it can be used to approximate the height of the cone beneath the doublet. This is shown in Figure 17 where the drain was pumped at such a rate that salt water was drawn into it. The source reinjected 50% of the produced fresh water and formed a well defined, stable bulge in the upconed interface. As can be seen in the figure, a small amount of salt water is dragged into the drain along the outer edge of the injection bulge. However, the actual amount of salt water produced is small. For this situation the computed interface does not describe the actual interface very well, especially not the configuration of the small amount of salt water drawn into the drain. It should be noted, however, that the height of the computed cone does not differ greatly from the observed position of the interface beneath the source. Thus it should be possible to estimate whether a particular doublet configuration would depress the interface or not.

Finite Penetration Interval of Doublet

The effect of changing the injection interval of the source and keeping the other parameters constant was investigated briefly. As with the finite interval of the drain, it was found that lengthening the injection interval over a considerable range had a negligible influence on the upconed interface. The differences in the computed interfaces were very small and it was difficult to distinguish

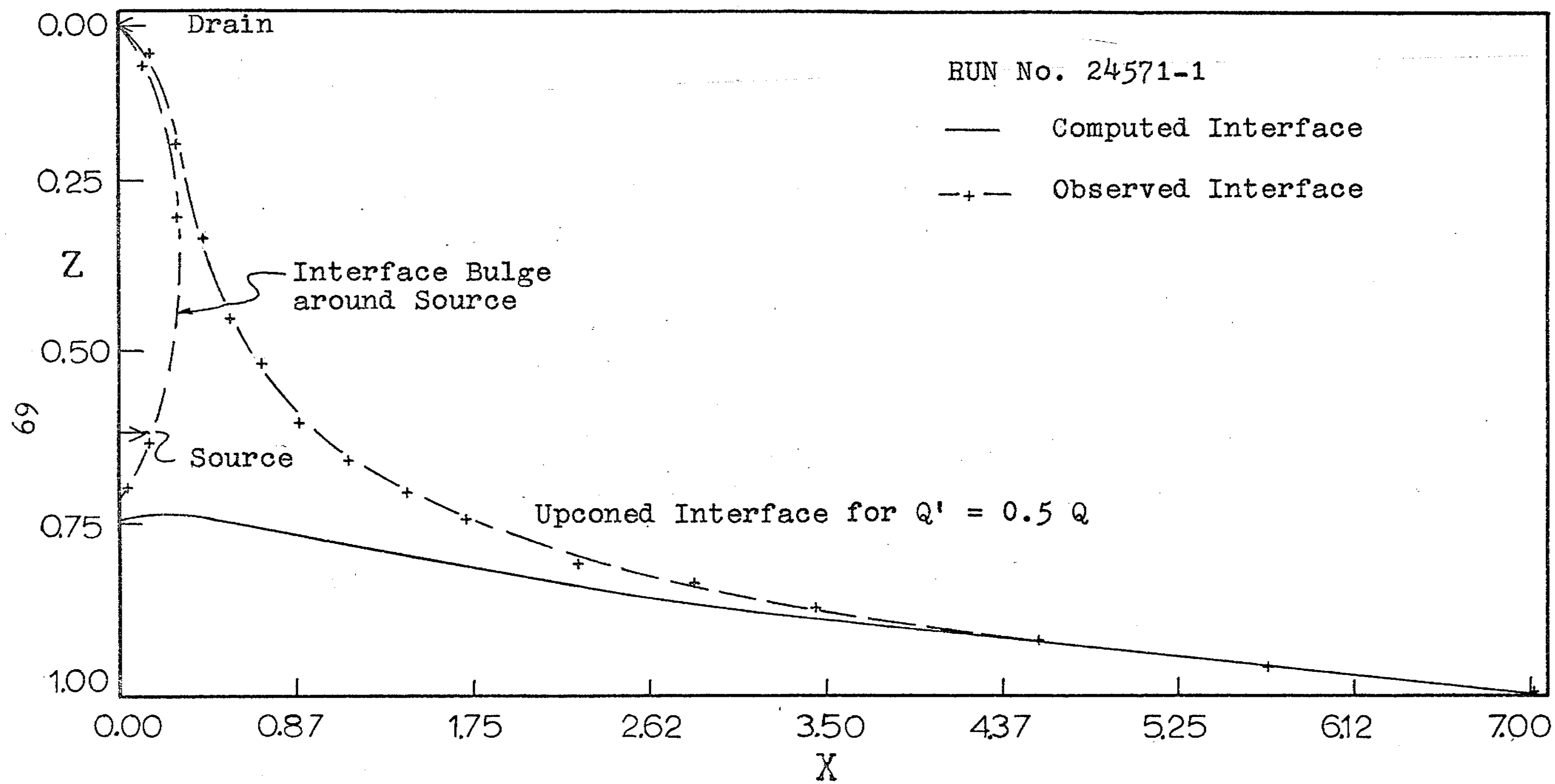


FIGURE 17: COMPARISON OF EXPERIMENTAL AND COMPUTED INTERFACES WHEN SALT WATER IS PUMPED INTO DRAIN WHILE OPERATING THE HYDRAULIC DOUBLET SYSTEM

between the actual change and an experimental error in the observed interfaces. In the further treatment of this problem the finite interval will be abandoned. Any source or sink in the system will be assumed to be a point located at the center of the actual open interval.

Summary of Doublet Results

While the experiments indicated that the doublet system was effective in depressing the upconed interface, the computational routine did not always describe the shape or position of the interface very well. The computed interface appeared to be sensitive only to the net producing rate from the aquifer, that is the pumping rate of the drain less the injection rate of the source. Changing the injection position or interval did not result in any great changes in the shape or position of the interface.

The reason why the computational routine does not calculate a good interface shape for the doublet system apparently lies in the way the interface is determined. For the drain only, the draw downs along the initial interface separating the two liquids seem to represent the potential differences in the aquifer quite well. However, the potential distribution in the region of flow is more complex for the doublet system. In this case the potential differences along the initial interface probably do not represent the complicated potentials in the entire flow region very well. Perhaps the interface can be computed more accurately along another line between the source and the drain. Care should be exercised when in the immediate vicinity of the source or drain computing the interface because this is where the greatest potential differences occur and a greater distortion of the computed interface could result.

This analysis of the potential distribution in the flow region might also be more suitable for the investigation of the finite penetration interval for the doublet system.

From the experimental observations it appears that the optimum injection position would be about 1/3 of the distance from the bottom of the drain to the initial interface. At this position with a moderate injection rate approximately 20-30% of the drain pumping rate, the source would form a stable bulge in the upconed interface and would greatly reduce the contamination of the fresh water by salt water. A small amount of salt water would still be pulled into the drain.

The quantitative evaluation of the effectiveness of the doublet system will have to be delayed until a more reliable method of computation of the interface shape and position is found.

The Scavenger Well

The treatment of the scavenger well differs from that of the doublet in that in this case the salt water is no longer stationary. Since it will also be produced, there will be a potential gradient in this zone also, which can not be neglected. For the purposes of this study it will be assumed that the density of the salt water is not much greater than that of the fresh water so that the density difference can be neglected. For the case of fresh and sea water this is a reasonable assumption. For the scavenger well then, the flow system of the entire, finite confined aquifer will be considered. The position and shape of the interface will be determined by the strength and location of two drains in a vertical plane. Experimentally the position of the interface between the two drains is difficult to determine since little or no flow occurs in this region and the interface becomes diffused and difficult to observe.

Pumping Rate of Scavenger Well

As in the case of the doublet system, an arbitrary location for the scavenger well was selected and then the pumping rate was varied while all other parameters were held fixed. The position of the scavenger well was chosen so that it would intercept the upconed salt water. The pumping rate of the drain was high enough so that the salt water would be drawn into it if the scavenger was not pumping. Figures 18 through 21 compare the experimental and computed interfaces. It can readily be seen that the scavenger well was successful in intercepting the upconed salt water and controlling the contamination of the produced fresh water.

