A LABORATORY STUDY OF STREAMING POTENTIAL

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MAJOR SYMBOLS AND DEFINITIONS

D	Dielectric Constant.
d	Diameter of the sand particle.
K	Hydraulic conductivity or permeability.
k	Specific conductance.
P	Head difference. Except in the $\frac{\pi}{3}$ model, the stream lines are horizontal. Only in the case of horizontal stream lines, the head difference is equal to the pressure difference. In the figures, the pressure is referred to head difference.
R	Resistance of sand bed.
S	Diameter of the cylinderical plug.
η	Dynamic viscosity of liquid.
V	Streaming potential.
ε	Porosity.
ζ	Zeta potential.
γ	Unit weight of water.

ABSTRACT

The streaming potential which is generated as water flows through sand has been studied as a function of head difference in the sand plug; but it has never been studied to determine the properties of an aquifer. The purpose of the present investigation is to establish the magnitude of the streaming potential generated by the flow in a water bearing stratum induced by a pumping well. Consequently, the streaming potential, V, generated by the head difference, P, in three different hydraulic models, was measured by means of reversible electrodes. The relationship between V and P was also investigated for five different sand grains and for a range of resistivities of water. A specially designed $\frac{\pi}{3}$ model tank sector served to represent conditions of a fully penetrating well in an unconfined aquifer. In every case, the streaming potential, V, was found to be directly proportional to the head difference, P. The ratio of V/P was independent of changes in the geometry of the models.

Experiments on sand grains show that the ratio of V/P decreases with increase of grain diameter in confirmation of the streaming potential equation derived by Zucker. It contradicts the results of Schriever. The experimental results are supported by a hypothesis based on double layer model.

From the study of the streaming potential with the grain size to permeablity, an empirical formula relating V, P, and K, was obtained. The formula is valid only in the range of the experimental data. The relationship between the streaming potential with permeablity shows that the change involved is not sufficient for application to the field studies.

Thorough analysis of all the data obtained in the course of the investigation indicates that extremely low field values of the streaming potential may be expected in the vicinity of a pumping well.

A LABORATORY STUDY OF STREAMING POTENTIAL

INTRODUCTION

Reuss (1809), first observed the flow of water through a porous diaphragm in response to a difference in electrical potential across its boundaries, an effect known as electro-osmosis. If an applied e.m.f. produces an electro-osmotic flow of liquid in sand or clay, it is reasonable to expect that the motion of a liquid in a porous plug due to mechanical forces will produce a detectable e.m.f. The voltage that arises when a liquid streams through the plug is called the streaming potential. Quincke (1859) first observed that this potential, developed at the ends of the glass capillary, is directly proportional to the head difference for any given solution. The phenomena of electro-kinetic potential was explained by Helmholtz (1879) in his classical theory of a rigid double layer. This theory describes a fixed double layer at a solid-liquid interface as analogous to the two plates of an electric condenser. The potential between the plates is termed Zeta potential (ζ). Gouy (1917) and Stern (1924) amended the Helmholtz concept to include a diffuse,

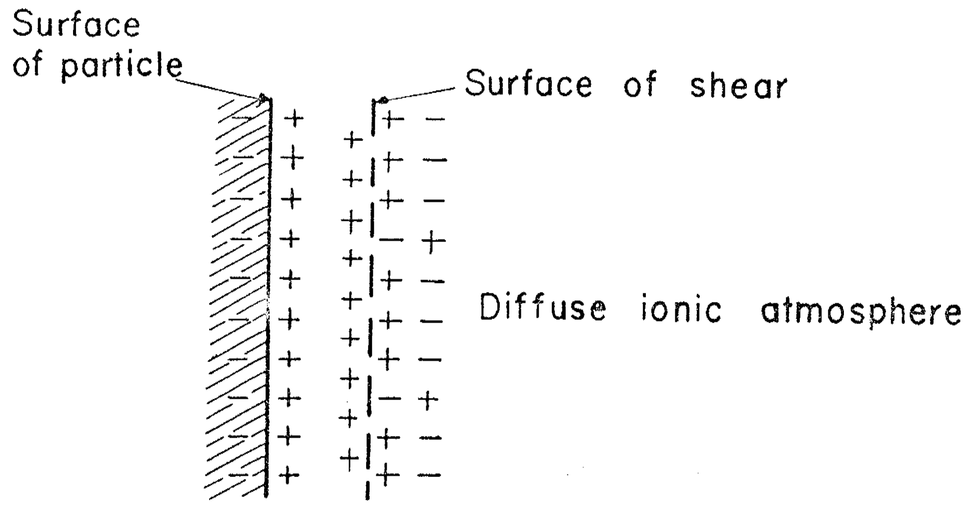

Fig. 1 - Diffuse Double Layer

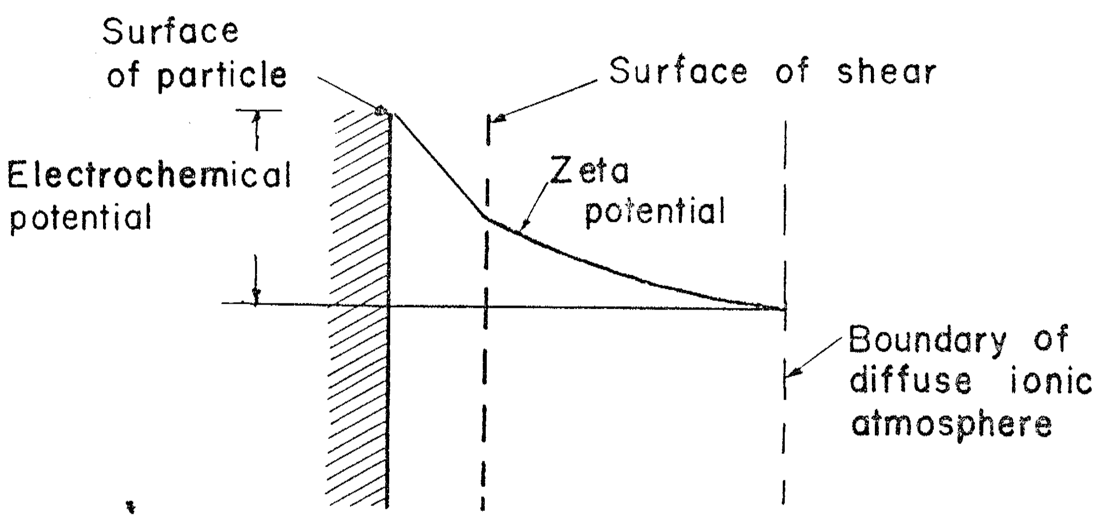

Fig. 2 - The Electric Potential as a Function of Distance from a Charged Particle in a Liquid Medium

mobile, ionic layer. Figures 1 and 2 illustrate the current concept, in which the Zeta potential represents only part of the total or electrochemical potential which exists between the solid interface and the bulk solution.

Briggs (1928), Fuerstenau (1956), and many others have measured the streaming potential in flow tubes in order to determine the Zeta potential, which is important in the fields of colloidal chemistry and mineral preparation.

The Schlumberger brothers (1932) described the potential observed in oil well logging as an electro-filtration potential, but Mounce (1944) showed later that the electro-chemical potential is the principal component measured in well logging. More recently, Wyllie (1951) and others have concluded that the streaming potential may have an influence on the self potential logs; but the last word on this subject is yet to be written.

Schriever and Bleil (1957) studied the effect of grain size, temperature, and solution composition on the streaming potential. They found the potential, V, to be directly proportional to the pressure difference, P, with the ratio of V/P independent of changes in the porosity and in the length of the sand column. They reported a great variation in the ratio of V/P with respect to grain diameter and observed that the streaming potential decreases with smaller grain diameters.

Much of the laboratory work previously reported on streaming potential measurement is subject to an important source of error. Zucker (1959) has criticized the interpretations drawn from earlier results for their invalidity, pointing out that electrodo potentials, induced by the motion of the electrolyte with respect to the electrode, mask the true streaming potential.

For hydrological purposes, Tsitsishvils (1955), and Corlick (1956) attempted field measurements of the streaming potential due to the flow of underground streams and to the flow induced by pumping wells. No quantitative results was reported.

All the laboratory work has utilized distilled water and high hydraulic head difference. The results of such laboratory investigation can not be applied to field problems where in general the pressures are low and the water conductive.

In order to apply laboratory results directly to hydrological problems, streaming potentials must be measured in the proper ranges of concentration, e. g., in water containing salts of 100 to 1000 ppm; head differences up to 30 feet; and grain sizes from 10 to 200 mesh.

The purpose of this investigation is to measure the streaming potential as a function of head difference, P, in different hydraulic models, in sands of different grain size using water of a salt concentration usually found in the field, to predict the magnitude of the streaming potential generated by pumping wells.

EXPERIMENTAL PROCEDURE

Most streaming potential measurements are to be made under field conditions in which the concentration of salts in the water is of the order 10^2 to 10^3 ppm. Since one of the purposes of this study is the simulation of field conditions, the concentration of ions in the models should approach that found in waters in the field. Small values of streaming potential are anticipated for these conditions. In order to record a true value of the potential, background interference, or variations in potential attributable to all other effects, must be minimized. Three precautions are necessary to achieve the measurements of small potentials. First, the voltmeter should not electrically load the sample. That is, excessive current should not be drawn during measurement; otherwise the value recorded will be reduced and the electrodes may be polarized. Second, in order to make use of the most sensitive range of the voltmeter, all background potentials should be minimized or neutralized. Third, measuring probes or electrodes should be of a type which will not produce any variable contact or masking potential. Accurate measurements may be achieved by judicious use of reversible electrodes in the presence of a sufficient concentration of the ions to which they are reversible. Silver chloride electrodes were prepared for this purpose and were tested for reversibility and reliability.

