NEW MEXICO INSTITUTE OF MINING AND TECHNOLOGY

ELECTROLYTIC MODEL STUDY FOR COLLECTOR WELLS UNDER RIVER BEDS

by

BRUCE E. DeBRINE

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ABSTRACT

Hantush and Papadopulos (1962) obtained analytical solutions for unsteady as well as steady distribution of drawdown around collector (horizontal) wells for several flow systems. In addition to the usual assumptions of a uniformly thick, homogeneous, and isotropic aquifer having constant formation coefficients, they assumed a uniform distribution of yield along the laterals. On the basis of the comparable theory of flow toward partially penetrating vertical wells, they concluded that their solutions approximate very closely those of a constant head along the laterals. An electrolytic model is set up that reproduces the flow to a single-lateral well located under a river bed and along which the head distribution is maintained constant. The results of this model study are compared with those of the analytical solution of Hantush and Papadopulos. This comparison is presented graphically in several figures. These figures show that the experimental data agree rather well with theory, the relative deviation being about 2.2 percent. According to this study, the conclusion of Hantush and Papadopulos has been verified, at least for the case investigated.

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ELECTROLYTIC MODEL STUDY FOR COLLECTOR WELLS UNDER RIVER BEDS

INTRODUCTION

General

Adequate water supplies for industry, domestic, and recreational purposes are becoming increasingly critical in many areas of the world. One source of these supplies is ground water. Ground water has certain desirable characteristics over surface water as a source of water supply. It has more uniform qualities than surface water in regard to temperature, mineralization, and bacterial content. Induced infiltration by means of wells located adjacent to large surface bodies of water such as lakes and large perennial streams is being utilized as a major source of ground water. The collector, or horizontal well, has been advanced in recent years as one of the principal methods for this type of operation (Klaer, 1953). Collector wells located in the flood plains of rivers afford an excellent means of securing ground water by induced infiltration (Kazmann, 1948). As early as 1953, more than 100 of these wells were reported in operation in the United States and about 20 in Europe (Klaer, 1953).

Description of a Collector Well

Figure 1 shows a typical set-up of a collector well. The descriptive material which follows is based on the paper by Kazmann (1948). Briefly, a collector well consists of a central caisson, generally 13 to 15 feet in diameter, constructed of concrete and sunk to a predetermined depth. A concrete plug seals the bottom of the shaft. Lateral or horizontal pipes, 6 to 8 inches in diameter are then jacked hydraulically through ports in the caisson into the aquifer. The location of these laterals is arbitrary, but generally symmetric location of the collectors is observed. In the case of a well located adjacent to a stream, the laterals are generally projected beneath or toward the stream bed rather than inland. The total length of the collectors is usually from 1000 to 1200 feet. Individual lengths are 100 to 450 feet, depending on aquifer conditions. The boring head of each lateral has holes in it which permit water to return through the pipe to the central caisson, washing out fines in the formation, leaving a natural gravel pack around the lateral. The laterals may either be pre-slotted or perforated after installation. The latter case is usually observed in European installations.

Collector wells which are intended to obtain most of their water from induced infiltration will work best near large rivers where periodic flood stages would tend to remove fine sand and silt deposits from the stream bottom. For this reason, collector wells which are designed to operate principally

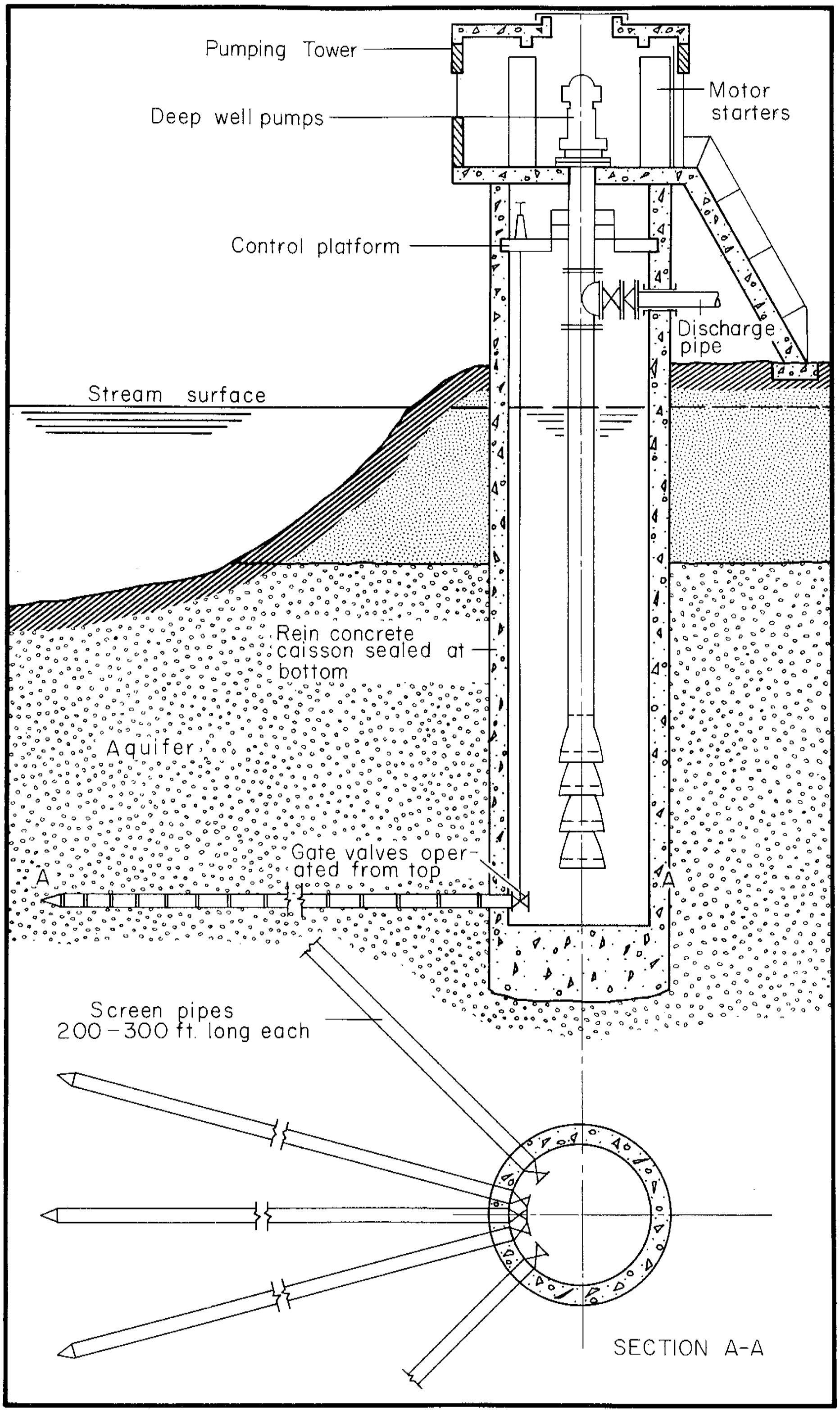

Fig. I A typical collector well located under a stream bed.

through induced infiltration, should not be located immediately upstream from obstructions to the natural flow of the stream. In these areas, silt deposition on the stream bottom would tend to be greater, whereby induced infiltration to the well would be decreased.

Yields of these wells have been reported to be as high as 13,900 gpm (Klaer, 1953). In highly permeable aquifers removed from surface water bodies, yields up to 2800 gpm have been reported (Todd, 1959).

Previous Work on the Hydraulics of Collector Wells

Although collector wells have been in general use for over twenty-five years, very little work had been done to determine their flow equations until the last fifteen years. The yield of collector wells has been generally estimated by using the Dupuit-Forchheimer well-discharge formula with an experimentally suggested "equivalent well radius". This "equivalent radius", depending on the geometry of the laterals, is about 3/4 of the average length of the laterals of the collector well (Hantush and Papadopulos, 1962). The "equivalent well" is a hypothetical vertical well completely penetrating the aquifer and having a radius such that the drawdown in this well is equal to the drawdown in the collector well. This method of approach may result in a fair estimate of the yield of the system, but it fails to determine water levels in and around the well.

Since 1948, many studies, both analytical and experimental, have been reported on various aspects of the hydraulics of collector wells (Milojevic, 1963). Most of these investigations have been concerned with the steady-state yield of special situations of collector wells, such as a single lateral well in an unconfined or an artesian aquifer that is infinite in areal extent.

Steady-state analytical studies have been reported by Cocchi and by Polubarinova-Kochina (Milojevic, 1963). Most studies have neglected the head losses caused by the flow into and through the laterals of the collector well. Cocchi (1953) assumed that the yield along the lateral may be expressed as a second-order polynomial. Polubarinova-Kochina (1955) assumed a constant yield along the laterals. She replaced the ellipsoidal equipotential surfaces that theory produces in regions very close to the lateral by cylindrical surfaces enclosing an equivalent volume. This resulted in a more or less constant head along the lateral. According to Milojevic, these are valid assumptions in the case of an infinitely deep aquifer.

Steady-state experimental studies on the hydraulics of collector wells are more numerous than their analytical counterparts. Milojevic (1963) cites three works as being the best and most up-to-date. The first of these is an investigation by Haefeli and Zeller (1953) on a hydraulic model of symmetric radial flow to a single lateral well in an unconfined, homogeneous, and isotropic aquifer of limited thickness and infinite in areal extent. The head loss along the laterals is neglected. An empirical formula, valid within certain limits, was presented by these authors. The second study, also a hydraulic model investigation, is by Nahrgang and Falcke (1954). In this work, the authors have considered the effect of well losses in the laterals on the flow to the well. They showed that the ratio between the head loss along the laterals and the total drawdown has an effect upon the relation between the capacity of and the drawdown in the well. The third work reported by Milojevic is a study by Kordas, using an electrodynamic analog model (1960). This author investigated a collector well with symmetrically located laterals in a homogeneous and isotropic artesian aquifer of infinite areal extent. Here again, head losses along the laterals are neglected. Kordas summarized his experiments in an empirical formula relating well discharge and drawdown with aquifer thickness, effective radius of influence, well diameter, and the length and position of the laterals. Milojevic (1961 and 1963) presented the results of a study similar to that of Kordas except that he treated the problem of a line of collector wells parallel to a river bank.

Unsteady as well as steady-state solutions have been advanced by Hantush and Papadopulos (1962). Their treatment is presently the only study that has dealt with unsteady flow to collector wells. They assumed that the yield of the well is evenly distributed along the symmetrically located laterals and that the lateral diameter is small relative to the aquifer thickness. They have obtained drawdown distributions around collector wells for several problems of flow in confined and unconfined aquifers. Hantush and Papadopulos concluded that although their work is based on a uniform distribution of yield along the laterals, the solutions approximate very closely (when properly used) those for collector wells of constant head along the laterals. These solutions provide a relatively simple means for determining the yield of collector wells. They also provide a means for describing the complex nature of the drawdown that exists around such wells.

Drawdown in Collector Wells Under Stream Beds: Hantush-Papadopulos Formula

Figure 2 shows a collector well under a stream bed. In obtaining their general formula for the drawdown in such a flow system, Hantush and Papadopulos assumed that (1) the aquifer is homogeneous, isotropic, uniformly thick, and has constant formation coefficients; (2) the capacity of the stream is large relative to the maximum discharge of ground water so that the effect of the slope of the surface stream on the ground water flow may be neglected; (3) the percentage

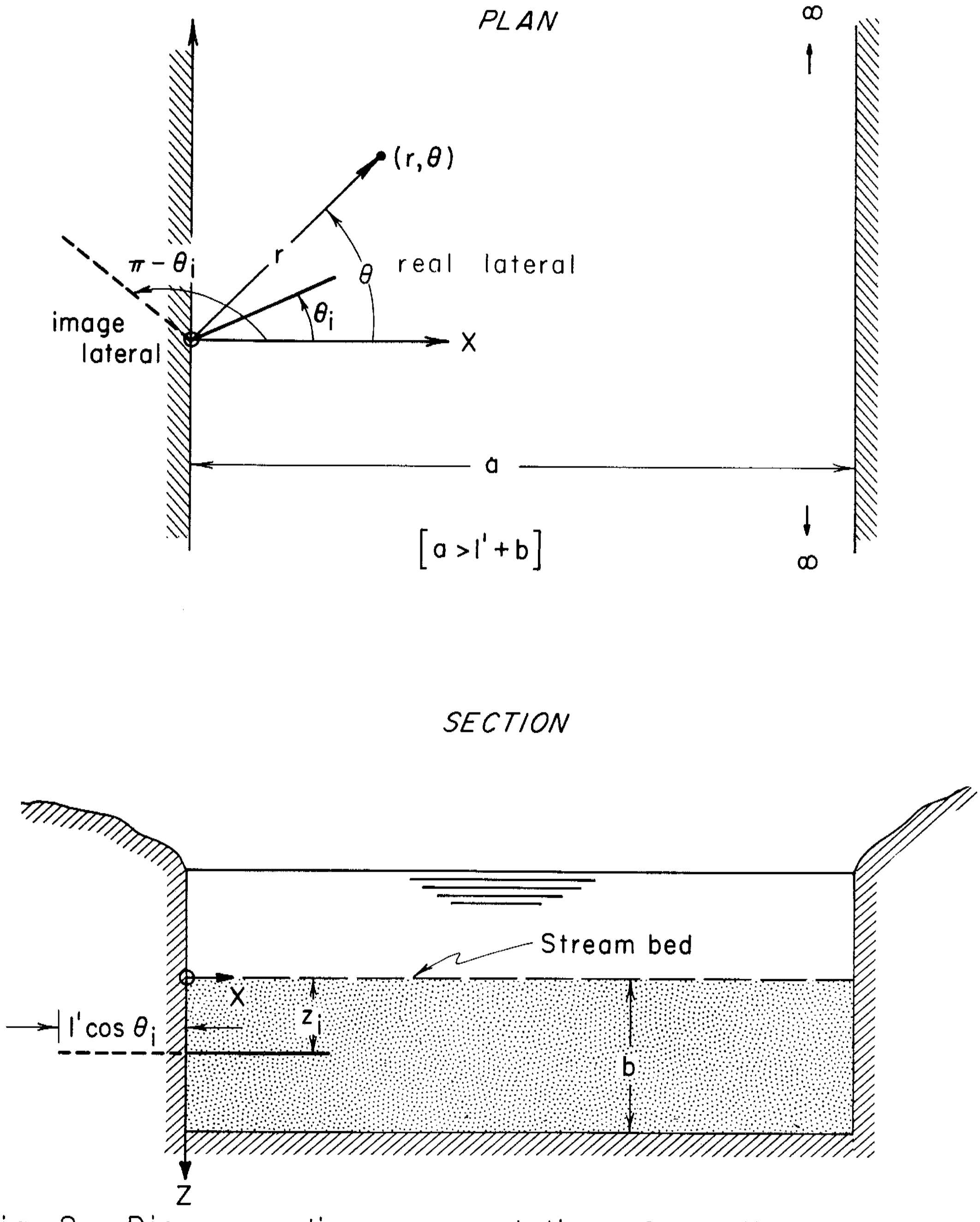

Fig. 2 Diagrammatic representation of a collector well under a stream bed (after Hantush and Papadopulos, 1962)

of the well discharge that originates from storage in the inland portion of the aquifer is small compared to that which comes from induced infiltration and from storage in the aquifer under the stream bed so that the stream banks of the effectively infinite and straight stream may be assumed to be vertical impermeable planes that cut completely through the aquifer; (4) the yield of a well having symmetrically located laterals is evenly distributed along the length of the laterals; (5) the radius of the laterals is small relative to other physical dimensions; and (6) the well losses are neglected.