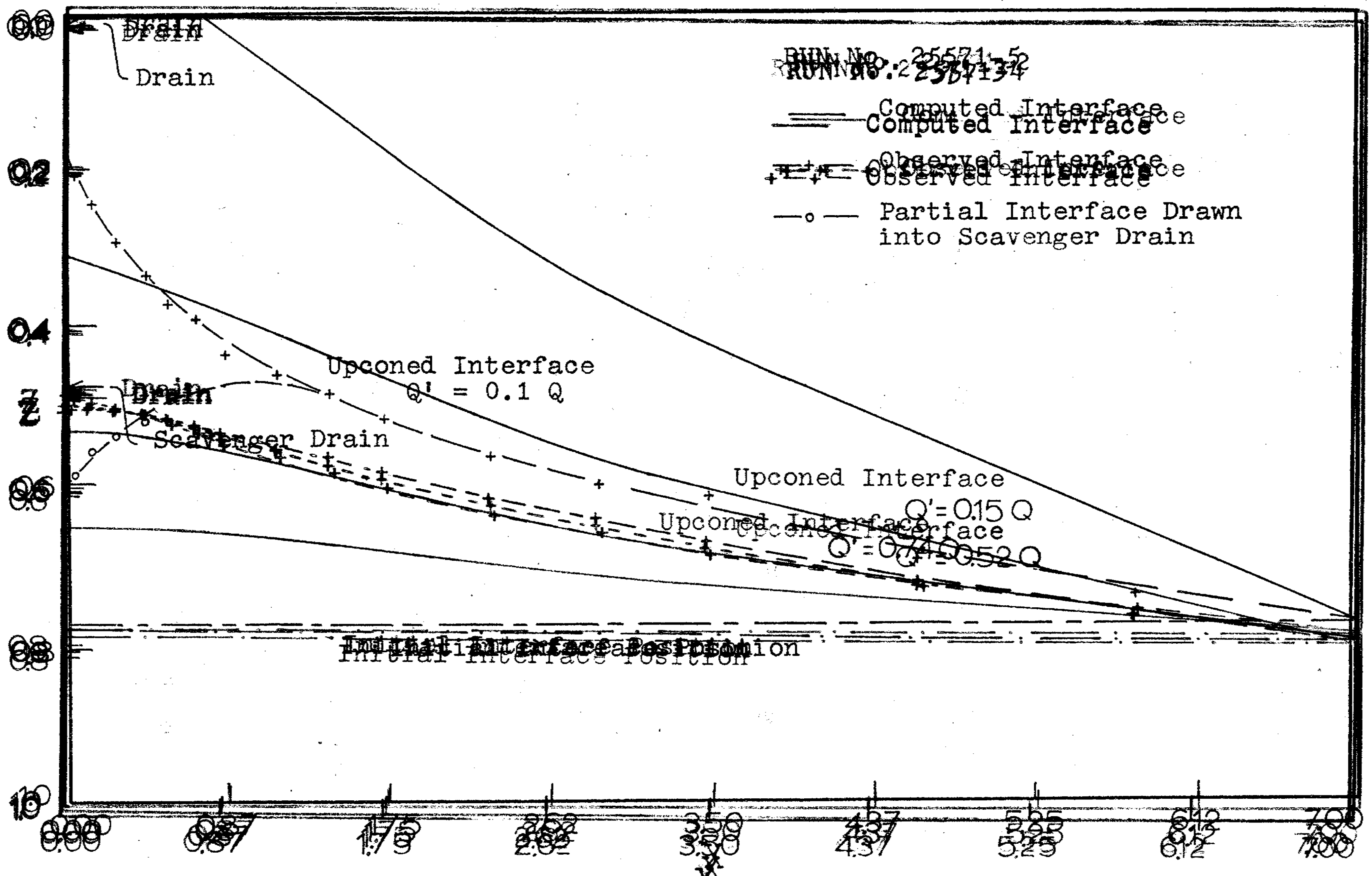


FIGURE 21: EFFECT OF VARYING PUMPING RATE ON THE SHAPE OF THE INTERFACE FOR THE SCAVENGER WELL SYSTEM

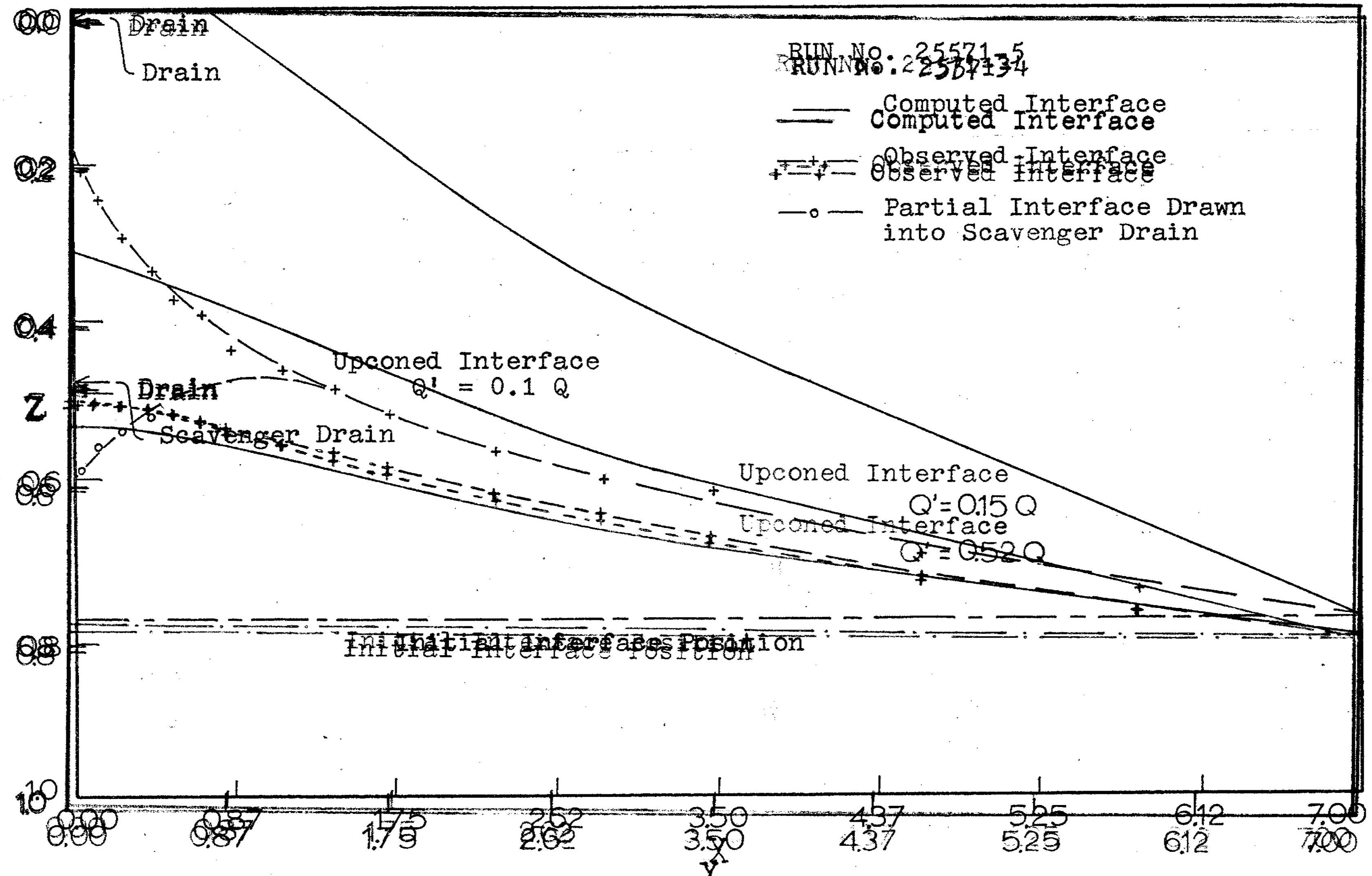


FIGURE 21: EFFECT OF VARYING PUMPING RATE ON THE SHAPE OF THE INTERFACE FOR THE SCAVENGER WELL SYSTEM

FIGURE 20: EFFECT OF VARYING PUMPING RATES ON THE SHAPE OF THE INTERFACE FOR THE SCAVENGER WELL SYSTEM

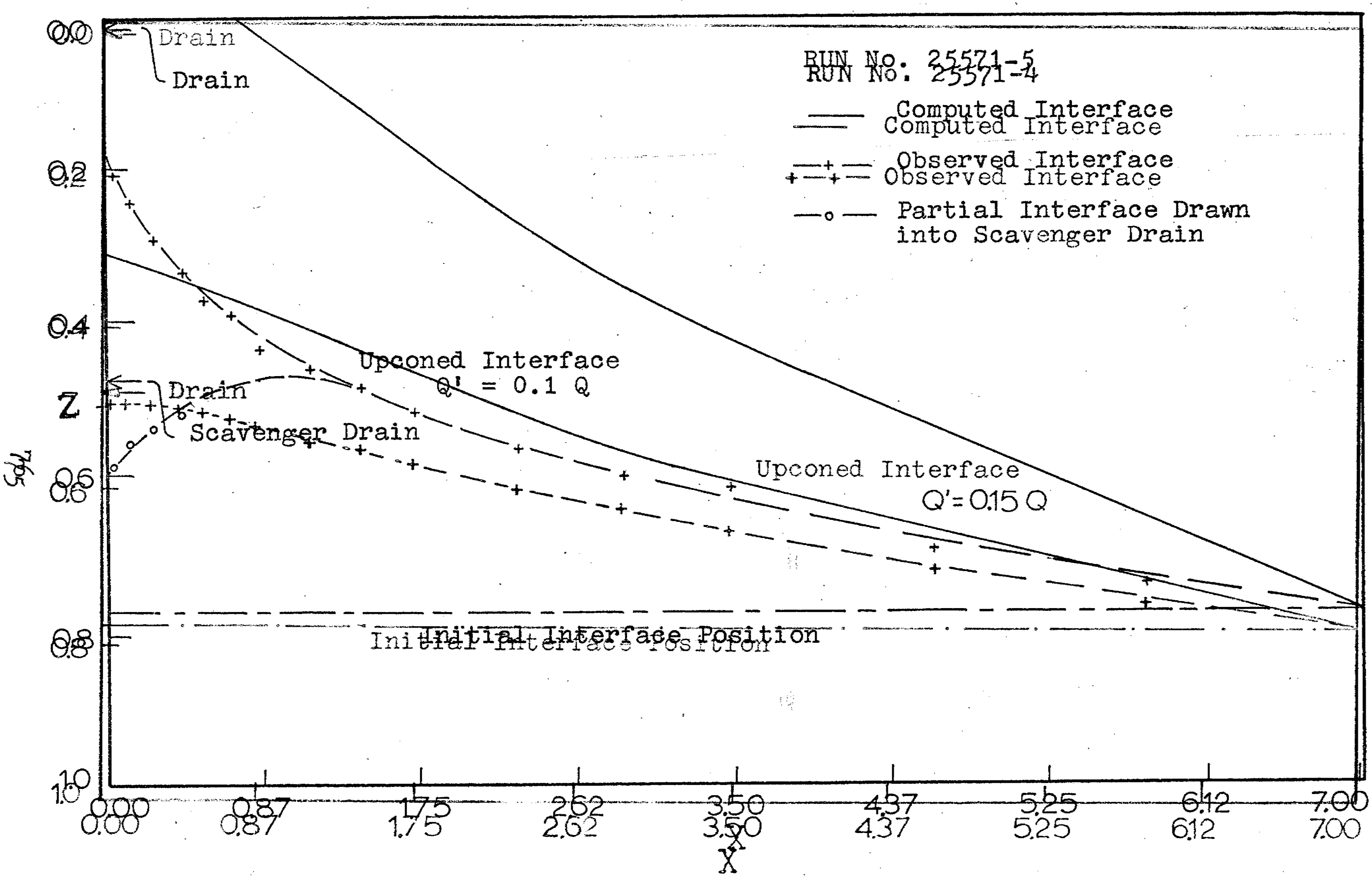


FIGURE 21: EFFECT OF VARYING PUMPING RATE ON THE SHAPE OF THE INTERFACE FOR THE SCAVENGER WELL SYSTEM

FIGURE 20: EFFECT OF VARYING PUMPING RATES ON THE SHAPE OF THE INTERFACE FOR THE SCAVENGER WELL SYSTEM

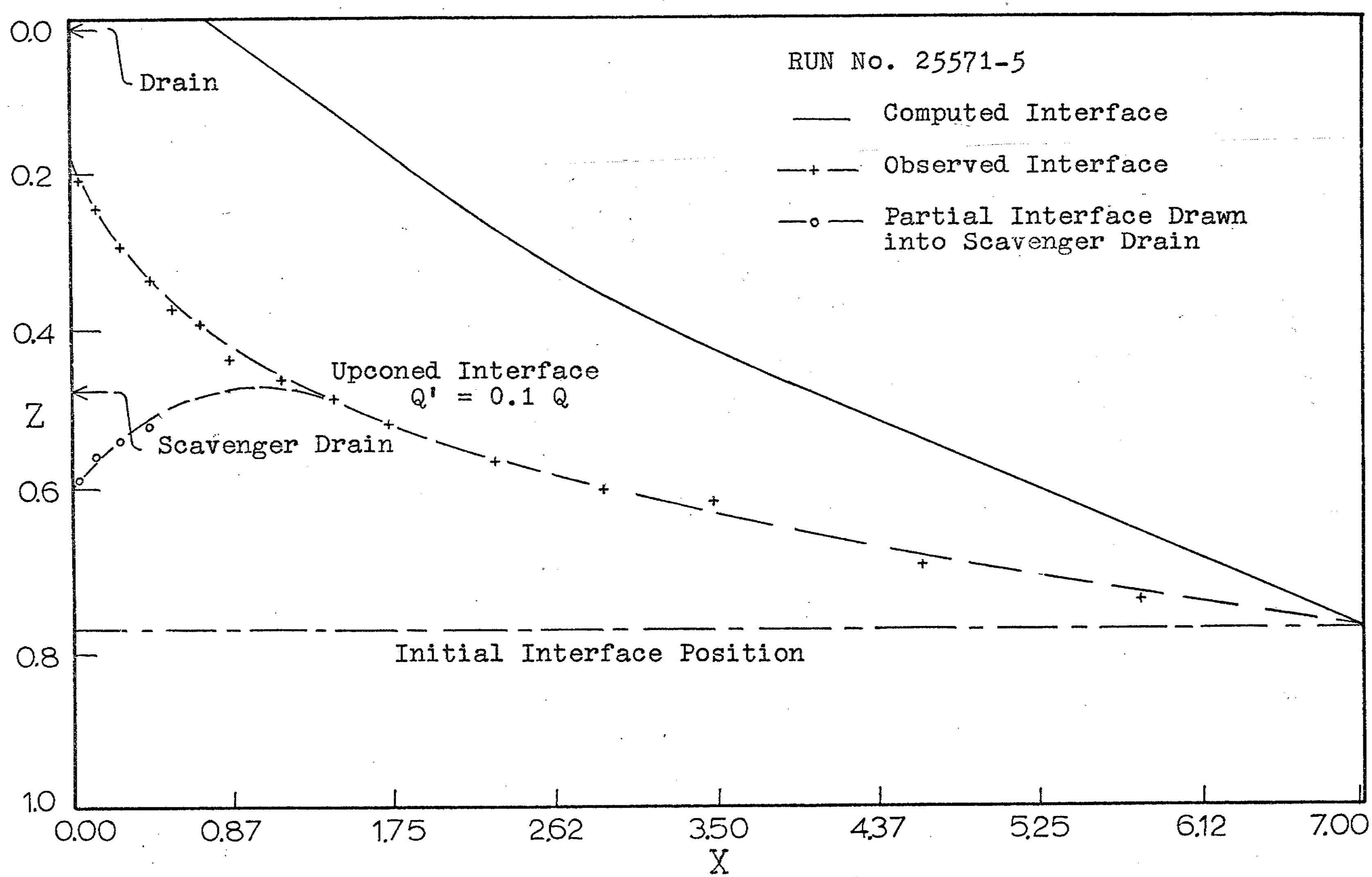


FIGURE 21: EFFECT OF VARYING PUMPING RATE ON THE SHAPE OF THE INTERFACE FOR THE SCAVENGER WELL SYSTEM

The computed interface, however, did not produce predictable cone shapes. This makes it very difficult to use the computational routine developed here to study the effect the scavenger well has on the interface. Inspection of the computed interfaces, however, did indicate again that the shape and position of the interface are dependent on the net pumping rates. In the case of the scavenger well these rates are combined so that the potential differences due to the two drains are increased and the meaning of the interface computed therefrom becomes confused.

Changing Position of the Scavenger Well

Even though the computed interface for the scavenger well was not reliable, the effect that a changing scavenger position might have on the shape of the interface when all other parameters were held fixed was investigated. Similar to the changing positions of the injection well in the doublet system, the computed effect of varying the location of the scavenger well was so small that it could not be presented graphically and it is thus summarized in Table V. As in the previous cases it is again apparent that the shape of the interface separating the two fluids is governed by the net flow rate from the system.