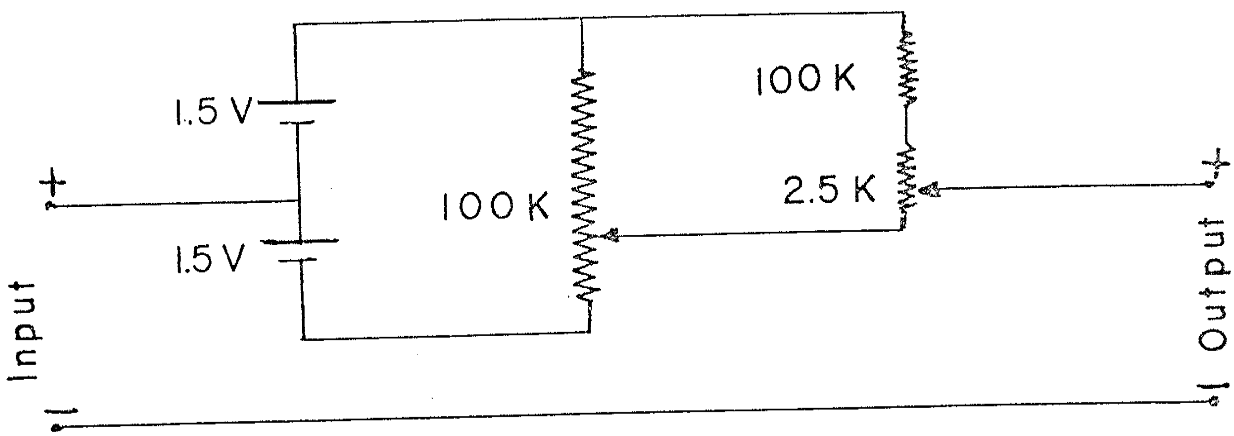

Fig. 3- Neutralizing Circuit to Balance the Steady D.C. Electrode Potential

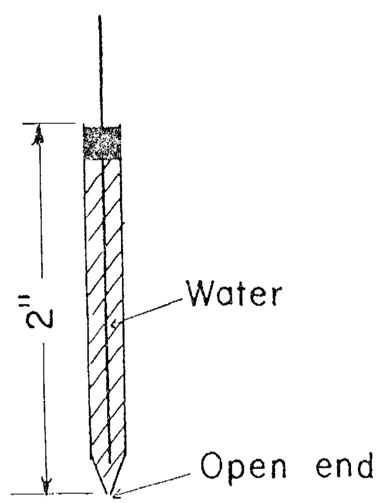

Fig. 4 - $AgCl_2$ Electrode Detail

Measuring Technique

The first requirement was satisfied with a chopper-stabilized D. C. microvoltmeter of high input impedance and low linear drift. In its microvolt ranges, the input impedance is 3.3 megohms per 1 millivolt to insure no loss of current due to the measurements. Taking into consideration the electrode surface area, a current of approximately 80 microamperes could be drawn without serious electrode polarization. For the range of values of streaming potential expected, the current drawn by the voltmeter was less than 0.0001 microamperes. A continuous and permanent record of the electro-kinetic potentials as a function of time was obtained by recording the output of this voltmeter on an Esterline-Angus recorder.

In order to meet the second requirement and make use of the microvolt ranges of the meter, it was necessary to eliminate the steady D. C. potential between the electrodes and the sand. A neutralizing circuit, as shown in Figure 3, was used to neutralize this D. C. potential. The other noise potentials were minimized by shielding all the wires. As far as possible everything was placed in a Faraday Cage to eliminate induced currents. Always the inflow end electrode was connected to a point at which the instrument ground was also connected. Only this point was grounded. This prevented unnecessary circulations of ground currents.

The third requirement was met by the use of reversible electrodes. Most of the early workers did not give sufficient attention to the problem of electrodes and used platinum electrodes as a medium of connection between voltmeter and the sand column. When the platinum electrodes were tried in running water and in a water sand column, no definite potential readings could be obtained. Several other kinds of electrodes were used, and similar behavior was observed. It became obvious that there is a 'flow potential' associated with the motion of electrolyte against the electrode which is indistinguishable from the streaming potential if irreversible electrodes are used. This phenomenon was observed by Gatty (1938), who called it the 'Motoelectric effect'.

Investigation shows that some reversible electrodes do not exhibit 'flow potential', whereas others, such as hydrogen-platinum electrodes or pH. sensitive glass electrodes, do exhibit flow potential.

Reversible electrodes of silver chloride were prepared as described below and are shown in Figure 4. Two pure silver foils of 6 cms long, and 0.3 cms wide were first cleaned with an abrasive cleaning compound and then washed with distilled water. Silver chloride was deposited on these foils by a single electrolysis using 0.75 normal solution of HCl. The coated foils were then cemented with beeswax into a small tube of 4 mm diameter filled with distilled water.

One end of the glass tube was drawn out to a capillary of 0.5 mm.

These electrodes were tested in a sample holder containing only running water, and no 'flow potential' was observed. Daily observation of these electrodes were taken in distilled water. A total drift of less than 1 mv was observed in 20 days. This low drift insured that true value of the potential would not be masked. A further test was applied, to check if the electrodes were reversible. In a small water-saturated sand box a very small current of few microamps, produced by a function generator, was passed through the current electrodes, and the potential was observed between these prepared silver chloride electrodes on an oscilloscope. No change in wave pattern of current and potential electrode was observed. This confirmed that the electrodes were reversible.

Thus, with all the above precautions, almost all the background potential was eliminated in order to record a real value of potential.

Model Experiments

Three different hydraulic models were used in the streaming potential measurements. (1) Channel flow was modelled in a rectangular box. (2) The flow tube simulated conduit or pipe flow. (3) A model tank of radial sector of $\frac{\pi}{3}$ radians simulated radial flow in an unconfined aquifer. They were constructed of non-metallic material in

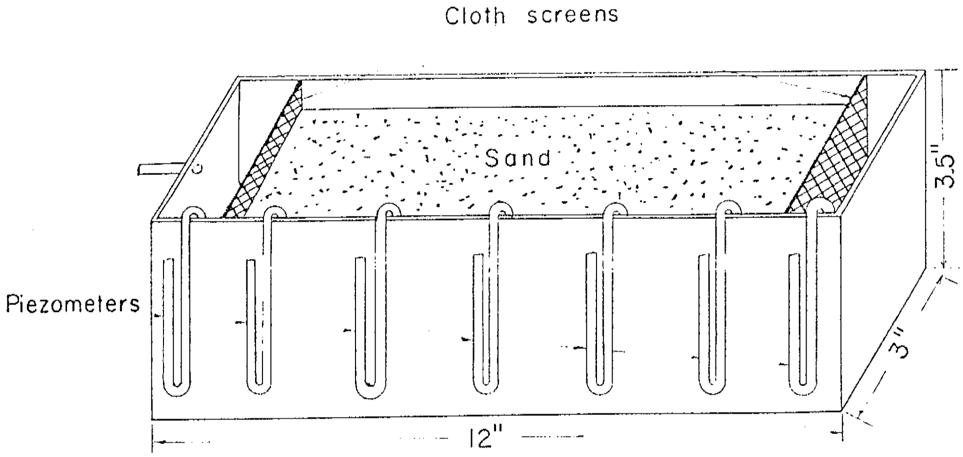

Fig. 5 - Rectangular Experiment Box

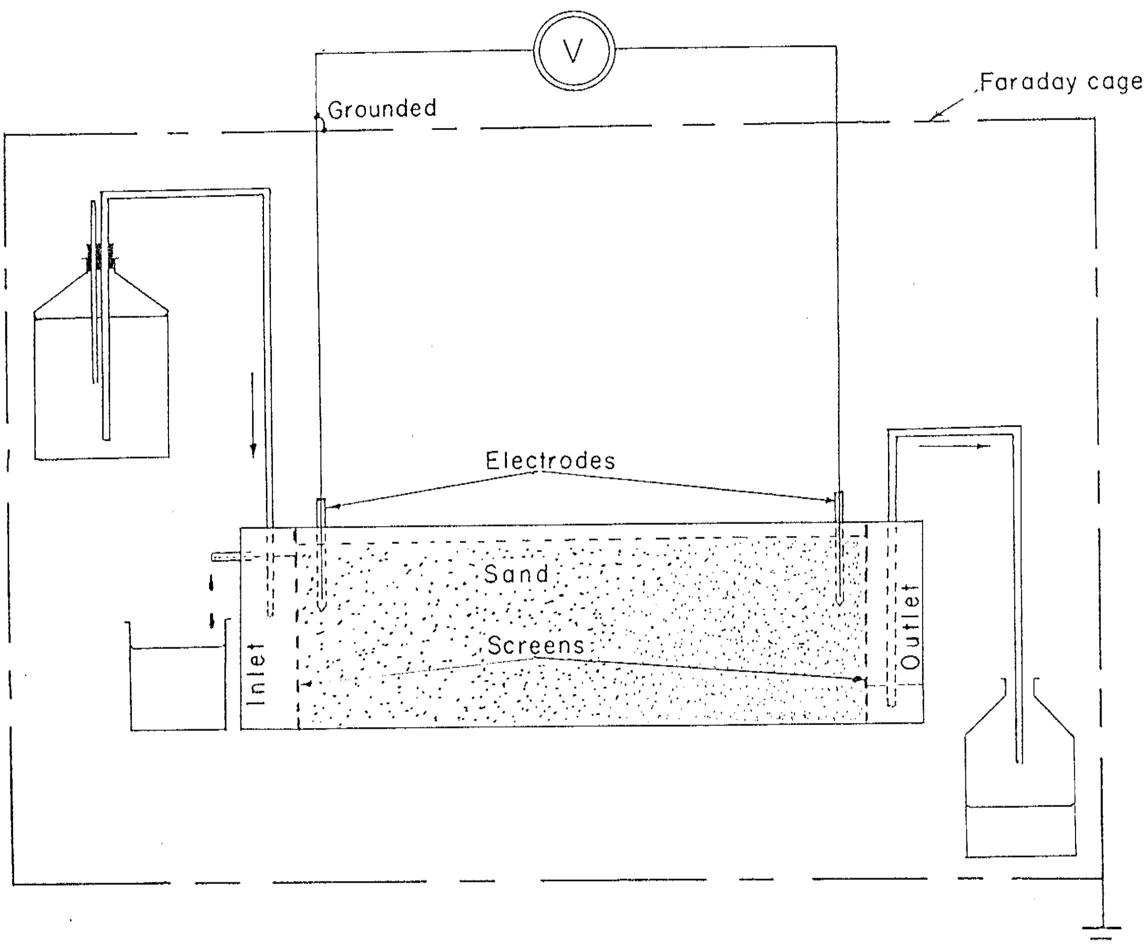

Fig. 6 - Schematic Representation of Experiment in Rectangular Box

order to eliminate electrochemical potentials.