When evaluated at $r = r_m$ (r_m is a point along the lateral where theory gives maximum drawdown), the Hantush-Papadopulos formula assumedly approximates the drawdown in a collector well in which the head along the laterals is maintained constant. The point of maximum drawdown for a well of four or more symmetrically located laterals can be taken at the face of the caisson, that is, at $r = r_c$, provided that $r_c < 0.05\ell$, where ℓ is the length of the lateral. For a well of a single lateral in an aquifer of infinite areal extent, this point occurs at the midpoint of the lateral (Hantush and Papadopulos, 1962).

A special case of the Hantush-Papadopulos solution is that of the steady-state drawdown in the well. For a well of a single lateral normal to the stream bank, the solution for a lateral extending from bank to bank (that is, $\ell = a$), if the well losses are neglected, is given (Hantush, 1964) by

$$s_{cs} = \frac{Q}{4\pi Ka} \ln \frac{\left[1-\cos\frac{\pi}{2b}(2z_i+r_w)\right]\left[1+\cos\frac{\pi}{2b}r_w\right]}{\left[1+\cos\frac{\pi}{2b}(2z_i+r_w)\right]\left[1-\cos\frac{\pi}{2b}r_w\right]} \qquad (1)$$

where a = width of the stream;

b = thickness of the aquifer;

K = hydraulic conductivity of the aquifer;

Q = total discharge of the collector well;

r_w = effective radius of the lateral;

s_{cs} = drawdown in the collector well during the steady state; and

z_i = vertical position of the lateral.

In this solution, the condition of constant head along the lateral is realized for all practical purposes, provided that the position of the lateral is not very close to either the upper or lower boundary and that the radius of the lateral is very small relative to other dimensions of the flow system. This is because the lateral extends from bank to bank and the variation of head around the circumference of the lateral is very small, resulting in a more or less purely radial flow in the immediate vicinity of the lateral. Quantitatively, this means that $r_w < b/\pi$ and that $r_w/y_i < 0.1$, where y_i is the smaller of the quantities z_i and $(b-z_i)$.

For a well of a single lateral normal to the stream bank, the solution for partially penetrating laterals, provided that a $0.5(b + 2r_c + \ell)$, is given (Hantush and Papadopulos,

1964) by:

$$s_{cs} = \frac{\Omega}{8\pi K\ell}\left\{\ell n\frac{\left[1-\cos\frac{\pi}{2b}(2z_i+r_w)\right]\left[1+\cos\frac{\pi}{2b}r_w\right]}{\left[1+\cos\frac{\pi}{2b}(2z_i+r_w)\right]\left[1-\cos\frac{\pi}{2b}r_w\right]}\right.$$

$$+\frac{16}{\pi}\sum_{0}^{M'}\frac{1}{2n+1}\left[L\left(\frac{2n+1}{2b}\pi\ell,0\right)+L\left(\frac{2n+1}{2b}\pi(2r_c+\ell),0\right)-\frac{\pi}{2}\right.$$

$$\left.\left.-L\left(\frac{2n+1}{2b}2\pi r_c,0\right)\right]\sin\frac{2n+1}{2b}\pi(z_i+r_w)\cdot\sin\frac{2n+1}{2b}\pi z_i\right\} \qquad (2)$$

where ℓ = length of lateral;

M' = an integer such that $M' > 0.5\ b/r_c$;

r_c = effective radius of the caisson; and

$L(u,0) = -L(-u,0) = \int_0^u K_o(y)\,dy$, a function that is available in tabular form (Hantush, 1964).

Other symbols are as previously defined.

Purpose of Study

Hantush and Papadopulos' treatment of the problem assumed uniform distribution of yield along the laterals. Notwithstanding this assumption, they concluded, on the basis of the comparable theory of flow toward partially penetrating vertical wells, that their solutions approximate very closely those of a constant head distribution along the laterals. The purpose of the present study is to test the validity of this conclusion by comparing the results of Equation (2) with those obtained experimentally from an electrolytic model that reproduces flow systems in which the head distribution along the lateral is maintained constant.

ELECTRICAL MODELS

History of Use in Studies of Problems of Flow Through Porous Media

Electrical models, whether of solid, liquid, or gelatin conductors, have been used for many years in studies of fluid flow through porous media. Pavlovsky used such models in investigating several problems of ground water flow as early as 1918 (Polubarinova-Kochina, 1962). Investigators in the petroleum industry have used such models in the study of water flooding and other problems as early as the 1930's. Among such investigators were Muskat (1932), Botset (1946), and Leverett et al (1946). Civil engineers have used these models to study seepage from canals and under dams, and in other related problems (Harza, 1935; Wyckoff and Reed, 1935; Ram et al, 1938; Selim, 1947; Johnson, 1955). Other problems of ground water movement have been studied, using electrical models, by Li et al (1954) and Todd and Bear (1959). These studies were concerned with two-dimensional flow problems. The adaptability of electrical models to three-dimensional flow problems has long been recognized, however. Two such studies were presented by Opsal (1955) and by Zee et al (1957). Milojevic (1961) and Kordas (1960) were among the first to use electrical models in the study of flow of ground water to collector wells where the flow is three-dimensional in nature (see page 6).

Analogy to Ground Water Flow

When laminar, the ground water motion is governed by Darcy's law, namely by

$$v_s = -K\frac{\partial \phi}{\partial s} \tag{3}$$

where v_s = bulk flow velocity;
K = hydraulic conductivity; and
ϕ = hydraulic head.

When Darcy's law is introduced in the continuity equation, the steady motion of ground water is described by Laplace's equation in terms of the hydraulic head, namely by

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \tag{4}$$

where x, y, and z are the rectangular coordinates.

The flow of electricity is described by Ohm's law, namely

$$i_s = -\sigma\frac{\partial V}{\partial s} \tag{5}$$

where i_s = current per unit area;
σ = conductivity of the conducting medium; and
V = electric potential.

When Equation (5) is introduced into the continuity equation, Laplace's equation in terms of the electric potential V results, namely

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \qquad (6)$$

The analogy between Darcy's and Ohm's laws and that between Equations (4) and (6) are quite apparent. Obviously, a correspondence exists between the bulk flow velocity and the current flow, between the hydraulic conductivity and the specific conductivity, between the hydraulic gradient and voltage gradient, between the hydraulic head and the electric potential, between the equipotential lines of the fluid flow system and the equipotential lines of the current flow system, and between the fluid flow lines and the current flow lines. Table I lists some of these analogous quantities.

Table I

Analogy Between Fluid and Current Flow

Fluid Flow	Current Flow
Bulk velocity = v_s	Current per unit area = i_s
Hydraulic head = ϕ	Electric potential = V
Hydraulic conductivity = K	Specific conductivity = σ
Darcy's law = $v_s = -K\frac{\partial \phi}{\partial s}$	Ohm's law = $i_s = -\sigma\frac{\partial V}{\partial s}$
Laplace's equation for fluid flow = $\nabla^2\phi = 0$	Laplace's equation for current flow = $\nabla^2 V = 0$

Thus, an unknown hydraulic head ϕ in a specified flow system with given boundary conditions can be defined by an electric potential V of an electrolytic model having the same geometric form and satisfying analogous boundary conditions. Measurements of the electric potential and the current flow in the model provide a solution for the distribution of the hydraulic head and the discharge in the porous medium, respectively, provided the appropriate scaling relations are used. For a discussion on model scaling, the reader is referred to Leverett et al (1946), Todd and Bear (1959), and Collins (1961).

ELECTROLYTIC MODEL FOR STEADY-STATE FLOW TO COLLECTOR WELLS UNDER RIVER BEDS

Basic Considerations

Equations (1) and (2) are analytical solutions that approximate the steady-state drawdown in a collector well under a stream bed of a single lateral along which the drawdown is maintained constant (Figure 2). In these steady-state solutions, the following assumptions must hold: (1) the aquifer is homogeneous, isotropic, uniformly thick, and has constant formation coefficients; (2) the capacity of the stream is large relative to the maximum discharge of ground water so that the effect of the slope of the surface stream on the ground water flow may be neglected whereby a constant head on the bed of the stream will be essentially realized; (3) the percentage of the well discharge that originates from the inland portion of the aquifer is small compared to that which comes from induced infiltration so that the stream banks may be assumed to be vertical, impermeable planes that cut completely through the aquifer; (4) the radius of the lateral is small relative to other physical dimensions, that is, $r_w < b/\pi$; (5) in the case of Equation (2), the width of the stream must satisfy the relation $a > 0.5(b + 2r_c + \ell)$; and (6) the river channel is fairly straight and effectively infinite in length.

These assumptions and conditions are reproduced in the model as follows: (1) the electrolyte represents the homogeneous, isotropic aquifer in which uniform thickness is realized by maintaining a constant liquid level, and the impermeable base of the aquifer is represented by the air-electrolyte interface; (2) a plate electrode to which a constant potential is applied reproduces the constant head on the bottom of the stream; (3) the sides of the model tank reproduce the impermeable planes that assumedly cut through the aquifer along the stream banks; (4) the lateral in the model is represented by an electrode maintained at a constant potential; (5) the radius of the lateral electrode is so chosen that it falls within the limits $r_w < b/\pi$; (6) for partially penetrating laterals, the electrodes are so chosen that they satisfy the relation $a > 0.5(b + 2r_c + \ell)$; and (7) the length of the model is so chosen that when the lateral electrode is positioned at one end of the tank, the voltage drop at the other end is essentially zero, thus fairly insuring that the flow in the model behaves as if the model were effectively infinite.

For an electrode extending from side to side in the model, the total voltage drop in an electrolytic model satisfying the previous conditions and assumptions, with the lateral electrode positioned at one end of the tank of the model, from Equation (1), is given by

$$V = \frac{2I}{4\pi\sigma a} \ln \frac{\left[1-\cos\frac{\pi}{2b}(2z_i+r_w)\right]\left[1+\cos\frac{\pi}{2b}r_w\right]}{\left[1+\cos\frac{\pi}{2b}(2z_i+r_w)\right]\left[1-\cos\frac{\pi}{2b}r_w\right]} \qquad (7)$$

and for partially penetrating lateral electrodes, from Equation (2), is given by

$$V = \frac{2I}{8\pi\sigma\ell}\left\{ \ell n \frac{\left[1-\cos\frac{\pi}{2b}(2z_i+r_w)\right]\left[1+\cos\frac{\pi}{2b}r_w\right]}{\left[1+\cos\frac{\pi}{2b}(2z_i+r_w)\right]\left[1-\cos\frac{\pi}{2b}r_w\right]} \right.$$

$$+ \frac{16}{\pi}\sum_{0}^{M'} \frac{1}{2n+1}\left[L\left(\frac{2n+1}{2b}\pi\ell, 0\right) + L\left(\frac{2n+1}{2b}\pi(2r_c+\ell), 0\right) - \frac{\pi}{2}\right.$$

$$\left.\left. - L\left(\frac{2n+1}{2b}2\pi r_c, 0\right)\right] \sin\frac{2n+1}{2b}\pi(z_i+r_w)\cdot\sin\frac{2n+1}{2b}\pi z_i \right\} \qquad (8)$$

where V is the potential difference between the electrodes, σ is the conductivity of the electrolyte, and I is the total current in the model, observing that in this set-up of the model, Q corresponds to 2I.

Description of the Model

The electrolytic model used in this investigation consists of several components, including the electrolytic tank, the electrical circuit, the conducting medium, and the lateral and plate electrodes. Photographic views of the model are shown in Figure 3. The individual components are discussed subsequently.

