NEW MEXICO INSTITUTE OF MINING AND TECHNOLOGY

HYDRAULICS OF CERTAIN STREAM-CONNECTED AQUIFER SYSTEMS

by

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Submitted to the faculty of New Mexico Institute of Mining and Technology in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

June 1962

PREFACE

As the demand for water rises, it becomes increasingly important to develop ground-water and surface-water resources conjunctively to distribute equitably the greatest quantity of water for the least cost. In order to plan for optimum use of water resources, it is necessary to know the quantitative relationships of ground-water storage to the flow of springs and streams under fluctuating natural conditions, and to be able to predict the effects on the hydrologic cycle of man's use of water and the land. An analytical approach is one of the most valuable methods of quantitative investigation.

The objectives of this investigation were to:

(1) investigate the basic principles involved in the concept of the quantitative interrelationship of ground water and surface water,

(2) set up systems of approximate differential equations to represent the nonsteady plane motion of ground water in recharged leaky systems of paired parallel aquifers,

(3) review existing solutions for special cases of the systems of differential equations, for various boundary conditions,

(4) determine the type and geometry of the stream-connected aquifer systems that are present in the Rio Grande drainage basin, and,

(5) state and solve new boundary-value problems applicable to some of the aquifer systems in the basin.

The writer was first introduced to qualitative concepts of stream-connected aquifer systems by Tolman's textbook (1937) and Bryan's (1938) explanation of the relationship of the Rio Grande to its ground-water reservoir. A terminology and classification of aquifer systems has been established herein (see Introduction and Appendices) in order to express the basic concepts of stream-connected aquifers more concisely and facilitate quantitative treatment of complicated geohydrologic situations.

The Laplace, Poisson, and heat conduction types of second order linear differential equations have been used by Muskat (1937) and others in the solution of a number of boundary-value problems involving steady flow, steady flow with recharge, and nonsteady flow, respectively. These and other equations with a term representing linear vertical leakage across semiconfining beds have been used by Jacob and Hantush (see List of References). Inasmuch as many aquifer systems require concurrent consideration of their storage, leakage, and recharge properties, approximate differential equations are derived for plane-parallel motion in an arbitrary system of two parallel, mutually leaky aquifers receiving areal recharge (see Derivations).

Differential equations for special cases of flow have the form of an equation for heat conduction given by Carslaw and Jaeger (1959). The analytical expressions of the various possible associated lateral boundary conditions in diffusion and heat conduction are tabulated and compared to analogous boundary conditions for stream-connected aquifers (see Appendix B). This table facilitates a review of solutions of boundary-value problems in diffusion and heat conduction that are analogous to problems of stream-connected aquifer systems, a review more fruitful than anticipated. More than one hundred solutions were found. Appendix E is an index to the source material for selected solutions.

The drainage basin of the Rio Grande above Rio Salado was selected for field study because this river basin is one of the largest and best examples of a stream-connected aquifer system. The optimum use of the water resources required to satisfy all water demands in the drainage basin cannot be achieved without increased knowledge and application of quantitative methods. Available geohydrologic data for the Rio Grande basin are reviewed and evaluated on the basis of the writer's previous geohydrologic investigations (see List of References). New field data were collected where required to provide a better basis for subdivision of the drainage basin and for analytical representation of certain aquifer systems. Portions of these observations have already been published (Spiegel, 1961a, 1961c). Although this paper is concerned primarily with solutions of the equations derived, a brief summary of the geology is given in connection with a description of geohydrologic provinces of part of the Rio Grande drainage basin.

The Sunshine Valley and Albuquerque areas were selected for detailed investigation both because of their importance in the water economy of the Rio Grande basin and because a considerable amount of hydrologic data was already available.

The writer expresses his deep appreciation to Dr. Mahdi S. Hantush for his advice in the analytical work, for his constant encouragement during the investigation, and for the excellent background provided by his formal courses in ground-water hydrology. Part of the work was done under a National Science Foundation Cooperative Fellowship at New Mexico Institute of Mining and Technology. Thanks also are extended to the New Mexico State Engineer Office and to the Water Resources Division of the U. S. Geological Survey for their encouragement and cooperation.

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ABSTRACT

Ground-water motion in certain stream-connected systems of non-leaky, leaky perched, or leaky semiperched aquifers is represented analytically by systems of second-order, linear partial differential equations, using boundary conditions representative of one or more of the following characteristics: (a) unlined channel, (b) leaky channel lining, (c) impermeable channel lining, (d) prescribed flux, and (e) finite-capacity reservoir. The differential equations are derived by integration over a prismatic element of a two-aquifer leaky system and contain terms for divergence, leakage, storage change, and areal recharge.

The principal pre-watercourse aquifers in the Rio Grande drainage basin of Colorado and New Mexico are in the Santa Fe group (river gravel, eolian sand, basalt, and portions of an alluvial fan facies). These aquifers are bounded on one or more sides by relatively impermeable rocks of Santa Fe (late Cenozoic) age, older rocks in fault or sedimentary contact, or intrusive rocks. Lake beds and other fine-grained sediments within the Santa Fe group locally cause leaky conditions. New analytic solutions are given for several cases of infinite closed strip and half strip leaky two aquifer systems, and for a rectangular aquifer closed on three sides and bounded by an unlined stream channel with uniform gradient.

HYDRAULICS OF CERTAIN STREAM-CONNECTED AQUIFER SYSTEMS

INTRODUCTION

STREAM-CONNECTED AQUIFER SYSTEMS

A stream-connected aquifer system, as defined and used herein, is any aquifer or combination of interconnected aquifers that is hydraulically related to a surface stream network. Most aquifers and aquifer systems are stream-connected. The direction of groundwater motion may be irreversible, as in the case of a perched stream or aquifer, but in general it is reversible. Two or more aquifers in a system may be interconnected in series, parallel, or mixed arrangement, as illustrated in Appendix C. Two or more adjoining surface stream networks, each with its own aquifer system, may be interrelated by means of hydraulic connections between aquifers of each system. The outflow from one or more aquifers in a stream basin is the principal source of the base flow of the streams connected to the aquifers within the basin, apart from surface-water storage releases or glacier melt-water.

Meinzer (1934) summarized the early history of thought regarding the origin of the base flow of streams connected to aquifer systems. The most important early scientific work appears to be that of Mariotte (1686), who actually measured the flow of a large stream and found that this flow could easily be accounted for by the infiltration of a part of the rainfall upon the land, by movement through the ground, and by discharge to springs feeding the rivers. Darcy (1856) established experimentally the linear law of the relation of the velocity of ground water to the hydraulic gradient. This law, which later came to bear his name, is the basis upon which Dupuit (1863) and others derived differential equations and their solutions for problems of ground-water flow to wells and streams. Maillet (1905) reviewed a number of analytical solutions by Boussinesq for the flow to streams in an important early attempt to put the study of stream base flow on a quantitative basis.

An exponential flow relation (Maillet, 1905; Horton, 1914, 1933) has been the basis for nearly all work done in this country on base-flow characteristics of streams and springs. Horton's ideas (Horton, 1933, p. 448) on the form of the base-flow curve were reported to have been postulated at about the same time that Maillet first presented his interpretation to the Academy of Sciences of Paris. Forchheimer (1914, p. 462), Horton (1933, p. 448), and Werner and Sundquist (1951, p. 203) discussed Maillet's work, but only so far as to state that he derived the relation $Q = A \exp(at)$ for the ground-water contribution to streams. They did not give Maillet due credit for considering a large number of specific stream connected aquifer problems, based on original analytic solutions as well as on those of Boussinesq and Dupuit. Among the later solutions is a second base-flow relation, $Q = b/(1 - at)^2$, b and a constant, due to Boussinesq (1903). Maillet's practical discussion, unfortunately, was limited to consideration of an exponential law derived using a process of integration over an aquifer. He did not use the many solutions obtained by Boussinesq and himself for specific aquifer geometries, nor discuss the geohydrologic character of the aquifers supplying the springs and streams. Many other aquifer system problems solved by early European hydrologists have not been acknowledged in the English literature on hydrology, perhaps partly because the emphasis of analytical investigations later shifted to the effects of withdrawals by wells on water levels in single aquifers. The effects of water withdrawals from wells on the flow of springs and streams and on other aquifers has received comparatively little attention.

In the 1930's, and again in the past few years, surface-water hydrologists have studied the base-flow or low-flow characteristics of streams, but the approach has been largely empirical (see publications of Horton and International Union of Geodesy and Geophysics in the List of References). The empirical approach alone is inadequate, particularly for prediction of stream flow and water levels under changed conditions. The behavior of aquifer systems must be investigated by analytical, model, or analog studies.

In recent years, interest has been revived in the analytical treatment of stream flow derived from ground water. Solutions to new problems of stream-connected aquifer systems have been given by Baumann, Hantush (1960, 1962), Haushild and Kruse, Jacob and Lohman, Maasland (1959), Polubarinova-Kotchina (1952), and in several papers by Werner (see List of References). Despite these excellent contributions, several recent papers have reported that most aquifer systems are too complex to be attacked by analytical methods. It is hoped that this viewpoint will be shown to be unduly pessimistic.

ANALYSIS AND TERMINOLOGY OF AQUIFER SYSTEMS

The detailed nature of the geologic framework in an area is the most important single class of knowledge required to understand the behavior of aquifer systems in that area. Detailed data on the discharge and chemical quality of natural springs and aquifer outflow, and maps of the water-level contours of each aquifer in a system comprise the second most important class. The fluctuations of water levels caused by natural and artificial factors is a third class.

The effects of many of the geologic factors on the recharge, motion, and discharge of ground water have been described extensively in the literature. The recharge W; the hydraulic conductivity K, storage coefficient S, and thickness m of aquifers; the vertical hydraulic conductivity K', and thickness m' of semi-confining beds; and the gross geometric parameters of the aquifer system are the principal quantitative characteristics that must be determined. Many of these characteristics can be estimated roughly from purely geologic investigations. Data of the second and third classes mentioned above are useful in refining the estimates of the aquifer characteristics.

The spatial and temporal values of areal recharge, and spatial values of the parameters related to leakage are probably the most difficult factors to evaluate quantitatively. Quantitative methods of evaluation of the recharge rate (infiltration) have been reported, primarily in the literature of the soil and agricultural sciences. The quantitative determination of leakage across semiconfining beds is a relatively new technique, although the concept of leaky artesian aquifers was recognized at least as early as 1885, and analytical work has been done since 1930 (Hantush, 1949, p. 1). A statement made by Gilbert (1875, p. 115) suggests that the possibility of artesian leakage was recognized much earlier.

Inasmuch as ground water in most aquifers (except cavernous limestone and basalt) moves at a rate given by Darcy's law (Hubbert, 1940, p. 791), $v = -K(d/ds)h$, the total quantity of ground water crossing a given section of an aquifer in unit time is $Q = v_n A$, where v_n designates the normal component of velocity in the direction of increasing space variable, and A is the area of the section. This means that if the aquifer coefficients, thickness, and water levels are known in the vicinity of a stream connected to the aquifer, the aquifer outflow to the stream or the aquifer inflow from the stream can be calculated. Similarly, the inflow or outflow can be calculated, analytically or otherwise, for any arbitrary gradient and thickness that may be assumed to occur at another time. The qualitative effects of streams and reservoirs on the shape of the water-level contours in adjacent aquifers were reviewed by Tolman (1937) and Muskat (1937).

The degree of penetration of an aquifer by a bounding stream is important in the vicinity of the stream. However, since most aquifers have large lateral dimensions compared to their thickness, and since the vertical components of flow caused by incomplete penetration are usually very small compared to lateral compenents in the aquifer, the effects of partial penetration are neglected herein. They must, however, be taken into account in most problems of drainage for agriculture or engineering works. A further justification for assuming full stream penetration is that most streams occupy valleys containing a thick fill of coarse, highly permeable material, and the entire watercourse (see definition in the next section) can usually be considered to be fully penetrating even if the stream channel alone is not.

The channels of streams, or the walls and bottoms of watercourses, can be classified as either unlined or lined. The water levels in aquifers immediately adjacent to an unlined channel are assumed to be equal to the level of the stream or reservoir. Lined channels can be impermeable or leaky, or transmit prescribed flux of water from the stream. The analytical expressions for these relations for both streams and reservoirs are given in the list of boundary conditions included in Appendix B, and graphical representations are given in Appendix C.

Most of the concepts discussed herein are referred to by the terms in common use by ground-water hydrologists in the United States, as reflected by recent publications (e.g., Todd, 1959; Ferris, 1959). Certain concepts introduced here or referred to by terms not in general usage are defined below. Units are indicated where appropriate. Parameters represented by letter symbols are listed in Appendix A.

Recharge (L/T):--areal recharge, or the water added to an aquifer by infiltration from direct precipitation upon the land surface, surface sheetflow, or flow in closely spaced drainage channels.

Stream recharge (L^2/T):--the lateral inflow to an aquifer caused by losses from well-defined stream channels. Includes the subclass of canal recharge.

Perched aquifer:--aquifer underlain by a less permeable layer that is in turn underlain by a lower unconfined aquifer. The perching layer may be leaky or nonleaky but must be underlain by an aerated zone.

Semi-perched aquifer:--aquifer underlain by a less permeable layer that is in turn underlain by an aquifer having water levels higher than the top of the confining bed (confined or semiconfined aquifer). The definitions given by Meinzer (1923) for perched and semi-perched aquifers did not explicitly include leaking aquifers as is done herein, but are logically extensible to them.

Inflow (L^2/T):--the water moving laterally inward across a vertical section of unit length at an aquifer boundary.

Outflow (L^2/T):--the water moving laterally outward across a vertical section of unit length at an aquifer boundary.

Aquifer outflow, total (L^3/T):--the total outflow of ground water to a stream or adjacent aquifer from the entire length of an aquifer.

Aquifer inflow, total (L^3/T):--the total inflow of ground water to the entire length of an aquifer, from an adjacent stream or aquifer.

Stream loss (L^2/T):--the loss of flow per unit length of a stream caused only by stream recharge to an adjacent aquifer system (both sides).

Stream loss, total (L^3/T):--total loss of flow to a given length of stream-connected aquifer system.

Stream gain (L^2/T):--the gain of flow of a stream derived from ground water in an adjacent aquifer system (both sides).

Stream gain, total (L^3/T):--total gain of flow from a given length of stream-connected aquifer system.

Potentiometric surface (for fresh water of unit density):--replaces the term "piezometric surface", that is the imaginary surface representing the levels (potentials) to which the water of an aquifer will rise in tubes or wells open to the aquifer at various points. Note that in an aquifer in which there is a vertical component of flow, the potentiometric surface is multivalued for points along a vertical line through the aquifer. The water level in a fully penetrating well represents the mean of the potentials in vertical sections. Unless otherwise noted, the term always designates the surface represented by the potentials corresponding to the mean-in-vertical of velocities in horizontal planes. If the water is saline or underlain by saline water, the true potential will be somewhat higher than that indicated by the water level.

Steady:--at the same level or value in time; not changing with time.

Nonsteady:--changing with time.

Uniform:--at the same level or value in space.

Transmissivity coefficient (L^2/T):--briefly, transmissivity; equivalent to the more commonly used term transmissibility coefficient, both denoted by the letter T (Appendix A).

DIFFUSION AND HEAT CONDUCTION ANALOGIES

The analogy of the nonsteady motion of ground water in porous media to the conduction of heat in solids was apparently first recognized by Boussinesq (1877, p. 257) in a footnote in his treatise on hydraulics of stream flow. Slichter (1899), unaware of Boussinesq' work, expressed surprise that the analogy had not been noted previously, and solved some steady-state problems using conformal mapping. Slichter also gave a long list of references to early literature on ground water. Theis (1935), Muskat (1937, p. 140), and Carslaw and Jaeger (1959, p. 29) also discussed the analogy of ground water to heat conduction in solids, and Crank (1956) recognized and applied the analogy of diffusion and heat conduction. A number of solutions of ground water problems, previously obtained for problems in heat conduction, have been published, principally by Bittinger, Brown, Glover, Luthin and Holmes, Moody, Rowe, and Theis (see List of References).

A system of linear, second-order differential equations is derived herein for the nonsteady plane-parallel motion of ground water in a leaky recharged aquifer system. These equations, (12k), have the same form as the equation given by Carslaw and Jaeger (1959, p. 32) for analogous problems in heat conduction. The analytical and graphical representations of lateral stream and reservoir boundaries, as well as of conditions at the interfaces between aquifers in series systems, are shown in Appendix B and Appendix C. The analogous differential equations and boundary conditions for diffusion and heat conduction are also shown in Appendix B. An index to the sources of selected solutions of problems applicable to stream-connected aquifer systems has been prepared (Appendix E), and, with Appendices B and C, will facilitate the quantitative evaluation of many kinds of aquifer systems elsewhere.

CLASSIFICATION OF AQUIFER SYSTEMS

Aquifer systems are classified herein as shown in the Appendices. The geometric classification (Appendix D) is a modification of earlier ones (Muskat, 1937, p. 727; Hantush and Jacob, 1954; and Carslaw and Jaeger, 1959). The Muskat-Hantush designation "infinite half-strip" was changed to semi-infinite strip to agree with the terminology of Carslaw and Jaeger, and to facilitate further subdivision of infinite-strip aquifers. The symmetrical infinite strip (infinite half-strip of the classification given herein) and the infinite strip with impermeable faces (infinite closed strip) occur so frequently in nature that they are given equal status with the more general case of the infinite strip. Carslaw and Jaeger's terminology of the linear rod is omitted because these problems can be classified as special cases of the semi-infinite or infinite strip aquifer. The analogy to radiation from the cylindrical rod surface is taken into account by the leakage term in the general differential equation. Carslaw and Jaeger's linear rod cases also have direct analogies in ground water (e.g., sandstone channels enclosed in clay).

Although this investigation was limited to the class of aquifers bounded by parallel or perpendicular planes, the principles and geometrical classification of aquifers can be extended to include those of cylinders, wedges, cones, and other shapes. For example, the upper basin of the Jemez River (see Geohydrology) can be considered approximately as a system consisting of a cylinder and concentric cylindrical shell.

Systems consisting of two or more aquifers are called series, parallel, or mixed systems, by analogy to electric circuits. Series systems designated as "homogeneous-composite" actually are single aquifers physically but have segments with differing recharge conditions; each segment may be represented by a different differential equation. In the more general case for series aquifers ("non-homogeneous composite") the properties of each of the series segments are different (see Appendix C). The boundary conditions appropriate to series aquifer systems are also shown in Appendix C.

Aquifers in parallel can be completely independent (nonleaky systems) or interrelated by linear leakage across semi-confining layers (leaky systems; see discussions of leaky artesian aquifers by Jacob, 1946; Hantush and Jacob, 1954). Individual aquifers within a parallel system can be said to be either perched or semiperched above underlying aquifers, depending upon the presence or absence of a zone of aeration below the perching layer. Although the terms perched and semiperched have generally been used with reference to nonleaky systems, the concepts hold just as well for leaky systems. The distinction between perched and semiperched conditions has greater hydrologic significance in leaky systems, for in nonleaky systems the aquifers are independent whether or not there is a zone of aeration below the perching layer. On the other hand, in a leaky system the rate of leakage from the upper aquifer is dependent upon the water levels of the lower aquifer in the perched case. The semiperched concept can be extended (analytically, at least) to the case where the potentiometric surface of the underlying artesian aquifer is higher than the water table in the upper aquifer--only the direction of leakage is reserved.

THE RIO GRANDE SYSTEM

The part of the Rio Grande included in this investigation (see Fig. 3) is separated from adjacent stream basins (Colorado, Arkansas, and Pecos rivers, and Estancia Valley) by rocks of very low permeability. The surface drainage divides bounding this part of the Rio Grande drainage basin generally coincide with the ground-water divides. All the aquifers in the region studied are integrated by the Rio Grande and its tributaries into a single complex stream-connected aquifer system that is essentially isolated except for the outflow past San Acacia. In southern New Mexico and western Texas the Rio Grande is part of a "super-system" extending from the Gulf of Mexico to the Gulf of California by way of the Mimbres and Animas basins (southwestern New Mexico) and the Gila River (New Mexico and Arizona).

Thomas (1951, p. 136 and Pl. I) used the term "watercourse" to designate a hydrologic unit consisting of a surface stream plus the underlying materials deposited by the stream, and pointed out that watercourses may be hydraulically connected to underlying ground-water reservoirs. Such combinations of ground-water reservoirs and streams or watercourses are a special class of stream-connected aquifer systems.

Although the watercourse concept is a useful one, in most of the Rio Grande valley, and in many other areas, the aquifers underlying and adjoining the "watercourse" (inner valley alluvium) are comparable to the watercourse aquifer in transmissivity and have even greater gross storage capacity. The more general concept of a stream-connected aquifer system must therefore be used in quantitative hydrologic studies.