Summary of Scavenger Well Results

Control of the upconed interface with a scavenger

TABLE V

COMPARISON OF INTERFACE SHAPE FOR VARYING SCAVENGER WELL POSITIONS WITH PUMPING RATES REMAINING CONSTANT

Initial interface position: Z = 0.7800

Scavenger positions	D' = 8218	D' = 8606	D' = 0.8993	D' = 0.9769
X	Z*	Z*	Z*	Z*
0.00	0.6682	0.6621	0.6589	0.6563
0.14	0.6558	0.6556	0.6551	0.6545
0.43	0.6490	0.6493	0.6495	0.6496
0.72	0.6520	0.6521	0.6521	0.6522
1.16	0.6622	0.6622	0.6622	0.6622
1.74	0.6780	0.6780	0.6780	0.6780
2.32	0.6934	0.6934	0.6934	0.6934
2.90	0.7077	0.7077	0.7077	0.7077
3.48	0.7178	0.7178	0.7178	0.7178
4.64	0.7364	0.7364	0.7364	0.7364
5.80	0.7550	0.7550	0.7550	0.7550
7.06	0.7753	0.7753	0.7753	0.7753

* Z coordinates of interface for the indicated scavenger well location.

well was very effective and simple to achieve and maintain in the model experiments. The computed interface did not correspond well with the observed interface. Again the computed interface appeared to be a function of the net pumping rate, which in the case of the scavenger well is even greater than the pumping rate of the drain alone. Due to the poor correspondence between the observed and computed interface, the computational routine could not be used to reliably determine the optimum scavenger drain position and pumping rate to control the upconing of the interface.

From the experiments that were run, it appears that the optimum scavenger position would be relatively close to the drain. This would allow the scavenger to operate at small pumping rates which would just intercept the upconed salt water, holding the amount produced to a minimum.