Electrolytic Tank. As the conducting medium was an electrolyte, a tank was constructed of 1/4-inch lucite plates to hold the liquid. The inside dimensions of the tank were approximately 83.3 cm by 50 cm by 28.5 cm deep. Acrylic cement,

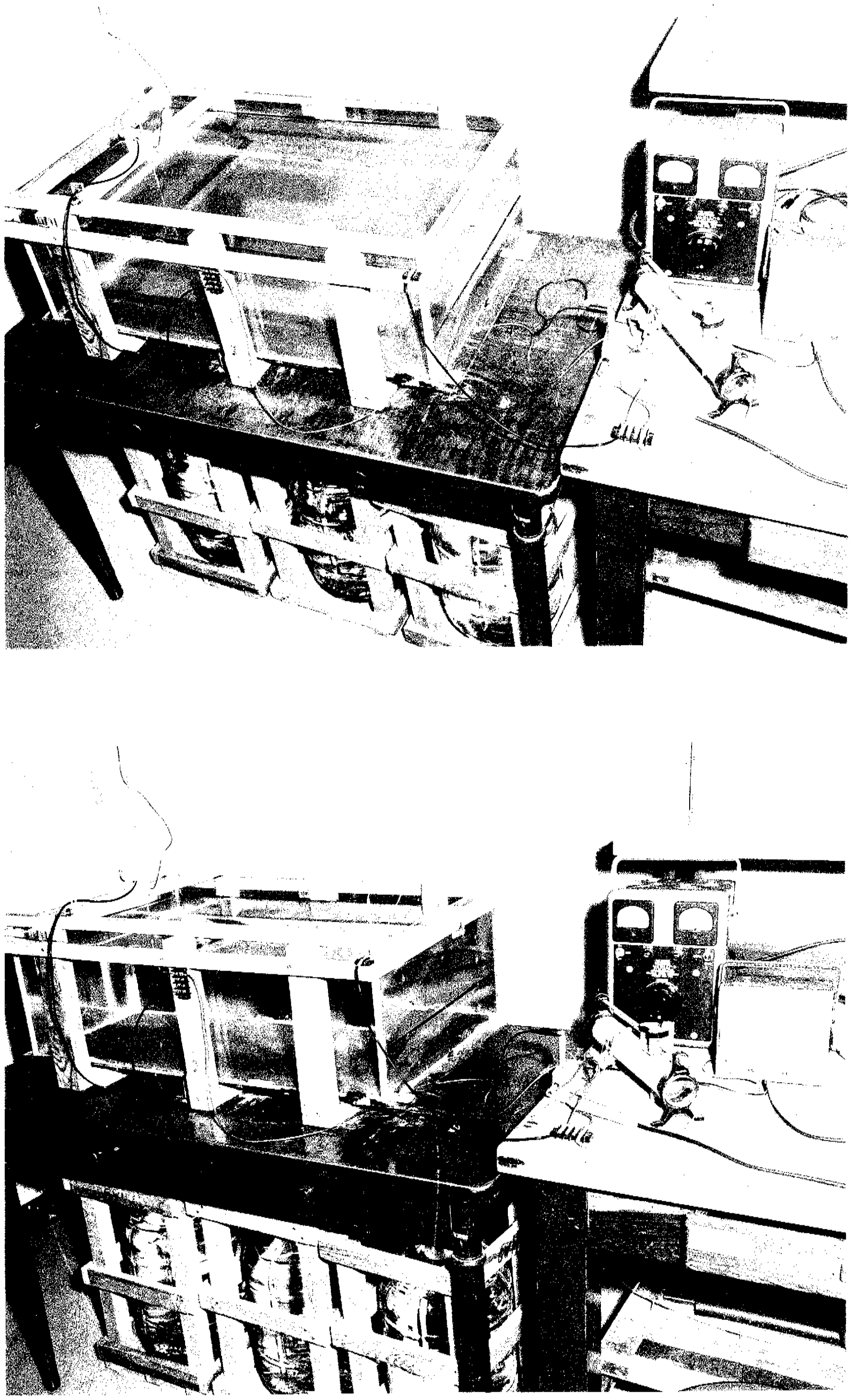

Fig. 3 Photographic views of the electrolytic model.

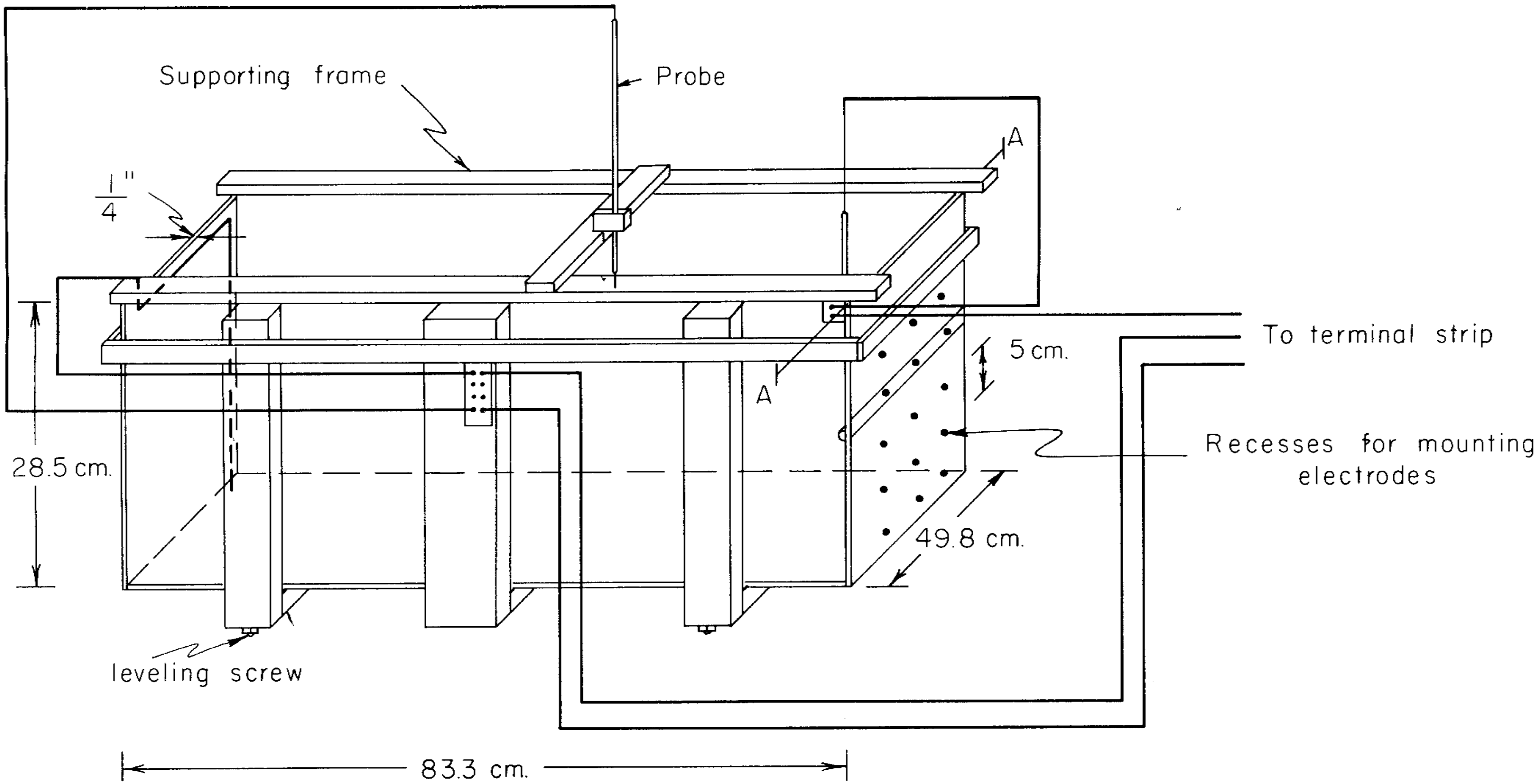

Fig. 4 Diagrammatic view of the model tank.

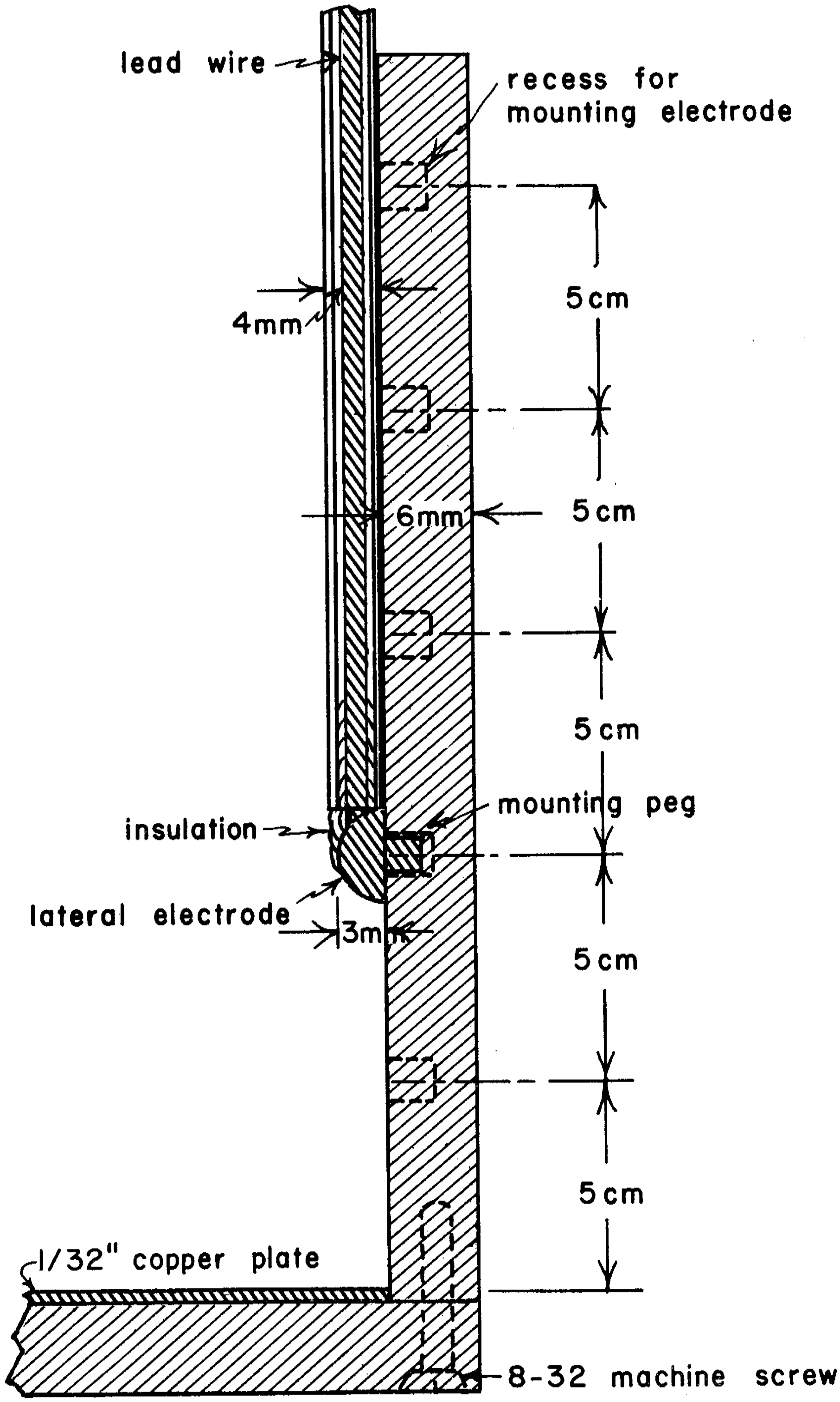

Fig. 5 Details of lateral electrode mounting and lead in wire (Sec. A–A of fig. 4, not to scale)

Duco cement, and stainless steel, flat head 3/4-inch 8-32 machine screws were used in the construction. Details of the tank are shown in Figure 4. A frame was made of 2 x 4 inch lumber to furnish support for the tank sides for the purpose of preventing the expected bowing of the sides of the filled tank. Leveling bolts were attached to the bottom of this frame, at each corner, to insure a level base in the tank and a uniform depth of electrolyte. Small-diameter recesses, 1/8-inch deep, were drilled in one end of the tank at 5 cm intervals in order to mount the lateral electrode (see Figure 5).

Electrical Circuit. The electrical circuit for the model is shown in Figure 6. A 60-cycle alternating current was connected to the electrolytic tank through a General Radio type W10MT3W, metered Variac, autotransformer. The meters on this Variac have an accuracy of ±3% of full scale. This Variac was used since variable frequency was not needed and because of the convenience of 60-cycle current. Two types of resistors connected in series with the tank were used in order to measure the current in the system. Tests using tap water as the electrolyte were run with various standard non-inductive resistors. The runs in which dilute copper sulfate solutions were used as electrolytes were conducted using a double wire-wound resistor. This resistor has a resistance, as measured on a Leeds and Northrup bridge (accuracy ±1%), of 108 ohms. The resistors were used rather than a milli-ammeter because the high internal impedance of the meter resulted in

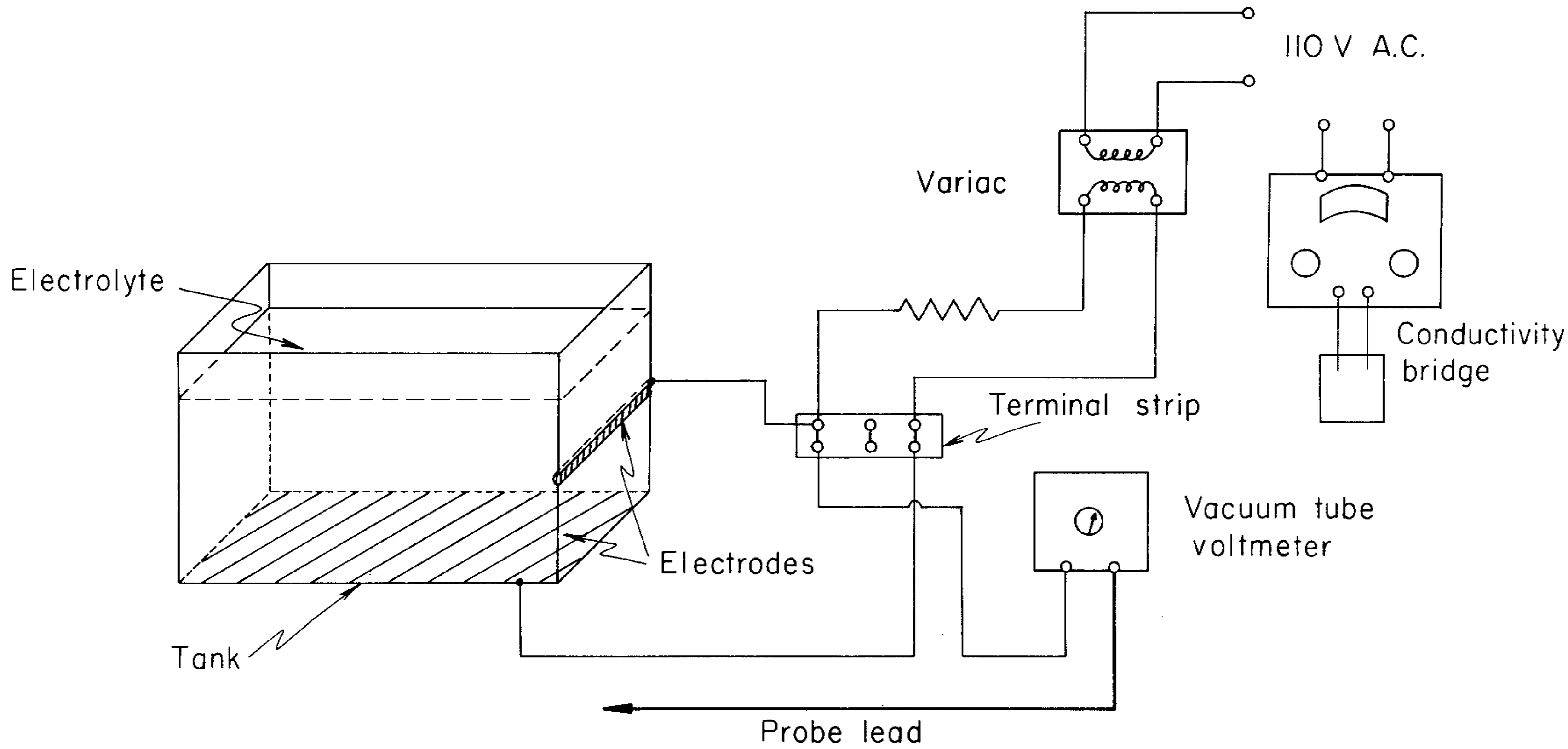

Fig. 6 Electrical circuit for the electrolytic model.

current readings that were too low. An RCA Senior Voltohmyst, vacuum tube voltmeter, type WV-98C was used to measure the potential drops in various parts of the circuit. This meter has an accuracy of ±3% of full scale.