DERIVATIONS OF DIFFERENTIAL EQUATIONS

CASE I. NONSTEADY VERTICAL MOTION IN LEAKY INFINITE CLOSED STRIPS

An elementary example of a leaky parallel-aquifer system with vertical motion is illustrated in Figure 1a. The upper aquifer in the system illustrated is semiperched; analysis is simplified for perched conditions. Such vertical flow systems occur naturally along many of the mountain streams tributary to the Rio Grande, and in many other places. If the lateral outflow from the upper aquifer (Region I) is relatively small, and the transmissivity of the lower aquifer (Region II) is very large, the water levels in each of the aquifers will be nearly uniform, and the direction of motion of ground water in the upper aquifer will be nearly vertical. If the aquifers are separated by a semi-confining bed of relatively low vertical hydraulic conductivity, the vertical variation of potential in the upper aquifer may be neglected (the potential at the base of the upper aquifer can be assumed equal to the water-table elevation h_1^*). The vertical leakage across the semi-confining bed is represented by the Darcy law (Darcy, 1856; Hubbert, 1940), quantity/unit area/unit time = $v_z = \left[K'(h_1^* - h_2)/m'\right]$ (all symbols used are identified in Appendix A, or explained at the point of use).

Let the upper aquifer be recharged at a rate represented by W, the quantity/unit area/unit time that enters the saturated zone. Since the recharge to the aquifer must be accounted for by the sum of leakage and the water going into storage, the law of conservation of matter can be written in terms of the elevation of water table, h_1^*, and the potentiometric surface of the semi-confined aquifer, h_2, (taking the time derivative positive for water-table rise) as

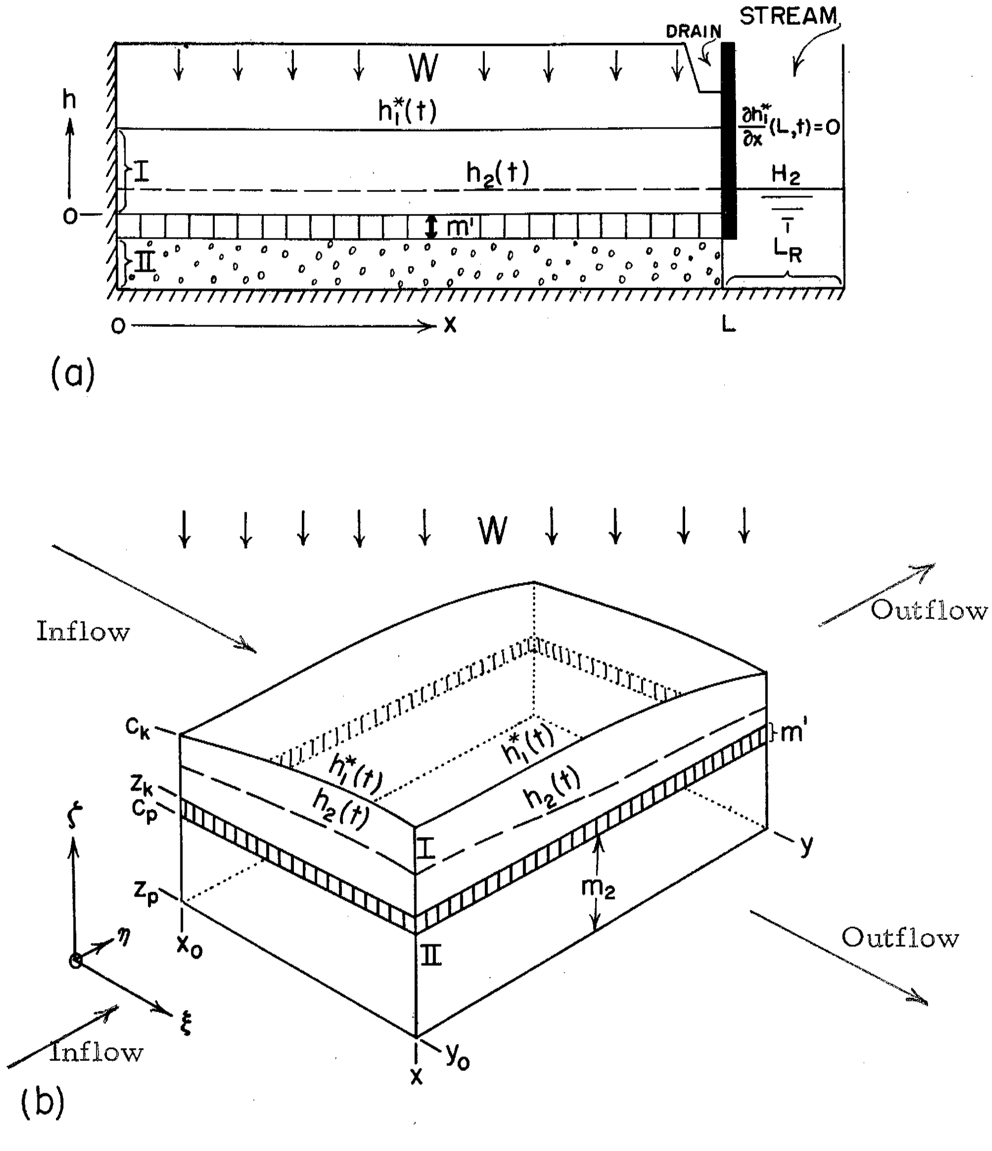


Fig. 1. Leaky parallel-aquifer systems.

(a) Case I, vertical motion (highly transmissive lower aquifer).

(b) Case II, lateral or plane-parallel motion.

(1) $W = S_w(dh_1^*/dt) + (K'/m')(h_1^* - h_2)$, or

$$(dh_1^*/dt) + (K'/m'S_w)h_1^* = (K'/m'S_w)h_2 + W/S_w,$$

which can be written in symbolic operator notation as

(2) $(D_t + K'/m'S_w)h_1^* = (K'/m'S_w)h_2 + W/S_w.$

Solutions of this equation are obtainable for many forms of variation of the rate of recharge W(t) or for independent gage-height fluctuations $h_2(t) = H(t)$ in the stream or reservoir connected to the lower aquifer. If the recharge ceases, equation (2) becomes simply

(3) $(D_t + K'/m'S_w)h_1^* = (K'/m'S_w)h_2.$

Where there is no zone of aeration between the upper and lower aquifers (the case shown in Fig. 1a), and the lower aquifer is connected to a reservoir of finite capacity (see Appendices B and C, boundary conditions III), the level of the reservoir and lower aquifer (assumed highly transmissive) may fluctuate in response to the leakage from the upper aquifer as well as to exterior influences on the stream. This mutual effect of one aquifer upon another can be taken into account by writing a pair of simultaneous conservation equations,

(4) $W = S_w(dh_1^*/dt) + (K'/m')(h_1^* - h_2)$ and

(5) $O = (S_a + L_r/L)(dh_2/dt) - (K'm')(h_1^* - h_2).$

The condition illustrated in Figure 1a and the condition where the water level in the lower aquifer is higher than in the upper aquifer are the same analytically. In the latter case, the direction of movement is reversed, and the stream penetrating the lower aquifer will lose water instead of gaining. On the other hand, in the perched condition, where the water level of the lower aquifer is at or below the base of the upper aquifer, the water level in the upper aquifer cannot be affected by that of the lower aquifer. Therefore the rate of vertical movement is governed by the height of the water table in the upper aquifer only, h_2 being replaced by atmospheric pressure, which can be assumed to be zero. An aquifer underlying a perched aquifer receiving steady uniform recharge may be treated as if it represented an independent aquifer receiving nonsteady uniform recharge. The requirement of a high transmissivity is no longer necessary, but the lateral flow must be taken into account.

If the water table in the upper aquifer reaches a flat land surface without a deep drain, a steady state may be maintained wherein the upward artesian discharge is balanced by surface overflow off the ponded land surface, or by evapotranspiration if the artesian movement is not too large. If the water level of the upper aquifer reaches a deep drain, lateral flow toward the drain will occur and the rate of water-table rise may no longer be represented by the vertical flow equations (1) or (4) and (5). The conditions of Case I are commonly encountered on stream terraces where irrigation projects or artificial recharge ponds are to be constructed, and the appropriate differential equations can be solved to determine the limits of recharge rate which would be imposed by the vertical hydraulic conductivity of the alluvial materials.

The differential equations for the falling-head permeameter (Wenzel, 1942, pp. 51-61) and for an inflow-outflow tank are special cases of equation (1). Equations for other cases of vertical flow downward into the soil from water-filled tubes or reservoirs were given by Polubarinova-Kotchina (1952, ch. 12).

The total leakage to the lower aquifer per unit length of aquifer strip is obtained from Darcy's law,

(6) $$q_L(t) = L(K'/m')(h_1^* - h_2).$$

If the transmissivity of the lower aquifer is very large, or if the aquifer strip is very narrow, equation (6) also represents the approximate rate of outflow from the lower aquifer to the stream.

CASE II. NONSTEADY LATERAL MOTION IN A LEAKY PARALLEL-AQUIFER SYSTEM RECEIVING AREAL RECHARGE

A general type of leaky parallel-aquifer system with dominantly lateral ground-water motion (Fig. 1b) is one in which the uppermost aquifer receives water by areal recharge and loses water by linear leakage across a semi-permeable bed into an underlying artesian aquifer. Jacob (1946) derived the differential equation for nonsteady radial flow to a well in a leaky confined aquifer for the special case where the upper aquifer is held at a steady uniform level. Solutions to this equation given in Jacob's paper and later papers by Jacob and Hantush can be extended to aquifer systems in which water is withdrawn from a well in the upper aquifer, the lower aquifer being maintained at a steady uniform level, providing that the upper aquifer is relatively thick. Polubarinova-Kotchina (1952) and De Wiest (1961) discussed leaky aquifer systems and summarized previous work.

In the more general case where the water levels in n parallel, series, or series parallel aquifers may be functions of time as well as position, and the upper aquifers receive areal recharge, ground-water motion in the system can be represented by a corresponding system of n simultaneous nonsteady differential equations. The n equations for the kth aquifer, where k = 1(1)n, are based on hydrologic conservation equations of the type

(7k) (Inflow - Outflow) - (Leakage) = (Storage change) - (Recharge).

The bracketed terms, or their equivalents in mathematical notation, can be called, respectively, the divergence, vertical leakage, storage (transient), and recharge terms. Where necessary for clarity the subscript k suffixed to the equation number will also be used to identify the variables and parameters of the corresponding aquifer. If the number of parallel aquifers exceeds two, the leakage must be represented by two terms, one for the (k + 1)th aquifer and one for the (k - 1)th aquifer. However, the discussion herein is limited to systems of parallel aquifers in pairs only. Therefore in the equations representing the kth aquifer the parallel aquifer is designated by the subscript p.

An integral representation is more readily related to the corresponding physical system than is the usual infinitesimal volume-element representation (see Page, 1955) and also aids in understanding the significance of the classical Dupuit (1863) approximation for water-table aquifers, as shown below. The integral form of equations (7k) is (see Fig. 1b)

$$\Big[\int_{y_0}^{y}\int_{z_k}^{z} v_x(x_0,\eta,\zeta)\,d\zeta\,d\eta - \int_{y_0}^{y}\int_{z_k}^{z} v_x(x,\eta,\zeta)\,d\zeta\,d\eta + \int_{x_0}^{x}\int_{z_k}^{z} v_y(\xi,y_0,\zeta)\,d\zeta\,d\xi$$

$$(8k)\qquad - \int_{x_0}^{x}\int_{z_k}^{z} v_y(\xi,y,\zeta)\,d\zeta\,d\xi\Big] - \int_{y_0}^{y}\int_{x_0}^{x} v_z'(\xi,\eta,z_{kc})\,d\xi\,d\eta$$

$$= \int_{x_0}^{x}\int_{y_0}^{y}\int_{z_k}^{z} S_{ks}\frac{\partial h_k}{\partial t}(\xi,\eta,\zeta)\,d\zeta\,d\eta\,d\xi - \int_{y_0}^{y}\int_{x_0}^{x} W_k(\xi,\eta)\,d\xi\,d\eta .$$

Each aquifer is assumed to be homogeneous and isotropic, and the density and viscosity of the water are assumed to be uniform and constant. The recharge rates W and the variables v_s, v'_s and h are assumed to be functions of time, but the hydraulic conductivities K characteristic of each aquifer have constant values.

Perform the following operations on equation (8k) to give the general integro-differential equation for two-dimensional flow in a leaky parallel aquifer system with areal recharge: (a) differentiate partially with respect to x, then with respect to y, using the Leibnitz rule for differentiation of integrals, (b) replace the velocities of vertical leakage by Darcy's law written for the confining bed, $v'_z = (K'/m')(h_{kc} - h_{pc})$, and (c) divide by $K_s = K_k$. The result is

$$-\frac{1}{K_k}\int_{z_k}^{z}\frac{\partial}{\partial x}v_x(x,y,\zeta)\,d\zeta - \frac{1}{K_k}\int_{z_k}^{z}\frac{\partial}{\partial y}v_y(x,y,\zeta)\,d\zeta - \frac{(K'/m')}{K_k}(h_{kc} - h_{pc})$$

(9k)

$$= \frac{1}{K_k}\int_{z_k}^{z} S_{ks}\frac{\partial h}{\partial t}(x,y,\zeta)\,d\zeta - \frac{W_k(x,y)}{K_k}.$$

The form of the final differential equations that can be derived from equations (8k) depends upon the way in which the potential h is related to the z-coordinate, and upon the values of the integral limits. Two forms are given below.

Case IIa. Aquifers of Uniform Thickness

The thickness of saturation is uniform for artesian aquifers semi-confined by parallel horizontal aquicludes, and approximately so for thick unconfined aquifers that have relatively low water-table gradients and thickness variations. Accordingly the upper limit of integration in (9k) is a constant, $z = c_k$. Dividing (9k) by the mean aquifer thickness $\bar{m}$ and interchanging the order of differentiation and integration gives

$$-\frac{1}{K_k}\frac{\partial}{\partial x}\left[\frac{1}{\bar{m}_k}\int_{z_k}^{c_k} v_x(x,y,\zeta)\,d\zeta\right] - \frac{1}{K_k}\frac{\partial}{\partial y}\left[\frac{1}{\bar{m}_k}\int_{z_k}^{c_k} v_y(x,y,\zeta)\,d\zeta\right] - \frac{(K'/m')}{K_k\bar{m}_k}\left(h_{kc}-h_{pc}\right)$$

(10k)

$$= \frac{1}{K_k\bar{m}_k}\frac{\partial}{\partial t}\left[\int_{z_k}^{c_k} S_{ks}\, h(x,y,\zeta)\,d\zeta\right] - \frac{W_k(x,y)}{K_k\bar{m}_k}.$$

The two bracketed expressions on the left of these equations represent the means $\bar{v}_{kx}$ and $\bar{v}_{ky}$ of the respective velocity components taken along vertical sections. The integral on the right side can be closely approximated by the product $(S_k \cdot h_{kav})$, whereby the differential equations can be written

$$-\frac{1}{K_k}\frac{\partial}{\partial x}\bar{v}_{kx} - \frac{1}{K_k}\frac{\partial}{\partial y}\bar{v}_{ky} - \frac{(K'/m')}{K_k\bar{m}_k}\left(h_{kc}-h_{pc}\right)$$

(11k)

$$= \frac{S_k}{K_k\bar{m}_k}\frac{\partial}{\partial t}h_{kav} - \frac{W_k(x,y)}{K_k\bar{m}_k}.$$

Now let h_{ke} represent effective average values of the potential corresponding to the mean velocities $\bar{v}_{kx} = -K_k(\partial h_{ke}/\partial x)$ and $\bar{v}_{ky} = -K_k(\partial h_{ke}/\partial y)$ and let $B_k^2 = K_k\bar{m}_k/(K'/m')$. The potentials h_{kav} and h_{kc} can also be closely approximated by the effective average potential h_{ke}, and h_{pc} can be represented by h_{pe}, provided that the assumptions of small vertical velocity components and nearly uniform aquifer thickness are met. Thus (11k) can be reduced to

$$\frac{\partial^2}{\partial x^2}h_{ke} + \frac{\partial^2}{\partial y^2}h_{ke} - \frac{1}{B_k^2}\left(h_{ke}-h_{pe}\right)$$

(12k)

$$= \frac{S_k}{K_k\bar{m}_k}\frac{\partial}{\partial t}h_{ke} - \frac{W_k(x,y)}{K_k\bar{m}_k}$$

which are valid for many unconfined aquifers (except near the seepage face) and for nearly all confined aquifers.

Case IIb. Aquifers of Non-uniform Thickness

For the general case where confined aquifers or unconfined aquifers have moderate thickness variations due to variations in the level of the top of the saturated zone, the upper limits of the integrals with respect to ζ in (8k) and (9k) are not constants. If the elevation of the top of the zone of saturation is represented by $z = h_k^*$ and if suitable average velocities $\bar{v}_{kx}$, $\bar{v}_{ky}$ are chosen, as in the preceding case, then in (8k) the inner integrals with respect to ζ can be represented by $(h_k^* - z_k)\bar{v}_{ks}$ on the left, and by $S_{ks}(h_k^* - z_k)\partial(h_{ke} - z_k)/\partial t$ on the right; thus the conservation equations for aquifers of non-uniform thickness can be written

$$\int_{y_0}^{y}(h_k^*-z_k)\bar{v}_x(x_0,\eta)d\eta - \int_{y_0}^{y}(h_k^*-z_k)\bar{v}_x(x,\eta)d\eta + \int_{x_0}^{x}(h_k^*-z_k)\bar{v}_y(\xi,y_0)d\xi$$

(13k)
$$-\int_{x_0}^{x}(h_k^*-z_k)\bar{v}_y(\xi,y)d\xi - \int_{y_0}^{y}\int_{x_0}^{x} v_z'(\xi,\eta,z_{ke})d\xi d\eta$$

$$= \int_{x_0}^{x}\int_{y_0}^{y} S\,(h_k^*-z_k)\frac{\partial}{\partial t}h_{ke}(\xi,\eta)d\eta d\xi - \int_{x_0}^{x}\int_{y_0}^{y} W_k(\xi,\eta)\,d\eta\,d\xi.$$

As in the preceding case, perform two repeated partial differentiations with respect to x and y, let $\bar{v}_{ks} = -K_k(\partial h_{ke}/\partial s)$ and $v'_z = (K'/m')(h_{kc} - h_{pc})$, and divide by K_k to give

$$\left[\left(\frac{\partial h_{ke}}{\partial x}\right)\left(\frac{\partial h_k^*}{\partial x}\right) + (h_k^*-z_k)\frac{\partial^2(h_{ke}-z_k)}{\partial x^2}\right] + \left[\left(\frac{\partial h_{ke}}{\partial y}\right)\left(\frac{\partial h_k^*}{\partial y}\right) + (h_k^*-z_k)\frac{\partial^2(h_{ke}-z_k)}{\partial y^2}\right]$$

(14k)

$$-\frac{(K'/m')}{K_k}(h_{ke}-h_{pc}) = \frac{S_{ks}}{K_k}(h_k^*-z_k)\frac{\partial(h_{ke}-z_k)}{\partial t} - \frac{W_k(x,y)}{K_k}.$$

Now if the vertical and horizontal components of ground-water flow are not too large, the variables h_k^* and h_{kc} can be approximated by the average effective function h_{ke}, and h_{pc} by h_{pe}, whereby the preceding equations can be simplified to

$$\frac{\partial^2(h_{ke}-z_k)^2}{\partial x^2}+\frac{\partial^2(h_{ke}-z_k)^2}{\partial y^2}-\frac{2(K'/m')}{K_k}(h_{ke}-h_{pe}) = \frac{S_{ks}}{K_k}\frac{\partial(h_{ke}-z_k)^2}{\partial t}-\frac{2W_k}{K_k}, \tag{15k}$$

where the products of derivatives in (14k) have been written in the form $(\partial h_{ke}/\partial s)(\partial h_k^*/\partial s) = (\partial h_{ke}/\partial s)^2$, $u\partial^2 u/\partial s^2 = (1/2)\partial^2 u^2/\partial s^2 -(\partial u/\partial s)^2$, and $u(\partial u/\partial t) = (1/2)\partial u^2/\partial t$.

For nonleaky aquifers, the leakage terms of (15k) are zero, and the remaining terms in the equations for the independent aquifers are linear in $(h_{ke}- z_k)^2$. For the more general case of slightly to moderately mutually-leaky parallel aquifers, the leakage term can be linearized to give an approximate system of differential equations by multiplying the leakage terms by

$$\left[h_{ke} + h_{pe} - (2h_{ke}z_k - 2h_{pe}z_p - z_k^2 + z_p^2)/(h_{ke} + h_{pe})\right] / \left[D_k + D_p - (2D_k z_k - 2D_p z_p - z_k^2 + z_p^2)/(D_k + D_p)\right],$$ to give

$$\frac{\partial^2(h_{ke}-z_k)^2}{\partial x^2}-\frac{\partial^2(h_{ke}-z_k)^2}{\partial y^2} - \frac{2(K'/m'K_k)\left[(h_e-z_k)^2-(h_{pe}-z_p)^2\right]}{\left[D_k+D_p-(2D_k z_k-2D_p z_p-z_k^2+z_p^2)/(D_k+D_p)\right]} = \frac{S_s}{K_k}\frac{\partial(h_e-z_k)^2}{\partial t}-\frac{2W_k}{K_k}, \tag{16k}$$

where D and D_p, respectively, represent average values of h_e and h_{pe}, taken over the aquifers.