Rectangular Box. The dimensions of the small rectangular Bakelite model were 30.5 cms long, 7.5 cms wide and 8.8 cms deep. Two cloth screens separated the box into three chambers of 3 cms, 25 cms, 2.5 cms. Several piezometers were fixed at equal intervals to obtain water level. Figure 5 shows a sketch of this hydraulic model. The middle chamber of the box was filled with 20 to 30 mesh pure silica sand, and the box was filled with distilled water of 2,700 ohm-meter resistivity. Observations were taken only after the electro-chemical equilibrium was attained between sand and water. Figure 6 illustrates the observational set-up. The head difference was produced across the sand filled chamber by siphoning water out of the outlet chamber. Voltmeter readings were noted before flow began. Constant head was maintained in the inlet chamber. Streaming potentials were recorded only after steady state conditions were reached. Then the outlet rubber tube was closed and water was allowed to come to its original equilibrium position. Streaming potential readings were accepted only when the voltmeter readings were the same before and after the flow measurements. The difference between the flow and no flow reading gave the value of the streaming potential for the values of head difference noted in the piezometers.

No difference in potential could be observed when both electrodes were placed together at either end of the sand water column.

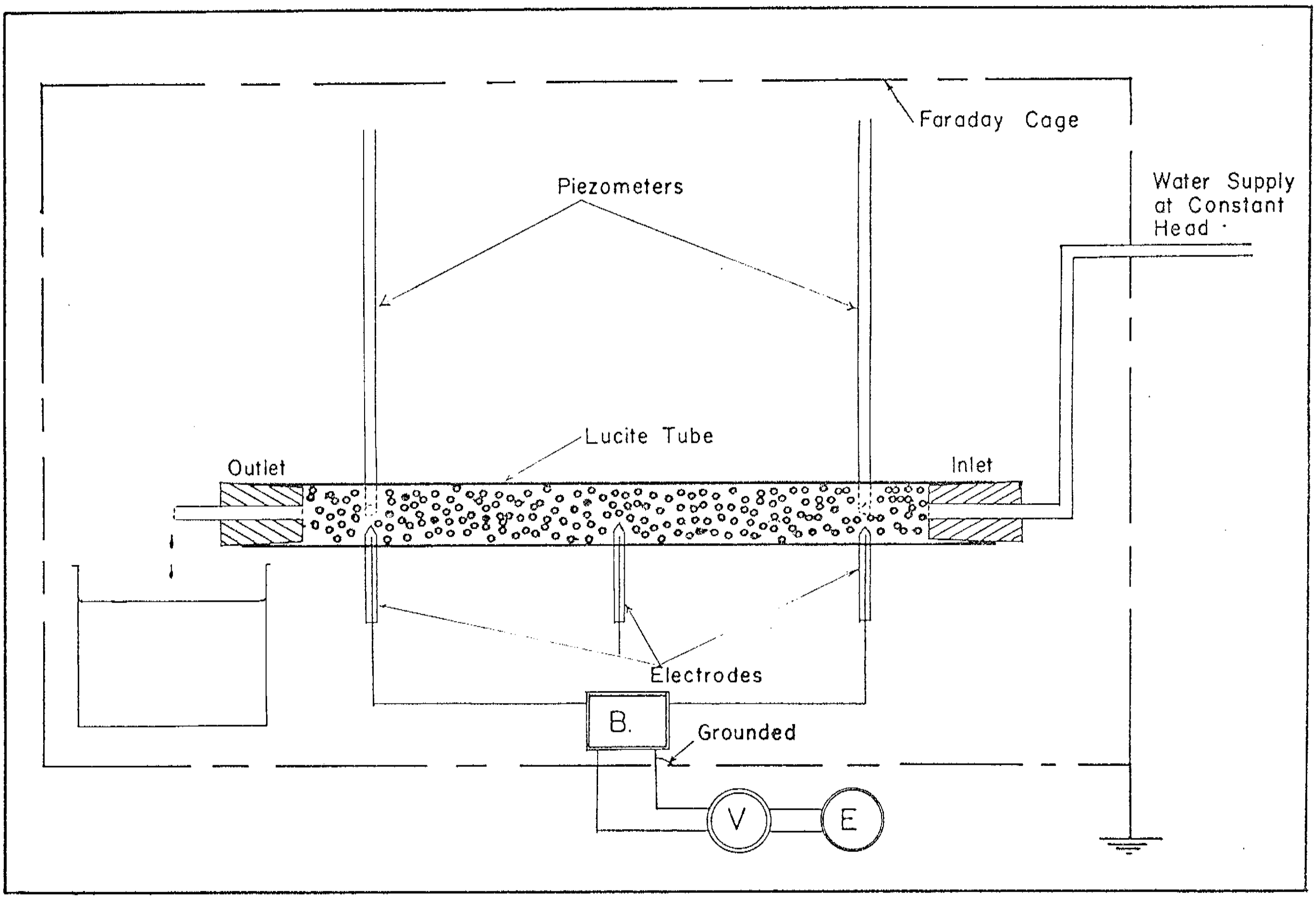

Fig. 7- SCHEMATIC REPRESENTATION OF EXPERIMENT IN FLOW TUBE

The observations were repeated for water resistivities of 1.250 ohm-meters, 680 ohm-meters, and 24 ohm-meters.

Flow Tube. The dependence of the streaming potential on grain size was studied in a Lucite tube of 1.6 cm diameter and 19 cm length (Fig. 7). Electrodes and piezometer tubes spaced 11.5 cms apart provided streaming potential and hydraulic head data. Two rubber corks at both ends kept the sand compactly in place. The observational procedure was the same as that in the previous experiments.

Tank. This tank was constructed of marine plywood of 60° sector, 30" by 30" (Fig. 8). No nails were used. Burlap was wrapped around a circular wooden frame to serve as a screen between sand and water reservoir. The tank was placed under compression by two iron frames and sealed on the inside with a coating of waterproof cement. 100 megohms resistance was measured between ground and the tank filled with water. Water level readings were taken on five tubes of laminated plastic (0.312" diameter) which served as observation wells. These were fixed at one side of the box and spaced at intervals of 2", 4", 8", 12", 16". These tubes were perforated all through and wrapped with cloth. A laminated plastic tube 34" long and 1.25" in diameter served as pumping well. The tube was perforated for a distance of 18" and wrapped with burlap. Due to a limited supply of pure silica sand only an 18" radius sector of the tank was used.

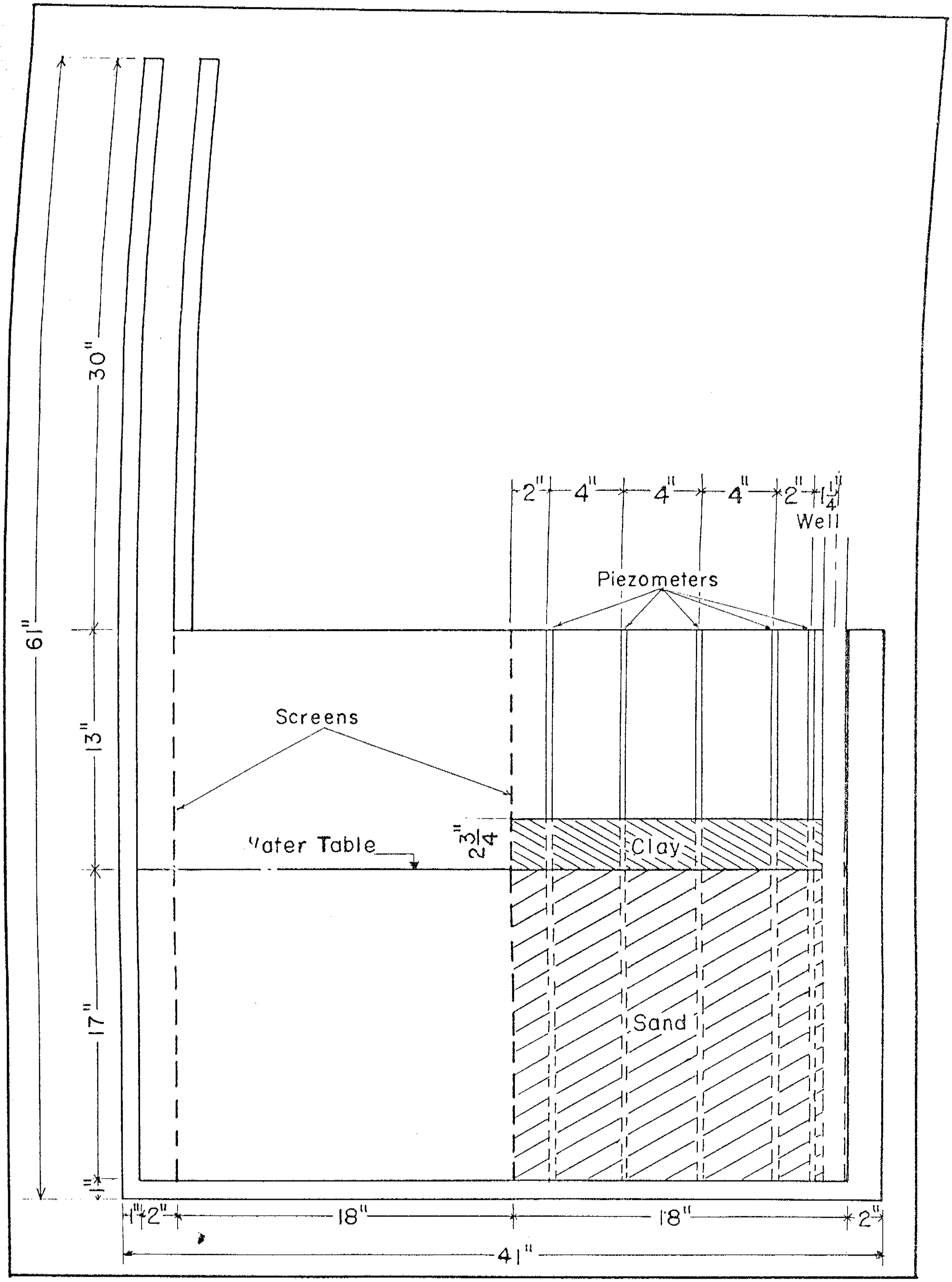

Fig. 8 . CROSS-SECTIONAL VIEW OF $\frac{\pi}{3}$ TANK

The first attempt to measure the streaming potential on sand of 80 - 120 mesh failed because the sand was of too small a permeability to permit water to flow through the pumping well at the allowable head differences across the sand section. Next the sand section was refilled with St. Peter Sand of 20 - 30 mesh in a compact bed 17" high. Electrochemical equilibrium condition was attained in the saturated sand section after several days of flushing water through the sand and pumping it out of the well. Streaming potentials were measured between two electrodes inserted about 3 cms into the sand adjacent to the screen and to the well. The observational set up is shown in Figure 9. A constant pressure head across the sand section was maintained during pumping by using two 5 gallon jars. One of the jars supplied water to maintain a constant level in the water reservoir behind the sand bed. The water was pumped into the water reservoir jar from the well by means of a siphon. The rate of flow of water through the sand section was varied by siphoning water from the pumping well through a range of tube sizes. By changing tube diameter from 0.25" to 0.65", the pumping rate was increased from 0.3 to 2.5 g.p.m. The four rates of discharges (produced by the siphon tubes of 0.25", 0.35", 0.5" and 0.65") resulted in an average head differences of 1", 4", 6" and 8" respectively.