Conducting Medium. Two types of solutions were used as electrolytes in this study. The initial series of readings were taken with ordinary tap water as the conducting medium. Its conductivity averaged about 372 micromhos per centimeter. Subsequent runs were made using dilute copper sulfate solutions. One series was made with a conductivity averaging about 995 micromhos per centimeter, a second series with a conductivity between 645 and 665 micromhos per centimeter, and a third series run with a solution having a conductivity between 1168 and 1178 micromhos per centimeter. Conductivities were frequently checked during the experiments with a conductance bridge (Industrial Instruments Model RC 16 B2 conductance bridge using a type G1 cell, which has an accuracy of ±1%).

Lateral Electrodes. Details of the lateral electrodes are shown in Figure 5. The electrodes are made of 6 mm copper rod machined in half with small knobs on the flat side for mounting purposes. The radius of the electrode is well within the criterion $r_w < b/\pi$, since $r_w = 0.3$ cm, and $b = 25$ cm. The ends and back sides of the electrodes are insulated with G-C Electronics Company Koloid K-29 High Volt Resin. The electrodes are connected to the rest of the electrical circuit by means of copper wires sealed with paraffin in glass tubing 4 mm in diameter. These leads are located at what would be considered the caisoon end of the lateral. The electrode lengths are 50 cm, 24.5 cm, and 10.8 cm, representing full penetration, one-half, and one-quarter penetration of the aquifer width, respectively. The longest of the partially penetrating electrodes satisfies the criterion $a > 0.5(b + 2r_c + \ell)$, since $a = 50$ cm, $b = 25$ cm, $r_c = 0.2$ cm, and $\ell = 24.5$ cm.

Plate Electrode. A 1/32-inch thick copper plate covers the bottom of the tank. The plate is sealed to the tank along its edges by paraffin to prevent electrolyte from seeping beneath the plate. A copper wire sealed in 3-mm glass tubing with paraffin provides the connection between the copper plate electrode and the rest of the electrical circuit. This lead is located in one corner of the tank at the opposite end from the lateral electrode so as to minimize any distortion of the electrical field in the electrolyte.

General Remarks

The model, as set up, is inverted from the actual flow problem. This inversion is to permit easier access to the interior of the tank, especially to facilitate placement of the lateral electrodes. Originally, a one-millimeter diameter wire, representing the lateral electrode, was mounted in the center of the long dimension of the tank. This setup was abandoned because the high impedance of the wire introduced unwanted electrode effects, and because the tank was found to be too short to reproduce an effectively long stream, since measurable voltage drops existed at the ends os the tank. The electrode effects are reduced by introducing a larger, three-millimeter radius electrode. The insufficient length of the tank is overcome by utilizing the symmetry of the system; that is, by placing the lateral electrode (a copper rod, machined in half) at one end of the tank. With such positioning of the electrode, the model reproduces one half of the complete system. With this arrangement, negligible voltages are found at the end of the tank opposite the lateral electrode.

EXPERIMENTAL PROCEDURES

Description of Experiments

The purpose of the present study is to compare experimental results from an electrolytic model simulating steady-state flow toward a collector well under a river bed with the analytic solutions of Hantush and Papadopulos represented by Equations (7) and (8). These equations state that the potential difference across the model tank is a function of the current flow, the conductivity of the electrolyte, the length, the vertical position, and the effective radius of the lateral electrode. Consequently, three experiments were set up using lateral electrodes of three different lengths to represent complete penetration, $\ell = a$, for use in Equation (7) and two partial penetration cases for use in Equation (8). Each experiment consisted of applying a certain potential difference between the plate and the lateral electrode, then measuring the conductivity of the electrolyte and the current in the system. Four electrode positions were used in each experiment, with six different potential differences applied across the model in each position. Each experiment was conducted in four different electrolytic media. In all, a total of 74 runs were made in the three experiments.

An experimental run consisted of measuring the current and the conductivity for each of six potential differences that were applied to the model with the lateral electrode fixed

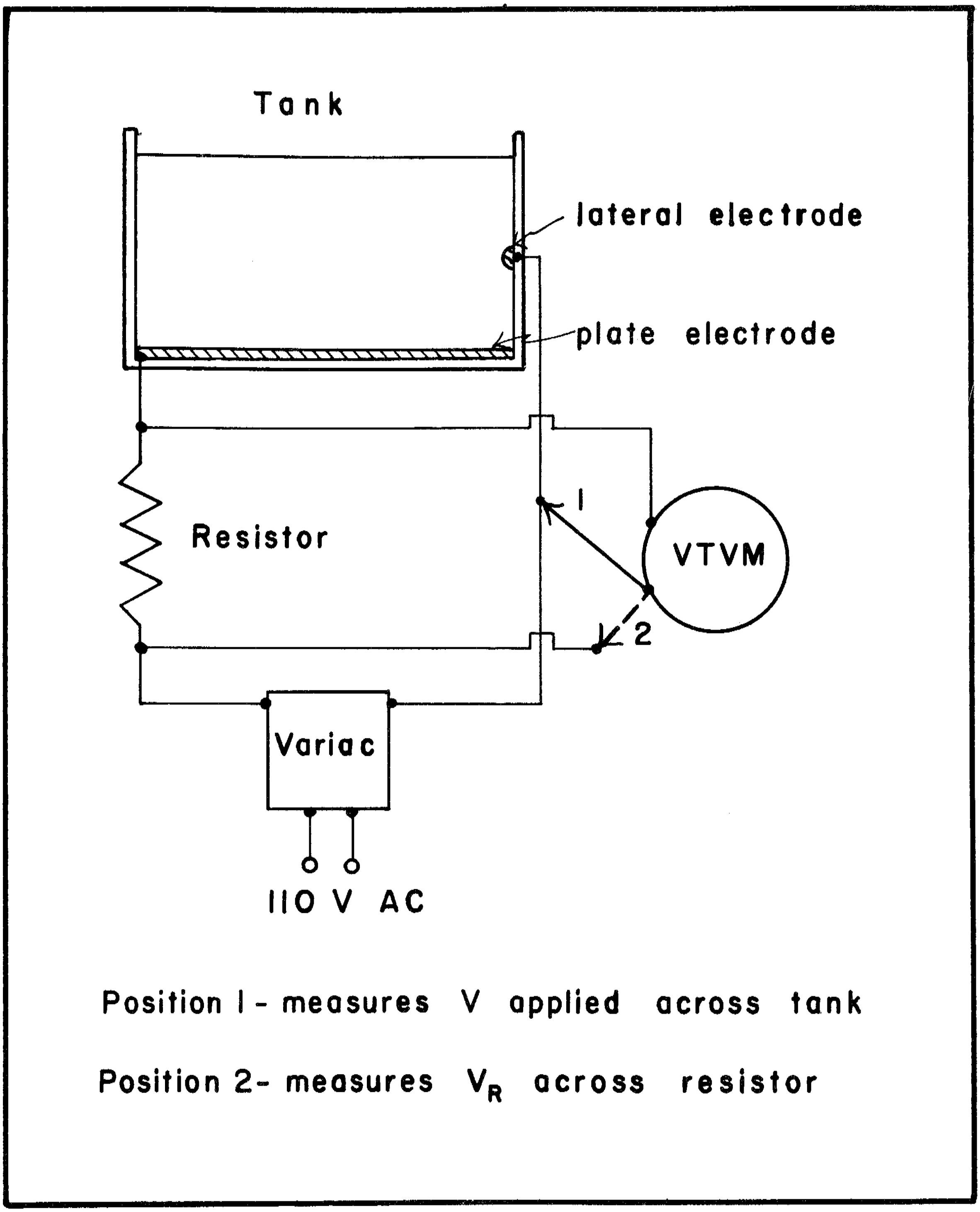

Fig. 7 Details of electrical circuit for measuring V_r and V.

elevations of the completely penetrating electrode. Twenty complete runs were made using tap water as the electrolyte. The experiment was repeated for the partially penetrating electrodes. The number of runs performed for the 24.5 cm and the 10.8 cm electrodes were twelve and six respectively.

The tank was then drained and cleaned and the dilute copper sulfate solution added for an electrolyte. The above process (describing a typical run) was repeated for each of the three electrodes, with four runs being made for each electrode in each of the three copper sulfate solutions. In the three experiments, thirty-six runs were made with the dilute copper sulfate electrolytes.

Probable Experimental Errors

In general, with proper instrumentation, a reasonable degree of accuracy can be obtained in an electrolytic model (Todd and Bear, 1959). Regardless of their magnitudes, errors will, nevertheless, occur in any type of model study. The errors in this study are contributed primarily by inaccuracies of the measuring instruments, by inaccuracies in the construction of the several components of the model, by inaccuracies of simulating in the model the boundary conditions of the actual flow problem, and by the effects of polarization that generally occur in studies using electrolytic models.

Polarization introduces a capacitive effect into electrolytic tanks, which in effect is a displacement of the electrodes (Liebmann, 1953), thus introducing some error. An error, however small it may be, is introduced by the fact that the electrolytic tank is finite in length, thus not simulating exactly the actual flow problem. Probable inaccuracies in the model construction and in the positioning of the electrodes are further sources of error of unknown magnitudes. Another source of error is introduced by what could be called a "conductivity cell effect". This effect is brought about by a deviation, that has been observed in practice, of the cell constant C of a conductivity cell from the theoretical straight-line relationship, with its resistance R expressed by $C = \sigma R$ (Shedlovsky, 1932). This relationship was checked for some of the experimental runs to determine if the conductivity had any appreciable effect on the results. The difference between the maximum and minimum values of σR for the range of R checked was about three percent which is within the overall experimental error.

Although the ranges of the error of the measuring instruments are known (see page 23), the overall error is not *a priori* quantitatively known. The overall error can, however, be determined approximately by calibrating the model through use of the data observed for the case where Equation (7) is applicable. The process of calibration is discussed subsequently.

Calibration of the Model

If there are no experimental errors, the results observed for the case of a lateral extending from bank to bank, should agree very closely with those calculated through use of Equation (7), provided, of course, that the condition imposed on the magnitude of the effective radius of the lateral r_w is realized; namely, that r_w is very small compared to other dimensions of the model (see page 10). Quantitatively speaking, r_w should satisfy the relations $r_w < b/\pi$ and $r_w < 0.1y_i$, where y_i is the smaller of the vertical distances from the lateral to the top or bottom of the tank. According to theory, a plot of $\frac{V}{2I/4\pi\sigma a}$ versus the logarithmic term of Equation (7) on linear scales should give a straight line with a slope equal to unity. Accordingly, the ratio between this theoretical slope and that obtained for the best fit straight line through the observed data is a measure of the overall relative error.

The data collected for the case of a completely penetrating lateral, $\ell = a$, are given in Table II in the Appendix. These data are collected for the purpose of calibrating the model. Table III gives the average values of $\frac{V}{2I/4\pi\sigma a}$ (from Table II) and the values of $F(z_i)$ which is the logarithmic term of Equation (7). Figure 8 is prepared using Table III. The relative error $\left[(m-m')/m' \times 100\right]$ as obtained from this plot is 3 percent, where m and m' are the slopes of the observed and theoretical curves respectively. This relative error is used subsequently to correct the data observed for the cases of the partially penetrating lateral electrodes.

TABLE III

Average Value of Reduced Data and Theoretical Logarithmic Terms for Model Calibration

z_i	$\left(\frac{V}{2I/4\pi\sigma a}\right)_A$	$F(z_i)$
5	7.5080	7.1438
10	8.9445	8.7294
15	10.3117	10.0075
20	12.0315	11.6422

$F(z_i)$ = logarithmic term of Equation (7).