Discussion

The general systems of equations derived for Cases IIa and IIb can be reduced to the corresponding equations for nonleaky or steady systems by letting the leakage or transient terms, respectively, approach zero. A negative value may be assigned to the recharge term to represent loss by evapotranspiration or closely spaced well withdrawals. Of course the recharge term would normally be zero for an equation assigned to a confined aquifer, and the appropriate type of storage coefficient must be used.

As described in the discussion of Case I, the direction of leakage in the semiperched system may be either up or down. Also for a perched aquifer, the leakage term of the differential equation has been shown to contain only one dependent variable.

Direct solutions to the systems of differential equations for Case IIb are much more difficult to obtain than for equivalent problems for Case IIa. However, the similarity in form of equations (12k) and (16k) for these cases (both are linear and of the second order) suggests that solutions of (16k) can be obtained from those for the case of uniform thickness (IIa) by replacing the variables h_{ke} and h_{pe} by $(h_{ke} - z_k)^2$ and $(h_{pe} - z_p)^2$ respectively, and the coefficients $1/B^2$, $S/K\bar{m}$, $W/K\bar{m}$ in (12k) by the new coefficients

$2(K'/m'K_k)/\left[D_k + D_p - (2D_k z_k - 2D_p z_p - z_k^2 + z_p^2)/(D_k + D_p)\right]$, S_{ks}/K_k, and $2W_k/K_k$, respectively (see Hantush, 1960, p. 3848, for an example of this procedure applied to a particular boundary-value problem).

The outflow to a stream, or inflow to an aquifer from a stream, is determined by applying the generalized Darcy law to the solution for potential and evaluating the result at the aquifer-stream interface.

GEOHYDROLOGIC PROVINCES OF THE RIO GRANDE BASIN

The Rio Grande unifies a large area of Colorado, New Mexico, Texas, and Mexico by serving as both surface-water drainage channel and ground-water drain for a large area that contains a great variety of geological and hydrological subdivisions and conditions. The early inhabitants in the area probably recognized the relative constancy, purity, and ease of control of natural ground-water outflow in many places even if they had not formulated a complete set of principles of ground-water occurrence. Most later investigations and developments of the water supplies failed to reveal fully the complex interrelationships of the natural surface-water runoff, ground-water reservoirs, and man-made changes in the ground-water phase of the hydrologic regime, especially in connection with irrigation diversions.

An understanding of the interrelationships between rocks of diverse age and character is essential to the elucidation of the movement of ground water. Such understanding also is necessary for the determination of the quantitative relationships of the various aquifers that comprise the great ground-water reservoir drained by the Rio Grande and its tributaries. Many parts of the Rio Grande valley have already been mapped geologically. Generally poor and scattered exposures, concealed faults, facies changes, and lack of continuity of outcrops from one locality to another have caused the map units and geologic interpretations to vary from area to area. Differences in viewpoint, purpose, and general background knowledge of the many individuals likewise have contributed to the lack of unity in nomenclature and interpretation.

The geographic names used in this report are those used on the U. S. Geological Survey topographic and base maps for New Mexico and Colorado, scale 1:500,000, 1955 edition. The locations of places or topographic features not shown on the published base maps are identified by section, township, and range. The geologic units of the Santa Fe group referred to in the discussion of various geohydrologic provinces are shown on the correlation chart, Figure 2. The names chosen for the various geohydrologic provinces are not proposed for formal usage in any way, but are used for convenience in discussion of this large and complicated river basin.

The part of the Rio Grande basin considered is a huge horseshoe-shaped upland mass enclosing a structural and erosional trough trending generally south. The Rio Grande trough is an erosional modification of a complex suite of structurally depressed soft sedimentary rocks and resistant lava interbeds that were deposited from late Miocene time to about middle Pleistocene time. These rocks are referred to collectively as the Santa Fe group (Baldwin, 1956; 1962, in press).

The structural trough was developed by folding and faulting of a complex assemblage of rocks ranging in age from Precambrian to middle Tertiary. These rocks now form the rim of the Rio Grande basin. One of the most important features of this assemblage, so far as the history and character of the later Cenozoic rocks is concerned, is the thick sequence of volcanic rocks that blanketed much of what is now the Rio Grande basin. These volcanic rocks, and the adjacent or underlying pre-volcanic assemblages, were deeply weathered and eroded as new or renewed uplifts occurred. The erosional products, locally accompanied by pyroclastic material and lava flows, were deposited in closed sedimentary basins, which although initially distinct, later coalesced into a much larger south-trending basin drained by a large aggrading river called the ancestral Rio Grande by Bryan (1938).

The sedimentary basins in which Santa Fe group rocks were deposited were much more extensive than the present Rio Grande

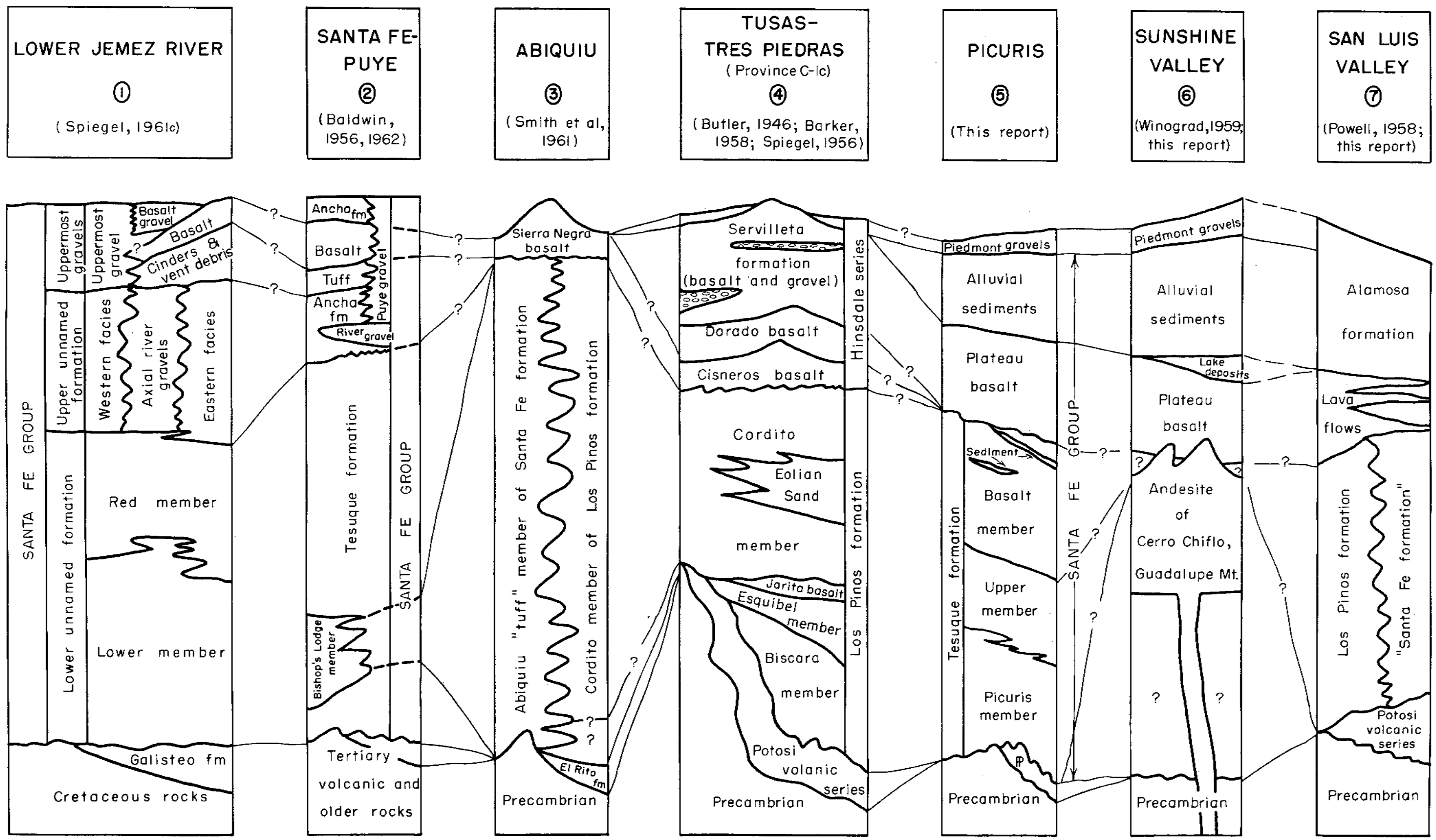


Fig. 2. Correlation of lithologic units of the Santa Fe group.

structural trough as usually defined by Kelley (1954). The basins apparently were formed initially by subsidence of local troughs interspersed among elongated uplifts consisting of en echelon step blocks, tilted blocks, and alternating horsts and grabens. Some of the principal sediment source-areas, especially for the Miocene and Pliocene section, appear to have been areas broadly domed by rejuvenated uplift of early Tertiary or older highlands (e.g., San Juan Mountains, Tusas-Tres Piedras area, Sangre de Cristo Mountains, Nacimiento-San Pedro mountains, Zuni Mountains, Pedernal Hills, and Ladron-Los Pinos mountains). Parts of these areas were capped or flooded by extensive middle Tertiary volcanic piles and related eruptive centers and intrusions (e.g., San Juan and Conejos Ranges, Cimarron Mountains, older volcanoes of the Jemez-Cerrillos region, Datil-Mogollon region). Some early faulting occurred in all the uplifts and within the sedimentary basins, and faulting continued intermittently during deposition, but most of the faults that outline the present basin-and-range topography of New Mexico are of Pleistocene age; sediments deposited during Miocene and Pliocene time are commonly found faulted down into and preserved in the present-day graben and tilt-block structures, but have been eroded off the uplifted blocks so that the original extent of the sediments and related volcanic rocks comprising the Santa Fe group must be inferred in many areas.

A correlation chart (Fig. 2) has been prepared to show the relationships of some of the sequences of lithologic units that have been reported in the Santa Fe group of the Rio Grande basin. Additional sequences and correlations have been given by Baldwin (1956, 1962) and Spiegel (1961c).

Although locally there are important bedrock aquifers in the mountainous rim of the Rio Grande basin, and the watercourses of the various streams of the area are important nearly everywhere, the various units of the Santa Fe group form by far the largest ground-water reservoir. Therefore the delineation of the geohydrologic provinces in Figure 3 is based primarily on the occurrence of ground water in the Santa Fe group.

A. UPLANDS

A-1. Mountain Rim

The mountain drainage basins of the Rio Grande headwaters and eastern tributaries vary somewhat in details of geology and hydrology, but in general the mountain areas are underlain by rocks of low permeability. Ground water occurs in fracture openings and weathered zones in the bedrock, and in glacial deposits, talus, and alluvium in the valleys. Fractures and weathered zones are also generally localized in the valleys, because the valleys were eroded in such areas. The areal extent and depth of the valley aquifers are small, and the specific yields of the aquifers are low, so that ground-water storage is small. The hydraulic conductivity of valley alluvium, however, may be very large. Ground water is recharged principally from snowmelt in late winter and spring; in some intervals practically no recharge occurs during the rest of the year.

The movement of ground water within the mountain drainage basins is complex in detail because of the variable precipitation, high relief, and irregular distribution of recharge and aquifers. The gross pattern consists of areal recharge to bedrock aquifers in interstream areas and minor stream valleys, and movement and outflow to alluvial or glacial drift aquifers in the main valleys (watercourses). Perennial streams in the valley watercourses are maintained by the aggregate flow of numerous springs from the alluvium or adjacent bedrock. Most of the ground water of the mountains becomes the base flow of the main surface streams within the mountain region itself. Relatively little ground water moves out of the mountain ranges by direct subsurface

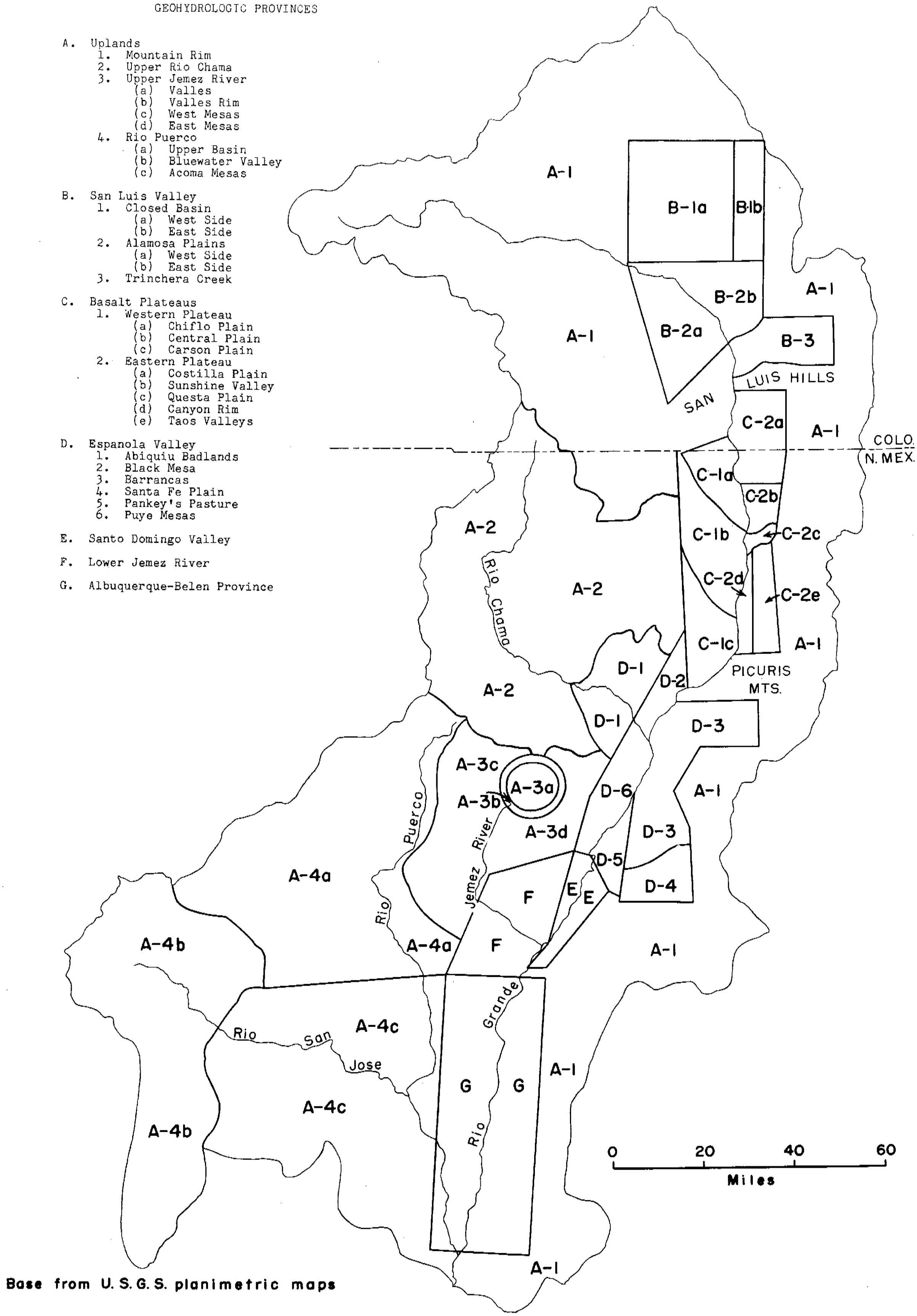


Fig. 3. Geohydrologic provinces of part of the Rio Grande stream basin.

outflow to adjoining aquifers in the Rio Grande trough. The ground-water yield of the mountains can be estimated from hydrographs of the gaged streams and the yield coefficients calculated from nearby gaged stream basins (Spiegel, 1962). The small amount of ground water that discharges into the Rio Grande trough by direct subsurface outflow from mountain-front gully areas can be estimated by applying unit ground-water yield data to areas not drained by perennial streams.

A-2. Upper Rio Chama

The Rio Chama basin above Abiquiu Dam (about 5 miles upstream from Abiquiu) is almost entirely underlain by stratified sedimentary rocks of Mesozoic age. Sandstone aquifers locally furnish water to artesian or contact springs, but yields are usually small. Variations in spring flow can be analyzed analytically by treating them as flowing wells at constant water level. The water-course aquifers in most of the area are capable of absorbing and carrying most of the outflow from bedrock springs, hence when there is no snowmelt or direct runoff, most tributaries of the Rio Chama are dry, except for short reaches below important bedrock springs.

A relatively small area (similar hydrologically to the northern part of the Mountain Rim discussed above) yields most of the base flow of the Rio Chama, principally to the Brazos River and Rio Chama above Chama. The headwaters of the Rio Ojo Caliente, tributary to the lower Rio Chama, and Rio de Los Pinos, tributary to the Rio Grande main stem, are included with the upper Rio Chama even though they contain extensive areas of equivalents of the Santa Fe group (Los Pinos Formation; see Fig. 2). This was done because of insufficient knowledge of the characteristics of the aquifers in this area.

A-3. Upper Jemez River

The upper Jemez River area is geologically complex, but can be divided into four general regions (Fig. 3). The principal aquifers are in the Valles area (A-3a), a circular region which is the caldera of a large volcano (Ross et al, 1961; Titus, 1961). The caldera filling is surrounded by a rim of volcanic rocks (Valles Rim, A-3b) representing the pre-caldera volcanic sequence. These two regions are drained by two headwater streams which form a nearly circular drain, and join in the southwestern part of the caldera rim to form the Jemez River.

Mesas flanking the caldera rim on the west are underlain principally by Mesozoic rocks that yield some water to localized springs. These mesas and the Precambrian cores of the adjoining San Pedro and Nacimiento mountains are classified as the West Mesas sub-province (A-3c). Mesas flanking the east and south sides (A-3d) of the caldera rim are underlain principally by volcanic rocks. Numerous springs furnish a small perennial flow to the many streams that incise the volcanic rocks of this area.

A-4. Rio Puerco

Most of the Rio Puerco drainage basin is underlain by Mesozoic sedimentary rocks, like the Rio Chama basin, but the area has fewer springs and perennial stream reaches because of its generally lower altitude and precipitation. Otherwise the occurrence of ground water in the Mesozoic rocks is similar to that of the Rio Chama basin. A few small uplifts are underlain by older rocks, and are similar hydrologically to the Mountain Rim province (A-1).

The Rio Puerco area is divided into three sub-provinces on the basis of the hydrologic characteristics of the Permian rocks that occur principally in the subsurface. In the Upper Basin (A-4a) the

Permian section, principally sandstone and siltstone called the Cutler formation, has generally low transmissivity. In Bluewater Valley (A-4b) a gypsiferous limestone facies called the San Andres formation is an important aquifer. The natural aquifer outflow is in the broad valley southeast of Grants. An extensive area of basalt flows in the southern part of this province also contributes ground-water outflow to this valley. The San Andres formation is also present in the southeastern part of the drainage basin, but in general contains highly saline water. The high salinity is due to restricted circulation and contamination from saline deposits in underlying beds of the Yeso formation. This area is called the Acoma Mesas (A-4c).

B. SAN LUIS VALLEY

All the important headwater tributaries of the upper Rio Grande in Colorado enter the river in San Luis Valley, which is here defined as the broad valley bounded by relatively impermeable rocks of the Sangre de Cristo Mountains on the northeast, San Juan Mountains on the west, and the San Luis Hills on the south. The Rio Grande watercourse breaches the impermeable barrier of the San Luis Hills.

The San Luis Valley is one of numerous tilted and downfaulted areas that originated by a complex system of en echelon normal faults, sets of step-faulted blocks, and alternate minor horsts and grabens. The zone of normal faulting, of which San Luis Valley is a part, extends northward through central Colorado and Wyoming, as well as southward. However, the region of continuous ground-water reservoirs in Pliocene and Pleistocene rocks integrated by the erosional valley of the Rio Grande ends at Poncha Pass, where the Sangre de Cristo Range is juxtaposed against a southern extension of the Sawatch Range.

The San Juan Mountains immediately west of San Luis Valley are essentially continuations of the belt of northwest-trending en echelon fault blocks represented in the Conejos Mountains (between Tres Piedras and Chama) of New Mexico. These fault blocks dip gently to the northeast and east, so that in travelling eastward, one rises in the section from Precambrian rocks through the thick sequence of Tertiary rocks shown in the generalized section for San Luis Valley (Fig. 2, col. 7). Sediments of the "Santa Fe formation" and part of the Alamosa formation form a leaky artesian aquifer system that underlies the entire San Luis Valley.

Use of both surface water and ground water in the San Luis Valley affects the quantity and quality of flow of the Rio Grande that is available for use downstream. Natural recharge from direct precipitation on the valley floor is very low; however a large area is irrigated by surface water, so that the unconfined ground-water reservoir underlying the valley is recharged principally by irrigation return and infiltration from stream channels debouching into the valley. Most of the recharge occurs during the late spring and early summer months (April to June) when the streamflow is derived from snowmelt runoff from the exceptionally high mountains rimming the valley.