The initial experiment failed to show any streaming potential with the electrodes placed at the surface of the sand, in spite of varying the pumping rate from minimum to the maximum. The failure to

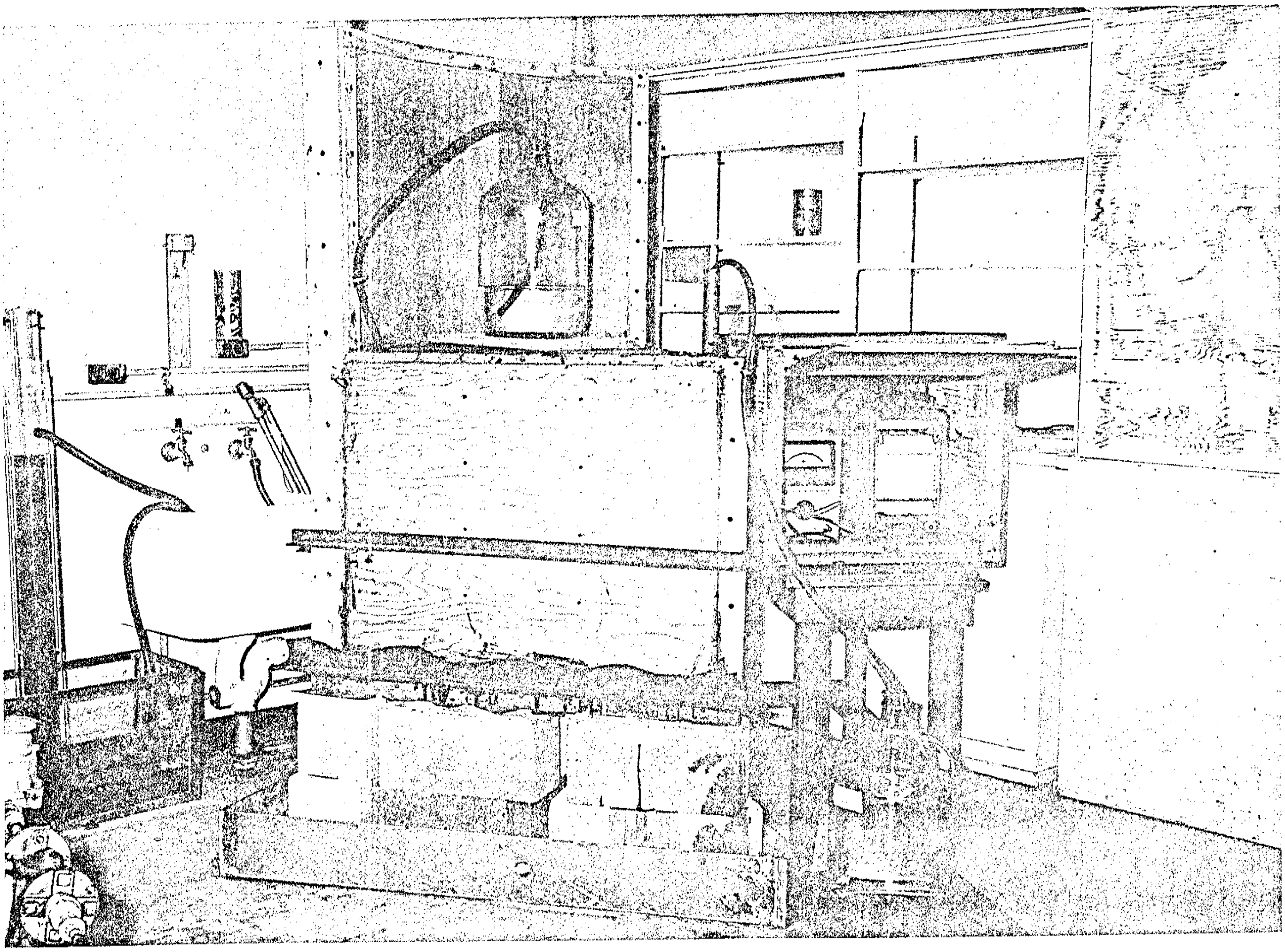

FIG. 9 EXPRIMENTAL SET UP
OF $\frac{\pi}{3}$ MODEL TANK

demonstrate measurable values of the streaming potential may be attributed to the change of environment of the electrode near the well. Upon pumping, as the water level lowered, the surroundings of the electrode changed and it polarized enough to reverse the sign of the streaming potential. The situation was corrected by burying the electrode near the well about nine inches beneath the sand surface. Before any observations were taken, a null point was obtained on the 1 mv scale of the voltmeter by balancing out spurious potentials with the neutralizing circuits. Next, potential drift was observed for five minutes, and then pumping was started. The water level was measured along the sand section by inserting wooden sticks into the observation wells near the electrodes. The sticks were coated with a water level indicator which turns red on contact with water. All measurements were taken with reference to the top of the box. Water level readings were noted approximately at the middle of pumping time on the assumption that the head difference remained more or less uniform during the duration of the potential measurements. The potential difference was recorded continuously with the Esterline-Angus recorder. The mean value of potential recorded was adopted as the representative value of the streaming potential. The experiment repeated without changing the water, until potential readings were recorded for a full set of pressure differences.

Under actual field conditions, the pumping well is cased and potentials can not ordinarily be measured by placing the electrode at the water level, but may be measured only at the ground surface. In order to simulate these field conditions in the laboratory and still maintain the surface electrode at stable equilibrium conditions during pumping, a clay layer of 2.75 inches thick (mostly kaolinite mixed with illite) was placed over the sand. The electrodes were placed inside the clay layer, one near the well and the other near the screen as before. The same technique of observation was followed as in the previous experiment. This procedure worked, but did not eliminate electrode drift. Potential readings were recorded only on flat portion of the drift curve.

PRESENTATION OF DATA

The experiment in the rectangular box provided data on the streaming potential as a function of head difference for four resistivities of water 2,700, 1,250, 680, 24 ohm-meters in 20 - 30 mesh pure silica sand. This data is presented in Table V and plotted in Figure 10. Figure 11 shows the ratio of V/P as a function of resistivities of water on semi-log paper — in other words, the effect of concentration of the electrolyte on the value of the streaming potential.

The flow tube experiment also supplied data on the streaming potential as a function of head difference for five different grain sizes. These results are presented in Table VI and illustrated in Figures 12 and 13. Table I shows the regular increase in value of V/P ratio with a decrease of grain size for a water of 24 ohm-meter resistivity.

TABLE I V/P RATIO AS A

FUNCTION OF GRAIN SIZES

Grain Sizes	V/P $\frac{mv}{inch}$
3 mm	0.10
20 to 30 mesh	0.13
35 to 42 mesh	0.14
80 to 120 mesh	0.16

Streaming potential was also measured for 6 mm and 3 mm glass beads, using distilled water of 5,600 ohm-meter resistivity. These values are plotted in Figure 13. In this case also the ratio of V/P increased regularly from 10 mv/inch for 6 mm glass beads to 28 mv/inch for 3 mm glass beads. A marked effect of the concentration of salts in the water is apparent on the value of the streaming potential for glass beads 3 mm size. The three natural pure silica sands which were used in this experiment were examined under the microscope in one field view. Their particle properties are summarized in Table II.

TABLE II MICROSCOPIC ANALYSIS

OF SAND GRAINS

Grain Size	20 to 30 mesh	35 to 42 mesh	80 to 120 mesh
Sphericity	0.7 to 0.9	0.7 to 0.9	0.3 to 0.5
Roundness	0.9	0.7 to 0.9	0.5 to 0.7
Texture	50% more frosted	80% more frosted	least frosted
Mineralogy	Pure quartz sands Impurities less than 1% of magnetite and pyrite	Pure quartz sands Very minor staining	Pure quartz sands Impurities less than 1%

Permeabilities were calculated for the four sizes of sand grains used in flow tubes by the following formula:

$$Q = K \frac{dP}{dl} \times A$$

Q = Quantity of flow　　　　dP = Head difference
A = Cross sectional area of sand　　dl = Length of the sand column
K = Permeability

The permeability figures are recorded in Table III.

TABLE III PERMEABILITIES OF SAND FOR FOUR GRAIN SIZES

Grain Sizes	Permeability
80 to 120 mesh	0.024 cm/sec
35 to 42 mesh	0.117 cm/sec
20 to 30 mesh	0.322 cm/sec
3 mm glass bead	8.767 cm/sec

In the $\frac{\Pi}{3}$ tank, two experiments were conducted as described in the previous section. The data are plotted in Figure 14 and presented in Table VII. For the single 17 inch thick sand layer, V was found to be proportional to, P, for the ranges of head differences used. The V/P ratio was 0.1 mv/inch for the single sand layer and 0.033 mv/inch for the sand-clay layer.