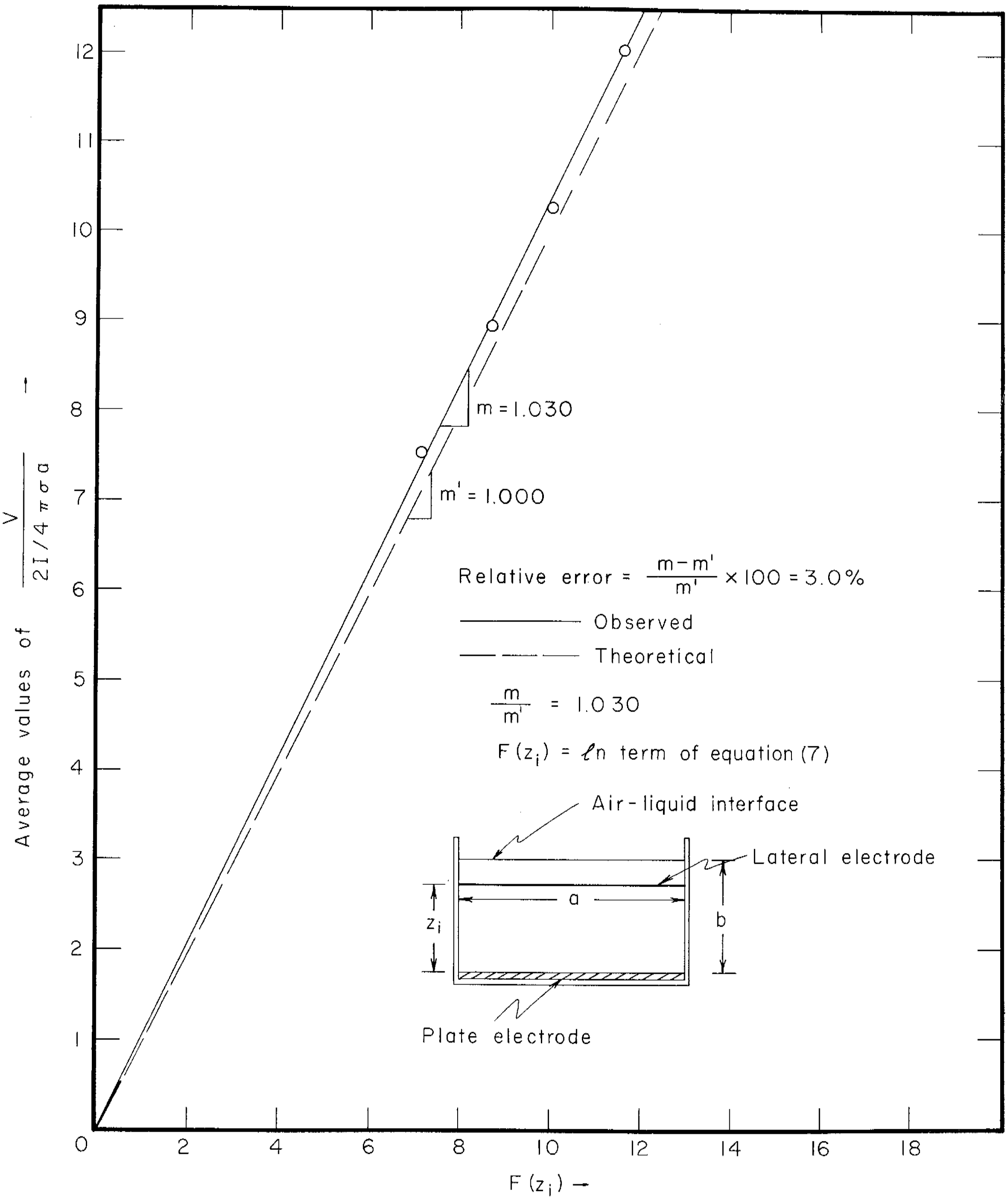

Fig. 8 Graphical presentation of reduced data for model calibration.

PRESENTATION OF DATA AND DISCUSSION OF RESULTS

The applied voltages, conductivities, currents, and the quantities $\frac{V}{2I/8\pi\sigma\ell}$ for the case of the 24.5 and 10.8 cm electrodes are given in the Appendix in Tables IV and V respectively. The last columns of these tables show the corrected values of $\frac{V}{2I/8\pi\sigma\ell}$, namely $\left(\frac{V}{2I/8\pi\sigma\ell}\right)_c$. The quantity $\left(\frac{V}{2I/8\pi\sigma\ell}\right)_c$ is equal to $\left(\frac{V}{2I/8\pi\sigma\ell}\right)/(1 + \epsilon)$, where ϵ is the relative error, 3.0%, as determined by the model calibration. The average values of $\left(\frac{V}{2I/8\pi\sigma\ell}\right)_c$ for both electrode lengths and the theoretical values of $f(z_i)$, the bracketed terms of Equation (8), are presented in Table VI. The theoretical term in this table is based on a value of $r_c = 0.2$ cm. This value of r_c is the average of $r_c = 0.4$ cm and $r_c = 0$. If the tubing containing the lead-in wire to the lateral had completely enveloped the end of the electrode (see Figure 5), 0.4 cm would have been the correct value for r_c. If the lead had been attached through the side of the tank, r_c would have been zero. Since neither of these cases existed, the average of these two values was chosen.

Figures 9, 10, and 11 are graphical representations of the data of Table VI. In these figures, the dashed lines represent the theoretical curves, with slopes of unity. The solid lines are those of the best fit straight lines through the average corrected experimental data. The relative deviations, as expressed in terms of the slopes of the observed and the theoretical curves, show that the experimental data agree rather well with theory. The relative deviations are 3.2 percent for the 24.5 cm electrode and 0.5 percent for the 10.8 cm electrode. The overall relative deviation for the composite plot shown in Figure 11 is 2.2 percent. This close agreement between the experimental and theoretical results supports the contention of Hantush and Papadopulos that their solutions, although based on the assumption of uniform flux along the lateral, may be used to approximate closely the corresponding solutions which assume a constant head along the lateral.

TABLE VI

Average Value of Corrected Reduced Data and Theoretical Terms for Partially Penetrating Electrodes

	z_i	$\left(\frac{V}{2I/8\pi\sigma\ell}\right)_A$	$f(z_i)$
24.5	5	13.0409	11.9988
24.5	10	15.3285	14.7481
24.5	15	17.4068	16.8223
24.5	20	19.6572	19.6323
10.8	5	11.8282	11.4170
10.8	10	13.6869	13.2898
10.8	15	14.4686	14.5583
10.8	20	16.2055	16.4593

$f(z_i)$ = bracketed terms of Equation (8).

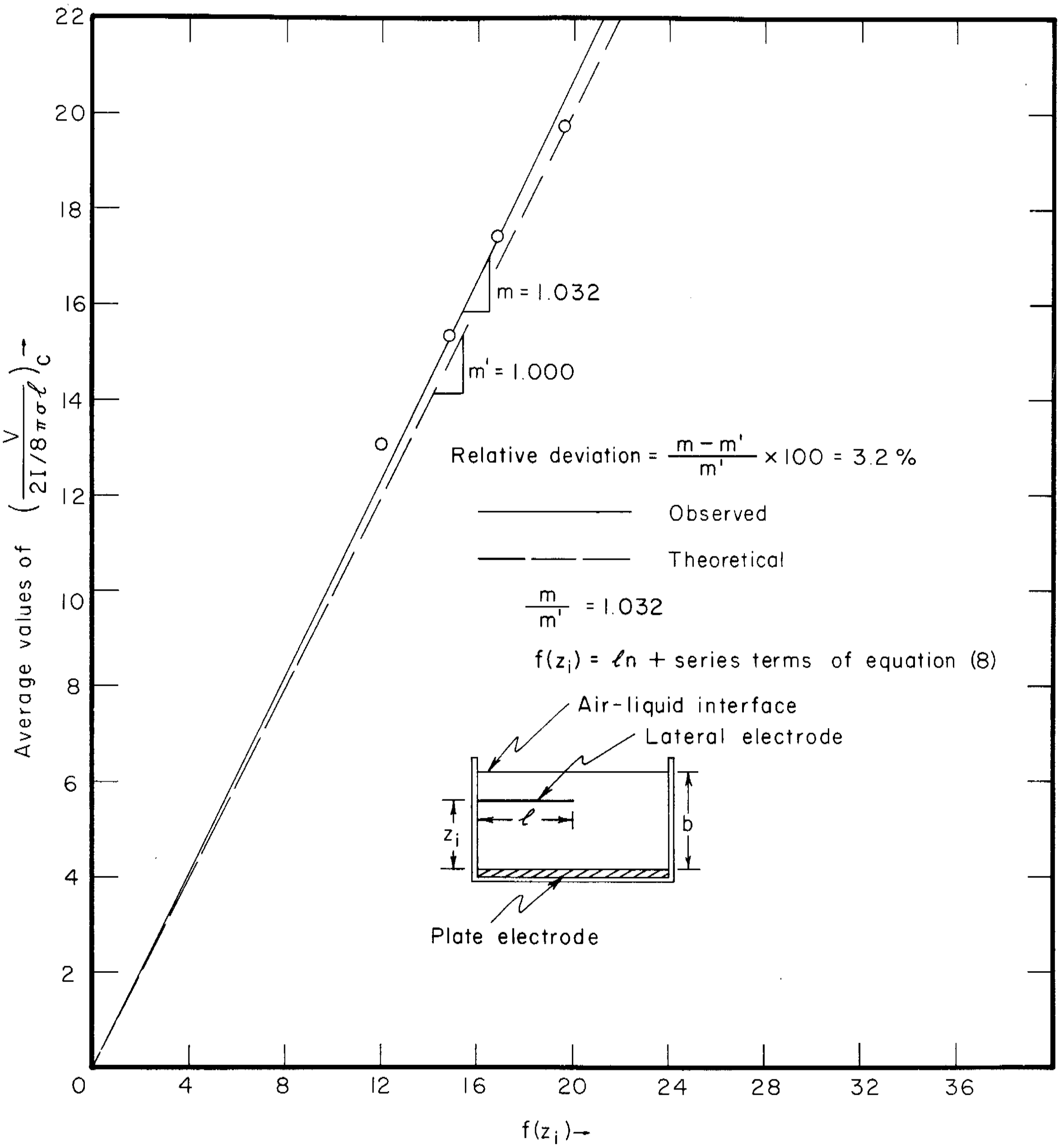

Fig. 9 Graphical representation of average $\left(\frac{V}{2I/8\pi\sigma\ell}\right)_c$ vs $f(z_i)$ for $\ell = 24.5$ cm.

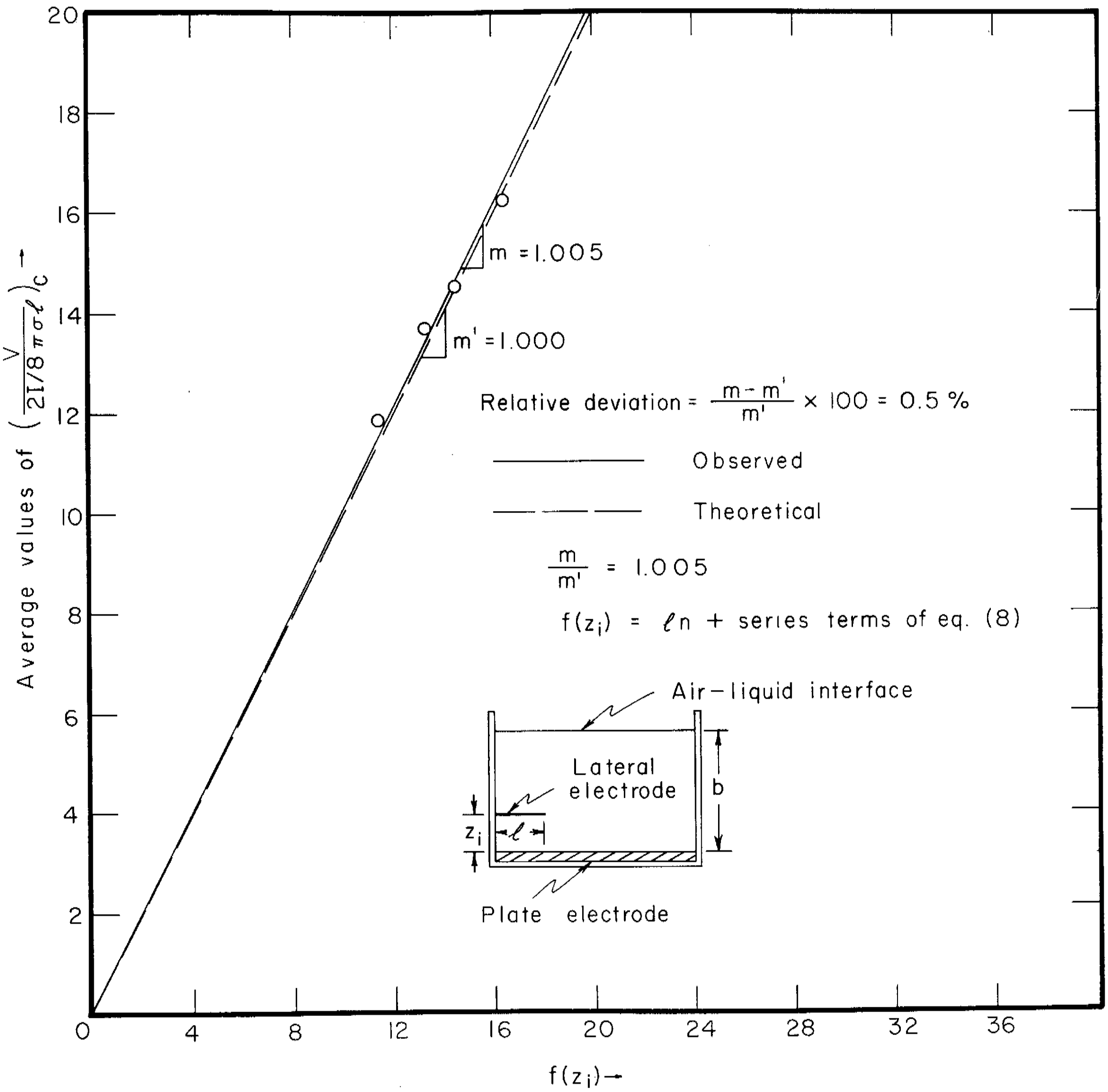

Fig. 10 Graphical representation of average $\left(\frac{V}{2I/8\pi\sigma\ell}\right)_c$ vs $f(z_i)$ for ℓ = 10.8 cm.