The direct tributaries of the Rio Grande that enter the San Luis Valley from the west are the Rio Grande main stem above Del Norte, Alamosa Creek, and Conejos River (including the Los Pinos and San Antonio Rivers which drain small areas of New Mexico); these three streams contribute the greater part of the water of the entire San Luis Valley. The upper part of the Alamosa formation is the principal unconfined aquifer of the San Luis Valley. Part of the Alamosa formation and the underlying "Santa Fe formation" are leaky artesian aquifers which are recharged by downward leakage from the upper part of the Alamosa formation in the alluvial fans of the San Luis Valley. The artesian aquifers discharge by upward leakage to springs and into the sediments underlying the lower slopes of the fans, and to flowing wells.

San Luis Valley can be divided into three subprovinces: two that contribute water to the Rio Grande (Alamosa Plains, B-2, and Trinchera Creek, B-3) and a third, the Closed Basin (B-1), that does not now contribute water to the Rio Grande. The cycle of precipitation, recharge and runoff, and ground-water discharge can be analyzed separately for each area, although they are hydraulically connected.

B-1. Closed Basin

The Closed Basin is that part of San Luis Valley north of the Rio Grande that is a closed topographic basin. It is also enclosed hydrologically along a streamline divide in the potentiometric surface of the unconfined aquifer (Powell, 1958, Pl. 8). This streamline divide nearly coincides with the topographic divide. The Closed Basin is a unique part of the Rio Grande trough in that it is a large shallow lake basin that has not yet been integrated with adjacent basins by aggradation or erosional breaching. Otherwise it is similar to older basins such as those in the Norin's Well and La Bajada areas (Spiegel, 1961c, p. 133), the Abiquiu area (Smith et al, 1961) and the Gabaldon Hills of eastern Valencia County, New Mexico (near T. 6 N., R. 2 W.; Wright, 1946).

Under present conditions, the Closed Basin does not contribute either surface water or ground water to the Rio Grande, and all water falling on or diverted into the area of interior drainage is discharged to the atmosphere by evapotranspiration, with the exception of minor amounts of waste irrigation water along the south border of the area.

Most of the flow of the Rio Grande above Alamosa is diverted northward onto the Rio Grande alluvial fan, the northern portion of which lies mostly within the Closed Basin. This artificial surface-water inflow and attending ground-water recharge to the Closed Basin have raised water levels greatly in both the artesian and shallow aquifers. This has in turn increased the flow of ground water to the lowest part of the basin, an area of about ten townships of shallow water table, saline soils, and lakes. This area is called the "sump area." Various plans have been proposed to drain surface water or ground water from the Closed Basin area into the Rio Grande so that the large area of saline soils in the "sump area" may be reclaimed for agriculture.

A north-south line through the axis of the "sump area" separates the Closed Basin into a western part (B-1a) and an eastern part (B-1b). Most of the ground-water recharge occurs in the western part, but water from the eastern part would also be removed from storage by a deep drain such as has been proposed by the Bureau of Reclamation (Powell, 1958). In the "sump area," the water levels in the artesian aquifer are above land surface, and are from 10 to 50 feet higher than in the shallow aquifer (Powell, 1958, Pls. 5 and 8). Although upward leakage from the artesian aquifer is discussed qualitatively in the report by Powell, the leakage was not taken into consideration in computing the steady drawdown and discharge of an open drain proposed for water salvage and land reclamation in the "sump area."

The nonsteady water levels and outflow from either aquifer can be predicted by solving the appropriate boundary-value problems for the eastern and western portions of the Closed Basin, using methods similar to those used in Problem G below. However, additional geological and geochemical information regarding the character of the artesian aquifer under the "sump area" is necessary.

B-2. Alamosa Plains

The Alamosa Plains extend from the northwest margin of the San Luis Hills to the low surface divide between the Rio Grande and the Closed Basin portion of San Luis Valley. Physiographically the area west of the Rio Grande is the southern portion of a single huge coalesced fan formed by the Conejos River, Alamosa Creek, and the Rio Grande.

Recharge occurs principally from surface-water irrigation, but inflow also is contributed by stream losses to the higher slopes of the alluvial fans. The water-table aquifer probably loses water to the artesian aquifer in the higher parts of the alluvial fans, but gains water in the lower parts. The artesian water discharges to three large springs, numerous small springs, hundreds of flowing wells, evapotranspiration areas along the streams, and by upward leakage into the shallow water-table aquifer, so that it is difficult to make quantitative estimates of the total artesian discharge. The gain in ground-water base flow of the Rio Grande from Del Norte to the gap through the San Luis Hills (below the mouth of the Conejos River) should reflect the excess of natural discharge over the sum of consumptive use and storage changes in the Alamosa Plains subprovince of the San Luis Valley. The western subprovince of the area can be represented analytically as a segment of a composite infinite half strip in which the western part is recharged. The eastern part discharges water by evapotranspiration and by direct outflow to the Rio Grande.

The drainage basin of Trinchera Creek is small and contributes very little surface water to the Rio Grande because the several headwater stream branches drain a low saddle area in the Sangre de Cristo Range to the east. Few ground-water data are available, but it is probable that artesian conditions are present, at least locally, in the reach from Fort Garland to the northern tip of the San Luis Hills. Outflow from the aquifers occurs in the lower part of this reach, but the quantity is probably small. The boundaries between ground water in Trinchera Creek subprovince and the adjacent Alamosa Plains subprovince (north) and the Basalt Plateaus (south) are probably streamlines controlled in part by local bedrock highs.

C. BASALT PLATEAUS

The Basalt Plateaus ground-water province is a large portion of the Rio Grande trough in southern Colorado and northern New Mexico in which sediments and thick basalt flows of late Santa Fe age filled an extensive basin formed by erosional modification of a downwarped area underlain by sediments and volcanic rocks of the Santa Fe group, older volcanic rocks, and an extensive Precambrian terrane. The sequence of basalt flows has been worked out in detail only in the extreme western part of the province (see Fig. 2, col. 4). The later basalts, called plateau basalts herein, are important aquifers only in the northern part of the province, where some of the later flows of the sequence are below the level of the Rio Grande or its tributaries. Earlier lava flows are apparently poor aquifers, and in much of the province the plateau basalts are relatively thin sheets overlying or interbedded with sediments that form the principal aquifers.

The northern boundary of the permeable plateau basalts appears to be the south margin of the San Luis Hills. Basalt is reported in the subsurface from Antonito to Alamosa, Colorado, but if it is an aquifer in that region, it is probably more closely related hydrologically to the San Luis Valley province than to the Basalt Plateaus.

The western limit of the Basalt Plateaus extends from near San Miguel, New Mexico, on the Los Pinos river, southward to about the middle of T. 30 N., R. 10 W., New Mexico, thence southeasterly along a fault zone that represents the western boundary of the Rio Grande structural trough, to near Tres Piedras. The fault zone apparently trends southward from Tres Piedras to the vicinity of Taos Junction, where it is offset westward almost to Rio Tusas and Ojo Caliente. The basalt flows are apparently above the water table south of the vicinity of Tres Piedras, but the underlying sediments of the Servilleta formation (see Fig. 2) transmit water southward from the northwest part of the province. The southwest boundary of the province is probably defined at least in part by a buried impermeable bedrock barrier that extends from near Ojo Caliente to Rinconada. Cerro Azul (Precambrian quartzite; see Winograd, 1959) in sec. 21, T. 24 N., R. 10 E., represents the highest peak of this buried ridge, and is the only known outcrop. Ground water is moving in a generally southerly direction in this area, so that a streamline of ground-water flow may be taken as the effective hydrologic limit of the southern part of the western subprovince, even if the physical constriction is only a partial one.

The plateau basalts extend several miles east of the Rio Grande in the entire province, from Culebra Creek, Colorado, to the Picuris Mountains (above Embudo, New Mexico). The flows dip eastward and are covered with a thick sequence of alluvial sediments that form important semiperched aquifers. The region east of the Rio Grande is considered as a separate subprovince because of the presence of the upper aquifers. The east limits of the basalts are not known, but the overlying sediments are cut off abruptly by the frontal fault of the Sangre de Cristo Mountains, and this fault is considered tentatively to be the limit of the basalts as well as the alluvial sediments.

The south boundary of the eastern subprovince is sharply defined by the north boundary faults of the Picuris Mountains, since the Precambrian rocks in the range south of the fault are a continuous barrier from the Sangre de Cristo Mountains proper to the Rio Grande. The Rio Grande divides the Basalt Plateaus into two parts, the Western and Eastern plateaus, each of which is divided into subprovinces on the basis of the characteristics of the aquifers and boundary conditions. The conclusions in this section are based in part upon the basic data accompanying a report by Winograd (1959).

C-1. Western Plateau

The zone of saturation of the Western Plateau is in the extensive basalt flows of the Servilleta formation (see Fig. 2, col. 4) in the northern part of the province (Chiflo Plain, C-la) and in sediments of the Servilleta formation and underlying Cordito member of the Los Pinos formation (see Fig. 2) in the Central (C-lb) and Carson (C-lc) plains. Locally, perched zones of saturation are present in the basalt sequence or recent alluvium.

This area is a lava plateau surmounted by volcano-like hills, most of which are older than the plateau lavas. Some hills are of older basalt and other volcanic rocks, but several are of Precambrian rock. The pre-basalt topography and structure of the province is unknown but is probably very complex. Ground-water movement in the basalt flows and underlying sediments is controlled by the pre-basalt topography, particularly Cerro Chiflo, Guadalupe Mountain, No Agua Mountain, Cerro Aire, Cerro Montoso, and Cerro Azul, and possibly Dormilion Peak, Cerro de los Taoses, and Tres Orejas Peak (see Winograd, 1959, Fig 7). The western subprovince is divided into three subareas separated by streamline divides that are primarily controlled by these and other pre-basalt ridges (see Fig. 3).

Cerro Chiflo is one of several hills of older rock (Winograd, 1959, pp. 13, 37) around which the plateau basalts flowed, but is apparently the only one into which the Rio Grande has eroded its canyon. Cerro Chiflo forms a natural barrier to ground-water movement from the northwest, although in most places along the streamline from the northwest that terminates at the mountain, the aquifer is probably continuous with that in the plateau basalts of the northeastern part of the Central Plain. The terminus of the streamline forming the south boundary of the Central Plain was selected at approximately the point where the base of the plateau basalts (basalts of the Servilleta formation) dips below the level of the Rio Grande.

The plateau basalts are present under the Carson Plain, but are above the main zone of saturation, which is in the underlying sediments. These sediments have not been mapped in the Carson Plain, but equivalents have been described by Butler (1946) and Winograd (1959, pp 35-37; samples from wells 25 N. 10 E. 4.222, 25 N. 10 E. 4.244, and 26 N. 10 E. 6.200 described on pp. 59-61). An eolian facies has been observed within the Santa Fe group and the Los Pinos formation in a few surface exposures near Ojo Caliente (Spiegel, 1956) and Vallecitos (Butler, 1946; Barker, 1958). Similar sand is more extensively exposed south of Ojo Caliente and near Embudo, on the west side of the Rio Grande. Wells in Vallecitos penetrated as much as 150 feet of fine-grained, well-sorted, orange-colored sand (Spiegel, 1954), and some of the sand described in the well logs by Winograd is probably also of eolian origin.

C-2. Eastern Plateau

The Eastern Plateau is crossed by a number of perennial streams draining the high Sangre de Cristo portion of the Mountain Rim province. Irrigation return and natural recharge (inflow) from these streams have produced extensive perched and semiperched zones of saturation (upper aquifer) in the thick alluvial sediments overlying the basalt sequence.

In the northermost subprovince (Costilla Plain, C-2a), the leakage is relatively large, apparently because of the large values of vertical hydraulic conductivity in this area; hence water levels are relatively deep, even in the upper aquifer. The recharge to the Costilla Plain and the northern part of the Chiflo Plain is insufficient to maintain the water levels in the basalt (lower aquifer) at or above the level of the Rio Grande and Costilla Creek, because of the extremely high transmissivity (hence low water-table gradients) of the plateau basalts in this area. The southern boundary of the Costilla Plain is a streamline terminating at the point at which the Rio Grande becomes a gaining stream (intersects the potentiometric surface of the basalt aquifer).

Sunshine Valley (C-2b) is similar to the Costilla Plain except that clay beds, probably deposited in a lake created by lava dams, form a semipermeable layer between the alluvial sediments and basalt aquifer (Winograd, 1959). The southern hydrologic boundary of Sunshine Valley is a streamline terminating at Guadalupe Mountain, and the mountain itself. Winograd (1959, p. 12) interprets Guadalupe Mountain as being one of the sources of the plateau basalts, but the log of well 29 N. 12 E. 21.411 given by Winograd (1959, pp. 63-64) indicates that part of the mountain consists of gray and pinkish-gray andesite like that at Cerro Chiflo. These rocks form an impermeable barrier separating Sunshine Valley from the Questa Plain (C-2c) to the south. Ground-water outflow from the highly permeable basalt aquifer occurs in springs in the Rio Grande canyon between the south line of T. 31 N. and Cerro Chiflo.

The Questa Plain (C-2c) is a small area south of Sunshine Valley that was included with the adjacent valley in an investigation by Winograd (1959). The Questa Plain is bounded by Guadalupe Mountain on the east, and Red River on the south. Ground-water outflow occurs to springs in the canyons of Red River and the Rio Grande below Cerro Chiflo. The water level contours given by Winograd (1959, p. 12) for both the upper aquifer and basalt aquifer were redrawn (Spiegel, 1960a) to take into account data on springs and wells along Cabresto Creek and Red River, as well as the impermeable core of Guadalupe Mountain. The resulting map is similar to that given by Winograd and is therefore not reproduced here. The principal modification is that the eastern part of the streamline terminating at the Cerro gaging station (sec. 20, T. 29 N., R. 12 E., opposite Cerro Chiflo) is shifted northward. This interpretation increases the size of the area inferred to contribute ground water to the springs in Red River and the Rio Grande below Cerro gaging station. The enlarged recharge area is more in accord with Winograd's (1959, p. 39) computation of the ground-water outflow to the various reaches of the Rio Grande and Red River. The occurrence of ground water under the Questa Plain is similar to that under Sunshine Valley, except that direct outflow from the upper aquifer is entirely below Cerro Chiflo.

The Canyon Rim subprovince (C-2d) of the Basalt Plateaus is a narrow strip of basalt from Red River to the Picuris Mountains. The section of basalt exposed in this area includes lava flows and interbedded sediments that are older and less permeable than those that form the basalt aquifer north of Red River. Therefore this area acts as a barrier to the movement of ground water in the adjacent area to the east, and only a few springs discharge from the basalt.

The Taos Valleys (D-2e) is the designation given to the area of alluvial sediments in the rectangular strip bounded by Red River, the Sangre de Cristo Mountains segment of the Mountain Rim (A-1), Picuris Mountains, and the Canyon Rim (C-2d). The area is crossed by numerous arroyos and several large streams and associated irrigation works. All the streams lose water to the alluvial sediments at the east edge of the area. Numerous large springs discharge ground water to Red River, Arroyo Hondo, and Rio Taos at the west edge of the area, but a complex irrigation system makes it very difficult to analyze the ground-water conditions quantitatively.

D. ESPANOLA VALLEY

The Espanola Valley province is the area occupied by the Santa Fe group in the lower Rio Chama valley and in the Rio Grande valley from Embudo to the mouth of Bland Canyon, about 6 miles above Cochiti. The principal pre-watercourse aquifer is the Tesuque formation (Fig. 2, col. 2), which is the lower part of the Santa Fe group in this area. More permeable beds higher in the section are generally located in interstream areas, above the water table.

D-1. Abiquiu Badlands

This area is underlain principally by a facies of the Santa Fe group derived from the deep weathering and erosion of a volcanic terrane. Locally the unit contains pyroclastic material, but most of the materials are fluvial and lake deposits characterized by a fine-grained matrix of clay and silt. The misnomer Abiquiu tuff applied to these deposits by Smith (1938) is the source of much confusion in interpretation, and the word "tuff" will be enclosed by quotation marks or replaced by the words facies, formation, or member in the following discussion (these remarks apply equally well to the Picuris "tuff" of Cabot, 1938, along the eastern side of the Espanola Valley province). The Abiquiu "tuff" becomes coarser northward and grades into the Los Pinos formation (see Fig. 2, col 3); it also interfingers eastward with nonvolcanic detritus probably equivalent to the Tesuque formation east of the Rio Grande. The Abiquiu "tuff" generally has very low transmissivity, and the watercourses incised into it are the most important aquifers in the province.

D-2. Black Mesa

The Black Mesa province is located between Rio Ojo Caliente and the Rio Grande, just northeast of the mouth of the Rio Chama. The thin basalt cap of Black Mesa probably does not contain ground water because the recharge supplied as the result of precipitation on the mesa is probably small, and the underlying sediments, in part eolian sand, are relatively permeable. Thus the area is a conduit for ground water moving southeastward from tributary watercourses of Rio Ojo Caliente to the Rio Grande. However, the saturated zone of the Santa Fe group (Tesuque equivalent) is inferred to have low transmissivity because of the moderately high water-level gradients in the area (Spiegel, 1961b). Also, the outcrops observed around the base of Black Mesa are generally fine-grained, poorly sorted sediments.

D-3. Barrancas

The Barrancas subprovince is a large area of nearly continuously exposed ancient alluvial fan deposits of the Tesuque formation. Most of the detritus is from Precambrian rocks, but locally there are basalt flows and lenses of fine-grained volcanic-derived sediments (the Picuris "tuff"; see discussion of the Abiquiu "tuff" in the Abiquiu Badlands) equivalent to the Bishop's Lodge member of the Tesuque formation (Fig. 2, col. 2). Some true pyroclastic material is present in the vicinity of Picuris, but elsewhere the volcanic-derived material was weathered and eroded from former exposures of tuffs that were spread over parts of the Sangre de Cristo Mountains during middle Tertiary time.

The north boundary of the subprovince is the Picuris Mountains, the east boundary is the Sangre de Cristo Mountains, and the west boundary is the Rio Grande, except in the extreme south. From San Ildefonso southward, the west boundary is drawn through a line of basalt volcanoes trending nearly southward to La Bajada Hill. This part of the boundary is inferred to be a leaky interface because of the discontinuous water levels in wells of the area caused by the basaltic intrusions (Spiegel, 1962, Pl. 22; see also compilation by Titus, 1961, Fig. 1). The south boundary of the province is a streamline located approximately under the long ridge just north of the city of Santa Fe.

A characteristic feature of this province is that the thick Tesuque formation dips westward at a steeper angle than the water-level gradients. A series of step-like discontinuities in water level is caused by the presence of tilted beds of alternating high and low transmissivity.

D-4. Santa Fe Plain

The Santa Fe Plain is hydrologically somewhat similar to the Barrancas province except that ground-water outflow occurs at an impermeable barrier exhumed by Santa Fe River. The Tesuque formation is the principal aquifer, even though it is overlain unconformably by the Ancha formation nearly everywhere, for the Ancha formation is generally above the water table (Spiegel, 1962).

D-5. Pankey's Pasture

The high mesa east of White Rock Canyon (on the Rio Grande between San Ildefonso and Cochiti), like the southern end of subprovince (D-6) described below, is underlain by a thick section of basalt flows and sediments. The area does not contribute much ground water to the Rio Grande. However, if the proposed Cochiti Dam were constructed at the lower end of White Rock Canyon, the area would have a large capacity for bank storage.

D-6. Puye Mesas

The exposed section in the mesas near Los Alamos consists of an extensive deposit of welded tuff overlying coarse-grained, well sorted (but not saturated) sediments of the upper part of the Santa Fe group. The Tesuque formation is exposed in the lower slopes of some of the mesas and is in the subsurface under most of the area; it is the principal aquifer of the area. A thick section of basalt is interbedded in the upper part of the Santa Fe group (Ancha equivalents, Fig. 2, col. 2) in the southern half of the province.

E. SANTO DOMINGO VALLEY

Nearly all of the Santo Domingo Valley province is underlain by a thick axial river gravel facies of the upper part of the Santa Fe group (Spiegel, 1961a; 1961c). This unit represents the deposits of an ancestral Rio Grande of late Pliocene to early Pleistocene (?) age. The axial gravels interfinger laterally with ancient alluvial fan deposits derived from the bordering uplifts. Numerous beds of white rhyolite pumice ash and cinders interbedded with the axial gravel and equivalent fan deposits indicate that volcanism occurred concurrently with the stream deposition. The great thickness (more than 500 feet) and vertical uniformity of the gravel unit also suggest that the basin was sinking at a rate nearly equal to the rate of sedimentation along the axis of the basin. The basalt sequence of White Rock Canyon is interbedded with the river gravel unit from Pena Blanca northward. The boundary between Santo Domingo Valley and Espanola Valley was chosen at a group of basalt intrusive centers near the mouth of Bland Canyon, about eight miles north of Cochiti. South of this boundary the sediments form the greater part of the section, but northward for about ten miles the basalts predominate. The rest of White Rock Canyon is mostly sediments with interbedded basalt flows.

The transmissivity of the axial gravel unit is so large that water-level gradients in it are very low. Consequently the Rio Grande loses water to the axial gravel unit. This inflow to the axial gravels moves westward and then follows southward along a trough in the water-level contours (Spiegel, 1961b; see also Titus, 1961). The ground water in this trough returns to the Rio Grande watercourse in the vicinity of Algodones, where the base of the axial gravel unit rises above river level on the north limb of a broad anticline involving the Santa Fe group (see Spiegel, 1961c, Fig. 1).