Analysis of Results

The computation of the shape of the upconed interface between fresh and salt water, when the former is pumped from a drain, was approximated by a computer program based on Muskat's approximation. This approximation was "corrected" empirically and could be used satisfactorily for an interface which penetrated up to 50% of the thickness of the original fresh water zone. Changing the interval of penetration of the drain did not alter the shape of the interface significantly. The extension of the drain over a larger producing interval affects the potential distribution in the immediate vicinity of the drain only; its effect on the potential distribution of the entire system is negligible.

Once a computational routine had been established which would compute the shape and position of the interface with some degree of accuracy over a considerable range of the thickness of the fresh water zone, it should also be useful to evaluate the effect the pumping of a doublet or scavenger system has on the upconed interface by the superposition of the potential distribution of two drains or a drain and an injection well. This was not to be the case; at best, only qualitative results could be obtained. No definite results were obtained for altering the position or the penetration interval for both of these cases. However, the shape and height of the computed interface was very sensitive to the net pumping rate; in other words, the production rate of the drain and the special technique combined. From this it appears that using the potential differences along the initial interface position yields satisfactory results for the drain alone but not for the special techniques for controlling the interface.

SUMMARY AND CONCLUSIONS

This report represents an attempt to devise an approximate mathematical means to study and analyze the effectiveness of a hydraulic doublet or a scavenger well system in controlling or depressing the upconed interface beneath a pumping drain. A technique was developed which utilizes a high speed digital computer to describe the shape and position of the upconed interface between two liquids of differing densities, such as fresh water and sea water in a porous medium when the lighter liquid is being pumped from a drain. The results obtained and the corresponding experiments apply to a homogeneous and isotropic porous medium for the two-dimensional flow into a drain. The computational technique was based on the Muskat approximation which was modified empirically to make it useful over a wider range of interface displacements. This technique was applicable for an upconed interface which penetrated up to 50% of the thickness of the zone of the upper liquid when the drain was near the top of the aquifer. Experiments conducted on a Hele-Shaw model confirmed the computed interface quite well for a height of the interface of up to 50% and a density difference of approximately 9%.

In reviewing the results of this report it appears that the greatest portion of the height of the upconed interface was due to the tilt resulting from the motion of the fresh water over the assumed stationary salt water. The vertical potential gradients along the initial interface beneath the pumping drain were small compared to the horizontal potential gradients due to the motion of the fresh water. However, these small differences in potential can cause a large change in interface position when the density contrast between the liquids is small.

The computational routine did not reliably describe the shape and position of the upconed interface for either the hydraulic doublet or the scavenger well systems. This routine would compute the position of the interface from the potential differences along the imaginary line of the initial horizontal interface. The poor results can be attributed in part to the horizontal potential differences due to the net flow of fluids being greater than the vertical potential differences in the plane of the drain. The influence of the drain and the special techniques on the vertical potential gradients diminished rapidly in the horizontal direction away from the drain. In most cases their effect did not extend more than two times the thickness of the zone of the lighter fluid. Only within this region did the upconing deviate from the "tilted" interface. Another reason for the poor results experienced for the computed interface position in the case of the special techniques is that superposition of the solutions for the potential differences for the drain and the special techniques is not strictly valid. To begin with, these individual solutions are obtained by approximations of the non-linear problem. Thus there is room for doubt that superposition of these solutions for the special pumping techniques can be justified.

One of the main sources of error in using a Hele-Shaw model is that the hydraulic conductivity of the model is a function of the square of the spacing between the plates. Small changes in this spacing due to warping of the plates could appreciably change the value of the hydraulic conductivity. Careful assembly of the model should minimize this source of error. The smooth curves of the observed upconed interface indicated that the plate spacing was fairly uniform throughout the model. The addition of Clorox to the liquids prevented organic growth which could also have influenced the hydraulic conductivity.

The actual location of the upconed interface beneath the drain often was difficult to determine experimentally. This is a point of stagnation in the flow system and the actual interface became diffused in the region beneath the drain and between the drain and the scavenger well. Because of the flow pattern in the doublet system, the interfaces usually were quite sharp.

The value of 4/3 used for the "correction" of Muskat's approximation was found to describe the height of the upconed interface the closest. An interface computed by this method would approximate an observed interface better than if it were calculated by the free surface analogy based on Dupuit assumptions. This value was established empirically by matching the computed results to the experimentally determined interface. Although this worked quite well for the system used, extension of this factor to other values for hydraulic conductivity or density contrasts is not justified without further investigation.

The effectiveness of the hydraulic doublet still is doubtful. If a portion of the produced fresh water must be injected in an attempt to produce the drain at a higher rate, then it appears that producing the drain at the net rate only will give the same results. Since this would be the simpler installation and operation of the system, it probably would also be the most economic.

The scavenger system can be used to obtain greater pumping rates of fresh water without contamination by upconing. When more fresh water is produced, a greater amount of salt water must also be produced. Locating the scavenger fairly close to the drain will keep the amount of salt water produced to a minimum. In the operation of this type of system however, one is faced with the problem of disposal of the unwanted salt water.

RECOMMENDATIONS FOR FURTHER INVESTIGATIONS

Only liquids of a limited density contrast were used in the experiments and calculations of this study. This density contrast is greater than that of the fresh water-sea water system and it might be advisable to verify the results obtained here with experiments using a density difference corresponding to the fresh water-sea water system. Although the results of this study did not indicate the computed interface to be very sensitive to the "correction factor" used, it is a distinct possibility that this factor could be dependent on the density difference of the liquids in the system.

It could well be that the computation of the interface should be based on the potential differences along some line other than the position of the initial interface. This could be at the position of the drain or the special technique or some other line which might have to be determined experimentally. Due to the singularity of the sinks and/or sources, computations directly to these points would not be possible. However, they could be carried out to the very close proximity of these points.

Another method which ought to be investigated further is to evaluate the effectiveness of the special techniques from the potential distribution or the stream line pattern in the region of flow instead of attempting to describe the shape and position of the upconed interface. A high-speed digital computer and a contouring routine would greatly facilitate this task.

An initially horizontal interface between liquids in a porous medium probably does not occur naturally. Generally at least one of the fluids is moving which causes the interface to be slanted. Any pumping from such a system will change the tilt of the interface and cause an additional upconing due to vertical potential gradients in the vicinity of the drain. This indicates that a more detailed investigation of the distribution of potentials could lead to a separation of the upconed portion from the tilted part of the interface. The problem of initiating the computation for the upconed interface would thus be avoided. One approach to the solution of this problem would be to use the computed value of ζ in the equation relating it to the specific discharge.

APPENDIX A

COMPUTER PROGRAM FOR CALCULATING AND PLOTTING OF UPCONED INTERFACE

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FORTRAN IV     MODEL 44  PS ,     VERSION 3, LEVEL 3   DATE  71130                    PAGE  0001

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      C     *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      C
      C     ADAPTIVE INTEGRATION  -  GREENS FUNCTION  -  DATA DECK
      C
      C         DEFINITIONS
      C
      C       HC             HYDRAULIC CONDUCTIVITY (K)
      C       HW             THICKNESS BETWEEN PLATES ON HELE-SHAW MODEL
      C       XD             INDEPENDANT VARIABLE OF MODEL DATA
      C       ZD             INTERFACE VALUES FROM MODEL
      C       FWZ            FRESH WATER THICKNESS OF AQUIFER
      C       WPR            WELL PUMPING RATE
      C       DPR            DOUBLET PUMPING RATE
      C       XIO            MAXIMUM VALUE OF XDC(USED IN DETERMINING XIHI)
      C       TOL            TOLERANCE FOR ADAPTIVE INTEGRATION
      C       NUMPLT         NUMBER OF VALUES TO BE CALCULATED FOR PLOTTING
      C       WTS,WBS        TOP AND BOTTOM SCREENS OF WELL
      C       DTS,DBS        TOP AND BOTTOM SCREENS OF DOUBLET
      C       XDC,ZDC        NORMALIZED VALUES OF XD AND ZD
      C       DSCHRG,CONA    WELL AND DOUBLET RATES TO BE PLACED ON GRAPH
      C       VISC1,VISC2    VISCOSITIES OF LIQUIDS 1 AND 2
      C       GAMMA1,GAMMA2  SPECIFIC GRAVITIES OF LIQUIDS 1 AND 2
      C       XIHI,XILOW,
      C       ZETAHI,ZETLOW  SCALING DETERMINANTS FOR PLOT ROUTINE
      C
      C     *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      C
            COMMON PI,CHI3,ZINFCK
            COMMON/DMAIN/WLCTN,DLCTN,A1W,A1D,BW,BD,TOL
            COMMON/GMAIN/OOKD,CONSTO,O1KD,O2KD,PID
            COMMON/GRAF/CONA,DSCHRG,WINVAL,DINVAL,N3,DATE(50),STINF1,SLOPE,XIO
            DIMENSION HDING(26),STINF(20)
            DIMENSION A(100),CHI(13),ZINF(4),XD(20),XDC(20),ZD(20),ZDC(20)
            DATA HDING/'         DATA INPUT - EXPERIMENTAL WELL FROM             ',
           'TO          ,RATE= DOUBLET FROM          TO          ,RATE= '/
            MAXPT=13
            READ(5,1000) (XD(M),M=1,MAXPT)
            READ(5,1001) XIHI,XILOW,ZETAHI,ZETLOW
      C
      C         READ IN AND DEFINE CONSTANTS
      C      WHEN USING SYMMETRY  --  Q=1/2*Q(WELL)
            READ (5,1003)HC, HW, TOL, VISC1, VISC2, GAMMA1, GAMMA2
            DO 16 N3=1,100
            READ (5,1002,END=17) (DATE(K),K=1,7)
            READ (5,1003) WPR, WTS, WBS, DPR, DTS, DBS
            READ(5,1000) (ZD(M),M=1,MAXPT)
            NCHECK=0
            PI = 3.14159
            PID = 1.0/(2.*PI)
            DSCHRG=WPR/HW
            FWZ=ZD(MAXPT)+3.5
            HW=SQRT(.12*HC/980.)
            PRINT 2000,(HDING(K),K=2,8),(HDING(K),K=9,17),WTS,WBS,WPR,(HDING
           (K),K=18,26),DTS,DBS,DPR
            PRINT 2001, HC, FWZ, HW
      C
      C         NORMALIZE DATA
            DO 2 MM=1,MAXPT
            XDC(MM)=XD(MM)/FWZ
          2 ZDC(MM)=(ZD(MM)+3.5)/FWZ
            XIO=XD(MAXPT)/FWZ
            AVINF=XIO*3.
      C
      C         SET UP VALUES TO BE USED IN PLOT ROUTINE
            NUMBER=XIO+.90
            XIHI=NUMBER
            NUMPLT=NUMBER*20+1
            WINVAL=0.0
            DINVAL=0.0
            WLCTN=(WTS+WBS)/(2.0*FWZ)
            DLCTN=(DTS+DBS)/(2.0*FWZ)
      C
      C         SET UP CONSTANTS TO COMPUTE INTERFACE