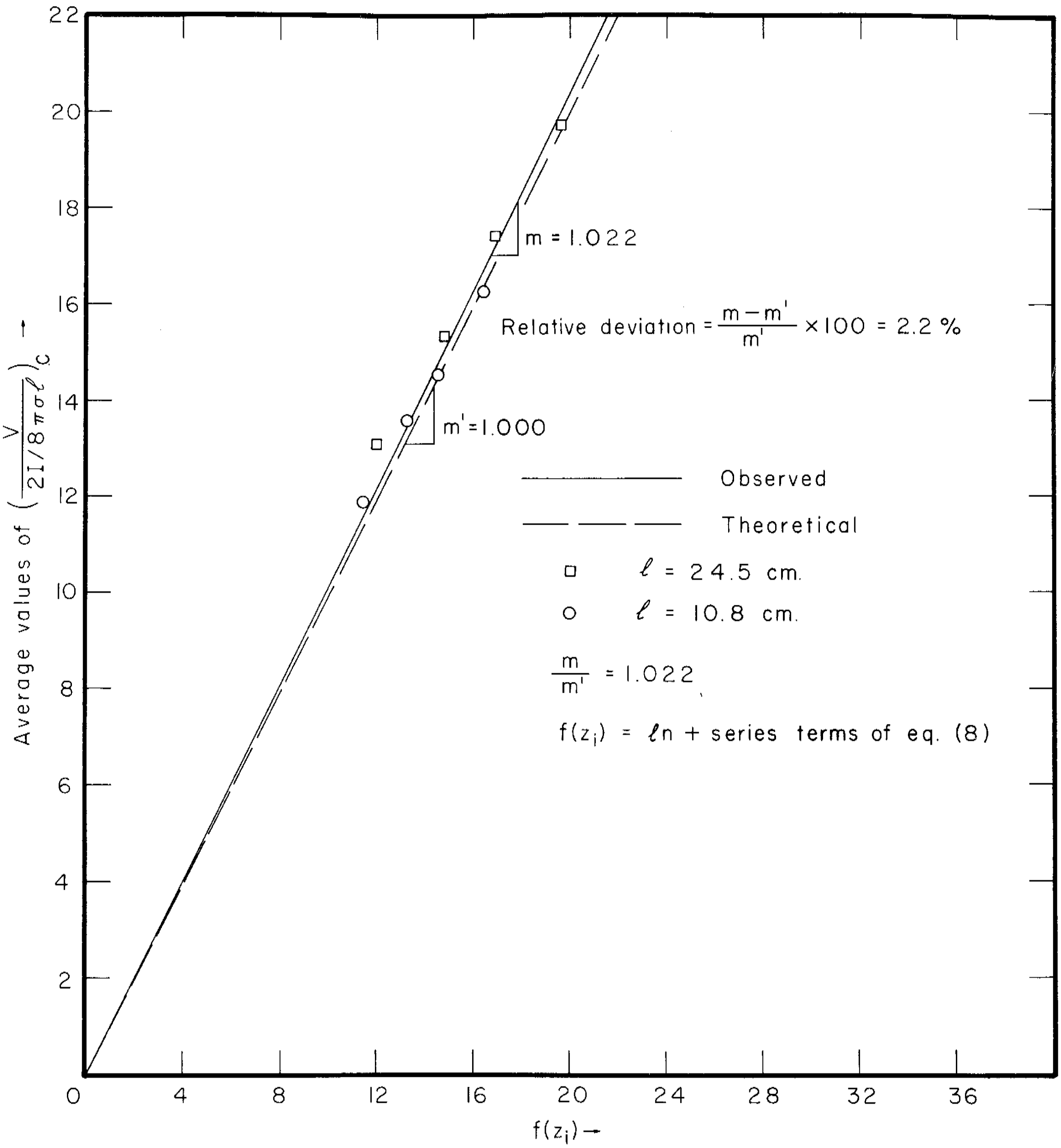

Fig. 11 Composite plot of average $\left(\frac{V}{2I/8\pi\sigma\ell}\right)_c$ vs. $f(z_i)$.

RECOMMENDATIONS FOR FUTURE WORK

Future investigators should work with a more precisely constructed tank. The tank should be longer and, if possible have a variable width, so as to permit more flexibility regarding boundary conditions. Thicker plexiglas should be used for the tank and reinforcement of the sides should be by means of transverse ribs. This would eliminate the wooden frame, permitting better visibility for probe and electrode positioning. The lateral electrode should be mounted so that the lead connections would be through the sides of the tank. Here again, flexibility could be introduced by means of some sort of grooved connection that would permit any positioning of the lateral electrode between $0 \leq z_i \leq b$ and $0 \leq \theta \leq 90^o$. Multiple laterals should also be considered.

The above suggestions, if achieved, would permit investigation of several analytical solutions from the study of Hantush and Papadopulos and would permit adaptation of the tank to solutions of other flow problems, not necessarily connected with collector wells. Tabulation of the $L_o(u,\pm w)$ and $L(u,\pm w)$ functions would be necessary to investigate other analytic solutions from Hantush and Papadopulos' work.

Effects of polarization should be determined quantitatively and deviations due to conductivity cell effects should be accurately determined. Profiles should be mapped along the lateral, if electrode effects can be overcome, so that the point of maximum drawdown, r_m, could be precisely determined.

CONCLUSION

The very close agreement between the experimental results of a constant head along the lateral and the theoretical results when based on Equation (2) leads to the conclusion that the solutions of Hantush and Papadopulos, although based on the assumption of a uniform flux along the lateral, will nevertheless approximate very closely the corresponding solutions where a uniform head distribution along the lateral is prevailing. This contention of Hantush and Papadopulos has been shown to be valid at least for the case studied in this work, namely, for steady-state flow to a collector well which has a single lateral located beneath a river bed.

APPENDIX I

TABLE II—EXPERIMENT I

Observed and Reduced Data for Model Calibration:

$\ell = a = 50.0$ cm and $r_w = 0.3$ cm

Run no.	z_i	σ	V	R	V_R	I	$\frac{V}{2I/4\pi\sigma a}$
1	5	377	2.0	52.63	1.56	0.0296	8.0032
	5	377	4.0	52.63	3.32	0.0631	7.5089
	5	377	6.0	52.63	5.05	0.0960	7.4028
	5	377	8.0	52.63	6.95	0.1321	7.1736
	5	377	10.0	52.63	8.85	0.1682	7.0422
	5	377	12.0	52.63	10.75	0.2043	6.9573
2	5	378	2.0	100.00	3.22	0.0322	7.3746
	5	378	4.0	100.00	6.70	0.0670	7.0897
	5	378	6.0	100.00	10.10	0.1010	7.0547
	5	378	8.0	100.00	13.90	0.1390	6.8347
	5	378	10.0	100.00	17.40	0.1740	6.8245
	5	378	12.0	100.00	21.20	0.2120	6.7216
3	5	373	2.0	150.00	4.95	0.0330	7.1023
	5	373	4.0	150.00	9.95	0.0663	7.0709
	5	373	6.0	150.00	15.20	0.1013	6.9420
	5	373	8.0	150.00	20.50	0.1367	6.8587
	5	373	10.0	150.00	25.80	0.1720	6.8138
	5	373	12.0	150.00	30.90	0.2060	6.8271
4	5	372	2.0	76.10	2.32	0.0305	7.6628
	5	372	4.0	76.10	4.62	0.0607	7.6997
	5	372	6.0	76.10	7.38	0.0970	7.2280
	5	372	8.0	76.10	9.98	0.1311	7.1301
	5	372	10.0	76.10	12.70	0.1669	7.0013

σ = conductivity of electrolyte in mmhos/cm

V = potential applied across tank in volts

R = value of resistor in series with tank in ohms

V_R = potential drop across R in volts

I = V_R/R = current flowing in system in amperes

z_i = electrode position in cm

TABLE II (cont'd)

Run no.	z_i	σ	V	R	V_R	I	$\frac{V}{2I/4\pi\sigma a}$
5	5	368.5	2.0	76.00	2.43	0.0320	7.2333
	5	368.5	4.0	76.00	5.00	0.0659	7.0262
	5	368.5	6.0	76.00	7.75	0.1020	6.8089
	5	368.5	8.0	76.00	10.55	0.1388	6.6717
	5	368.5	10.0	76.00	13.50	0.1776	6.5176
	5	368.5	12.0	76.00	16.60	0.2184	6.3600
6	5	1003	2.0	108.00	7.75	0.0718	8.7758
	5	1003	4.0	108.00	18.20	0.1685	7.4794
	5	1003	6.0	108.00	26.80	0.2481	7.6200
	5	1003	8.0	108.00	35.40	0.3278	7.6901
	5	1003	10.0	108.00	45.10	0.4176	7.5455
	5	1003	12.0	108.00	55.50	0.5139	7.3579
7	5	665	2.0	108.00	5.35	0.0495	8.4388
	5	665	4.0	108.00	11.10	0.1028	8.1284
	5	665	6.0	108.00	17.40	0.1611	7.7801
	5	665	8.0	108.00	24.10	0.2232	7.4878
	5	665	10.0	108.00	30.20	0.2796	7.4716
	5	665	12.0	108.00	37.20	0.3444	7.2789
8	5	1179	2.0	108.00	7.80	0.0722	10.2617
	5	1179	4.0	108.00	17.20	0.1593	9.3002
	5	1179	6.0	108.00	27.00	0.2500	8.8889
	5	1179	8.0	108.00	37.00	0.3426	8.6486
	5	1179	10.0	108.00	45.60	0.4222	8.7735
	5	1179	12.0	108.00	53.30	0.4935	9.0070
					average value:		7.5080
9	10	376	2.0	52.63	1.30	0.0247	9.5648
	10	376	4.0	52.63	2.74	0.0521	9.0662
	10	376	6.0	52.63	4.05	0.0770	9.2024
	10	376	8.0	52.63	5.55	0.1054	8.9636
	10	376	10.0	52.63	7.10	0.1349	8.7550
	10	376	12.0	52.63	8.60	0.1634	8.6730
10	10	378	2.0	100.00	2.64	0.0264	8.9968
	10	378	4.0	100.00	5.45	0.0545	8.7165
	10	378	6.0	100.00	8.20	0.0820	8.6894
	10	378	8.0	100.00	11.00	0.1100	8.6365
	10	378	10.0	100.00	13.90	0.1390	8.5434
	10	378	12.0	100.00	16.90	0.1690	8.4317
11	10	373	2.0	150.00	4.10	0.0273	8.5874
	10	373	4.0	150.00	8.10	0.0540	8.6806
	10	373	6.0	150.00	12.20	0.0813	8.6493
	10	373	8.0	150.00	16.30	0.1087	8.6253
	10	373	10.0	150.00	20.60	0.1373	8.5361
	10	373	12.0	150.00	25.00	0.1667	8.4364

TABLE II (cont'd)

Run no.	z_i	σ	V	R	V_R	I	$\frac{V}{2I/4\pi\sigma a}$
12	10	372	2.0	76.10	1.84	0.0242	9.6572
	10	372	4.0	76.10	3.95	0.0519	9.0050
	10	372	6.0	76.10	5.98	0.0786	8.9193
	10	372	8.0	76.10	8.05	0.1058	8.8359
	10	372	10.0	76.10	10.20	0.1340	8.7199
	10	372	12.0	76.10	12.38	0.1627	8.6182
13	10	369	2.0	76.00	2.03	0.0267	8.6806
	10	369	4.0	76.00	4.08	0.0537	8.6337
	10	369	6.0	76.00	6.35	0.0836	8.3183
	10	369	8.0	76.00	8.53	0.1122	8.2636
	10	369	10.0	76.00	10.82	0.1424	8.1394
	10	369	12.0	76.00	13.25	0.1743	7.9792
14	10	1001	2.0	108.00	7.22	0.0668	9.4162
	10	1001	4.0	108.00	14.62	0.1354	9.2915
	10	1001	6.0	108.00	22.20	0.2056	9.1785
	10	1001	8.0	108.00	28.90	0.2676	9.4018
	10	1001	10.0	108.00	36.20	0.3352	9.3826
	10	1001	12.0	108.00	43.40	0.4018	9.3926
15	10	665	2.0	108.00	5.08	0.0470	8.8889
	10	665	4.0	108.00	10.20	0.0944	8.8515
	10	665	6.0	108.00	15.50	0.1435	8.7349
	10	665	8.0	108.00	20.60	0.1907	8.7633
	10	665	10.0	108.00	25.70	0.2380	8.7773
	10	665	12.0	108.00	30.50	0.2824	8.8770
16	10	1175.5	2.0	108.00	8.05	0.0745	9.9157
	10	1175.5	4.0	108.00	15.15	0.1403	10.5291
	10	1175.5	6.0	108.00	24.05	0.2227	9.9502
	10	1175.5	8.0	108.00	33.00	0.3056	9.6677
	10	1175.5	10.0	108.00	42.00	0.3889	9.4958
	10	1175.5	12.0	108.00	51.50	0.4768	9.2944
					average	value:	8.9445
17	15	375	2.0	52.63	1.12	0.0213	11.0620
	15	375	4.0	52.63	2.36	0.0448	10.5180
	15	375	6.0	52.63	3.45	0.0656	10.7739
	15	375	8.0	52.63	4.75	0.0902	10.4480
	15	375	10.0	52.63	6.05	0.1150	10.2438
	15	375	12.0	52.63	7.40	0.1406	10.0545
18	15	378	2.0	100.00	2.27	0.0227	10.4602
	15	378	4.0	100.00	4.61	0.0461	10.3040
	15	378	6.0	100.00	7.00	0.0700	10.1781
	15	378	8.0	100.00	9.38	0.0938	10.1279
	15	378	10.0	100.00	12.00	0.1200	9.8961
	15	378	12.0	100.00	14.40	0.1440	9.8961

TABLE II (cont'd)