F. LOWER JEMEZ RIVER

The geology of the Lower Jemez River area was mapped (Spiegel, 1961c) in order to determine the reason for the close spacing of water-level contours (Bjorklund and Maxwell, 1961, Pl. lb) near Bernalillo. A section of more than 500 feet of fine-grained sediments, called the red member of the unnamed lower formation of the Santa Fe group (see Fig. 2, col. 1), is folded into a broad southeast-plunging anticline which impedes the southward movement of ground water in the overlying sediments of the Santa Fe group in the area upstream from the anticline. The upper unnamed formation has been removed by erosion in most of the Jemez River valley proper, and the principal pre-watercourse aquifer is the lower member of the lower unnamed formation.

The Jemez River loses water into the lower member throughout its course from San Ysidro to Jemez Dam. North of Jemez River this ground water moves eastward under Santa Ana Mesa (the region between Borrego Arroyo and the Jemez River) into the Rio Grande watercourse, apparently by upward leakage across the red member. South of Jemez River, the ground water replenished by Jemez River moves southeastward and apparently leaks upward into the upper part of the Santa Fe group and the Rio Grande watercourse.

G. ALBUQUERQUE-BELEN PROVINCE

The Albuquerque-Belen province is defined herein as the segment of the Rio Grande trough from the vicinity of Sandia Pueblo southward to the Rio Salado. Apart from the watercourse of the Rio Grande itself, the principal aquifer is the upper unnamed formation of the Santa Fe group, (Spiegel, 1961c), which consists of an axial river gravel deposit and related eastern and western facies (Fig. 2, col. 1). The importance of these three related sedimentary facies to the hydrology and geologic history of the Albuquerque-Belen province had not been recognized by previous workers, although equivalents of the river gravels were reported in several localities (Herrick, 1898; Bryan, 1938; Denny, 1940a, 1940b).

The Albuquerque-Belen valley was separated from the Santo Domingo valley by Bryan (1938) on the basis of the valley constriction caused by erosional resistance of the basalt flows of Santa Ana Mesa. However, the geohydrologic boundary between the Albuquerque-Belen and Santo Domingo valleys is actually formed by the red member of the Santa Fe group described in the preceding section. This unit, although equivalent to red sediments mapped by Bryan and McCann (1937) along the Rio Puerco west of Bernalillo, was apparently not recognized by them along the Rio Grande. The south boundary of the province is formed by relatively impermeable rocks in fault blocks that constrict the valley near the mouth of Rio Salado (Spiegel, 1955).

The Albuquerque-Belen geohydrologic province can be considered approximately as a closed rectangular-strip aquifer system divided into a number of north-trending substrips parallel to the Rio Grande, which flows through the province from north to south. The hydrology of this province is discussed in greater detail in a later section.

SOLUTIONS OF SELECTED PROBLEMS

Many of the aquifer systems in the Rio Grande basin can be represented, at least to a good approximation, by one or more of the boundary-value problems previously solved (see Appendices). For two important areas, Sunshine Valley and the Albuquerque-Belen Valley, the existing solutions are not sufficiently good approximations. Boundary-value problems applicable to these two areas are stated and solved below.

SUNSHINE VALLEY

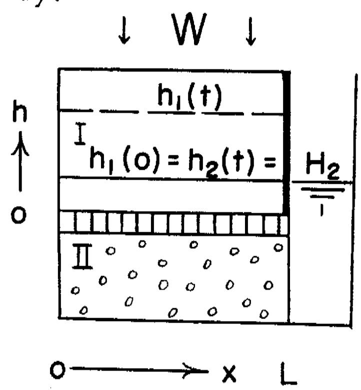
The geohydrologic data available suggest that in Sunshine Valley the upper aquifer is semiperched on the underlying lake clays and basalt aquifer and that the east side of the aquifers is bounded by Precambrian rocks. Although some outflow occurs from both aquifers at the west side of the strip, the exact nature of lateral out-flow from the upper aquifer is unknown. According to the classification presented in Appendices C and D, the area can be represented analytically as a segment of an infinite half-strip parallel-aquifer system. Before discussing the present conditions, however, approximations to the conditions that probably existed in prehistoric times are considered in Problems A to D.

Discussion of Problems A and B

In the natural state, recharge to the upper aquifer was principally by infiltration of streams debouching onto fans along the eastern part of the aquifer strip, and principally from local precipitation in the western part. The configuration of the natural water table in the upper aquifer was dependent upon the areal distribution of values of recharge, aquifer transmissivity, and leakage coeficient. Inasmuch as the natural recharge was probably much less than the recharge incidental to the present surface-water irrigation systems, the water levels in both aquifers, in the natural state, were much lower and flatter than at present. The top of the zone of saturation in the upper aquifer may not have been high enough to permit direct outflow of ground water into the basalt upon which the alluvium rests on the west margin of the valley. The ground water in the upper aquifer probably moved nearly vertically downward across the underlying lake beds, and laterally through the highly transmissive lower aquifer. This system can be approximated by Problems A and B, in which the transmissivity of the lower aquifer is considered very large (infinite); that is, the slight slope and fluctuations of the potentiometric surface are neglected. Letter designations of the Problems are omitted in the equation numbers, but are retained in cross-references and subsidiary equations in the text.

PROBLEM A

Nonsteady vertical motion in a steadily, uniformly recharged, leaky, semiperched aquifer overlying a semiconfined highly transmissive aquifer, after initially zero head difference; water level of aquifer maintained steady.



$(h_1(t)$ rising)

(1a) $(d/dt)h_1 + (K'/m'S_w)h_1 = (K'/m'S_w)h_2 + W/S_w$

(1b) $h_1(o) = H_2$

Nonsteady solution:

(2) $h_1(t) = H_2 + (Wm'/K')(1 - \exp\left[-(K'/m'S_w)t\right])$

(3) $q_{1z}(t) = LW(1 - \exp\left[-(K'/m'S_w)t\right]) = q_2(L,t)$

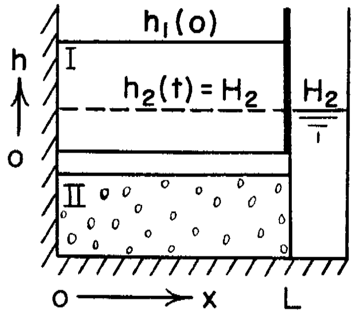
Steady-state solution:

(4) $h_{1s} = h_1(\infty) = H_2 + Wm'/K'$

(5) $q_{1zs} = q_2(L,\infty) = LW$

PROBLEM B

Nonsteady vertical motion in a leaky semiperched aquifer overlying a semiconfined highly transmissive aquifer, after cessation of steady uniform recharge; water level of lower aquifer maintained steady.



($h_1(t)$ declining)

(1a) $(d/dt)h_1 + (K'/m'S_w)(h_1 - h_2) = O$

(1b) $h_1(o) = H_2 + W_o m'/K'$

Nonsteady solution:

(2) $h_1(t) = H_2 + (W_o m'/K')\exp\left[-(k'/m'S_w)t\right]$

(3) $q_{1z}(t) = LW_o \cdot \exp\left[-(K'/m'S_w)t\right] = q_2(L,t)$

Steady-state solution:

(4) $h_{1s} = h_1(\infty) = H_2$

(5) $q_{1zs} = q_2(L,\infty) = O$

Problems A and B give solutions of the approximate differential equation derived in equation (1) of Case I of the Derivations for, respectively, the rate of rise of the water table during steady uniform recharge, and for the decline of the water table after the cessation of steady uniform recharge. In both problems the solutions are obtained by determining particular solutions H_p by the symbolic operator method (Miller, 1941) and noting that the general form of solution of equations (A, B-1a) is $h_1 = c_1 \cdot \exp(m_i x) + H_p$. The expressions (A, B-3) for vertical leakage are obtained by using Darcy's Law in the form $q_{1z}(t) = LK'(h_1 - h_2)/m'$. Steady-state solutions for aquifer water level (A, B-4) and outflow (A, B-5) are obtained by letting the time approach infinity in the corresponding nonsteady expressions.

The type of aquifer system treated in Problems A and B is also present in many of the stream valleys of the Mountain Rim of the Rio Grande, and in many other streams with terraced alluvial valleys filling erosional or fault-block troughs in relatively impermeable bedrock.

Problems A and B using differential equation (1) in Case I of the Derivations are equivalent to boundary-value problems using the differential equation (12k) in Case IIa, with the boundary conditions $(\partial/\partial x)h(o) = (\partial/\partial x)h(L) = O$, since under these conditions $(\partial^2/\partial x^2)h(x) = O$ if h_{pe} is uniform.

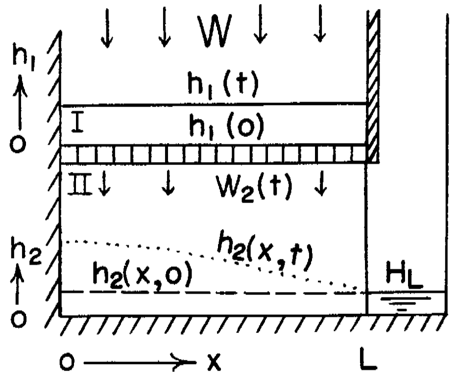
Discussion of Problems C and D

Many of the irrigated aquifer systems in the Rio Grande Basin, especially in the Mountain Rim, consist of a closed-strip perched upper aquifer that leaks downward to an unconfined infinite half-strip aquifer in which the transmissivity cannot be assumed infinite. In such cases, the leakage from the upper closed-strip aquifer may be considered to be a form of nonsteady recharge to the lower aquifer. Solutions have been obtained for two nonsteady cases, one for an increasing amount of leakage (Problem C), the other for a decreasing amount (Problem D). These solutions can also be used to approximate the outflow of the basalt aquifer in Sunshine Valley if the lateral outflow and areal leakage variations of the upper aquifer can be neglected.

For example, if the upper aquifer is recharged uniformly at a steady rate, the downward leakage, or replenishment to the lower aquifer, will be proportional to the exponential function of time given by equation (A-3). Problem C represents this case. The nonsteady configuration of the water table in the lower aquifer is given by equation (C-3), and the resulting increasing outflow to a stream incised into the lower aquifer is given by equation (C-4). As time becomes very large, each of the infinite series in (C-3,4) approaches zero; the remaining steady-state solutions are the same as for a single aquifer recharged at a steady uniform rate (Jacob, 1943).

PROBLEM C

Nonsteady motion in a perched-aquifer system consisting of a leaky, perched, infinite closed strip receiving steady uniform recharge, and a lower infinite half strip replenished by leakage from the perched aquifer; lower aquifer connected to a stream at steady level, upper aquifer initially dry.



$W = W_o$ (steady)

$W_2(t) = W_o\left[1 - \exp(-Mt)\right]$

where $M = (K'/m'S_w)$; see Problem A

Upper aquifer:

(1ab) $(d/dt)h_1 + (K'/m'S_w)h_1 = W/S_w$; $h_1(o) = h_p = O$

Solution of (1ab) given by (A-2,3), where $H_2 = h_p = O$

Lower aquifer:

(2a) $(\partial^2/\partial x^2)h_2 = (1/k)(\partial/\partial t)h_2 - (W_2/T_2)$, $W_2 = W_c\left[1 - \exp(-Mt)\right]$

where $M = (K'/m'S_w)$

(2bcd) $h_2(x,o) = H_L$; $h_2(L,t) = H_L$; $(\partial/\partial x)h_2(o,t) = O$

PROBLEM C-Continued

Nonsteady solution for lower aquifer:

(3) $$h_2(x,t) = H_L + (W_0/2T)/L^2\left\{1 - \frac{x^2}{L^2} - \frac{32}{\pi^3}\sum_{m=0}^{\infty}\frac{(-)^m \cos Nx \cdot \exp[-N^2 k t]}{(2m+1)^3 (1 - N^2 k/M)}\right\}$$

$$+ \frac{4W_0 \cdot \exp[-Mt]}{MS\pi}\sum_{m=0}^{\infty}\frac{(-)^m \cos Nx}{(1 - N^2 k/M)}$$

(4) $$q_2(L,t) = W_0 L\left\{1 - \frac{8}{\pi^2}\sum_{m=0}^{\infty}\frac{\exp[-N^2 k t]}{(2m+1)^2 (1 - N^2 k/M)}\right\}$$

$$- \frac{W_0 T \exp[-Mt]}{LMS}\sum_{m=0}^{\infty}\frac{(2m+1)}{(1 - N^2 k/M)}$$

Note: $N = \frac{(2m+1)\pi}{2L}$, $M = (K'/m'S_w)$

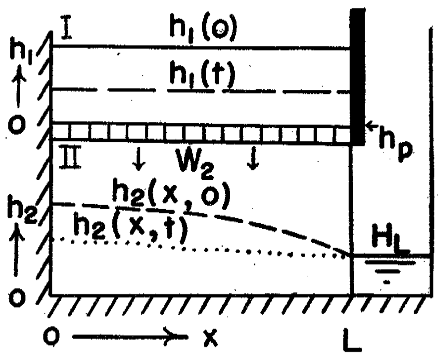
Aquifer outflow for large M:

(5a) $$q_2(L,t) = W_0 L\left\{1 - \frac{8}{\pi^2}\sum_{m=0}^{\infty}\frac{\exp[-N^2 k t]}{(2m+1)^2}\right\}$$

(5b) $$q_2(L,t) = 2W_0 (kt)^{1/2}\sum_{m=0}^{\infty}(-)^m\left\{\pi^{-1/2} - i\mathrm{erfc}\left[\frac{(2m+1)L}{(kt)^{1/2}}\right]\right\}$$

PROBLEM D

Nonsteady motion in a perched-aquifer system consisting of a leaky, perched, infinite closed strip (after the cessation of steady uniform recharge) and a lower infinite half strip replenished by leakage from the perched aquifer and connected to a stream at steady level; both aquifers initially at a steady state in equilibrium with steady uniform recharge.



Upper aquifer:

(1ab) $(d/dt)h_1 + (K'/m'S_w)(h_1 - h_p) = O$; $h_1(o) = W_o(m'/K')$

Solution of (1ab) given by (B-2,3), where $h_p = H_2 = O$

Lower aquifer:

(2a) $(\partial^2/\partial x^2)h_2 = (1/k)(\partial/\partial t)h - (W_o/T)\exp(-Mt)$

(2b) $h_2(x,o) = h_s(x) = H_L + W_o(L^2 - x^2)/2T$

(2cd) $h_2(L,t) = H_L$; $(\partial/\partial K)h_2(o,t) = O$

Steady-state solution for lower aquifer:

(3) $h_2(x,\infty) = h_{2s} = H_L$

(4) $q_2(L,\infty) = q_{zs} = O$

PROBLEM D-Continued

Solution of (D-2) is the sum of solutions of (D-5) and (D-6) below:

(5ab) $(\partial^2/\partial x^2)u = (1/k)(\partial/\partial t)u$; $u(L,t) = H_L$

(5cd) $u(x,o) = u_s(x) = H_L + W_o(L^2 - x^2)/2T$; $(\partial/\partial x)u(o,t) = O$

(Solution given by Jacob, 1943; Carslaw and Jaeger, 1959, p. 98)

(6ab) $(\partial^2/\partial x^2)v = (1/k)(\partial/\partial t)v - (W_o/T)\exp(-Mt)$; $v(L,o) = O$

(6cd) $v(L,t) = O$; $(\partial/\partial x)v(o,t) = O$

Solution given by Carslaw and Jaeger, 1959, p. 132)

Outflow solution for (D-2):

(7)
$$q_2(L,t) = \frac{8W_oL}{\pi^2}\sum_{m=0}^{\infty}\frac{(-)^m \exp(-N^2kt)}{(2m+1)^2}$$

$$+ W_o(k/M)^{1/2}\cdot \tan L(M/k)^{1/2}\cdot \exp(-Mt)$$

$$+ \frac{2W_o}{TL}\sum_{m=0}^{\infty}\frac{(-)^m \exp(-N^2kt)}{M/k - N^2}$$

where $N = (2n+1)\pi/2L$ and $M = (K'/m'S)$.

Similarly, if the leakance coefficient (K'/m') of the perching layer is very large, the recharge approaches a constant value rapidly, and the nonsteady solution (C-3) reduces to that given by Werner (1953) for steady uniform recharge. Carslaw and Jaeger (1959, p. 130-131) give the solution for steady recharge in a form equivalent to that of Werner's, and also in a form more convenient for computation for small values of time, as well as in a non-dimensional graph. Equations (C-5a) and (C-5b) give the outflow in both forms for this steady recharge condition (M very large).

The solution for the more general case of Problem C is obtained from a general integral form (Carslaw and Jaeger, 1959. p. 131, eqn. (9) corrected by the factor (1/4) in the exponential term) by using the non-steady recharge expression $W_2(t') = W_o(1 - \exp\left[-(K'/m'S)t'\right])$ in place of the general function $A(t')$. This solution is given by equations (C-3, 4).

Problem D represents the conditions in the lower aquifer in a system of the type described in the previous problem, after both the upper and lower aquifers have reached a steady state and the uniform recharge on the upper aquifer has ceased. As shown in Problem B, the water level and vertical leakage of the upper aquifer decrease at a negative exponential rate determined by the values of the leakance and storage coefficients and the previous recharge rate. If the upper aquifer is not recharged again, ultimately all water will drain out of it, as well as from the lower aquifer, and the equations reduce to the final steady equations (B-4, 5) and (D-e, 4).

This problem, stated in equations (D-1, 2), is solved by summing the solutions of the two problems given by equations (D-5, 6). Values of the former solution are given in nondimensional graphical form by Carslaw and Jaeger (1959, Fig. 10d). The nonsteady outflow to the stream is given by equation (D-7). Note that in the limit as M approaches infinity, the outflow solution for Problem (D-1), given by equation (D-7), approaches that for (D-5), since the second and third terms of (D-7) approach zero.

Thus the first series of (D-7) is the outflow solution for the problem given by equations (D-5).

In many areas the natural cycles of seasonal increase and waning of precipitation or irrigation water supply can be approximated by exponential curves of the type employed in problems C and D. Hence in such areas the solutions of these problems can be applied to single aquifers of infinite half-strip type by choosing appropriate values of the coefficients W_o and M.

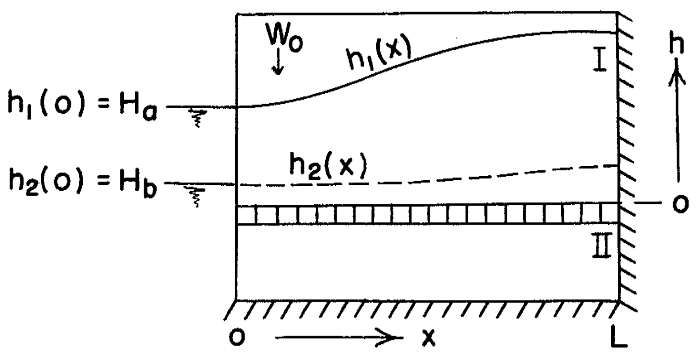
Discussion of Problem E

This problem closely resembles the actual parallel-aquifer system of Sunshine Valley as it was before the completion of irrigation wells, and after a long period of operation of the surface irrigation system. The principal unknown physical feature of the Sunshine Valley aquifer system is the nature of the west boundary of the upper aquifer. The hydrologic features related to the physical problem are (a) the amount and paths of the lateral outflow where the upper aquifer overlaps onto the basalt, and (b) the amount of water lost by evaportranspiration from the area of shallow ground water along the western margin of the valley.

The validity of estimates of aquifer coefficients and areal recharge made on the basis of well performance and surface stream inflow can be checked by using the estimated values in the solution of this problem (equations E-2ab) and comparing the calculated values of water levels along sections across the aquifers with actual water levels. In the Sunshine Valley aquifer system the two expressions for discharge, equations (E-3ab), can not be used separately to check the aquifer coefficients in this manner because the outflow of the upper aquifer, other than that due to evaportranspiration, is underground.

PROBLEM E

Steady flow in a leaky infinite half-strip parallel-aquifer system consisting of a uniformly-recharged unconfined aquifer overlying a leaky artesian aquifer. The water levels of the two aquifers are maintained at steady nonequal levels at the stream boundary.