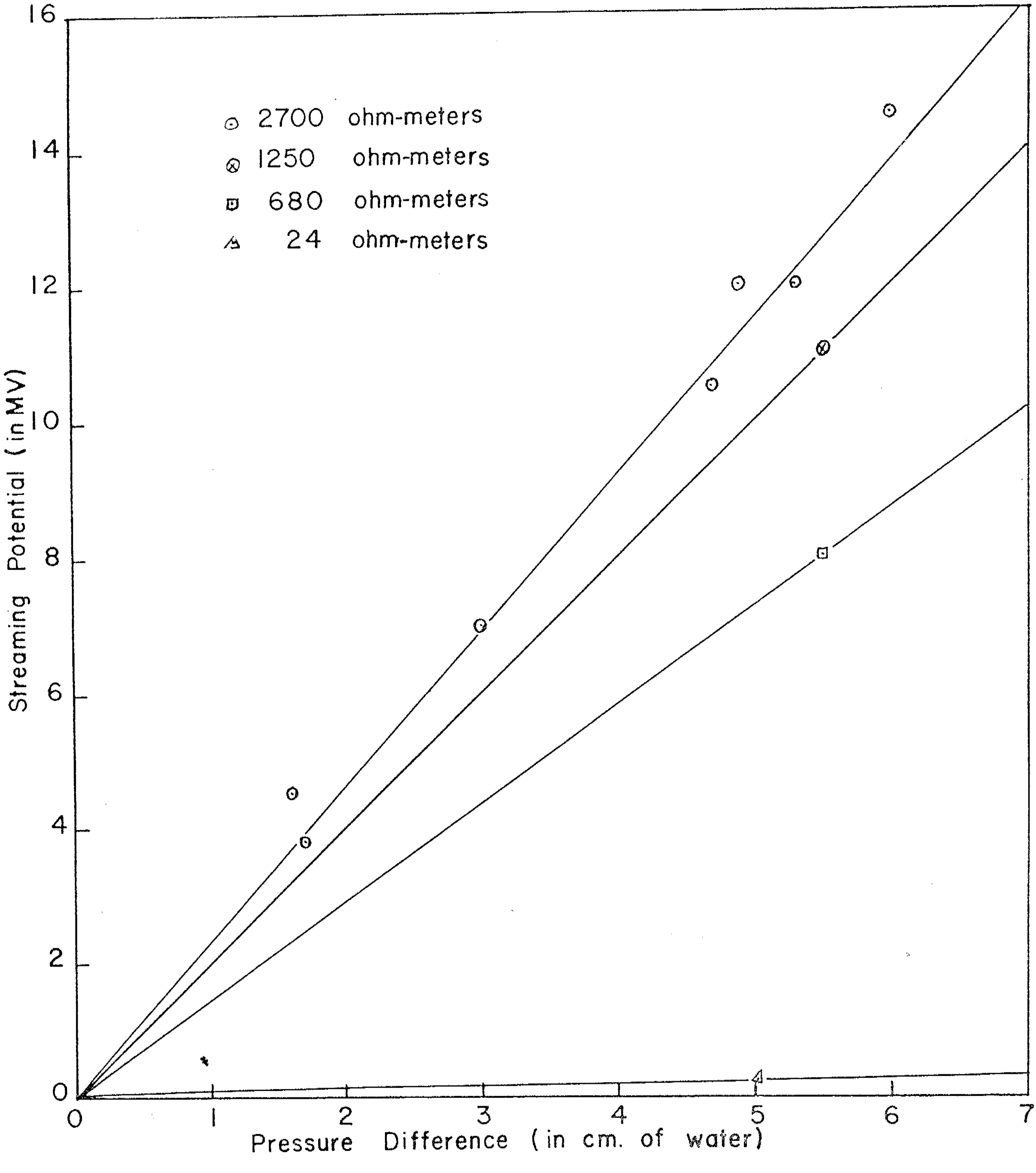

Fig. 10 - Streaming Potential as a Function of Pressure Difference for Four Resistivities of Water for 20-30 Mesh Sand

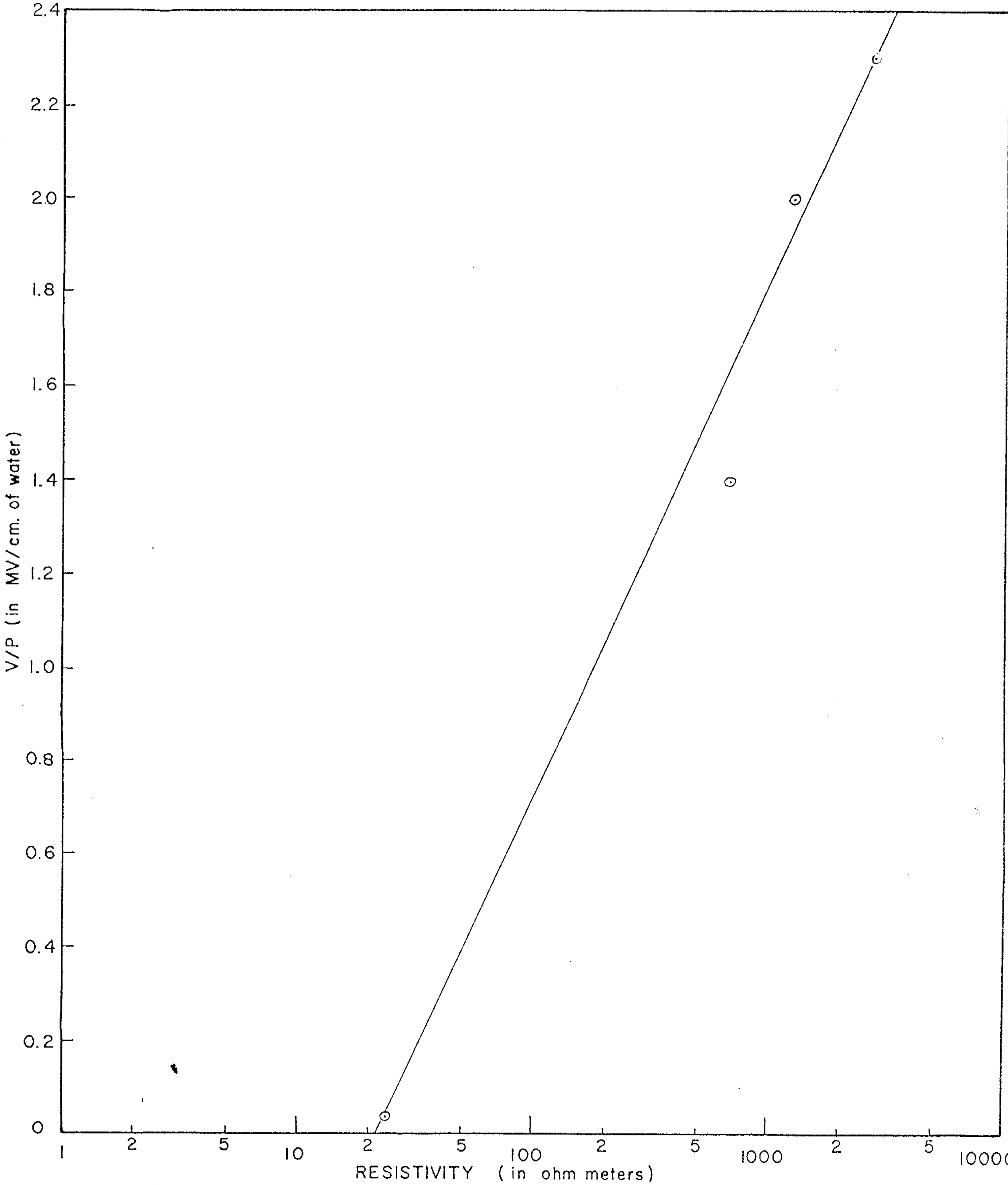

Fig. 11 - Effect of Resistivities of Water on Streaming Potential per Unit of Pressure Difference for 20-30 Mesh Sand

DISCUSSION OF THE RESULTS

The range of sizes of silica sands used in the streaming potential measurements in the hydraulic models was so chosen as to cover the range of grain size of sands generally found in the field.

The three different types of hydraulic models were selected to study the effect on the streaming potential of the different flow patterns which are controlled by the geometry of the model. The rectangular box provided unidirectional unconfined flow, the flow tube-unidirectional artesian condition, and the $\frac{\pi}{3}$ tank -- an unconfined aquifer representing radial flow. Using the same 20 - 30 mesh sand and 24 ohm-meter water, the V/P ratio was 0.10 mv/inch in rectangular box; 0.10 mv/inch in the $\frac{\pi}{3}$ tank but 0.13 mv/inch in the flow tube. This small difference may be attributed either to the geometry of the model which controls the flow pattern or to the difference of grain packing. In this connection it would be advisable to examine the fundamental equation of the streaming potential for geometrical effects. The fundamental equation is based on the concept of double layer, and as described previously, is equivalent to an electric parallel-plate condenser of separation l, each carrying a charge e coulombs per square cm. The difference of potential between the plates may be taken as the Zeta potential. If D is the dielectic constant of the medium between the plates, the Zeta potential can be expressed by:

$$= \frac{4 \pi e l}{D} \qquad (1)$$

This is the basis for the quantitative treatment of the streaming potential. If the flow of water is considered in a single capillary tube of small diameter, a double layer would be formed at the solid-liquid interface in this capillary tube. One side of the double layer is mechanically sheared by the flow of water. This zone of shear is shown in Figure 1. The positive charges are carried along with the liquid. As a consequence of this transport of charge, a streaming potential is generated. Using Poiseuille's Law for the flow of a liquid in a tube, the streaming potential may be expressed in terms of pressure as:

$$V = \frac{-P \zeta D}{4 k \pi \eta} \qquad (2)$$

P = Head difference
ζ = Zeta potential
k = Specific conductance
D = Dielectic constant
η = Viscosity of liquid

Earlier workers have abundantly verified this equation experimentally and found that it holds for capillary tubes as well as porous plugs. They have found the V/P ratio is independent of length and size of capillary tube, but only if the radius is not too small.