Run no.	z_i	σ	V	R	V_R	I	$\frac{V}{2I/4\pi\sigma a}$
19	15	372.5	2.0	150.00	3.50	0.0233	10.0452
	15	372.5	4.0	150.00	6.80	0.0453	10.3306
	15	372.5	6.0	150.00	10.40	0.0693	10.1300
	15	372.5	8.0	150.00	14.20	0.0947	9.8839
	15	372.5	10.0	150.00	17.70	0.1180	9.9157
	15	372.5	12.0	150.00	21.40	0.1427	9.8393
20	15	372	2.0	76.10	1.64	0.0216	10.8225
	15	372	4.0	76.10	3.40	0.0447	10.4575
	15	372	6.0	76.10	5.15	0.0677	10.3555
	15	372	8.0	76.10	6.98	0.0917	10.1937
	15	372	10.0	76.10	8.80	0.1156	10.1082
	15	372	12.0	76.10	10.62	0.1396	10.0444
21	15	369	2.0	76.00	1.72	0.0226	10.2564
	15	369	4.0	76.00	3.53	0.0464	9.9925
	15	369	6.0	76.00	5.42	0.0713	9.7529
	15	369	8.0	76.00	7.40	0.0974	9.5193
	15	369	10.0	76.00	9.35	0.1230	9.4233
	15	369	12.0	76.00	11.38	0.1497	9.2908
22	15	998	2.0	108.00	6.30	0.0583	10.7585
	15	998	4.0	108.00	12.90	0.1194	10.5042
	15	998	6.0	108.00	19.70	0.1824	10.3146
	15	998	8.0	108.00	26.10	0.2417	10.3788
	15	998	10.0	108.00	32.00	0.2963	10.5820
	15	998	12.0	108.00	38.10	0.3528	10.6648
23	15	665	2.0	108.00	4.42	0.0409	10.2145
	15	665	4.0	108.00	9.05	0.0838	9.9701
	15	665	6.0	108.00	13.40	0.1241	10.0993
	15	665	8.0	108.00	18.10	0.1676	9.9713
	15	665	10.0	108.00	22.50	0.2083	10.0291
	15	665	12.0	108.00	26.80	0.2482	10.1002
24	15	1177	2.0	108.00	6.78	0.0628	11.7786
	15	1177	4.0	108.00	13.78	0.1276	11.5908
	15	1177	6.0	108.00	21.30	0.1972	11.2507
	15	1177	8.0	108.00	29.20	0.2704	10.9394
	15	1177	10.0	108.00	37.00	0.3426	10.7921
	15	1177	12.0	108.00	44.80	0.4148	10.6971
					average	value:	10.3117
25	20	374	2.0	52.63	1.00	0.0190	12.3686
	20	374	4.0	52.63	2.04	0.0388	12.1139
	20	374	6.0	52.63	2.95	0.0560	12.5892
	20	374	8.0	52.63	4.02	0.0764	12.3039
	20	374	10.0	52.63	5.20	0.0988	11.8934
	20	374	12.0	52.63	6.30	0.1197	11.7797

TABLE II (cont'd)

Run no.	z_i	σ	V	R	V_R	I	$\frac{V}{2I/4\pi\sigma a}$
26	20	378	2.0	100.00	1.98	0.0198	11.9976
	20	378	4.0	100.00	3.98	0.0398	11.9332
	20	378	6.0	100.00	6.00	0.0600	11.8741
	20	378	8.0	100.00	8.04	0.0804	11.8168
	20	378	10.0	100.00	10.15	0.1015	11.7000
	20	378	12.0	100.00	12.25	0.1225	11.6324
27	20	372.5	2.0	150.00	2.90	0.0193	12.1212
	20	372.5	4.0	150.00	5.90	0.0393	11.9083
	20	372.5	6.0	150.00	9.00	0.0600	11.7005
	20	372.5	8.0	150.00	12.10	0.0807	11.5992
	20	372.5	10.0	150.00	15.10	0.1007	11.6184
	20	372.5	12.0	150.00	18.30	0.1220	11.5086
28	20	372.5	2.0	76.10	1.48	0.0194	12.0627
	20	372.5	4.0	76.10	3.00	0.0394	11.8765
	20	372.5	6.0	76.10	4.38	0.0576	12.1877
	20	372.5	8.0	76.10	5.98	0.0786	11.9083
	20	372.5	10.0	76.10	7.56	0.0993	11.7827
	20	372.5	12.0	76.10	9.10	0.1196	11.7394
29	20	369.5	2.0	76.00	1.52	0.0200	11.6077
	20	369.5	4.0	76.00	3.10	0.0408	11.3830
	20	369.5	6.0	76.00	4.58	0.0603	11.5518
	20	369.5	8.0	76.00	6.35	0.0836	11.1096
	20	369.5	10.0	76.00	7.95	0.1046	11.0988
	20	369.5	12.0	76.00	9.65	0.1270	10.9699
30	20	997	2.0	108.00	5.20	0.0481	13.0208
	20	997	4.0	108.00	11.10	0.1028	12.1877
	20	997	6.0	108.00	16.20	0.1500	12.5287
	20	997	8.0	108.00	22.00	0.2037	12.3001
	20	997	10.0	108.00	27.10	0.2509	12.4828
	20	997	12.0	108.00	32.40	0.3000	12.5287
31	20	665	2.0	108.00	3.75	0.0347	12.0409
	20	665	4.0	108.00	7.50	0.0694	12.0409
	20	665	6.0	108.00	11.25	0.1042	12.0289
	20	665	8.0	108.00	15.20	0.1407	11.8782
	20	665	10.0	108.00	19.20	0.1778	11.7495
	20	665	12.0	108.00	23.20	0.2148	11.6709
32	20	1171.5	2.0	108.00	6.05	0.0560	13.1406
	20	1171.5	4.0	108.00	12.08	0.1118	13.1666
	20	1171.5	6.0	108.00	18.30	0.1694	13.0350
	20	1171.5	8.0	108.00	24.80	0.2296	12.8246
	20	1171.5	10.0	108.00	31.40	0.2907	12.6614
	20	1171.5	12.0	108.00	38.20	0.3537	12.4870

average value: 12.0315

TABLE IV--EXPERIMENT 2

Observed and Reduced Data for Partially Penetrating Electrode: ℓ = 24.5 cm, r_w = 0.3 cm, and r_c = 0.2 cm

Run no.	z_i	σ	V	R	V_R	I	$\frac{V}{2I/8\pi\sigma\ell}$	$\left(\frac{V}{2I/8\pi\sigma\ell}\right)_c$
1	5	377	2.0	82.5	1.40	0.0170	13.6519	13.2543
	5	377	4.0	82.5	2.94	0.0356	13.0378	12.6581
	5	377	6.0	82.5	4.28	0.0519	13.4168	13.0260
	5	377	8.0	82.5	5.85	0.0709	13.0954	12.7140
	5	377	10.0	82.5	7.42	0.0899	12.9082	12.5322
	5	377	12.0	82.5	8.95	0.1085	12.8356	12.4618
2	5	377	2.0	100.0	1.72	0.0172	13.4953	13.1022
	5	377	4.0	100.0	3.53	0.0353	13.1492	12.7662
	5	377	6.0	100.0	5.25	0.0525	13.2626	12.8763
	5	377	8.0	100.0	7.15	0.0715	12.9849	12.6067
	5	377	10.0	100.0	9.00	0.0900	12.8949	12.5193
	5	377	12.0	100.0	10.85	0.1085	12.8356	12.4618
3	5	377	2.0	150.0	2.62	0.0175	13.2626	12.8763
	5	377	4.0	150.0	5.38	0.0359	12.9282	12.5516
	5	377	6.0	150.0	8.00	0.0533	13.0634	12.6829
	5	377	8.0	150.0	10.75	0.0717	12.9492	12.5720
	5	377	10.0	150.0	13.45	0.0897	12.9383	12.5615
	5	377	12.0	150.0	16.40	0.1093	12.7416	12.3705
4	5	377	2.0	118.0	2.02	0.0171	13.5685	13.1733
	5	377	4.0	118.0	4.14	0.0351	13.2231	12.8380
	5	377	6.0	118.0	6.30	0.0534	13.0378	12.6581
	5	377	8.0	118.0	8.45	0.0716	12.9660	12.5884
	5	377	10.0	118.0	10.62	0.0900	12.8949	12.5193
	5	377	12.0	118.0	12.98	0.1100	12.6596	12.2909
5	5	994	2.0	108.0	4.65	0.0431	14.2046	13.7909
	5	994	4.0	108.0	9.55	0.0884	13.8504	13.4470
	5	994	6.0	108.0	14.10	0.1306	14.0614	13.6518
	5	994	8.0	108.0	19.30	0.1787	13.7010	13.3019
	5	994	10.0	108.0	23.80	0.2204	13.8870	13.4825
	5	994	12.0	108.0	29.00	0.2685	13.6783	13.2799

σ = conductivity of electrolyte in mmhos/cm

V = potential applied across tank in volts

R = value of resistor in series with tank in ohms

V_R = potential drop across R in volts

I = V_R/R = current flowing in system in amperes

z_i = electrode position in cm

TABLE IV (cont'd)

Run no.	z_i	σ	V	R	V_R	I	$\frac{V}{2I/8\pi\sigma l}$	$\left(\frac{V}{2I/8\pi\sigma l}\right)_c$
6	5	646	2.0	108.0	2.95	0.0273	14.5666	14.1423
	5	646	4.0	108.0	6.10	0.0565	14.0796	13.6695
	5	646	6.0	108.0	9.35	0.0866	13.7804	13.3790
	5	646	8.0	108.0	12.85	0.1190	13.3712	12.9818
	5	646	10.0	108.0	16.70	0.1546	12.8667	12.4919
	5	646	12.0	108.0	20.30	0.1880	12.6957	12.3259
7	5	1169	2.0	108.0	5.18	0.0480	14.9925	14.5558
	5	1169	4.0	108.0	10.58	0.0980	14.6897	14.2618
	5	1169	6.0	108.0	16.10	0.1491	14.4823	14.0605
	5	1169	8.0	108.0	22.25	0.2060	13.9762	13.5691
	5	1169	10.0	108.0	28.20	0.2611	13.7836	13.3821
	5	1169	12.0	108.0	34.10	0.3157	13.6799	13.2815
						average	value:	13.0409
8	10	377	2.0	100.0	1.48	0.0148	15.7853	15.3255
	10	377	4.0	100.0	3.01	0.0301	15.4739	15.0232
	10	377	6.0	100.0	4.40	0.0440	15.4719	15.0213
	10	377	8.0	100.0	6.00	0.0600	15.3965	14.9476
	10	377	10.0	100.0	7.62	0.0762	15.3304	14.8839
	10	377	12.0	100.0	9.22	0.0922	15.2536	14.8093
9	10	377	2.0	150.0	2.20	0.0147	15.6863	15.2294
	10	377	4.0	150.0	4.50	0.0300	15.4202	14.9711
	10	377	6.0	150.0	6.75	0.0450	15.8228	15.3619
	10	377	8.0	150.0	9.05	0.0603	15.4739	15.0232
	10	377	10.0	150.0	11.35	0.0757	15.2300	14.7864
	10	377	12.0	150.0	13.70	0.0913	15.1038	14.6639
10	10	377	2.0	118.0	1.72	0.0146	15.9362	15.4720
	10	377	4.0	118.0	3.58	0.0303	15.3610	14.9136
	10	377	6.0	118.0	5.30	0.0449	15.5481	15.0952
	10	377	8.0	118.0	7.15	0.0606	15.3610	14.9136
	10	377	10.0	118.0	9.02	0.0764	15.2300	14.7864
	10	377	12.0	118.0	10.90	0.0924	15.1095	14.6694
11	10	378	2.0	131.5	1.90	0.0144	16.1551	15.6846
	10	378	4.0	131.5	3.85	0.0293	15.8856	15.4229
	10	378	6.0	131.5	5.80	0.0441	15.8311	15.3700
	10	378	8.0	131.5	7.90	0.0601	15.4889	15.0378
	10	378	10.0	131.5	9.82	0.0747	15.5763	15.1226
	10	378	12.0	131.5	12.00	0.0912	15.3100	14.8641
12	10	993	2.0	108.0	3.98	0.0368	16.6113	16.1275
	10	993	4.0	108.0	8.15	0.0755	16.1943	15.7226
	10	993	6.0	108.0	12.10	0.1120	16.3756	15.8986
	10	993	8.0	108.0	16.40	0.1518	16.1095	15.6403
	10	993	10.0	108.0	20.60	0.1907	16.0308	15.5639
	10	993	12.0	108.0	24.80	0.2296	15.9766	15.5113

TABLE IV (cont'd)

Run no.	z_i	σ	V	R	V_R	I	$\frac{V}{2I/8\pi\sigma l}$	$\left(\frac{V}{2I/8\pi\sigma l}\right)_c$
13	10	646	2.0	108.0	2.72	0.0252	15.7853	15.3255
	10	646	4.0	108.0	5.45	0.0505	15.7542	15.2953
	10	646	6.0	108.0	8.10	0.0750	15.9190	15.4533
	10	646	8.0	108.0	11.30	0.1046	15.2120	15.7398
	10	646	10.0	108.0	14.15	0.1310	15.1837	14.7415
	10	646	12.0	108.0	17.20	0.1593	14.9831	14.5467
14	10	1168	2.0	108.0	4.53	0.0419	17.1674	16.6674
	10	1168	4.0	108.0	9.10	0.0843	17.0648	16.5678
	10	1168	6.0	108.0	13.85	0.1282	16.8303	16.3401
	10	1168	8.0	108.0	19.00	0.1759	16.3532	15.8769
	10	1168	10.0	108.0	23.90	0.2213	16.2498	15.7765
	10	1168	12.0	108.0	29.00	0.2685	16.0707	15.6026
						average value:		15.3285
15	15	376	2.0	100.0	1.28	0.0128	18.0832	17.5566
	15	376	4.0	100.0	2.68	0.0268	17.2786	16.7753
	15	376	6.0	100.0	3.85	0.0385	18.0397	17.5046
	15	376	8.0	100.0	5.22	0.0522	17.7384	17.2218
	15	376	10.0	100.0	6.65	0.0665	17.4064	16.8994
	15	376	12.0	100.0	8.05	0.0805	17.2538	16.7513
16	15	376	2.0	150.0	1.95	0.0130	17.8094	17.2907
	15	376	4.0	150.0	3.92	0.0261	17.7384	17.2218
	15	376	6.0	150.0	5.90	0.0393	17.6730	17.1582
	15	376	8.0	150.0	7.90	0.0527	17.5708	17.0590
	15	376	10.0	150.0	9.95	0.0663	17.4581	16.9496
	15	376	12.0	150.0	11.95	0.0797	17.4267	16.9191
17	15	992.5	2.0	108.0	3.55	0.0329	18.5701	18.0292
	15	992.5	4.0	108.0	7.20	0.0667	18.3234	17.7897
	15	992.5	6.0	108.0	10.60	0.0981	18.6858	18.1416
	15	992.5	8.0	108.0	14.30	0.1324	18.4630	17.9252
	15	992.5	10.0	108.0	17.70	0.1639	18.6428	18.0998
	15	992.5	12.0	108.0	21.40	0.1981	18.5099	17.9708
18	15	646	2.0	108.0	2.40	0.0222	17.9212	17.3992
	15	646	4.0	108.0	4.85	0.0449	17.7226	17.2064
	15	646	6.0	108.0	7.30	0.0676	17.6522	17.1381
	15	646	8.0	108.0	9.80	0.0907	17.5439	17.0329
	15	646	10.0	108.0	12.25	0.1134	17.5408	17.0299
	15	646	12.0	108.0	15.20	0.1407	16.9635	16.4694
19	15	1168	2.0	108.0	4.25	0.0394	18.2482	17.7167
	15	1168	4.0	108.0	8.45	0.0782	18.3908	17.8552
	15	1168	6.0	108.0	12.70	0.1176	18.3486	17.8142
	15	1168	8.0	108.0	17.10	0.1583	18.1736	17.6443
	15	1168	10.0	108.0	21.80	0.2018	17.8190	17.3000
	15	1168	12.0	108.0	26.10	0.2417	17.8545	17.3345
						average value:		17.4068