FORTRAN IV     MODEL 44  PS   ,   VERSION 3, LEVEL 3   DATE  71130                    PAGE  0002

            CONA=DPR/HW
            DELGAM=GAMMA2-GAMMA1
            DELTA=DELGAM/GAMMA1
            IF (GAMMA2.LT.0.01)DELTA = 1.0
            PRIMEK=HC*DELTA
            CONSTO=ALOG(COSH(PI*XIO))*2.0
            O1KD=WPR/(HW*HC*FWZ*DELTA)
            O2KD=DPR/(HW*HC*FWZ*DELTA)
            OOKD=O1KD+O2KD
            A1W=WTS/FWZ
            IF(WTS.EQ.0.0) A1W=0.00001
            BW=WBS/FWZ
            A1D=DTS/FWZ
            BD=DBS/FWZ
      C
      C         CALCULATE STRAIGHT LINE INTERFACE
            CHI3=1.0
            SLOPE=OOKD
            STINF(1)=1.0-SLOPE*XDC(MAXPT)
            STINF1=STINF(1)
            DO 3 I=2,MAXPT
          3 STINF(I)=SLOPE*XDC(I)+STINF(1)
      C
      C         GENERATE XI AND CHI GRID
            DO 13 L=1,2
            DO 4 I=1,MAXPT
          4 CHI(I)= 1.0
            PRINT 3000
            SMDZSQ=0.0
            IF(L.EQ.2) GO TO 9
            DO 8 I=1,MAXPT
            CHI3 = CHI(I)
            DO 5 J=1,4
            CHI1=CHI(I)
      C
      C         COMPUTE INTERFACE
            ZINF(J)=G(XDC(I),CHI1)
      C         ITERATE INTERFACE
          5 CHI(I)=ZINF(J)
            DIFF= ZINF(1)-STINF(I)
            IF(DIFF.GE.0.00001) GO TO 7
            IF(NCHECK.EQ.1) GO TO 7
            NCHECK=1
            XCK=XDC(I-1)
          6 XCK=XCK+1.0E-02
            ZINFCK=G(XCK,1.0)
            STCK=SLOPE*XCK+STINF1
            DIFF=ZINFCK-STCK
            IF(DIFF.GE.0.00001) GO TO 6
          7 DELZ=ZDC(I)-ZINF(1)
            DELZSQ=DELZ**2
            PRINT 4000,XDC(I),ZDC(I),(ZINF(KK),KK=1,4),DELZ,DELZSQ
          8 SMDZSQ=SMDZSQ+DELZSQ
            GO TO 12
          9 DO 11 I=1,MAXPT
            CHI3 = CHI(I)
            DO 10 J=1,4
            CHI1=CHI(I)
      C
      C         COMPUTE INTERFACE
            ZINF(J)=G(XDC(I),CHI1)
      C         ITERATE INTERFACE
            ZINF(J)=ZINF(J)-(1.-ZINF(J))*(XIO-XDC(I))/AVINF
         10 CHI(I)=ZINF(J)
            DELZ=ZDC(I)-ZINF(1)
            DELZSQ=DELZ**2
            PRINT 4000,XDC(I), ZDC(I),(ZINF(KK),KK=1,4),DELZ,DELZSQ
         11 SMDZSQ=SMDZSQ+DELZSQ
         12 PRINT 5000,SMDZSQ
         13 CONTINUE
            CALL GRAPH(XIHI,XILOW,ZETAHI,ZETLOW,1,NUMPLT,XDC,ZDC)
         16 CONTINUE
         17 PRINT 6000
       1000 FORMAT(13F6.3)
```

```
0100      1001 FORMAT(4F5.2)
0101      1002 FORMAT (15A4)
0102      1003 FORMAT (7F10.5)
0103      2000 FORMAT('1',4X,7A4///2(15X,9A4,T29,F6.3,T39,F6.3,T52,F8.4//))
0104      2001 FORMAT ('+',15X,'HYD. COND.=',F5.2,3X,'DEPTH=',F5.2,3X,'HW=',F6.4
              ///)
0105      3000 FORMAT(7X,'X',6X,'Z(EXP)',5X,'ZINF1',5X,'ZINF2',5X,'ZINF3',5X,
              'ZINF4',4X,'DELTAZ',2X,'DELTAZ**2')
0106      4000 FORMAT(9F10.4)
0107      5000 FORMAT(' ',70X,'----------'/56X,'SUM DELTAZ**2=',F10.4//////)
0108      6000 FORMAT('1           ')
0109           CALL EXIT
0110           END