If, however, a porous plug is used, as in our sand models, equation (2) may still be applied from the basic consideration because the porous plug behave as a bundle of capillaries. This equation predicts that the streaming potential created by the applied pressure is independent of the length of flow and geometry of the sand model. This result is verified by all the different geometrical models used in this study. The wide range of geometrical sizes and proportions amongst flow tube, rectangular box, and model tank -- all of which produced almost the same values of the ratio of V/P -- is striking experimental evidence that geometry plays little or no part in the streaming potential generation.

The dependence of the streaming potential on head difference for different grain sizes was investigated using five different sands. Three natural grain sizes 20 to 30 mesh, 35 to 42 mesh, 80 to 120 mesh and artificial 3 mm spherical glass beads were used in the flow tube with water of the same resistivity under similar temperature conditions and for similar packing. These results are depicted in Figures 12 and 13 which show, V, as a function of, P, for each sand grain size. An inspection of Figure 12 indicates a small but definite increase in the ratio of V/P with decrease of the grain size. The spread between the 3 mm glass beads and the finest grain sand (80 - 120 mesh) is 0.06 mv/inch. The maximum observed drift is ± 0.04 mv in 3 hours during measurement, is not sufficient to explain this trend. There may be other factors not yet determined.

The three natural sands were examined under the microscope

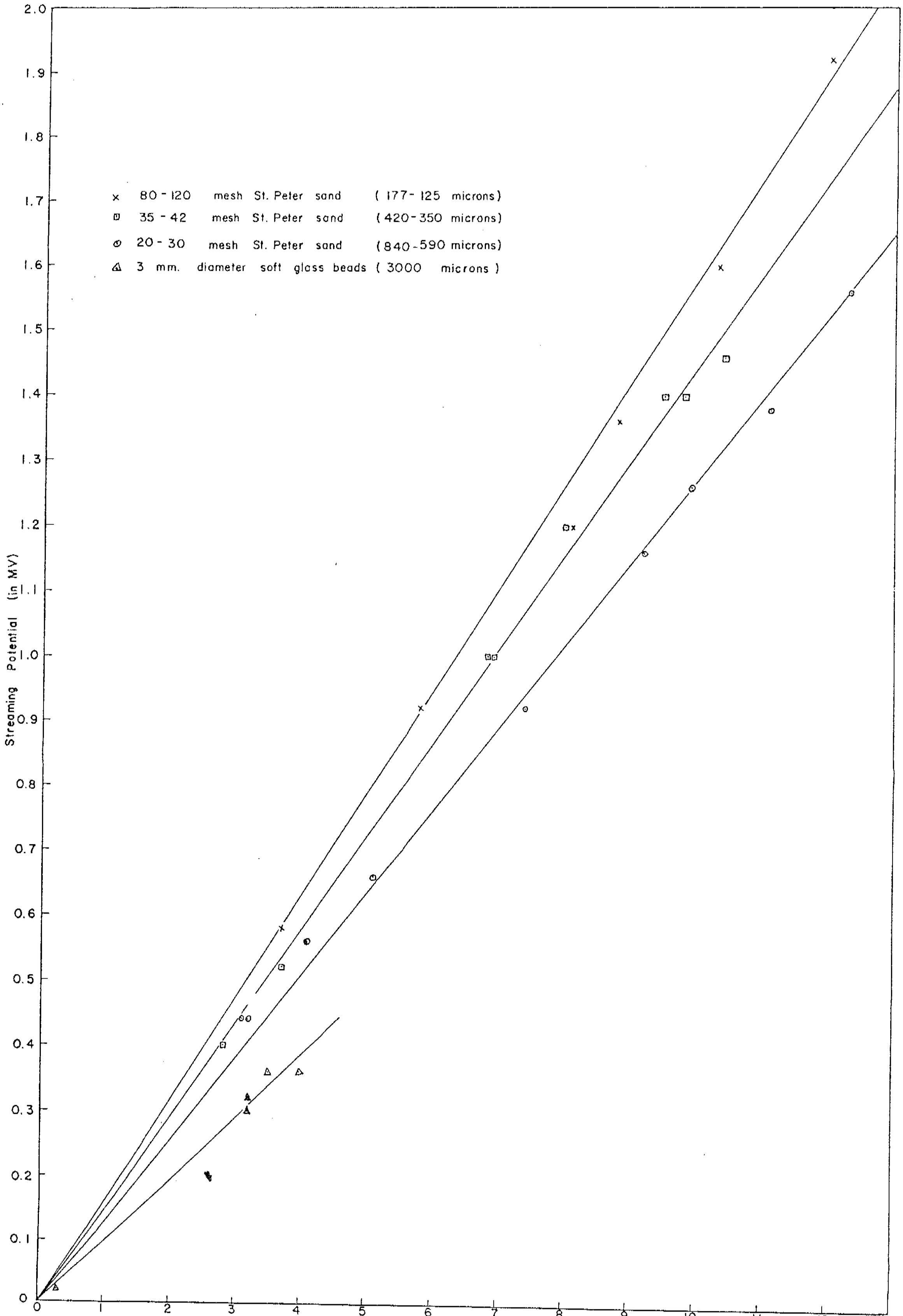

Fig. 12 - Streaming Potential as a Function of Pressure Difference for Four Grain Sizes and Water of 24 Ohm-m. Resistivity

for mineralogical and textural analysis, and the microscopic analysis, shown in Table II, did not indicate any obvious mineralogical difference. The only difference is in the size and in the surface area.

This increase in the V/P ratio with decrease in size of sand grain, contradicts the results found previously by Schriever and Bleil. They used four different grain sizes from 147 to 300 microns and distilled water, and found that V/P ratio greatly varied with the grain size, but decreased with the decrease of grain size. Bull (1932) reported the same trend, but found much smaller variation of V/P with size.

To study further the effects of grain size on the ratio of V/P, soft glass beads of 3 mm and 6 mm diameter were placed in the flow tube with distilled water of 5,600 ohm-meter resistivity. The ratio of V/P again may be observed to increase significantly with the decrease of the grain size (Fig. 13). However, there is much more scatter in the plotted data. V does not appear to decrease linearly with P. Similar observations have been reported in the past and during this investigation, but not explained. The scatter may be the result of the change in the resistivity of water with time. For the purpose of this investigation, a straight line variation has been assumed following previous investigators.

The discrepancy between this data and that reported by Schiever may be the result of the technique of measurement. The electrodes

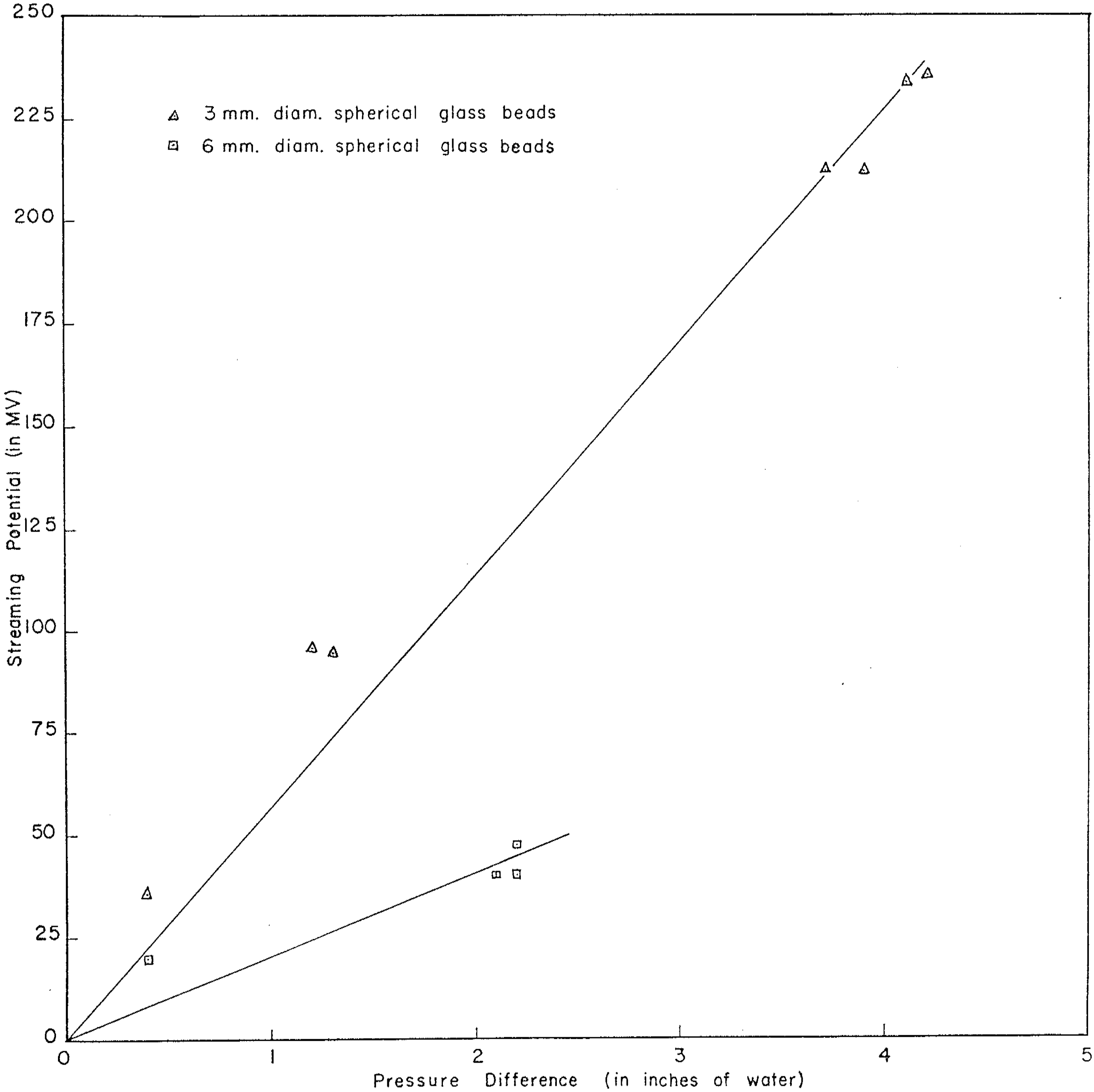

Fig. 13 - Streaming Potential as a Function of Pressure Difference for Two Sizes of Glass Beads for Distilled Water of 5600 Ohm-meters Resistivity

used by Schriever were bright platinum electrodes. As mentioned earlier, Zucker demonstrated that significant potential is developed on a bright platinum electrode simply because the flow of water at 1 cm^3/minute in an empty plug can produce as much as 50 mv of flow potential and may change the streaming potential significantly from the true value. Similar 'flow potentials' were observed during the early stages of the experiments with other nonreversible electrodes.