TABLE IV (cont'd)

Run no.	z_i	σ	V	R	V_R	I	$\frac{V}{2I/8\pi\sigma}$	$\left(\frac{V}{2I/8\pi\sigma}\right)_c$
20	20	376	2.0	150.0	1.69	0.0113	20.4918	19.8950
	20	376	4.0	150.0	3.47	0.0231	20.0401	19.4564
	20	376	6.0	150.0	5.12	0.0341	20.3666	19.7734
	20	376	8.0	150.0	6.98	0.0465	19.9154	19.3353
	20	376	10.0	150.0	8.75	0.0583	19.8531	19.2749
	20	376	12.0	150.0	10.55	0.0703	19.7596	19.1841
21	20	376	2.0	183.5	2.07	0.0113	20.4918	19.8950
	20	376	4.0	183.5	4.13	0.0225	20.5761	19.9768
	20	376	6.0	183.5	6.18	0.0337	20.6115	20.0112
	20	376	8.0	183.5	8.25	0.0450	20.5761	19.9768
	20	376	10.0	183.5	10.65	0.0580	19.9561	19.3748
	20	376	12.0	183.5	12.85	0.0700	19.8413	19.2634
22	20	991.5	2.0	108.0	3.15	0.0292	20.9205	20.3112
	20	991.5	4.0	108.0	6.38	0.0591	20.6612	20.0594
	20	991.5	6.0	108.0	9.45	0.0875	20.9351	20.3253
	20	991.5	8.0	108.0	12.65	0.1171	20.8551	20.2477
	20	991.5	10.0	108.0	16.20	0.1500	20.3500	19.7573
	20	991.5	12.0	108.0	19.60	0.1815	20.1816	19.5938
23	20	646	2.0	108.0	2.15	0.0199	19.9800	19.3981
	20	646	4.0	108.0	4.27	0.0395	20.1410	19.5544
	20	646	6.0	108.0	6.58	0.0609	19.5950	19.0243
	20	646	8.0	108.0	8.90	0.0824	19.3097	18.7473
	20	646	10.0	108.0	11.35	0.1051	18.9250	18.3738
	20	646	12.0	108.0	13.50	0.1250	19.0961	18.5399
24	20	1168	2.0	108.0	3.73	0.0345	20.8550	20.2476
	20	1168	4.0	108.0	7.55	0.0699	20.5761	19.9768
	20	1168	6.0	108.0	11.03	0.1021	21.1342	20.5186
	20	1168	8.0	108.0	15.00	0.1389	20.7093	20.1061
	20	1168	10.0	108.0	19.00	0.1759	20.4415	19.8461
	20	1168	12.0	108.0	23.00	0.2130	20.2600	19.6699

average value: 19.6572

TABLE V--EXPERIMENT 3

Observed and Reduced Data for Partially Penetrating

Electrode: $\ell = 10.8$ cm, $r_w = 0.3$ cm, and $r_c = 0.2$ cm

Run no.	z_i	σ	V	R	V_R	I	$\frac{V}{2I/8\pi\sigma\ell}$	$\left(\frac{V}{2I/8\pi\sigma\ell}\right)_c$
1	5	377	2.0	240	2.05	0.0085	12.0337	11.6832
	5	377	4.0	240	4.27	0.0178	11.4942	11.1594
	5	377	6.0	240	6.52	0.0272	11.2824	10.9538
	5	377	8.0	240	8.75	0.0365	11.2108	10.8843
	5	377	10.0	240	11.05	0.0460	11.1198	10.7959
	5	377	12.0	240	13.42	0.0559	10.9800	10.6602
2	5	989	2.0	108	2.20	0.0204	13.1579	12.7747
	5	989	4.0	108	4.42	0.0409	11.6959	11.3552
	5	989	6.0	108	6.62	0.0613	13.1348	12.7522
	5	989	8.0	108	9.12	0.0844	12.7206	12.3501
	5	989	10.0	108	11.65	0.1079	12.4378	12.0755
	5	989	12.0	108	14.25	0.1319	12.2088	11.8532
3	5	648	2.0	108	1.50	0.0139	12.6582	12.2895
	5	648	4.0	108	3.00	0.0278	12.6542	12.2856
	5	648	6.0	108	4.45	0.0412	12.8096	12.4365
	5	648	8.0	108	6.20	0.0574	12.2587	11.9016
	5	648	10.0	108	7.85	0.0727	12.0977	11.7453
	5	648	12.0	108	9.60	0.0889	11.8718	11.5260
4	5	1168	2.0	108	2.57	0.0238	13.3156	12.9278
	5	1168	4.0	108	5.38	0.0498	12.7307	12.3599
	5	1168	6.0	108	8.20	0.0759	12.5287	12.1638
	5	1168	8.0	108	11.20	0.1037	12.2268	11.9107
	5	1168	10.0	108	14.20	0.1315	12.0540	11.7029
	5	1168	12.0	108	17.60	0.1630	11.6686	11.3287
							average value:	11.8282

σ = conductivity of electrolyte in mmhos/cm

V = potential applied across tank in volts

R = value of resistor in series with tank in ohms

V_R = potential drop across R in volts

I = V_R/R = current flowing in system in amperes

z_i = electrode position in cm

TABLE V (cont'd)

Run no.	z_i	σ	V	R	V_R	I	$\frac{V}{2I/8\pi\sigma l}$	$\left(\frac{V}{2I/8\pi\sigma l}\right)_c$
5	10	377	2.0	240	1.78	0.0074	13.8217	13.4191
	10	377	4.0	240	3.72	0.0155	13.2013	12.8168
	10	377	6.0	240	5.60	0.0233	13.1723	12.7886
	10	377	8.0	240	7.62	0.0318	12.8679	12.4931
	10	377	10.0	240	9.58	0.0399	12.8189	12.4455
	10	377	12.0	240	11.60	0.0483	12.7078	12.3377
6	10	987	2.0	108	1.88	0.0174	15.3965	14.9481
	10	987	4.0	108	4.00	0.0370	14.4823	14.0605
	10	987	6.0	108	6.00	0.0556	14.4544	14.0334
	10	987	8.0	108	8.10	0.0750	14.2883	13.8721
	10	987	10.0	108	10.25	0.0949	14.1143	13.7032
	10	987	12.0	108	12.50	0.1157	13.8921	13.4875
7	10	647	2.0	108	1.30	0.0120	14.6306	14.2045
	10	647	4.0	108	2.10	0.0194	18.0996	17.5724
	10	647	6.0	108	3.60	0.0333	15.8186	15.3579
	10	647	8.0	108	5.20	0.0482	14.5720	14.1476
	10	647	10.0	108	6.75	0.0625	14.0489	13.6397
	10	647	12.0	108	8.25	0.0764	13.7899	13.3882
8	10	1168	2.0	108	2.45	0.0227	13.9665	13.5597
	10	1168	4.0	108	4.93	0.0456	13.9034	13.4984
	10	1168	6.0	108	7.45	0.0690	13.7836	13.3821
	10	1168	8.0	108	10.00	0.0926	13.6939	13.2950
	10	1168	10.0	108	12.70	0.1176	13.4771	13.0846
	10	1168	12.0	108	15.40	0.1426	13.3378	12.9493
						average value:		13.6869
9	15	376.5	2.0	282	1.88	0.0067	15.2555	14.8112
	15	376.5	4.0	282	3.90	0.0138	14.5985	14.1733
	15	376.5	6.0	282	5.92	0.0210	14.3266	13.9093
	15	376.5	8.0	282	8.05	0.0285	14.1443	13.7323
	15	376.5	10.0	282	10.20	0.0362	13.9997	13.5919
	15	376.5	12.0	282	12.38	0.0439	13.9357	13.5298
10	15	376.5	2.0	303	2.04	0.0067	15.2555	14.8112
	15	376.5	4.0	303	4.24	0.0140	14.8093	14.3780
	15	376.5	6.0	303	6.50	0.0214	14.5985	14.1733
	15	376.5	8.0	303	8.75	0.0289	14.3446	13.9268
	15	376.5	10.0	303	11.05	0.0365	14.1163	13.7051
	15	376.5	12.0	303	13.35	0.0440	13.9681	13.5613
11	15	986	2.0	108	1.68	0.0156	17.1527	16.6531
	15	986	4.0	108	3.70	0.0343	15.6006	15.1462
	15	986	6.0	108	5.50	0.0509	15.7729	15.3135
	15	986	8.0	108	7.45	0.0690	15.5129	15.0611
	15	986	10.0	108	9.35	0.0866	15.4512	15.0012
	15	986	12.0	108	11.42	0.1057	15.1899	14.7475

TABLE V (cont'd)

Run no.	z_i	σ	V	R	V_R	I	$\frac{V}{2I/8\pi\sigma l}$	$\left(\frac{V}{2I/8\pi\sigma l}\right)_c$
12	15	647	2.0	108	1.18	0.0109	16.1031	15.6341
	15	647	4.0	108	2.58	0.0239	14.6951	14.2671
	15	647	6.0	108	3.78	0.0305	15.0527	14.6143
	15	647	8.0	108	5.18	0.0480	14.6332	14.2070
	15	647	10.0	108	6.62	0.0613	14.3225	13.9053
	15	647	12.0	108	8.00	0.0741	14.2180	13.8039
13	15	1168	2.0	108	2.20	0.0204	15.5400	15.0874
	15	1168	4.0	108	4.45	0.0412	15.3905	14.9422
	15	1168	6.0	108	6.85	0.0634	15.0000	14.5631
	15	1168	8.0	108	9.20	0.0852	14.8837	14.4502
	15	1168	10.0	108	11.65	0.1079	14.6886	14.2608
	15	1168	12.0	108	14.14	0.1310	14.5190	14.0961
						average	value:	14.4686
14	20	376	2.0	303	1.82	0.0060	18.2315	17.7005
	20	376	4.0	303	3.82	0.0126	16.6044	16.1208
	20	376	6.0	303	5.75	0.0190	16.2911	15.8166
	20	376	8.0	303	7.78	0.0257	15.8919	15.4290
	20	376	10.0	303	9.82	0.0324	15.8053	15.3450
	20	376	12.0	303	11.95	0.0394	15.6270	15.1718
15	20	376	2.0	341	1.92	0.0056	17.0213	16.5255
	20	376	4.0	341	4.18	0.0123	16.2075	15.7354
	20	376	6.0	341	6.42	0.0188	16.1204	15.6509
	20	376	8.0	341	8.75	0.0257	15.8919	15.4290
	20	376	10.0	341	11.02	0.0323	15.7555	15.2966
	20	376	12.0	341	13.35	0.0392	15.5481	15.0952
16	20	984	2.0	108	1.55	0.0144	18.5529	18.0125
	20	984	4.0	108	3.25	0.0301	17.7462	17.2293
	20	984	6.0	108	4.90	0.0454	17.6471	17.1331
	20	984	8.0	108	6.58	0.0609	17.5439	17.0329
	20	984	10.0	108	8.45	0.0782	17.0765	16.5791
	20	984	12.0	108	10.15	0.0940	17.0479	16.5514
17	20	647	2.0	108	1.06	0.0098	17.9212	17.3992
	20	647	4.0	108	2.28	0.0211	16.6459	16.1611
	20	647	6.0	108	3.32	0.0307	17.1576	16.6579
	20	647	8.0	108	4.60	0.0426	16.4880	16.0078
	20	647	10.0	108	5.90	0.0546	16.0798	15.6115
	20	647	12.0	108	7.15	0.0662	15.9151	15.4516
18	20	1168	2.0	108	1.98	0.0183	17.3160	16.8116
	20	1168	4.0	108	4.02	0.0372	17.0430	16.5466
	20	1168	6.0	108	6.20	0.0574	16.5654	16.0829
	20	1168	8.0	108	8.32	0.0770	16.4677	15.9881
	20	1168	10.0	108	10.48	0.0970	16.3399	15.8640
	20	1168	12.0	108	12.68	0.1174	16.2009	15.7290
						average	value:	16.2055

APPENDIX II

Major Symbols and Definitions Used in the Text

a: width of stream, L;

b: uniform thickness of aquifer and electrolyte depth in model tank, L;

i_s: electric current per unit area, QT^{-1};

K: hydraulic conductivity, LT^{-1};

$L(u,0) = -L(-u,0) = \int_0^u K_o(y)\,dy$, tabular values for which are available (Hantush, 1964);

ℓ: length of collector well lateral, L;

ℓ': $\ell + r_c$, L;

M': an integar such that $M' > \frac{b}{2r_c}$;

Q: total discharge of a collector well, L^3T^{-1};

$r = \sqrt{x^2 + y^2}$, L;

r_c: effective radius of collector well caisson, L;

r_m: point of maximum drawdown along a lateral, L;

r_w: effective radius of collector well lateral, L;

s_{cs}: drawdown in the collector well during the steady state, L;

V: electric potential, $ML^2T^{-2}Q^{-1}$;

v_s: bulk velocity of ground water flow, LT^{-1};

x,y,z: rectangular coordinates;

z_i: vertical position of collector well lateral or lateral electrode in model, L;

θ: angular coordinate of a polar system;

θ_i: angular coordinate of the position of a lateral;

σ: conductivity of electrolyte, $TQ^2M^{-1}L^{-3}$; and

ϕ: hydraulic head, L.

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This thesis is accepted on behalf of the faculty of the Institute by the following committee:

Mahdi Hantush

Charles R Holmes

Gerardo Wolfgang Gross

[illegible]

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