 TOTAL MEMORY REQUIREMENTS 0011B0 BYTES
```



```
0001           SUBROUTINE GRAPH(XHI,XLOW,YHI,YLOW,N,NUMPLT,XGRAF,YGRAF)
0002           COMMON/GRAF/CONA,DSCHRG,WINVAL,DINVAL,N3,DATE(50),STINF1,SLOPE,XIO
0003           COMMON/DMAIN/WLCTN,DLCTN,AIW,AID,BW,BD,TOL
0004           COMMON PI,CHI3,ZINFCK
0005           DIMENSION XGRAF(20),YGRAF(20),ZINF(200,4),CHI(200),STINF(200)
          C       SET BOUNDARIES AND SCALING FOR GRAPH
0006           XSPACE=(XHI-XLOW)/8.0
0007           YSPACE=(YHI-YLOW)/5.0
0008           XSCALE=1./XSPACE
0009           YSCALE=1./YSPACE
0010           XREF=XLOW*XSCALE
0011           YREF=YLOW*YSCALE
0012           IF(N3.EQ.1) CALL PLOT (15.0,-25.0,-3)
          C       OUTLINE PAGE (8.5IN.X11.0IN.)
0013           CALL PLOT (0.0,0.0,-3)
0014           CALL PLOT (0.0,11.0, 2)
0015           CALL PLOT (8.5,11.0 2)
0016           CALL PLOT (8.5,0.0, 2)
0017           CALL PLOT (0.0,0.0, 2)
0018           IF(N3.GT.1)GO TO 3
0019           CALL SYMBOL (-0.2,4.0,0.2,'HAUBOLD',90.0,7)
0020           CALL PLOT (-1.25,0.0,3)
0021           CALL PLOT (-1.25,11.0,2)
          C       OUTLINE PLOT AND ESTABLISH NEW REFERENCE
0022         3 CALL PLOT (2.0,2.0,-3)
0023           CALL PLOT (0.0,8.0,2)
0024           CALL PLOT (5.0,8.0,2)
0025           CALL PLOT (0.0,0.0,3)
0026           CALL AXIS (0.0,0.0,'ZETA',-4,5.0,0.0,YLOW,YSPACE,10.0)
0027           CALL AXIS (5.0,0.0,'XI',-2,8.0,90.0,XLOW,XSPACE,10.0)
0028           CALL SYMBOL(1.0,4.0,0.1,DATE,90.0,28)
          C       PLACE WELL AND DOUBLET POSITIONS ON GRAPH
0029           PINVAL=WINVAL*YSCALE-YREF
0030           GINVAL=DINVAL*YSCALE-YREF
0031           ZETA1G=ZETA1*YSCALE-YREF-0.05
0032           ZETA2G=ZETA2*YSCALE-YREF-0.05
0033           CALL SYMBOL (ZETA1G,0.0,0.2,63,0.0,-1)
0034           ZETA1G=ZETA1G+0.05
0035           CALL PLOT (ZETA1G,0.160,3)
0036           ZETA1G=ZETA1G+PINVAL
0037           CALL PLOT (ZETA1G,0.160,2)
0038           ZETA1G=ZETA1G-0.05
0039           CALL SYMBOL (ZETA1G,0.0,0.2,63,0.0,-1)
0040           ZETA1G=ZETA1G+0.10-(PINVAL/2.0)
0041           CALL SYMBOL (ZETA1G,0.3,0.1,'DRAIN, Q=',90.0,9)
0042           CALL NUMBER (ZETA1G,1.2,0.1,DSCHRG,90.0,2)
0043           IF(CONA.EQ.0.0) GO TO 7
0044           CALL SYMBOL (ZETA2G,0.0,0.2,62,0.0,-1)
0045           ZETA2G=ZETA2G+0.05
0046           CALL PLOT (ZETA2G,0.160,3)
0047           ZETA2G=ZETA2G+GINVAL
0048           CALL PLOT (ZETA2G,0.160,2)
0049           ZETA2G=ZETA2G-0.05
0050           CALL SYMBOL (ZETA2G,0.0,0.2,62,0.0,-1)
0051           ZETA2G=ZETA2G+0.10-(GINVAL/2.0)
0052           CALL SYMBOL (ZETA2G,0.3,0.1,'DOUBLET, Q=',90.0,11)
0053           CALL NUMBER (ZETA2G,1.3,0.1,CONA,90.0,2)
          C       GENERATE AND PLOT ORIGINAL INTERFACE POSITION
0054         7 XGEN=0.0
0055           IF(YSCALE.GT.5.0) GO TO 18
0056           XCTOFF=YSCALE
0057           YCTOFF=0.0
0058           CALL PLOT (XCTOFF,YCTOFF,3)
          C
0059           DO 17 I=1,31,2
0060           BI=I
0061           CI=I+1
0062           YSOLID=BI*.25
0063           YSPACE=CI*.25
0064           CALL PLOT (XCTOFF,YSOLID,2)
0065        17 CALL PLOT (XCTOFF,YSPACE,3)
0066        18 N1=N
0067           DO 13 L=1,1
0068           DO 9 I=1,NUMPLT
```

```
    9 CHI(I)=1.0
      CHI3=1.0
      AVINF=XI0*3.
      DO 12 J=1,4
C       MOVE PEN TO FIRST INTERFACE VALUE
      XPLOT=0.0
      ZINF(1,J)=G(0.0,CHI(1))
      ZINF(1,J)=ZINF(1,J)-(1.-ZINF(1,J))/3.
      YPLOT=ZINF(1,J)*YSCALE-YREF
      CALL PLOT (YPLOT,XPLOT,3)
      IF(N1.EQ.0)N=N+1
C       GENERATE AND PLOT EXACT INTERFACE POSITION
      XI=0.05
      DO 12 I=2,NUMPLT
      STINF(I)=STINF1+SLOPE*XI
      ZINF(I,J)=G(XI,CHI(I))
      ZINF(I,J)=ZINF(I,J)-(1.-ZINF(I,J))*(XI0-XI)/AVINF
      IF (J.NE.N)GO TO 2
      IF(ZINF(I,J).GT.1.0) GO TO 2
    1 YPLOT=ZINF(I,J)*YSCALE-YREF
      XPLOT=XI*XSCALE-XREF
      CALL PLOT (YPLOT,XPLOT,2)
    2 CHI(I)=ZINF(I,J)
   12 XI=XI+0.05
   13  CONTINUE
C       PLOT VALUES OF INTERFACE AS TAKEN FROM MODEL
      DO 697 I=1,13
      YPLOT=YGRAF(I)*YSCALE-YREF
      XPLOT=XGRAF(I)*XSCALE-XREF
      CALL SYMBOL(YPLOT,XPLOT,0.1,03,0.0,-1)
  697 CONTINUE
      CALL PLOT(7.75,-2.0,-3)
      CALL PLOT(0.0,11.0,2)
      CALL PLOT(1.25,0.0,-3)
      RETURN
      END
```

TOTAL MEMORY REQUIREMENTS 001F00 BYTES

FORTRAN IV MODEL 44 PS VERSION 3, LEVEL 3 DATE 71130 PAGE 0001

```
      FUNCTION F(Z)
      COMMON/DF/CHI1,COSHX
      COMMON PI,CHI3,ZINFCK
       Z1 = (CHI1-Z)*PI
       Z2 = (CHI1+Z)*PI
        COS1 = COS(Z1)
        COS2 = COS(Z2)
       AGMNT = (COSHX-COS1)*(COSHX-COS2)
      F = ALOG(AGMNT)
      RETURN
      END
```

TOTAL MEMORY REQUIREMENTS 000218 BYTES

```
FORTRAN IV     MODEL 44  PS        VERSION 3,  LEVEL 3   DATE  71130                    PAGE  0001

              FUNCTION D(CHI2,X,ZETA )
              EXTERNAL F
              COMMON PI,CHI3,ZINFCK
              COMMON/DF/CHI1,COSHX
              COMMON/DMAIN/WLCTN,DLCTN,A1W,A1D,BW,BD,TOL
              CHI1=CHI2
              COSHX=COSH(PI*X)
            1 Z=WLCTN
              IF(ZETA.EQ.1.0)Z=DLCTN
              D=F(Z)
              RETURN
              END

  TOTAL MEMORY REQUIREMENTS 000238 BYTES
```

```
FORTRAN IV     MODEL 44  PS        VERSION 3,  LEVEL 3   DATE  71130                    PAGE  0001

C
C      * * * * * * * * * * * * * * * * * * * * * * *
C
C
C          FUNCTION TO COMPUTE WELL OR DOUBLET POTENTIALS
C
              FUNCTION G(X,CHI1)
              COMMON PI,CHI3,ZINFCK
              COMMON/GMAIN/Q0KD,CONSTO,Q1KD,Q2KD,PID
              ZETA1=0.0
              ZETA2=1.0
              PART1=(Q1KD*CONSTO)*PID
              PART2=Q1KD*D(CHI1,X,ZETA1)*PID
              IF(ABS(Q2KD).LT.0.00001) GO TO 7
              PART3=Q2KD*D(CHI1,X,ZETA2)*PID
              GO TO 8
            7 PART3=0.0
            8 G=CHI3-PART1+PART2+PART3
              RETURN
              END

  TOTAL MEMORY REQUIREMENTS 000270 BYTES
COMPILER HIGHEST SEVERITY CODE WAS  0
// EXEC LOADER                                                         16.03.43
LOADER HIGHEST SEVERITY WAS  0 --  EXECUTION                          LOAD TIME    0 MIN   9 SEC
```

```
      FUNCTION ADINTG(F,A1,B,TOL)
C FROM CACM
C ALGORITHM 182
C NONRECURSIVE ADAPTIVE INTEGRATION
C W. M. MC KEEMAN AND LARRY TESLER
C STANFORD UNIVERSITY, STANFORD, CALIFORNIA
C TRANSLATED FROM ALGOL TO FORTRAN FOR NMINT BY JOHN MATZEK
C
C A, B    = LIMITS OF INTEGRATION
C EPS     = TOLERANCE
C F       = USER SUPPLIED FUNCTION SUBROUTINE(EXTERNAL TO MAIN)
      DIMENSION DX(30),EPSP(30),X2(30),X3(30),F2(30),F3(30),F4(30),
     C          FMP(30),FBP(30),EST2(30),EST3(30),PVAL(30,3)
      INTEGER RTRN(30)
C
C
      A=A1
      EPS=TOL
      LVL = 0
      ABAREA = 1.0
      EST = 1.0
      DA = B - A
      FA = F(A)
      FM = 4.0*F((A+B)/2.0)
      FB = F(B)
C RETURN HERE IF NOT YET LEVEL 30
  100 CONTINUE
      LVL=LVL+1
      DO 101 LVE=LVL,30
      DX(LVE) = DA/3.0
      SX = DX(LVE)/6.0
      X2(LVE) = A + DX(LVE)
      X3(LVE) = X2(LVE) + DX(LVE)
      F1 = 4.0*F(A + DX(LVE)/2.0)
      F2(LVE) = F(X2(LVE))
      F3(LVE) = F(X3(LVE))
      F4(LVE) = 4.0*F(X3(LVE) + DX(LVE)/2.0)
      FMP(LVE) = FM
      FBP(LVE) = FB
      EPSP(LVE) = EPS
      EST1 = (FA + F1 + F2(LVE))*SX
      EST2(LVE) = (F2(LVE) + F3(LVE) + FM)*SX
      EST3(LVE) = (F3(LVE) + F4(LVE) + FB)*SX
      SUM = EST1 + EST2(LVE) + EST3(LVE)
      ABAREA = ABAREA - ABS(EST) + ABS(EST1) + ABS(EST2(LVE))
     C              + ABS(EST3(LVE))
C
      IF         (ABS(EST-SUM) .LE. EPSP(LVE)*ABAREA
     C       .AND. EST .NE. 1.0
     C       .OR. LVE .GE. 30) GO TO 901
      DA = DX(LVE)
      RTRN(LVE) = 1
      FM = F1
      FB = F2(LVE)
      EPS = EPSP(LVE)/1.7
  101 EST = EST1
C
  901 LVL=LVE
900   CONTINUE                                                          UP
      LVL = LVL - 1
      PVAL(LVL,RTRN(LVL)) = SUM
      I = RTRN(LVL)
      IF(I-2) 200,300,400
C
200   CONTINUE                                                          R1
      RTRN(LVL) = 2
      DA = DX(LVL)
      FA = F2(LVL)
      FM = FMP(LVL)
      FB = F3(LVL)
      EPS = EPSP(LVL)/1.7
      EST = EST2(LVL)
      A = X2(LVL)
      GO TO 100
```