The variation of V/P shown with the grain size is partly supported by an equation derived from theoretical consideration. Zucker (1959) presented an equation for the Zeta potential in terms of bed diameter and grain size. He considered a cylindrical tube through which liquid was flowing at a distance r from the axis of the tube, and considered the tube filled with N sand particles and n channels of average length L and radius a. From a consideration of movement of charges in such channels, equating the surface area of the particle with the channel and pore volume of the bed (porosity x volume), he obtained:

$$\zeta = -925 \cdot \frac{V}{PR} \quad \frac{d^2}{(1-\varepsilon)^2 \; S^2} \qquad (3)$$

R = Resistance of the bed in ohms
ε = Porosity of sand
S = Diameter of the bed in cms
V = Streaming potential
P = Head difference in cms of mercury
ζ = Zeta potential
d = Diameter of the sand particles in cms

If an assumption is made that Zeta potential is constant for different grain sizes similar in all other characteristics, (3) shows that the streaming potential is inversely proportional to the square of the diameter of the particle. Consequently, where the flow tube experiment is used for different sands and all remaining parameters are constant, the streaming potential observed should follow equation (3). However, the variation in the streaming potential with plug diameter, S, implied under the assumption of constant Zeta potential in equation (3), was not found to hold.

Earlier workers found a contradictory decrease of V/P ratio with the decrease of grain size. In this connection, it would be of some interest to mention the reasons propounded by the earlier workers for the change of the ratio V/P with the grain size. As mentioned earlier, the streaming potential for a liquid flowing through a capillary tube is independent of the length and the radius of the tube, provided the radius is not too small.

TABLE IV STREAMING POTENTIAL AS A FUNCTION OF CAPILLARY SIZE

Diameter of the Capillary in mm	$\frac{V}{P}$ $\frac{mv}{cm\ of\ Hg}$
0.0964	31.0
0.0405	32.0
0.0390	31.0
0.0058	1.6
0.0055	5.8
0.0053	0.0

Table IV reproduced from the work of White, Urban, and Van Atta (1932) shows the effect of dimensions of the capillary on the streaming potential. These data indicate that the value of the streaming potential per unit head difference is independent of the size of the tube so long as the size is not too small. But when the capillaries are of a diameter of 0.0058 mm or less, the streaming potential was less measurable, and decreased to zero. Factors affecting the lower values of the streaming potential for the smaller capillaries are not yet known.

From the observation that the streaming potential is independent of the diameter of the capillary provided the size is not too small, one may suggest that V/P for a given sand column should also be independent of the diameter of the sand grains and of the length of the column. Actually, equation (2) predicts it. Bull (1932), first, noted the decrease in the V/P ratio with grain size for sand column. He tried to explain his results on the basis of the conductance of the liquid filled matrix. Later workers have tried to explain the decrease of V/P with grain size in terms of the variation with the capillary sizes of the Zeta potential, the surface conductance, and the viscosity. Schriever attempted but could not obtain a satisfactory explanation for the change on the basis of surface conductance. The explanations of various investigators would suggest that the V/P ratio should decrease with the decrease of the grain size, whereas in these experiments the reverse phenomenon is obtained.

If the experimental data of this study is accepted on its face value, a reasonable explanation to justify these results is proposed on the basis of a double layer model. Figure 1 illustrates the distribution of charges surrounding a negatively charged particle, such as quartz, which gives rise to an electric double layer. The double layer consists of three parts -- (1) negative charges on the surface of the particle which is a part of the lattice; (2) counter positive ions held close to the surface, which occupy a plane known as Stern plane, of only a few angstroms thick; (3) counter ions of a pair of negative and positive charges forming the diffuse ionic atmosphere (Gouy layer). More positive charges are found towards the surface of the particle due to the preferential attraction of the surface. The thickness of the diffuse ionic atmosphere is about 1,000 Å thick in 10^{-5} molar solution. The potential distribution is indicated in Figure 2. There is a sharp fall of potential in the fixed part (surface of the particle to the surface of shear) followed by a gradual change of potential of exponential type across the double layer. When water moves relative to the solid surface, shear takes place outside the Stern plane in the Gouy layer. The position of shear plane is probably determined by the diameter of the capillary formed by the sand grain particles. If the size of the sand grain is small, the size of capillaries formed is also small. For very fine grained sand, the increase of flow velocity because of constrictions in the flow path forces the shear zone further into the Gouy layer towards the solid surface, although the average velocity of flow of water is less in a column of fine grain sand than in a column of large grain sand. In the fine grain sand column, as shown in Figures 1 and 2, the non-linear character of the Zeta potential in the Gouy layer will produce a greater than average potential from a capillary of the same average diameter. The maximum number of charges would be stripped off and a consequently larger potential would be generated. But in case of large grain sand particles, where the capillary size is large, the surface of shear would move away from the surface of the particle. Keeping this picture in mind, one would naturally expect more streaming potential for a given pressure difference in small grain sizes. This physical model attempts to explain the results obtained in this experiment.

As stated earlier, the streaming potential is inversely proportional to the square of the diameter of the grain if all other particle characteristics are the same. Permeability also depends on the grain size, and if grain size decreases, permeability decreases. According to Jacobs (1946), this dependence may be represented as:

$$K = cd^2 \frac{\gamma}{u} \qquad (4)$$

K = Hydraulic conductivity
c = Dimension-less constant
d = Effective grain diameter
γ = Unit weight of fluid
u = Dynamic viscosity

Thus some relationship should exist between the streaming potential and the hydraulic conductivity. In fact, from (4) and (3) the relationship may be written:

$$V = \frac{\text{constant}}{K}$$

Permeabilities or hydraulic conductivities calculated from the flow tube experiment (Table III) were plotted against the ratio of V/P on ordinary graph, semi-log paper, and a straight line was obtained on log log paper (Fig. 15). The following empirical relationship was found:

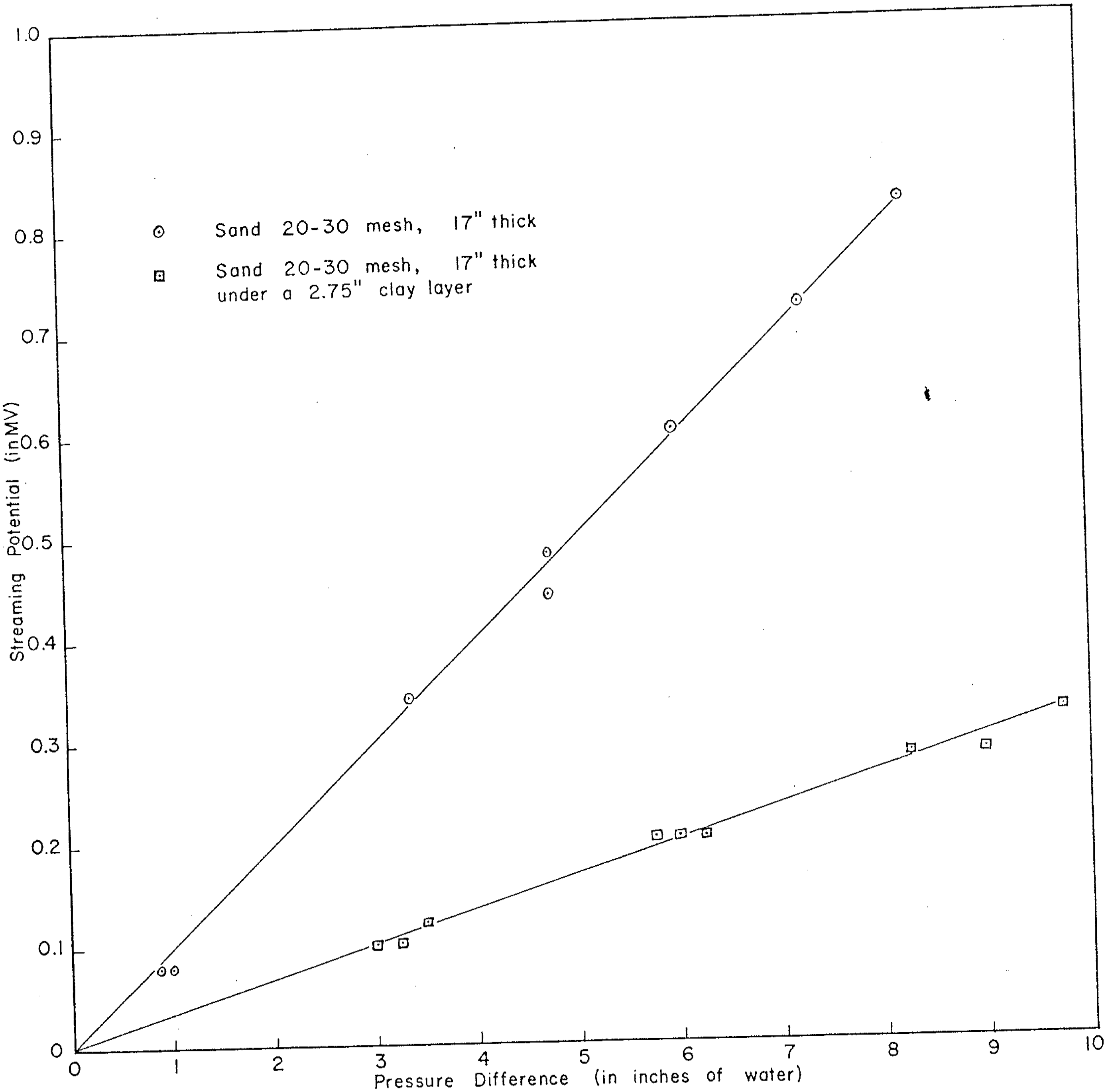

Fig. 14 -. Streaming Potential as a Function of Pressure Difference in $\frac{\pi}{3}$ Tank