(1a) $(d^2/dx^2)h_1 - (h_1 - h_2)/B_1^2 = -W_o/T_1$

(1b) $(d^2/dx^2)h_2 + (h_1 - h_2)/B_2^2 = 0$

(1cd) $(d/dx)h_1(L) = 0$; $(d/dx)h_2(L) = 0$

(1ef) $h_1(o) = H_a$; $h_2(o) = H_b$

PROBLEM E-Continued

Solutions:

(2a) $$h_1(x) = \left[(H_a - H_b) - \left(\frac{B_1^2 B_2^2}{B_1^2 + B_2^2}\right)\frac{W_0}{T_1}\right]\left(\frac{B_2^2}{B_1^2 + B_2^2}\right) \frac{\cosh\left(\frac{1}{B_1^2} + \frac{1}{B_2^2}\right)^{1/2}(L-x)}{\cosh\left(\frac{1}{B_1^2} + \frac{1}{B_2^2}\right)^{1/2} L}$$

$$- \left(\frac{B_1^2}{B_1^2 + B_2^2}\right)\frac{W_0}{T_1}\left[\frac{x^2}{2} - Lx\right] + \left[(H_a - H_b) + \left(\frac{B_2^4}{B_1^2 + B_2^2}\right)\frac{W_0}{T_1}\right]\left(\frac{B_1^2}{B_1^2 + B_2^2}\right) + H_b$$

(2b) $$h_2(x) = \left[(H_b - H_a) + \left(\frac{B_1^2 B_2^2}{B_1^2 + B_2^2}\right)\frac{W_0}{T_1}\right]\left(\frac{B_1^2}{B_1^2 + B_2^2}\right) \frac{\cosh\left(\frac{1}{B_1^2} + \frac{1}{B_2^2}\right)^{1/2}(L-x)}{\cosh\left(\frac{1}{B_1^2} + \frac{1}{B_2^2}\right)^{1/2} L}$$

$$- \left(\frac{B_1^2}{B_1^2 + B_2^2}\right)\frac{W_0}{T_1}\left[\frac{x^2}{2} - Lx\right] - \left[(H_b - H_a) + \left(\frac{B_1^2 B_2^2}{B_1^2 + B_2^2}\right)\frac{W_0}{T_1}\right]\left(\frac{B_1^2}{B_1^2 + B_2^2}\right) + H_b$$

Outflow solutions:

(3a) $$q_1(0) = -T_1\left[(H_a - H_b) - \left(\frac{B_1^2 B_2^2}{B_1^2 + B_2^2}\right)\left(\frac{W_0}{T_1}\right)\right]\left(\frac{B_2^2}{B_1^2 + B_2^2}\right)\left(\frac{B_1^2 + B_2^2}{B_1^2 B_2^2}\right)^{1/2}$$

$$\cdot \tanh\left(\frac{B_1^2 + B_2^2}{B_1^2 B_2^2}\right)^{1/2} L$$

(3b) $$q_2(0) = -T_2\left[(H_b - H_a) + \left(\frac{B_1^2 B_2^2}{B_1^2 + B_2^2}\right)\left(\frac{W_0}{T_1}\right)\right]\left(\frac{B_1^2}{B_1^2 + B_2^2}\right)\left(\frac{B_1^2 + B_2^2}{B_1^2 B_2^2}\right)^{1/2}$$

$$\cdot \tanh\left(\frac{B_1^2 + B_2^2}{B_1^2 B_2^2}\right)^{1/2} L$$

The solution of Problem E is obtained by writing (E-1a) and (E-1b) in symbolic operator notation, eliminating h_1, and expressing the resulting equation in h_2 as

$$(E\text{-}4) \qquad D^2\left[D^2 - (1/B_1^2 + 1/B_2^2)\right] h_2 = (W/T_1 B_2^2),$$

which has a solution in the form

$$(E\text{-}5) \quad h_2 = a_1 \sinh m_1(L - x) + a_2 \cosh m_2(L - x) + a_3 + a_4 x + H_p,$$

where H_p is a particular solution of (E-4) and the m_i are roots of the bracketed expression in (E-4). A similar expression for h_1 is found by substituting (E-5) in (E-1b). The four coefficients a_i are evaluated by using the four boundary condition equations (E-1cdef).

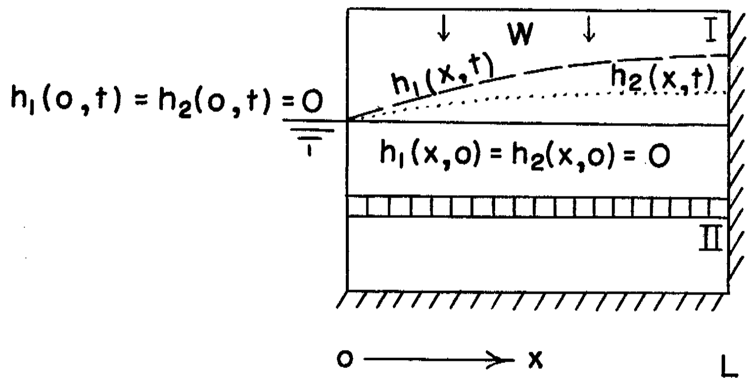
Discussion of Problem F

A physical situation represented by Problem F may arise where a buried stream channel penetrates two parallel aquifers and the inter-layered semi-confining bed, and the stream valley has been refilled by coarse channel deposits to above the top of the semi confining bed. The Rio Grande, as well as most other streams draining regions glaciated during Pleistocene time, does have highly permeable channel filling. However, the presence of an extensive parallel-aquifer system of the type illustrated in Problem F is not known on the main course of the Rio Grande, although short reaches of the Rio Chama and other tributaries probably fulfill the conditions required by this problem. On the other hand, Problem F can be combined with Problem E to give Problem G, which represents present nonsteady conditions in the Sunshine Valley aquifer system.

The analytical representation of Problem F is given by equations (F-1). The pair of equations (F-1ab) is an example of the system of general second order linear differential equations (12k) in Case IIa above, in which the recharge term is zero for the lower aquifer (Region II in Fig. 1b and Problem F). Taking the Laplace transformation of (F-1ab) with respect to the time variable, under conditions (F-1cd), results in (F-2ab), where the transformed dependent variables are indicated by $\bar{h}_1(x,p)$ and $\bar{h}_2(x,p)$.

PROBLEM F

Nonsteady motion in a leaky infinite half-strip parallel-aquifer system consisting of a steadily and uniformly recharged aquifer overlying a leaky artesian aquifer, the bounding stream and both aquifers initially at uniform level.



(1a) $(\partial^2/\partial x^2)h_1 - (h_1 - h_2)/B_1^2 = (1/k_1)(\partial/\partial t)h_1 - W/T_1$

(1b) $(\partial^2/\partial x^2)h_2 + (h_1 - h_2)/B_2^2 = (1/k_2)(\partial/\partial t)h_2$

(1cd) $h_1(x, o) = O$; $h_2(x, o) = O$

(1ef) $(\partial/\partial x)h_1(L, t) = O$; $(\partial/\partial x)h_2(L, t) = O$

(1gh) $h_1(o, t) = O$; $h_2(o, t) = O$

PROBLEM F-Continued

$\mathcal{L}\{(F\text{-}1ab)\}$ using (F-1cd), in operator notation:

$$(2a)\quad \left[D^2 - (p + k_1/B_1^2)/k_1\right]\bar{h}_1 + (1/B_1^2)\,\bar{h}_2 = -W/pT_1$$

$$(2b)\quad (1/B_2^2)\bar{h}_1 + \left[D^2 - (p + k_2/B_2^2)/k_2\right]\bar{h}_2 = 0$$

$\mathcal{L}\{(F\text{-}1)\}$, with (F-2ab) solved for $\bar{h}_1$ and $\bar{h}_2$, in operator notation:

$$(3a)\quad \left[(A_1A_2 - 1/B_1^2B_2^2) - (A_1 + A_2)\,D^2 + D^4\right]\bar{h}_1 = \left[D^2 - A_2\right](-W/pT_1)$$

$$= A_2W/pT_1$$

$$(3b)\quad \left[(A_1A_2 - 1/B_1^2B_2^2) - (A_1 + A_2)\,D^2 + D^4\right]\bar{h}_2 = (W/pT_1B_2^2)$$

$$(3cd)\quad (\partial/\partial x)\,\bar{h}_1(L, p) = 0 \quad ; \quad (\partial/\partial x)\,\bar{h}_2\,(L, p) = 0$$

$$(3ef)\quad \bar{h}_1(o, p) = 0 \quad ; \quad \bar{h}_2(o, p) = 0$$

where $A_1 = (p + k_1/B_1^2)\,k_1$,

$A_2 = (p + k_2/B_2^2)/k_2$

PROBLEM F-Continued

General form of solutions of (F-3ab):

(4a) $$\bar{h}_1(x,p) = a_1 \sinh m_1(L-x) + a_2 \cosh m_2(L-x) + a_3 \sinh m_3(L-x) + a_4 \cosh m_4(L-x) + \bar{H}_1$$

(4b) $$\bar{h}_2(x,p) = c_1 \sinh m_1(L-x) + c_2 \cosh m_2(L-x) + c_3 \sinh m_3(L-x) + c_4 \cosh m_4(L-x) + \bar{H}_2$$

where

$$m_1 = -m_2 = (1/2)^{1/2} \left\{ (A_1 + A_2) + \left[(A_1 - A_2)^2 + 4/B_1^2 B_2^2 \right]^{1/2} \right\}^{1/2}$$

$$m_3 = -m_4 = (1/2)^{1/2} \left\{ (A_1 + A_2) - \left[(A_1 - A_2)^2 + 4/B_1^2 B_2^2 \right]^{1/2} \right\}^{1/2}$$

$$\bar{H}_1 = \left[P - QD^2 + D^4 \right]^{-1} (A_2 W/pT_1) = (A_2 W/pT_1)/(A_1 A_2 - 1/B_1^2 B_2^2)$$

$$\bar{H}_2 = \left[P - QD^2 + D^4 \right]^{-1} (W/pT_1 B_2^2) = (W/pT_1 B_2^2)/(A_1 A_2 - 1/B_1^2 B_2^2)$$

$$P = (A_1 A_2 - 1/B_1^2 B_2^2), \quad Q = (A_1 + A_2)$$

Coefficients in (F-4ab):

(5a) $a_1 = a_3 = c_1 = c_3 = 0$

(5b) $c_2 = -m_4^2 \bar{H}_2/(m_4^2 - m_2^2) \cosh (m_2 L)$

(5c) $c_4 = m_2^2 \bar{H}_2/(m_4^2 - m_2^2) \cosh (m_4 L)$

(5d) $a_2 = -B_2^2(m_2^2 - A_2)\, c_2$

(5e) $a_4 = -B_2^2(m_4^2 - A_2)\, c_4$

PROBLEM F-Continued

Solutions of (3):

$$\text{(6a)}\quad \bar{h}_1(x,p) = \frac{(B_2^2 \bar{H}_2 m_2^2 m_4^2 - A_2 B_2^2 \bar{H} m_4^2)}{(m_4^2 - m_2^2)} \frac{\cosh m_2 (L-x)}{\cosh(m_2 L)}$$

$$+ \frac{(A_2 B_2^2 \bar{H}_2 m_2^2 - B_2^2 \bar{H}_2 m_2^2 m_4^2)}{(m_4^2 - m_2^2)} \frac{\cosh m_4 (L-x)}{\cosh(m_4 L)}$$

$$+ A_2 B_2^2 \bar{H}_2$$

$$\text{(6b)}\quad \bar{h}_2(x,p) = \bar{H}_2 - \frac{m_4^2 \bar{H}_2 \cosh m_2 (L-x)}{(m_4^2 - m_2^2)\cosh(m_2 L)} + \frac{m_2^2 \bar{H}_2 \cosh m_4 (L-x)}{(m_4^2 - m_2^2)\cosh(m_4 L)}$$

Outflow solutions of (3):

$$\text{(7a)}\quad \bar{q}_1(0,p) = -T_1[(m_4^2 m_2 A_2 B_2^2 \bar{H}_2)/(m_4^2 - m_2^2)] \tanh(m_2 L)$$

$$+ T_1[(m_2^3 m_4^2 B_2^2 \bar{H}_2)/(m_4^2 - m_2^2)] \tanh(m_2 L)$$

$$+ T_1[(m_4 m_2^2 A_2 B_2^2 \bar{H}_2)/(m_4^2 - m_2^2)] \tanh(m_4 L)$$

$$- T_1[(m_4^3 m_2^2 B_2^2 \bar{H}_2)/(m_4^2 - m_2^2)] \tanh(m_4 L)$$

$$\text{(7b)}\quad \bar{q}_2(0,p) = -T_2[(m_4^2 m_2 \bar{H}_2)/(m_4^2 - m_2^2)] \tanh(m_2 L)$$

$$+ T_2[(m_2^2 m_4 \bar{H}_2)/(m_4^2 - m_2^2)] \tanh(m_4 L)$$

PROBLEM F-Continued

Approximate outflow solution of (F-1) for small t:

(8a) $$q_1(o,t) = -\left(\frac{2W}{Lbk_3}\right)\left\{\left[ck_1/k_2 - a\right]\sum_{m=0}^{\infty}\left[\frac{1-\exp[-(a+N_1)t]}{(a+N_1)}\right]\right.$$

$$\left. -\left[(b-c)k_1/k_2 - (b-a)\right]\cdot e^{-bt}\cdot\sum_{m=0}^{\infty}\left[\frac{1-\exp[-(a-b+N_1)t]}{(a-b+N_1)}\right]\right\}$$

(8b) $$q_2(o,t) = \left(\frac{-2T_2Wk_2}{LT_1B_2^2bk_3}\right)\left\{\sum_{m=0}^{\infty}\left[\frac{1-\exp[-(c+N_2)t]}{(c+N_2)}\right] - e^{-bt}\cdot\sum_{m=0}^{\infty}\left[\frac{1-\exp[-(c-b+N_2)t]}{(c-b+N_2)}\right]\right\}$$

$$+\left(\frac{2T_2Wk_1}{LT_1B_2^2bk_3}\right)\left\{\sum_{m=0}^{\infty}\left[\frac{1-\exp[-(a+N_1)t]}{(a+N_1)}\right] - e^{-bt}\cdot\sum_{m=0}^{\infty}\left[\frac{1-\exp[-(a-b+N_1)t]}{(a-b+N_1)}\right]\right\}$$

Approximate outflow solution of (F-1) for large t:

(9a) $$q_1(o,t) = \left(\frac{-WB_1B_2}{2Lk_2k_4}\right)\left(2+k_2k_4\right)\left\{\exp(-gt)+\exp(-ft)\right\}\theta_1(1/2, i\pi t/2L^2k_4)$$

$$-\left(\frac{WB_1B_2}{Lk_1k_4}\right)\left\{(2c+gk_2k_4)\sum_{m=0}^{\infty}\left[\frac{1-\exp[-(g+N_4)t]}{(g+N_4)}\right] + (2c+fk_2k_4)\sum_{m=0}^{\infty}\left[\frac{1-\exp[-(f+N_4)t]}{(f+N_4)}\right]\right\}$$

(9b) $$q_2(o,t) = \left(\frac{-2T_2WB_1}{LT_1B_2k_4}\right)\left\{\sum_{m=0}^{\infty}\left[\frac{1-\exp[-(g+N_4)t]}{(g+N_4)}\right] - \sum_{m=0}^{\infty}\left[\frac{1-\exp[-(f+N_4)t]}{(f+N_4)}\right]\right\}$$

Note: $N_1 = (2m+1)^2\pi^2k_1/4L^2$

$N_2 = (2m+1)^2\pi^2k_2/4L^2$

$N_4 = (2m+1)^2\pi^2/2L^2k_4$

Equations (F-2) can be solved for $\bar{h}_1$ and $\bar{h}_2$ to give the fourth order linear differential equations (F-3ab), to which the boundary conditions (F-lefgh), transformed, can be adjoined to give the boundary value problem (F-3). For values of A_1, A_2, B_1, and B_2 that yield four real and distinct roots m_i of the auxiliary equations for the homogeneous equations corresponding to (F-3a) and (F-3b), the solutions of equations (F-3a) and (F-3b) can be written in the form (F-4a) and (F-4b), respectively.

The particular solutions $\bar{H}_1$ and $\bar{H}_2$ of the nonhomogeneous equations (F-3ab) were obtained by expanding the inverse operator by long division in the symbolic operator method (Miller, 1941), as indicated in (F-4). Note that $\bar{H}_1 = A_2 B_2^2 \bar{H}_2$.

The unknown coefficients a_i and c_i in the solutions (F-4ab) were evaluated by using conditions (F-3cd), where $a_1 = a_3 = c_1 = c_3 = 0$. Using these values, the expressions (F-4ab) for $\bar{h}_1$ and $\bar{h}_2$ were substituted into equation (F-3b) to give an equation from which relationships between a_2 and c_2, and between a_4 and c_4 were evaluated by the use of conditions (F-3ef). All the coefficients are listed in equations (F-5), and the solutions of the transformed problem (F-3) are given by equations (F-6ab).

The lateral outflow from each of the aquifers was obtained by applying Darcy's law to the solutions of the transformed problem, giving the outflow solutions (F-7ab). The inverse transforms of both the water-level and the outflow expressions are very difficult to obtain because the quantities m_1 and m_2 contain fourth roots of the variable p. Some simplifying assumptions are made to facilitate the inverse transformations, and only the outflow solutions are considered.

The roots m_1 and m_2 can be simplified in two ways, one for large values of the variable p (corresponding to small values of time) and another for small values of p (corresponding to large values of time). The inverse transforms of the resulting approximate forms of the solution are obtained by use of tabulated transform pairs and operations.

Approximation of (F-7ab) for Small Values of Time

From the form of the roots m_2 and m_4 in (F-4) it is apparent that for sufficiently large values of p, corresponding to small values of time, the constant term $(4/B_1^2B_2^2)$ can be neglected. Approximate values of the roots are then simply $m_2 = -(A_1)^{1/2} = -\left[1/k_1(p = k_1/B_1^2)\right]^{1/2}$. The denominator of each of the terms in the outflow solutions (F-7) then becomes $(m_4^2 - m_2^2) = -(A_1 - A_2) = -k_3(p + b)$, where k_3 and b are defined in (F-7c) below.

The first term of equation (F-7a), represented by $\bar{q}_1'(o, p)$, becomes

(F-7c) $$\bar{q}_1'(o,p) = \frac{Wk_1^{1/2}}{k_3 k_2} \cdot \frac{(p+c)}{p(p+b)} \cdot \frac{\tanh\left[(L/k_1^{1/2})(p+a)^{1/2}\right]}{(p+a)^{1/2}}$$

where $k_3 = (1/k_1 - 1/k_2)$, $a = k_1/B_1^2$, $b = (1/B_1^2 - 1/B_2^2)/k_3$, and $c = k_2/B_2^2$. The other terms are obtained similarly. The inverse transform of (F-7c) is obtained by the use of the convolution property (Churchill, 1959, p. 323, op. 7) on the functions $f_1(p) = (p + c)/p(p + b)$, and $f_2(p) = \left[\tanh\,(Lk_1^{1/2})(p + a)^{1/2}\right]/(p + a)^{1/2}$. The inverse transform of the function $f_1(p)$ is given by Erdelyi (1954, pair 5.2(5)). The inverse of $f_2(p)$ is obtained by the use of the delay property (Churchill, 1959, p. 323, op. 11) in conjunction with the inverse transform given by Erdelyi (1954, pair 5.9(34)).

The transform of the term given by (F-7c) is

(F-7d) $$q_1'(0,t) = \frac{W k_1^{1/2}}{k_3 k_2 b} \int_0^t \left[c + (b-c)e^{-b(t-t')}\right] \cdot \left[(-k_1^{1/2}/L) \cdot e^{-at'} \cdot \theta_1\left(\tfrac{1}{2}, i\pi k_1 t'/L^2\right)\right] dt'$$

where $\theta_1(\frac{1}{2})$ is the theta function (Whittaker and Watson, 1927, Ch. 21)

(F-7e) $$\theta_1\left(\tfrac{1}{2}\right) = 2\sum_{n=0}^{\infty} \exp\left[-(2n+1)^2 \pi^2 k_1 t/4L^2\right].$$

After performing the integration indicated in equation (F-7d) this term can be written as a sum of two infinite series. These series added to those obtained by using the above process on the three remaining terms of (F-7a) gives the result shown in (F-8a).

The approximate inverse transform of (F-7b) is obtained in a similar manner, using the inverse transform given by Churchill (1959, pair 12) instead of Erdelyi's pair 5.2(5). The result is given by equation (F-8b).

Approximation of (F-7ab) for Large Values of Time

From the form of the roots m_2 and m_4 in (F-4) it is apparent that for sufficiently small values of p, corresponding to large values of time, the term $(A_1 - A_2)^2$ in the inner radicals of the roots can be neglected. Approximate values of the roots are then $m_2 = -\left[\frac{1}{2}(A_1 + A_2 + 2/B_1B_2)\right]^{1/2} = -\left[(k_4/2)(p + g)\right]^{1/2}$, and $m_4 = -\left[(k_4/2)(p + f)\right]^{1/2}$, where $k_4 = (1/k_1 + 1/k_2)$, $g = (1/B_1 + 1/B_2)^2/k_4$, and $f = (1/B_1 - 1/B_2)^2 k_4$. The denominator of each of the terms in the outflow solutions (F-7) thus becomes $(m_4^2 - m_2^2) = -2/B_1B_2$. The first term of equation (F-7a) becomes

$$\text{(F-8c)} \qquad \bar{q}_1'(0,p) = \frac{WB_1B_2}{k_2(2k_4)^{1/2}} \cdot \frac{(p+c)}{p} \cdot \frac{\tanh\left[L(k_4/2)^{1/2}(p+g)^{1/2}\right]}{(p+g)^{1/2}}.$$

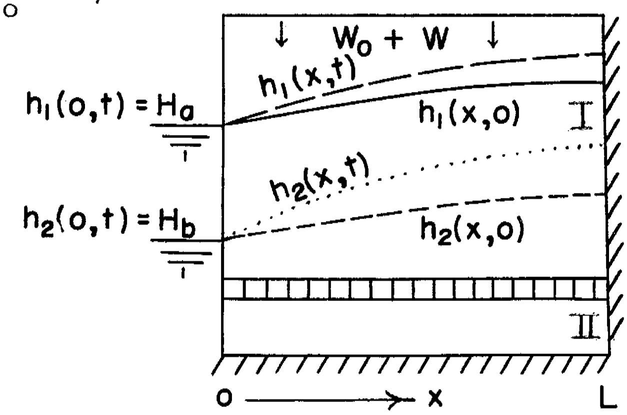
The other terms are obtained similarly. The inverse transform of (F-8c) is the sum of two parts, the first obtained directly from a tabulated inverse given by Erdeyli (1954, pair 5.9(34)) using the delay property. The second part of the inverse transform is obtained by the use of the convolution property on the functions $f_1(p) = 1/p$ and $f_2(p) = \left[\tanh L(k_4/2)^{1/2}(p + g)^{1/2}\right]/(p + g)^{1/2}$, with the delay property. These two methods are used to obtain the inverses of the remaining terms of (F-7a), and the second method is used for both terms of (F-7b). The resulting solutions are given by (F-10).