```
C
300   CONTINUE                                                          R2
      RTRN(LVL) = 3
      DA = DX(LVL)
      FA = F3(LVL)
      FM = F4(LVL)
      FB = FBP(LVL)
      EPS = EPSP(LVL)/1.7
      EST = EST3(LVL)
      A = X3(LVL)
      GO TO 100
C
400   CONTINUE                                                          R3
      SUM = PVAL(LVL,1) + PVAL(LVL,2) + PVAL(LVL,3)
      IF (LVL .GT. 1) GO TO 900
      ADINTG = SUM
      RETURN
      END
```

TOTAL MEMORY REQUIREMENTS 0002C BYTES

COMPILER HIGHEST SEVERITY CODE WAS 0
// EXEC LOADER   04.06.03

LOADER HIGHEST SEVERITY WAS 0 -- EXECUTION   LOAD TIME   0 MIN   9 SEC

APPENDIX B

COMPUTER PROGRAM FOR GENERATING

POTENTIAL VALUES IN THE FLOW REGION

```
C
C
C     ADAPTIVE INTEGRATION  -  POTENTIAL FUNCTION
C
      EXTERNAL F
      COMMON COSHX, CHII, PI
      DIMENSION A(100), XI(30), CHI(70), PHIW(30,70)
      READ (5,1001) (A(K), K=1,90)
      PRINT 1002, (A(K), K=1,15)
      READ (5,1010) [illegible]
      READ (5,1012) XSTEP1,XSTEP2,XSTEP3,XSTEP4,IA,IB,IC,IF,LPA
      READ (5,1013) ZSTEP1, JROW
      A1 = 0.00001
      PI = 3.14159
C
      B = 0.05
      DO 710 LP=1,LPA
      DIFF = B-A1
C
      DO 700 II=1,2
      I1 = (II-1)*9+1
      I2 = I1+8
      LPII = LP+II
      IF (LPII.LE.2) GO TO 50
      PRINT 1020, (A(K), K=16,30)
   50 PRINT 1021, (A(K), K=31,45)
      PRINT 1022, A1, B
      PRINT 1030, (A(K), K=46,60)
C
      XI(1) = 0.1
      CHI(1)= 0.00
      JROWA = JROW/5
C
      DO 110 I=I1,I2
      IX1 = I+1
      XSTEP = XSTEP1
      IF (I.GE.IA) XSTEP = XSTEP2
      IF (I.GE.IB) XSTEP = XSTEP3
  110 XI(IX1) = XI(I) + XSTEP
      PRINT 1040, (XI(I), I=I1,I2)
C
      DO 610 JJ=1,JROWA
      JJA = 5
      IF (JJ.EQ.1) JJA=1
      JROW1 = (JJ-2)*5+2
      IF (JJ.EQ.1) JROW1=1
      JROW2 = JROW1 + 4
      IF (JJ.EQ.1) JROW2=1
C
      DO 606 JA=1,JJA
      J = (JJ-2)*5+1+JA
      IF (JJ.EQ.1) J=JA
      J1 = J+1
      CHII = CHI(J)
C
      DO 510 I=I1,I2
      IX1 = I+1
      XITT = XI(I)*PI
      COSHX = COSH(XITT)
      ANSWER = ADINTG(F,A1,B,TOL)
      PHIW(I,J) = ANSWER/DIFF
      XSTEP = XSTEP1
      IF (I.GE.IA) XSTEP = XSTEP2
      IF (I.GE.IB) XSTEP = XSTEP3
  510 XI(IX1) = XI(I) + XSTEP
  606 CHI(J1) = CHI(J) + ZSTEP1
      PRINT 1199
  610 PRINT 1200, (CHI(J),(PHIW(I,J),I=I1,I2),J=JROW1,JROW2)
  700 PRINT 1300
  710 B = B + 0.05
C
 1001 FORMAT (15A4)
 1002 FORMAT ('9',' .',109X,'.'///////////////22X,15A4/22X,15A4)
 1010 FORMAT (7F10.5)
 1012 FORMAT (4F5.2,5I5)
 1013 FORMAT (F5.2,I5)
 1020 FORMAT ('9',' .',109X,'.'///////////////22X,15A4//)
 1021 FORMAT (22X,15A4)
```



```
 1022 FORMAT (35X,2F11.2)
 1030 FORMAT (/12X,15A4)
 1040 FORMAT (/18X, 9F9.2)
 1199 FORMAT (' ')
 1200 FORMAT (14X,F4.2, 9F9.3)
 1300 FORMAT ('7',///' .',109X,'.'///)
      CALL EXIT
      END
```

```
0001              FUNCTION F(Z)
0002              COMMON COSHX, CHI1, PI
0003               Z1 = (CHI1-Z)*PI
0004               Z2 = (CHI1+Z)*PI
0005                COS1 = COS(Z1)
0006                COS2 = COS(Z2)
0007               AGMNT = (COSHX-COS1)*(COSHX-COS2)
0008              F = ALOG(AGMNT)
0009              RETURN
0010              END
```



```
0001              FUNCTION ADINTG(F,A1,B,TOL)
            C FROM CACM
            C ALGORITHM 182
            C NONRECURSIVE ADAPTIVE INTEGRATION
            C W. M. MC KEEMAN AND LARRY TESLER
            C STANFORD UNIVERSITY, STANFORD, CALIFORNIA
            C TRANSLATED FROM ALGOL TO FORTRAN FOR NMIMT BY JOHN MATZEK
            C
            C A, B    = LIMITS OF INTEGRATION
            C EPS     = TOLERANCE
            C F       = USER SUPPLIED FUNCTION SUBROUTINE(EXTERNAL TO MAIN)
0002              DIMENSION DX(30),EPSP(30),X2(30),X3(30),F2(30),F3(30),F4(30),
                 C          FMP(30),FBP(30),EST2(30),EST3(30),PVAL(30,3)
0003              INTEGER RTRN(30)
            C
            C
0004              A=A1
0005              EPS=TOL
0006              LVL = 0
0007              ABAREA = 1.0
0008              EST = 1.0
0009              DA = B - A
0010              FA = F(A)
0011              FM = 4.0*F((A+B)/2.0)
0012              FB = F(B)
            C RETURN HERE IF NOT YET LEVEL 30
0013          100 CONTINUE
0014              LVL=LVL+1
0015              DO 101 LVE=LVL,30
0016              DX(LVE) = DA/3.0
0017              SX = DX(LVE)/6.0
0018              X2(LVE) = A + DX(LVE)
0019              X3(LVE) = X2(LVE) + DX(LVE)
0020              F1 = 4.0*F(A + DX(LVE)/2.0)
0021              F2(LVE) = F(X2(LVE))
0022              F3(LVE) = F(X3(LVE))
0023              F4(LVE) = 4.0*F(X3(LVE) + DX(LVE)/2.0)
0024              FMP(LVE) = FM
0025              FBP(LVE) = FB
0026              EPSP(LVE) = EPS
0027              EST1 = (FA + F1 + F2(LVE))*SX
0028              EST2(LVE) = (F2(LVE) + F3(LVE) + FM)*SX
0029              EST3(LVE) = (F3(LVE) + F4(LVE) + FB)*SX
0030              SUM = EST1 + EST2(LVE) + EST3(LVE)
0031              ABAREA = ABAREA - ABS(EST) + ABS(EST1) + ABS(EST2(LVE))
                 C            + ABS(EST3(LVE))
            C
0032                 IF         (ABS(EST-SUM) .LE. EPSP(LVE)*ABAREA
                 C        .AND. EST .NE. 1.0
                 C        .OR. LVE .EQ. 30) GO TO 901
0033              RTRN(LVE) = 1
0034              DA = DX(LVE)
0035              FM = F1
0036              FB = F2(LVE)
0037              EPS = EPSP(LVE)/1.7
0038          101 EST = EST1
            C
0039          901 LVL=LVE
            C
0040        900   CONTINUE                                              UP
0041              LVL = LVL - 1
0042              PVAL(LVL,RTRN(LVL)) = SUM
0043              I = RTRN(LVL)
0044              IF(I-2) 200,300,400
            C
0045        200   CONTINUE                                              R1
0046              RTRN(LVL) = 2
0047              DA = DX(LVL)
0048              FA = F2(LVL)
0049              FM = FMP(LVL)
0050              FB = F3(LVL)
0051              EPS = EPSP(LVL)/1.7
0052              EST = EST2(LVL)
0053              A = X2(LVL)
0054              GO TO 100
            C
0055        300   CONTINUE                                              R2
0056              RTRN(LVL) = 3
```