Discussion of Problem G

An increase in the average annual recharge to the Sunshine Valley aquifer system, as would occur if surface reservoirs were constructed in the adjacent mountains or if artificial spreading works were constructed, would cause a rise in water levels and an increase in the aquifer outflow. A knowledge of the rate at which the increased aquifer outflow would occur would be useful to assist in planning for increased water uses downstream. Problem G is the analytical representation of the Sunshine Valley aquifer system during such a period of increased recharge. The nonsteady solution of Problem G is obtained by adding the transient solution of Problem F to the steady-state solution of Problem E.

PROBLEM G

Nonsteady motion in a leaky infinite half-strip parallel-aquifer system consisting of an unconfined aquifer semiperched on a leaky artesian aquifer. The water levels of the two aquifers initially are at a steady state in equilibrium with recharge W_o, and are maintained at steady non-equal levels at the stream boundary during steady uniform recharge at the rate $(W_o + W)$.



(1a) $(\partial^2/\partial x^2)h_1 - (h_1 - h_2)/B_1^2 = (1/k_1)(\partial/\partial t)h_1 - (W_o + W)/T_1$

(1b) $(\partial^2/\partial x^2)h_2 + (h_1 - h_2)/B_2^2 = (1/k_2)(\partial/\partial t)h_2$

(1cd) $h_1(x, o) = h_{1s}$ (eqn. E-2a); $h_2(x, o) = h_{2s}$ (eqn. E-2b)

(1ef) $(\partial/\partial x)h_1(L, t) = O$; $(\partial/\partial x)h_2(L, t) = O$

(1gh) $h_1(o, t) = H_a$; $h_2(o, t) = H_b$

ALBUQUERQUE-BELEN PROVINCE

Comparison of the contours of water levels (Spiegel, 1960b; Bjorklund and Maxwell, 1961) with the geology (Spiegel, 1961c) of the lower Jemez River region suggests the interpretation that the close spacing of the water level contours north of Corrales and Sandia Pueblo is due to the low transmissivity of the red member of the Santa Fe group in this area. Although numerous faults were found in the area, the outcrop pattern of the red member of the Santa Fe group is controlled principally by a broad anticline, the axis of which plunges gently to the southeast along the Jemez River. The broad outcrop belt mapped on the south side of the Jemez River is the trace of the south limb of the anticline on the gentle slopes in that area.

The Rio Grande watercourse crosses the subcrop of the basal contact on the highly transmissive upper unnamed formation of the Santa Fe group in the area between Corrales and Sandia Pueblo. The abrupt change in spacing and shape of the water-level contours nearby suggests that the subcrop of the contact continues eastward beneath the younger fan deposits and westward beneath the terrace deposits and cover west of the Rio Grande. South of the area mapped the outcrops are generally poor and widely scattered, but from (a) the southerly dips of the exposures mapped west of Corrales, (b) the work of Bryan and McCann (1937), Wright (1946), Denny (1940), and Spiegel (1955), and (c) the flat-lying attitude of nearly all the Santa Fe outcrops observed along the Rio Grande valley in the Albuquerque-Belen province, it is concluded that the upper unnamed formation extends southward to the Rio Salado along both sides of the Rio Grande valley.

Although the eastern and western facies of the unnamed upper formation are poorly exposed in the Albuquerque-Belen province, the axial river gravel member is locally well exposed and is sufficiently distinctive in lithology to trace along the east bluffs and tributary arroyos of the Rio

Grande valley. The presence of the river gravel is also inferred from well data east of the Rio Grande valley floor. The axial river gravels apparently interfinger with the eastern facies (alluvial fan deposits) near the eastern boundary of the province, and with the western facies near the west side of the Rio Grande watercourse. The western facies in the Albuquerque-Belen province represents the coalesced deposits of broad alluvial fans that descended into the ancestral Rio Grande valley from the northwest. These deposits are generally better sorted than those from the smaller and steeper fans comprising the eastern facies, and may have been deposited by perennial streams tributary to the ancestral Rio Grande. The western facies and axial river gravel facies and Rio Grande watercourse form a single highly transmissive aquifer called the Albuquerque aquifer for convenience in discussions below.

A system of agricultural drains transects the Albuquerque-Belen province parallel to the Rio Grande, forming mutual hydrologic boundaries of the various sub-strips into which the drains divide the province. In general, the principal drains are (a) the riverside drains, one on each side of the river, and (b) the interior drains, one on each side of the valley floor, between the riverside drains and the bluffs or side-slopes bordering the inner valley floor (Nat. Res. Comm., 1938). Locally there are other drains, and the interior drains connect with the riverside drains at intervals. The riverside drains are the principal controls of the water-table elevation in the Rio Grande valley floor. The slope of the drains is approximately equal to or somewhat less than that of the river and the valley floor itself, that is, about $4\frac{1}{2}$ feet per mile to the south. The level of the drains fluctuates only slightly during the winter when the entire flow is ground-water outflow from the Rio Grande watercourse. At times during the summer the drain level may rise two or three feet because of local storm runoff and irrigation waste water.

The strip between the riverside drains (the river channel itself) is sometimes completely dry when the upstream flow of the Rio Grande is entirely diverted at Angostura Dam (near Algodones), but usually there is some surface flow covering a portion of the channel. The areal recharge to this strip afforded by infiltration of the river water moves laterally to the riverside drains, which have been excavated to a depth of several feet below the adjacent river channel. Steady flow in segments of each of the two halves of this narrow strip can be treated approximately by the method given by Kirkham (1950) if the slope of the river and associated drains is neglected. For example, during the spring snowmelt, water covers most of the channel for several months. If the drain is assumed fully penetrating, the nonsteady solutions given by Brown (1959) and Glover (1960a, p. 21) can be used to determine the approximate rate of recession of the water table in the channel after the cessation of surface flow (additional references are given in Appendix E, under the classification of infinite half strip with initially uniform water level).

The narrow strips of land between each riverside drain and the corresponding interior drain can also be considered approximately as pairs of infinite half strips. These strips are, in general, irrigated during 8 months of the year, and an approximately steady state is attained during most of this time, even though the irrigation applications are intermittent. If, as in the previous paragraph, the slope of the drains is neglected, the component of nonsteady flow in the direction of the drain corresponding to each half-strip can be treated by the methods given by Maasland (1959).

Most of the Albuquerque-Belen province, and hence most of the water in storage in the Albuquerque aquifer, is included in the two outer rectangles between the interior drains and the fault boundaries on the east and west sides of the province. These outer rectangles have small recharge and do not contribute much water to the drains in the inner valley under natural conditions. However, they could contribute significant quantities of water under artificial conditions (e.g., increased recharge from lawn irrigation, or lowered water levels in the valley floor).

Theis (1938, pp. 283-284) applied a solution of a problem in heat conduction in a semi-infinite plate to the case of rapid drawdown of water levels caused by construction of the interior drains in the Rio Grande valley. This method of calculating the approximate outflow from the aquifer neglects the slope and partial penetration of the drains and the effect of other hydrologic boundaries. These assumptions are valid for the short period (less than ten years) in which the indrease of drainflow is a significant quantity, if the outflow from only the side of the interior drains away from the Rio Grande (mesa side) is considered. The actual flow of the interior drains in the Rio Grande valley, however, is the sum of the outflow from the ground water on the valley floor between the interior drains plus that from the mesa side of the interior drains. The method of analysis of the drainage of ground water presented by Brown (1959) can be applied to calculate the drainflow derived from the valley side of the interior drains. Brown's method of analysis was intended to be used in the case of a rectangular aquifer bounded by four drains at uniform level; such areas are also present in the inner valley of the Rio Grande.

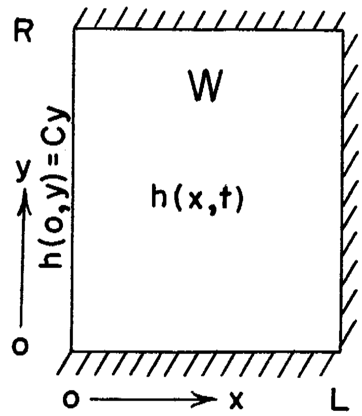
No two-dimensional solutions were found for the boundary conditions that apply to the rectangular Albuquerque aquifer as a whole, although the methods discussed above can be used for some portions of the province. Some steady solutions were obtained for the case of a rectangular aquifer closed on three sides and bounded by a stream with a constant gradient. Two of these solutions are given in the following problems.

Discussion of Problem H

Equations (H-1) give the analytical representation of two-dimensional ground-water motion in the Albuquerque aquifer east of the east side interior drains. The solution is obtained by summing the Fourier series solution of equation (H-1a, reduced) obtained by the method of separation of variables, with a particular solution of (H-1a) obtained by the symbolic operator method (Miller, 1941). The solution for the boundary conditions (H-1bcde) is given by equation (H-2), and the expression for the outflow per unit length of stream is given by equation (H-3).

PROBLEM H

Steady two-dimensional motion in a rectangular aquifer with three impermeable faces and one face bounded by a stream with constant gradient, during steady, uniform recharge.



(1a) $(\partial^2/\partial x^2)h + (\partial^2/\partial y^2)h = -W/T$

(1bc) $(\partial/\partial x)h(L,y) = 0; \qquad (\partial/\partial y)h(o,x) = 0$

(1de) $(\partial/\partial y)h(x,R) = 0; \qquad h(o,y) = Cy$

(2)
$$h(x,y) = \frac{R}{2}\left(C - \frac{2\,WR}{3T}\right) + \frac{W}{T}\left(Ry - y^2/2\right) + \frac{2R}{\pi^2}\sum_{m=1}^{\infty}\left[(-)^m C - \left(C - \frac{WR}{T}\right)\right]\frac{1}{m^2}\cdot\frac{\cosh m\pi(L-x)/R}{\cosh m\pi L/R}\cdot\cos m\pi y/R$$

(3)
$$q(o,y) = -\frac{2T}{\pi}\sum_{m=1}^{\infty}\left[(-)^m C - \left(C - \frac{WR}{T}\right)\right]\frac{1}{m}\cdot\tanh m\pi L/R\cdot\cos m\pi y/R$$

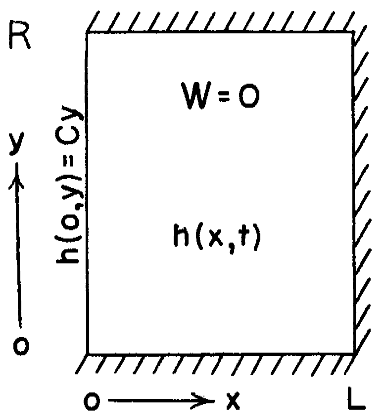
Discussion of Problem I

Equations (I-1) give an approximate analytical representation of two-dimensional ground-water motion in the Albuquerque aquifer west of the Rio Grande, for in most of this area the recharge is extremely small. The solutions (I-2) and (I-3) are obtained by the use of Fourier series or by letting $W = O$ in Problem H. The water-level contours are anti-symmetrical about the line $y = R/2$ because of the cosine factor in the infinite series. Thus the stream loses water in the reach $R>y>R/2$ and gains water in the reach $R/2>y>O$. The amount of water circulated from the losing reach to the gaining reach is obtained by integrating (I-3) over one of the halves of the stream course, giving the expression (I-4).

Unfortunately the solutions for this problem (and especially for the preceding one) are difficult to use for more detailed analysis of the water levels and stream gains or losses in specific reaches. A large number of type curves plotted for various values of the aquifer parameters would be required to use field water-level contours and streamgaging data to substantiate the order of magnitude of aquifer coefficients computed from well tests.

PROBLEM I

Steady two-dimensional motion in a rectangular aquifer with three impermeable faces and one face bounded by a stream with constant gradient, in the absence of recharge.



(1a) $(\partial^2/\partial x^2)h + (\partial^2/\partial y^2)h = 0$

(1bc) $(\partial/\partial x)h(L,y) = 0; \quad (\partial/\partial y)h(o,x) = 0$

(1de) $(\partial/\partial y)h(x,R) = 0; \quad h(o,y) = Cy$

(2) $$h(x,y) = \frac{RC}{2} + \frac{2RC}{\pi^2}\sum_{m=1}^{\infty}\left[(-)^m - 1\right]\frac{1}{m^2}\cdot\frac{\cosh m\pi(L-x)/R}{\cosh m\pi L/R}\cdot\cos m\pi y/R$$

(3) $$q(0,y) = -\frac{2CT}{\pi}\sum_{m=1}^{\infty}\left[1-(-)^m\right]\frac{1}{m}\cdot\tanh m\pi L/R\cdot\cos m\pi y/R$$

(4) $$Q(0,y)\Big]_0^{R/2} = \frac{2TRC}{\pi^2}\sum_{m=1}^{\infty}\left[1-(-)^m\right]\frac{1}{m^2}\cdot\tanh m\pi L/R$$

SUMMARY

The three most important contributions of this work are given below.

(a) Analytic representation: Published solutions to diffusion, heat conduction, and ground-water problems which are relevant to stream-connected aquifer systems are indexed, and a new general approach to analytic representation of aquifer systems is presented. Tables, figures, and clarifying definitions are provided to facilitate discussion of aquifer systems and the selection of existing solutions applicable to new field problems. Systems of approximate differential equations for leaky plane-parallel flow in systems of paired aquifers are derived from hydrologic considerations and related to equations of diffusion and heat conduction.

(b) Geohydrology: The author's studies of the ground-water geology of aquifer systems in the Rio Grande basin of Colorado and northern New Mexico are summarized. Geohydrologic provinces of the Rio Grande basin are delineated and described, principally on the basis of the occurrence of ground water in watercourses and the Santa Fe group. The presence of the highly permeable watercourse aquifers along most of the streams gives them a sufficiently great effective penetration into subjacent aquifers to permit the assumption of plane-parallel motion in the analytic investigations.

(c) Solutions of new analytical problems: Nine new analytical problems pertaining to stream-connected aquifer systems are stated and solved. The relation of each of these problems to a portion of the Rio Grande basin is discussed.

APPENDIX A. LIST OF SYMBOLS

Subscripts	Explanation
a, b	Denotes that the potential is measured at a boundary.
c	Signifies that the potential is measured at the face of the semiconfining bed.
e	Signifies that the potential is the "effective average" (potential corresponding to the mean velocity in vertical section).
i	The ith quantity of a set.
k = 1(1)n	Denotes the potential or aquifer coefficient pertaining to the kth of n parallel aquifers.
L, o	Value at a fixed point.
p	Denotes the potential or aquifer coefficient pertaining to the aquifer in parallel with the kth aquifer of a pair of mutually leaky aquifers; for a perched aquifer, denotes the atmospheric pressure (zero) at the lower face of the perching layer.
s	General space coordinate; in Problems, signifies steady-state solution.
t	Time.
x, y, z	Coordinate axes.

Symbols; units	Explanation
a	Defined in equation (F-7c).
a_i	Unknown coefficients.
A_1, A_2	Defined in equation (F-3).
b	Defined in equation (F-7c).
B_1, B_2; (L)	Leakage coefficient.
C	Constant.
c	Defined in equation (F-7c).
c_k	Upper limit of saturated zone (in Derivations, Case II).
c_i	Unknown coefficients.
D, D_t	Partial derivative operator.
f	Defined above equation (F-8c).
g	Defined above equation (F-8c).
H; (L)	Hydraulic potential at a stream boundary; particular solutions.
$\bar{H}$	Laplace transform of a particular solution.
h; (L)	Hydraulic potential, head.
$\bar{h}$	Laplace transform of potential.
h^*; (L)	Potential at the water table.

Symbols; units	Explanation
h_{av}; (L)	Mean potential in a vertical section.
i	$(-1)^{1/2}$.
K, K'; (L/T)	Hydraulic conductivity of an aquifer and semiconfining bed, respectively.
k; (L^2/T)	Hydraulic diffusivity, (T/S) or (K/S_s).
L; (L)	Width of an aquifer.
L_R; (L)	Width of a reservoir.
M	Defined in equation (D-7).
m, m'; (L)	Thickness of an aquifer and semiconfining bed, respectively.
$\bar{m}$; (L)	Mean thickness of saturation of an aquifer.
m_i	Roots of an auxiliary equation.
N_i	Defined in equation (F-9).
p	Image of the time variable in Laplace transforms.
Q, Q_t; (L^3/T)	Total aquifer inflow or outflow.
q; (L^2/T)	Aquifer inflow or outflow.
R; (L)	Point on the y-axis.

Symbols; units	Explanation
S, S_a, S_w	Aquifer storage coefficients (dimensionless). S, general coefficient; S_a, storage derived from expansion of aquifer and water; S_w, storage coefficient for unconfined aquifers.
S_s; (1/L)	Specific storage (storage coefficient for aquifer of unit thickness).
T; (L^2/T)	Aquifer transmissivity (transmissibility), equal to (Km).
t; (T)	Time.
t'; (T)	Current time variable of integration.
u, v; (L)	Hydraulic potentials (Problem D).
v; (L/T)	Effective or bulk velocity of a fluid in a porous medium, defined as the volume of fluid passing a unit area of gross cross-section per unit time.
v'_z; (L/T)	Vertical velocity in a semiconfining bed.
W; (L/T)	Areal recharge.
x, y, z	Coordinates in the rectangular Cartesian system.
ξ, η, ζ	Xi, Eta, Zeta; Cartesian coordinate axes used in Case II of Derivations.
Θ	Theta function.

APPENDIX B. ANALYTIC REPRESENTATION OF GROUND-WATER FLOW, HEAT CONDUCTION, AND DIFFUSION

	GROUND WATER	Units	HEAT CONDUCTION (Carslaw and Jaeger, 1959)	Units	DIFFUSION (Crank, 1956)	Units
BASIC LAWS	Effective velocity:		Rate of heat transfer:		Rate of mass transfer	
	$v_s = -K\ (\partial h/\partial s)$ (Darcy's Law)	L/T	$f = -K\ (\partial v/\partial n)$ (Fourier's Law)	$\frac{\varrho L \theta}{T}$	$F = -D\ (\partial C/\partial n)$ (Fick's Law)	ϱ L/T
GENERAL LINEAR DIFFERENTIAL EQUATIONS (Plane-parallel flow)	$\nabla^2_{xy}\ h_k - \frac{(h_k - h_p)}{B^2} = \frac{1}{k}\ \frac{h_k}{t} - \frac{W}{T}$ (see Derivations)	1/L	$\nabla^2_{xy}\ v - hv = \frac{1}{\varkappa}\ \frac{\partial v}{\partial t} - \frac{A}{K}$	θ/L^2	$\nabla^2_{xy}\ C - hC = \frac{1}{D}\ \frac{\partial C}{\partial t} - A$	ϱ/L^2
PHYSICAL QUANTITIES IN BASIC LAWS AND DIFFERENTIAL EQUATIONS	h, hydraulic potential; H, boundary potential	L	v, temperature; V, boundary temperature	θ	C, concentration; C_o, boundary concentration	ϱ
	K, hydraulic conductivity	L/T	K, thermal conductivity	$\varrho L^2/T$	D, diffusion coefficient	L^2/T
	v = Q/At, flux, bulk velocity	L/T	f = Q/At, heat flux	ϱ L /T	F = N/At, material flux	ϱ L/T
	Q_T = Q/t = vA = quantity of water per unit of time	L^3/T	Q, quantity of heat	$\varrho L^3\theta$	M, mass of diffusant	ϱL^3
	k = km/S, hydraulic diffusivity; m, aquifer thickness S, storage coefficient	L^2/T L None	$\varkappa = K/\varrho C$, thermal diffusivity ϱ, density C, specific heat	L^2/T ϱ ϱ	D, diffusivity	L^2/T
Note: primes refer to semi-confining bed.	$\frac{1}{B^2} = \frac{K'/m'}{Km}$, inverse square of leakage coefficient	$1/L^2$	h, $(\nu/\varkappa)$, b = H/K	$1/L^2$	h = k/D, first order reaction coefficient	$1/L^2$
	K'/m', leakance coefficient	1/T	H, surface heat transfer	ϱ/T	k	1/T
	W/T, recharge term	1/L	A/K, heat generation term	ϱ/L^2	A	ϱ/L^2

APPENDIX B - Continued

	GROUND WATER	Units	HEAT CONDUCTION (Carslaw and Jaeger, 1959)	Units	DIFFUSION (Crank, 1956)	Units
BOUNDARY CONDITIONS						
I. UNLINED CHANNEL, PRESCRIBED LEVEL						
a. Steady level	$h = H_c$	L	$v = V_o$	θ	$C = C_o$	e
b. Unsteady level	$h = H(x,y,z,t)$	L	$v = V\ (x,y,z,t)$	θ	$C = C\ (x,y,z,t)$	e
II. LINED CHANNEL						
a. Impermeable	$\partial h/\partial s = 0$	—	$\partial v/\partial n = 0$ (insulated)	e/L	$(\partial C/\partial n) = 0$	e/L
b. Prescribed flux	(per unit aquifer length)					
1. Steady	$q/m = -K\ (\partial h/\partial s)$	L/T	$f = -K\ (\partial v/\partial n)$	$\frac{eL\theta}{T}$	$F = -D\ (\partial C/\partial n)$	L/T
2. Nonsteady	$q/m = q\ (t)$	L/T	$f = f\ (t)$		$F = F\ (t)$	
c. Lateral leakage (linear)	$(\partial h/\partial s) = \frac{-(K_b/m_b)(H-h)}{K}$ (Leaky channel)	None	$(\partial v/\partial n) = h_b\ (v-V)$ (Newton radiation)	θ/L	$(\partial C/\partial n) = -h_b\ (C-C_o)$ (Surface evaporation)	e/L
III. FINITE RESERVOIR (Constant inflow; outflow by linear leakage; porous, $S_R < 1$; open, $S_R = 1$)	$L_R \cdot S_R\ (dH_R/dt) = -K\int_m(\partial h_b/\partial s)\,dm + Q_R + C_R\ (H_o - H_R)$	L^2/T	$Mc'(dV/dt) = -K\int_m(\partial v/\partial n)\,dm + Q_1 + H_1\ (v_o - V)$		(By analogy) $V_R\ (dC_R/dt) = -D(\partial C/\partial n) + M_I/t + E_R\ (C_E - C_R)$	eL^3/T
	Notation for unit length:					
	L_R = width of reservoir	L	M, reservoir mass	eL^3	V_R, reservoir volume	L^3
	S_R = storage coefficient	None	c', res. sp. heat	cal.	C_R, reservoir concentration	e
(a) Unlined, $h_b = H_R$	H_R = reservoir level	L	V, res. temperature	θ		
	h_b = aquifer level at face	L	v, temperature at face	θ		
(b) Lined, $(\partial/\partial x)h_b = -(K_b/Km_b)(H_R-h_b)$	Q_R = surface flow gain	L^2/T	Q, heat supplied/unit time	cal.	M_I, diffusant inflow	eL^3
	C_R = overflow coefficient	L/T	H_1, loss coeff.	eL/T	E_R, overflow coefficient	L^3/T
	H_o = spillway crest level	L	v_o, exterior temp.	θ	C_E, exterior concentration	e
	dm = element of boundary height	L	dm, element of boundary (Contact with perfect conduction or finite reservoir of well-stirred fluid; per unit area)	L	(Contact with finite reservoir of well-stirred diffusant; for unit area)	

APPENDIX B - Continued

	GROUND WATER	Units	HEAT CONDUCTION (Carslaw and Jaeger, 1959)	Units	DIFFUSION (Crank, 1956)	Units
IV. AQUIFER INTERFACES						
(a) Continuous water levels	$K_1\ (\partial h_1/\partial s) = K_2\ (\partial h_2/\partial s)$	L/T	$K_1\ (\partial v_1/\partial n) = K_2\ (\partial v_2/\partial n)$	$\varrho L\theta/T$	$D_1\ (\partial C_1/\partial n) = D_2\ (\partial C_2/\partial n)$	L/T
	$h_1 = h_2$	L	$v_1 = v_2$	θ	$C_1 = C_2$	None
(b) Leaky interface	$K_1\ (\partial h_1/\partial s) = K_2\ (\partial h_2/\partial s)$ $-K_1\ (\partial h_1/\partial s) = (K_b/m_b)(h_1-h_2)$	L/T	$K_1\ (\partial v_1/\partial n) = K_2\ (\partial v_2/\partial n)$ $-K_1\ (\partial v_1/\partial n) = H_b\ (v_1 - v_2)$	L /T	$D_1\ (\partial C_1/\partial n) = D_2\ (\partial C_2/\partial n)$ $-D_1\ (\partial C_1/\partial n) = k_b\ (C_1-C_2)$	L/T
PHYSICAL QUANTITIES IN BOUNDARY CONDITIONS (not explained above)						
<u>Note</u>: Subscript b refers to boundary membrane.	(K_b/m_b) , boundary leakance coefficient	1/T	$H_b = K_b/d$, boundary surface heat transfer coefficient; d, thickness of boundary layer	ϱ L/T	$k_b = D'/\delta$, mass transfer coefficient; D; membrane diffusity; δ, membrane thickness	L/T
	$\frac{(K_b/m_b)}{K} = \frac{1}{B_b}$, inverse of boundary leakage coefficient.	1/L	$h_b = H_b/K$	1/L	$h_b = k_b/D$	1/L

Note: Mass M is expressed in terms of density, ϱ , and volume, L^3, for convenience in comparison of parameters.

SINGLE-AQUIFER SYSTEMS

MULTIPLE-AQUIFER SYSTEMS

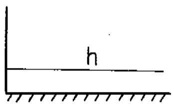
CONFINEMENT CONDITIONS

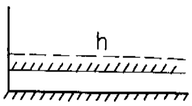
Single-aquifer systems

(Unconfined) (Confined)

Nonleaky

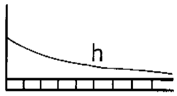
$\frac{1}{B^2} = 0$

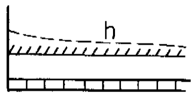




Leaky (linear)

$\frac{1}{B^2}(h)$

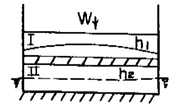




Multiple-aquifer systems

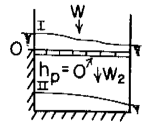
Nonleaky Perched

$\frac{1}{B^2} = 0$



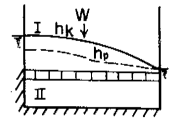
Leaky Perched

$\frac{1}{B_1^2}(h_1)$; $w_2 = w(x)$



Leaky Semi-perched

$\frac{1}{B_k^2}(h_k - h_p)$

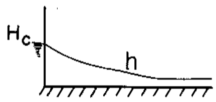


BOUNDARY CONDITIONS

I UNLINED CHANNEL

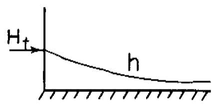
(a) Steady level H_c

$h(o,y,z) = H_c$



(b) Unsteady level

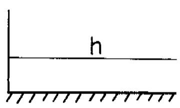
$h(o,y,z,t) = H_t$



II LINED CHANNEL

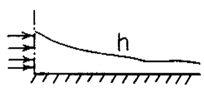
(a) Impermeable

$\frac{\partial h}{\partial x} = 0$



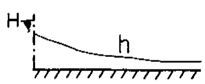
(b) Prescribed Flux

$q = -K \frac{\partial h}{\partial x}(o,y)$



(c) Linear Lateral Leakage

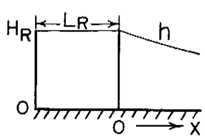
$-\frac{\partial h}{\partial x} = \frac{K_b/m_b}{K}(H_c - h)$



III FINITE RESERVOIR

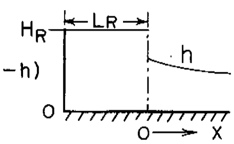
(a) Unlined

$h = H_R, t > 0$



(b) Leaky lining

$-\frac{\partial h}{\partial x} = \frac{K_b/m_b}{K}(H_R - h)$



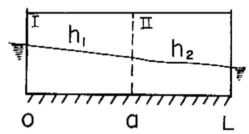
IV INTERFACES BETWEEN AQUIFERS IN SERIES

$$K_1 \frac{\partial h_1}{\partial x}(a) = K_2 \frac{\partial h_2}{\partial x}(a)$$

(a) Continuous levels at a

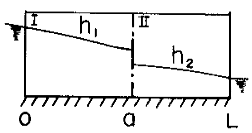
Homogeneous $(K_1 = K_2)$

Nonhomogeneous $(K_1 \neq K_2)$



$h_1(a) = h_2(a)$

(b) Leaky interface at a



$$-K_1 \frac{\partial h_1}{\partial s} = (K_b/m_b)(h_1 - h_2)\rfloor_a$$

V INITIAL CONDITIONS

(a) Upper aquifer (I)

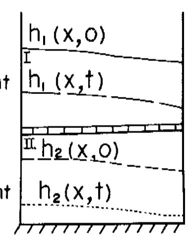
(b) Lower aquifer (II)

Initial

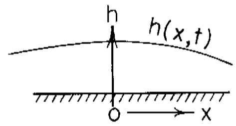
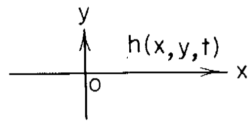
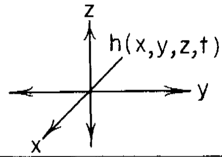
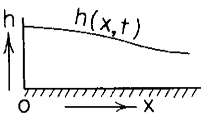
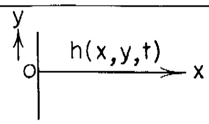
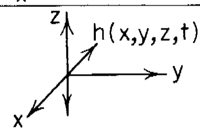
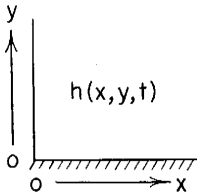
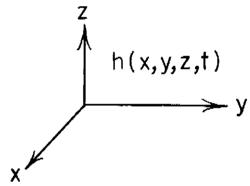
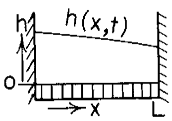
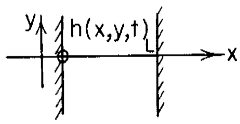
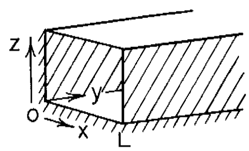
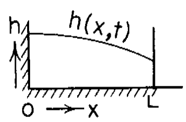
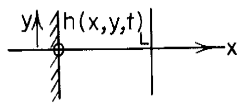
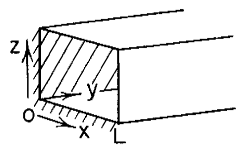
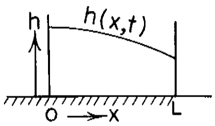
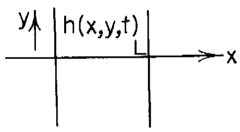
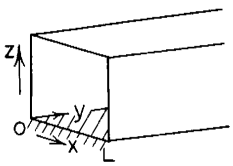
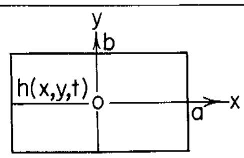
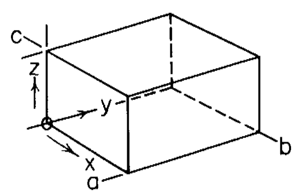
Current

Initial

Current



APPENDIX C. BOUNDARY AND CONFINEMENT CONDITIONS

CLASS	TYPE OF FLOW		
	LINEAR (Section)	PLANAR (Plan view)	3-DIMENSIONAL
Infinite			
Semi-infinite			
Infinite Quadrant	——		
Infinite Closed Strip	 (Leaky)		Infinite Closed Prism 
Infinite Half Strip			Infinite Half Prism 
Infinite Strip			Infinite Prism 
Rectangle	——	 Note: This class can be used to represent two-dimensional flow in vertical section.	Rectangular Parallelopiped 

APPENDIX D. GEOMETRIC CLASSES OF AQUIFER SYSTEMS

APPENDIX E. INDEX OF SELECTED ANALYTIC SOLUTIONS APPLICABLE TO STREAM-CONNECTED AQUIFER SYSTEMS

EXPLANATION OF INDEX NOTATION

Differential Equation:

$$\nabla^2_N h_k - a_k(h_k - h_p) = c_k \, \partial h_k / \partial t - W_k/T_k, \text{ where } a_k = 1/B_k^2, \; c_k = S_k/T_k$$

Aquifer Shape:

SI	Semi-infinite
ICS	Infinite closed strip
IHS	Infinite half strip
IS	Infinite strip
SIS	Semi-infinite strip
(C)	Composite (nonhomogeneous)
(HC)	Composite (homogeneous)
R	Rectangle

Flow Type:

N	Nonsteady	$(c_k \neq 0)$
S	Steady	$(c_k = 0)$
L	Leaky	$(a_k \neq 0)$

Recharge:

W_o	Initial value
W_c, W_1, W_2	Steady values

Initial Conditions:

C, H_o, H_L	Uniform (constant) values of water levels
$h(x)$	Nonuniform values

References (abbreviations):
(See Appendix F)

CJ	Carslaw and Jaeger, 1959
Ch	Churchill, 1958
Cr	Crank, 1956

Section references to above texts are identified by numbers.
Parentheses enclose equation numbers.

Boundary Conditions:

H_o, H_L, H_1	Steady levels
$H(t)$	Nonsteady level
F_o	Impermeable
F_c	Steady flux
$F(t)$	Nonsteady flux
L_B	Linear lateral leakage

APPENDIX E - Continued

INDEX

AQUIFER SHAPE	FLOW TYPE	DIFFERENTIAL EQUATION SUBSCRIPTS k	N	a; h_p	c	W	INITIAL CONDITIONS h(x,o)	BOUNDARY CONDITIONS	REFERENCES
SI	N	-	x	0	c	W_c	0	F_o; 0	Ch - prob. 48-9.
SI	N	-	x	0	c	W_t	0	F_o; 0	Ch - prob. 48-8.
SI	NL	-	x	a; 0	c	0	0	H_c; F_o	Ch - 47; prob. 48-4.
SI	NL	-	x	a; 0	c	0	C	0; F_o	Ch, prob. 48-12.
SI	N	-	x	0	c	0	h(x)	0; F_o	Ch, prob. 44-9.
SI	N	-	x	0	c	0	$h_o + Cx$	0; F_o	CJ - 2.4iii (13).
SI	N	-	x	0	c	0	0	H_t; F_o	Ch - 44.
SI	SL	-	x	a; 0	0	0	-	H_c; F_o	CJ - 4.3 (1).
SI	N	-	x	0	c	0	C	H_c; F_o	CJ - 12.4 Ii(5); 2.4(3); 2.4i(10); graph, fig. 5; table I in App. II. Ferris, 1955, p. 55.
SI	N	-	x	0	c	0	0	H_o, $o < t < t'$, H_1, $t > t'$; F_o	CJ - 2.5i (2,3).
SI	N	-	x	0	c	0	0	Ct; F_o	Cr - 3.3ii (3.16,3.17); graph,p.32. Rowe, 1960. Hantush, 1961.
SI	N	-	x	0	c	0	0	$Ct^{\frac{1}{2}}$; F_o	CJ - 2.5iii (6,7). Rowe, 1960. Hantush, 1961.
SI	N	-	x	0	c	0	0	F_o; $Ct^{m/2}$, m = 1,2...	CJ - 2.5iv (8); 12.4 Iii.
SI	N	-	x	0	c	0	0	F_o; $Ct(t_1-t)$	Rowe, 1960. Hantush, 1961.
SI	N	-	x	0	c	0	0	F_o; exp (Ct)	CJ - 2.5v (9); 4.2 (7).
SI	N	-	x	0	c	0	0	F_o; 2AsinCt	CJ - 2.6, 12.7. Werner, 1946.
SI	N	-	x	0	c	0	h(x)	F_o; H(t)	Ch - 44; prob. 44-10.
SI	N	-	x	0	c	0	0	F_o; H_c, $o < t < t_o$ o , $t > t_o$	Ch - 44; prob. 44-1.

APPENDIX E - Continued

INDEX

AQUIFER SHAPE	FLOW TYPE	DIFFERENTIAL EQUATION SUBSCRIPTS k	N	a; h_p	c	W	INITIAL CONDITIONS h(x,o)	BOUNDARY CONDITIONS	REFERENCES
SI	N	-	x	0	c	0	0	F_o; F_t	Ch - 43; prob. 44-6.
SI	N	-	x	0	c	0	0	F_o; F_c	CJ - 43 (4).
(C)SI	N	1	x	0	c	0	0	$h_1(-L,t) = H_c$; $F_1(o) = F_2(o)$	CJ - 12.8I.
		2	x	0	c	0	0	$h_1(o) = h_2(o)$ $F(\infty) = 0$	Cr - 3.5; graph, p. 40.
(C)SI	N	1	x	0	c	W_c	0	$h_1(-L,t) = 0$; $F_1(o) = F_2(o)$ $h_1(o) = h_2(0)$ $F(\infty) = 0$	CJ - 12.8 II.
ICS	NL	-	x	a; 0	c	0	h(x)	F_o; F_o	CJ - 4.7 (6).
IHS	N	-	x	0	c	W(t)	0	F_o; 0	CJ - 3.14ii(9). Ch - prob. 48-11.
IHS	N	-	x	0	c	W_o exp(-Ct)	0	F_o; 0	CJ - 3.14vi(14).
IHS	N	-	x	0	c	A sinCt	0	F_o; L_B	Neth. Inst., 1948, p. 52.
IHS	N	-	x	0	c	W_o AcosCt	0	F_o; 0	Werner & Noren, 1951.
IHS	N	-	x	0	c	W(x)	0	F_o; 0	CJ - 3.14iii(10).
IHS	S	-	x	0	0	W(x)	0	F_o; 0	CJ - 3.14, p. 132.
IHS	N	-	x	0	c	W_c	H_L	F_o; H_L + 2AsinCt	Werner & Noren, 1951.
IHS	NL	1	x	a; H_2	c	W_c	$C = H_L$	F_o; H_L	CJ - 4.14 (3,7)
IHS	N	-	x	0	c	W_c	$C = H_L$	F_o; H_L	CJ - 3.14i(7,8); graph, fig. 20. Werner, 1953, 1957.
IHS	S	-	x	0	0	W_c	$C = H_L$	F_o; H_L	Jacob, 1943, p. 566.
IHS	NL	-	x	a; H_L	c	0	Steady unif. recharge	F_o; H_L	CJ - 4.14.
IHS	NL	-	x	a; 0	c	0	h(x)	F_o; H(t)	CJ - 4.7 (5).

APPENDIX E - <u>Continued</u>

INDEX

AQUIFER SHAPE	FLOW TYPE	DIFFERENTIAL EQUATION SUBSCRIPTS k	N	$a;\ h_p$	c	W	INITIAL CONDITIONS h(x,o)	BOUNDARY CONDITIONS	REFERENCES
IHS	N	-	x	0	c	0	Parabola	$F_o;\ H_L$	CJ - 3.3v(16,17); graph, p. 98. Jacob, 1943.
IHS	N	-	x	0	c	0	Linear	$F_o;\ 0$	CJ - 3.3iv;graph, fig. 10b. Ch - prob. 48-7.
IHS	NL	-	x	a; 0	c	0	0	$F_o;\ H_L$	CJ - 4.2 (8).
IHS	N	-	x	0	c	0	0	$F_o;\ H_L$	CJ - 3.4(2); graphs, pp. 101-102.
IHS	N	-	x	0	c	0	C	$F_o;\ 0$	CJ - 3.3iii(8,9,13);graph,fig 10a. Ch - 45; probs. 75-6, 75-8. Glover, 1960a.
(C)IHS	SL	1	x	a_1; C	c_1	W_1	-	$H_o;\ F_1(a) = F_2(a)$	CJ - 4.13iv(10,11).
		2	x	a_2; C	c_2	W_2	-	$h_1(a) = h_2(a);\ F_o$	
IS	NL	-	x	a; 0	c	0	h(x)	$H_o(t);\ H_L(t)$	CJ - 4.7 (4).
IS	N	-	x	0	c	0	$C= H_o$	$H_o;\ H_L$	Ch - 46, 69. Werner, 1946.
IS	SL	-	x	a; C	c	0	-	$H_o;\ H_L$	CJ - 4.5 (1,3,4,6).
IS	N	-	x	0	c	0	0	0; Ct	Ch - 75.
(C)IS	N	1	x	0	c_1	0	0	$H_L;\ F_1(o) = F_2\ (o)$	CJ - 12.8 III.
		2	x	0	c_2	0	0	$h_1(o) = h_2(o);\ 0$	
(C)IS	SL	1	x	a_1; 0	c_1	W_1	-	$H_o;\ F_1(a) = F_2(a)$	CJ - 4.13ii(8).
		2	x	a_2; 0	c_2	W_2	-	$h_1(a) = h_2(a);\ H_o$	
(HC)IS	S	1	x	0	c	0	Linear	$H_o;\ F_1(a) = F_2(a)$	Werner, 1946.
		2	x	0	c	0	slope	$h_1(a) = h_2(a) = H_a;\ 0$	
SIS	N	-	x,y	0	c	0	1	h(o,o) = h(x,o) = h(x,1) = 0, t o	Ch - prob. 79-11.
R	N	-	x,y	0	c	0	C	h = 0 on all sides, t o	Brown, 1959.

APPENDIX F. LIST OF REFERENCES

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Date: May 30, 1962