```
          DA = DX(LVL)
          FA = F3(LVL)
          FM = F4(LVL)
          FB = FBP(LVL)
          EPS = EPSP(LVL)/1.7
          EST = EST3(LVL)
          A = X3(LVL)
          GO TO 100
C
  400     CONTINUE                                                R3
          SUM = PVAL(LVL,1) + PVAL(LVL,2) + PVAL(LVL,3)
          IF (LVL .GT. 1) GO TO 900
          ADINTG = SUM
          RETURN
          END
```

APPENDIX C

EXPERIMENTAL DATA SHOWN IN GRAPHS

Observed Position of Interface, (z), at Various Values of (x) and Differing Drain Pumping Rates

Ambient air temperature: 22° C   Specific gravity of salt water: 1.0920

Model Parameters: drain interval: z = 0.9 to 1.9 cm
initial interface position z = 39.0 cm

Run No.	09271-1	09272-2	09271-3	09271-4	09271-5	09271-6	09271-7	09271-8
Q(cc/sec)	2.28	4.06	6.08	8.14	9.96	11.9	13.8	15.6
x(cm)	z(cm)	z(cm)	z(cm)	z(cm)	z(cm)	z(cm)	z(cm)	z(cm)
0.0	36.6	34.8	32.6	30.2	28.2	26.0	23.7	21.3
5.0	36.8	35.2	33.0	30.6	28.7	26.3	24.0	21.6
10.0	36.9	35.3	33.2	30.9	29.0	26.7	24.2	22.2
20.0	32.0	35.5	33.6	31.4	29.6	27.4	25.0	23.3
30.0	37.1	35.7	34.0	31.9	30.0	28.1	25.8	24.1
40.0	37.2	35.8	34.1	32.1	30.4	28.6	26.3	24.8
60.0	37.3	36.2	34.7	32.8	31.4	30.0	27.6	26.6
80.0	37.5	36.5	35.2	33.5	32.2	31.0	28.9	28.1
100.0	37.6	36.6	35.5	34.0	32.9	31.9	30.2	29.7
120.0	37.8	37.0	36.2	35.0	34.2	33.4	32.0	31.7
160.0	38.3	38.1	37.1	36.5	36.1	35.7	34.6	34.6
200.0	38.7	38.8	38.3	37.9	37.7	37.5	37.1	37.1
243.5	39.0	39.0	39.0	30.0	39.0	39.0	39.0	39.0

Observed Position of Interface, (z), at Various Values of (x) for Differing Pumping Rates and Penetration Intervals.

Specific gravity of salt water was 1.0920

Run No.	17271-1	25271-7	26371-3	14471-3
Ambient air temp. (^{o}C)	21	21	23	22
Initial interface position (z,cm)	39	39	35	35
Drain interval (z,cm)	0.9 1.9	0.9 5.0	0.9 5.0	0.9 15.0
Injection interval (z,cm)	21.0 22.0	21.0 22.0	21.0 22.0	22.3 23.3
Q(cc/sec)	8.19	18.87	8.04	8.03
x(cm)	z(cm)	z(cm)	z(cm)	z(cm)
0.0	29.9	17.3	24.9	25.8
5.0	30.3	17.6	25.5	26.2
10.0	30.6	18.2	25.7	26.3
20.0	31.1	19.7	26.4	26.9
30.0	31.5	20.7	27.0	27.7
40.0	31.8	21.7	27.4	27.9
60.0	32.5	23.7	28.4	29.0
80.0	33.2	25.6	29.1	29.6
100.0	33.7	27.6	29.9	30.0
120.0	34.7	30.3	31.5	31.8
160.0	36.2	31.9	32.5	32.8
200.0	37.7	36.7	34.0	34.4
243.5	39.0	39.0	35.0	35.0

Observed Position of Interface, (z), at Various Values of (x) for Operation of the Hydraulic Doublet System.

Specific gravity of salt water was 1.0920

Ambient air temp.: 21° C

Model parameters: drain interval: z = 1.0 to 8.0 cm
injection interval: z = 20.0 to 23.3
initial interface position: 35 cm

Run No.	17471-2	17471-4	17471-5	17471-7
Q(cc/sec)	8.06	8.06	11.82	11.82
Q' (cc/sec)	0.0	1.26	0.00	0.90
x(cm)	z(cm)	z(cm)	z(cm)	z(cm)
0.0	26.2	27.9	20.8	22.1
5.0	26.5	27.2	21.1	22.4
10.0	26.4	28.1	21.0	22.4
20.0	26.8	28.4	21.6	23.1
30.0	27.7	29.3	22.6	23.8
40.0	28.0	29.3	23.1	24.2
60.0	29.1	29.9	24.5	25.0
80.0	29.8	30.1	25.7	25.9
100.0	30.5	30.7	27.1	27.1
120.00	32.0	32.0	29.2	29.2
160.0	33.2	33.0	31.4	31.4
200.0	34.7	34.7	33.8	33.8
243.0	35.0	35.0	35.0	35.0

Observed Position of the Interface, z, at Various Values of x when the Salt Water was Pumped into the Drain while Operating the Doublet System.

Model parameters: Run No. 24571-1

Ambient air temperature: 23°C
Specific gravity of salt water: 1.0927
Initial interface position: z = 6.5 cm
Interval of fresh water drain: z = 43.2 to 45.1 cm
Injection interval: z = 22.3 to 24.2 cm
Drain pumping rate: 1.33 cc/sec
Injection rate: 0.66 cc/sec

Note: Due to model reconstruction the grid for this experiment has been altered. The z-axis now is positive upwards with the origin at the bottom of the model.

x(cm)	z(cm)	z(cm)(1)	z(cm)(2)
0.0	40.6	40.5	17.0
2.0	40.3	40.2	17.3
5.0	39.0	38.8	19.5
10.0	35.0	31.2	27.0 at x=9.0
15.0	30.0		
20.0	26.0		
25.0	23.1		
30.0	20.1		
40.0	18.5		
50.0	17.0		
60.0	15.5		
80.0	13.3		
100.0	12.3		
120.0	11.0		
160.0	9.3		
200.0	8.0		
243.5	6.5		

(1) The values in this column are the coordinates for the upper portion of the injection bulge.
(2) The values in this column are the coordinates for the lower portion of the injection bulge. No value could be obtained for x=10 and thus the value at x=9 was read.

Observed Position of Interface, z, at Various Values of x for the Scavenger Well System.

Model parameters:
Ambient air temperature: 23°C
Specific gravity of salt water: 1.0927
Initial interface position: z = 6.5 cm
Interval of fresh water drain: z = 43.2 to 45.1 cm
Interval of salt water drain: z = 22.3 to 24.2 cm

Note: Due to model reconstruction the grid for this experiment has been altered. The z-axis now is positive upwards with the origin at the bottom of the model.

Run No.	25571-2	25571-3	25571-4	25571-5	
Q (cc/sec)	1.33	1.33	1.33	2.30	
Q' (cc/sec)	0.99	0.69	0.20	0.20	
x(cm)	z(cm)	z(cm)	z(cm)	z(cm)(1)	z(cm)(2)
0.0	19.3	19.2	18.8	(3)	20.0
2.0	19.2	19.1	19.0	32.0	20.9
5.0	19.0	19.0	19.0	29.8	21.9
10.0	18.7	18.9	19.0	27.8	22.5
15.0	18.4	18.7	18.8	26.1	23.0
20.0	17.9	18.3	18.6	24.5	23.3
25.0	17.3	17.5	18.2	(4)	23.6
30.0	16.7	17.4	17.8		21.8
40.0	16.0	16.6	17.0		20.4
50.0	15.9	15.9	16.4		19.5
60.0	14.3	15.3	15.7		18.0
80.0	12.9	13.7	14.3		15.9
100.0	12.0	12.5	13.2		14.3
120.0	10.7	11.4	12.0		13.7
160.0	9.0	9.5	9.9		10.0
200.0	7.9	7.9	8.0		8.2
243.5	6.5	6.5	6.5		6.5

(1) This column shows the portion of the interface which was drawn into the drain. The salt water had a much weaker color at the interface.
(2) This column shows the interface position with a strong color contrast.
(3) The interface entered the salt water drain.
(4) No separation of the interface could be detected.

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