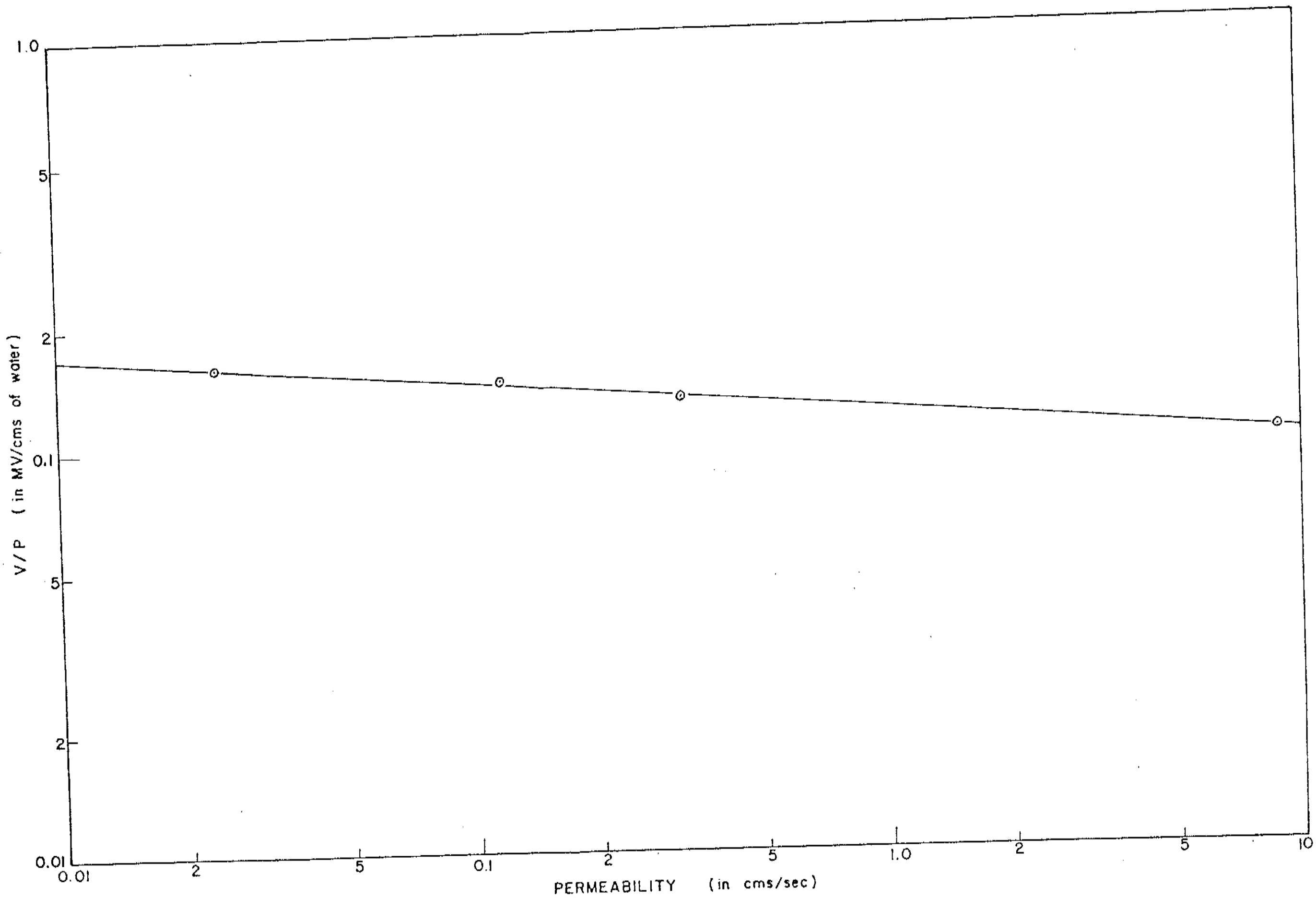

FIG. 15 - STREAMING POTENTIAL PER UNIT OF PRESSURE DIFFERENCE AS A FUNCTION OF PERMEABILITY FOR UNCONSOLIDATED SAND

$$V/P = 0.119\ K^{-0.077}$$

This empirical relation is in contradiction of $V = \frac{\text{constant}}{K}$ which was obtained on the assumption that $V \propto \frac{1}{d^2}$. Thus the variation of V is very far from it. An inspection of Figure 15 shows that the smaller the permeability, the greater the streaming potential. Permeability is the ease with which the water flows in a grain sand. In fine sand although molecular, capillary, and viscous forces slow the net flow of water, there is an increase of shear at zones of constriction in the capillaries. As discussed on page 38, the net result should be to increase the streaming potential. The relationship of V/P with permeability further supports the results of the previous discussion.

It is worthwhile to mention that the above relationship holds for permeabilities ranging from 0.024 to 8 cm/sec in pure silica sand. The scope of this relationship may not be extended beyond this experiment inasmuch as the mineralogical character of the sand used in the investigation was similar. The experiment does suggest that the variation of the streaming potential with the permeability is very small. As a result, the streaming potential measurements established by experiment will provide little clue to changes of permeability in the field.

Most of the previous work on the streaming potential was conducted with distilled water, and comparatively high values of the potentials were obtained. In the rectangular-box experiment, water of four different resistivities 2,700, 1,250, 680, and 24 ohm-meters was used with 20 -

30 mesh sand. The experiment was done for the purpose of observing the effect of the concentration of the electrolyte on the streaming potential. On the assumption that V/P varies with the logarithm of the resistivity of water Figure 11 was prepared. If the concentration of electrolyte increases, a decrease of the streaming potential results. As generally accepted, if the concentration of the electrolyte increases, the thickness of the double layer decreases primarily in the region of the diffuse Gouy layer. The Interionic Attraction Theory may be applied in explanation of the decrease of streaming potential with concentration of electrolytes. A salt dissolved in water is 100% ionized, but as the concentration increases, the conductance of the solution does not increase in the same proportion. This is explained by the fact that the increase in interionic attraction produces a decrease of ionic velocities.

McCardell (1953) has explained the abnormal conductivities found in certain shaly oil reservoir rocks on the basis of excess double layer conductivity, even though the mobilities of the positive ions in the double layer are not as high as those in the free solution.

The more dilute the solution, the greater the excess of positive ions in the diffuse zone of the double layer relative to those in the bulk solution. When the concentration of ions in the double layer is high in comparison to the number of ions in the remainder of the pore zone, the charge separation that results in the shear zone of the fluid adjacent to the solid surface produces a maximum value of the streaming potential.

This maximum value of the streaming potential occurs because the shorting action by the free ion in the bulk solution is relatively small. In other words, the more electro-negative the solid surface and the higher the ionic concentration in the shear zone relative to that in the free solution, the greater the streaming potential that will be produced by a unit head difference.

The previous discussion of the data was based on the results of the model experiments. Before an attempt is made to predict the value of the streaming potential in the field, the experimental results of the tank should be examined minutely. The model represents hydrologically a water table aquifer in which the well is fully penetrating the total depth of the aquifer. The first experiment simulates an unconfined aquifer overlying an impermeable layer. Electrically, it compares to a single resistive sand layer overlying a highly resistive layer. The second tank experiment represents a top conductive clay layer overlying a permeable resistive sand layer which, in turn, overlies a highly resistive layer.

In the first case, one electrode near the well was buried 9" inside the sand layer and the other on the surface near the screen. In the second case, both electrodes were placed in the top clay layer and disposed in the same vertical line through the sand section in each case. In both cases, average values of the hydraulic head were measured in the observation well. For a single sand layer, the V/P ratio was 0.1 mv per inch, but decreased to 0.033 mv per inch for the clay-sand layer.

Three reasons may be assigned to the decrease of the V/P ratio for the second case.

The most probable cause is the head distribution found in case of radial flow as illustrated by section in Figure 16. In case of a water table or free aquifer well, pumping causes a lowering (cone of depression) of the water table in the vicinity of the well. The bottom streamlines or flow lines are horizontal, but at higher elevation they slope towards the well. The lines of equihead, shown in the Figure 16, are curved lines perpendicular to the flow lines. The position of the electrodes with respect to the equihead lines are also shown in Figure 16. For a given hydraulic head surface the two electrodes are on two different equihead lines.

The electrical response to this hydrological condition, that is, the equipotential lines for the streaming current, is given in Figure 17. As the streaming potential is directly proportional to the head difference, the equihead lines (Fig. 16) may also be represented as equipotential lines shown in Figure 17. The potential difference measured with electrode in top position is less than that measured when electrode is in lower position.

The second cause of the decrease of potential measured is the higher resistance in the sand layer (Fig. 17). Due to the pumping of

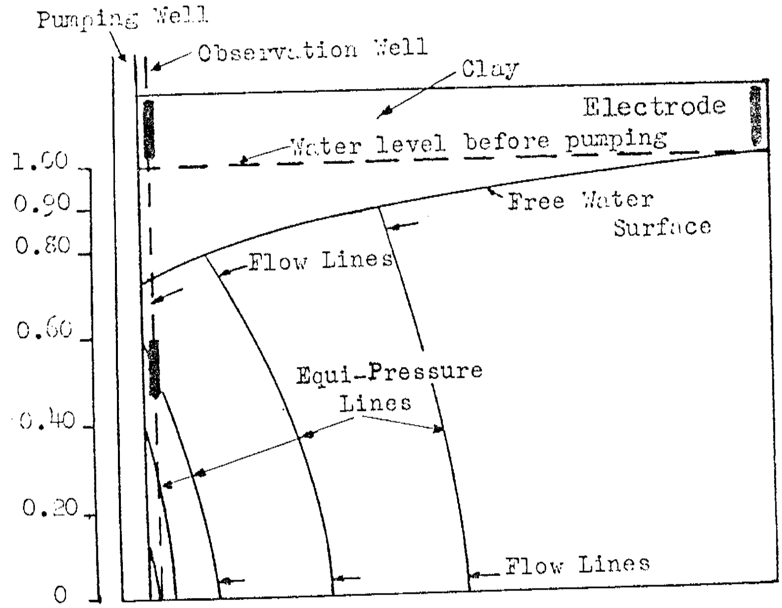

Fig. 16- Pressure distribution around a pumping well in an unconfined aquifer. (after Boulton)

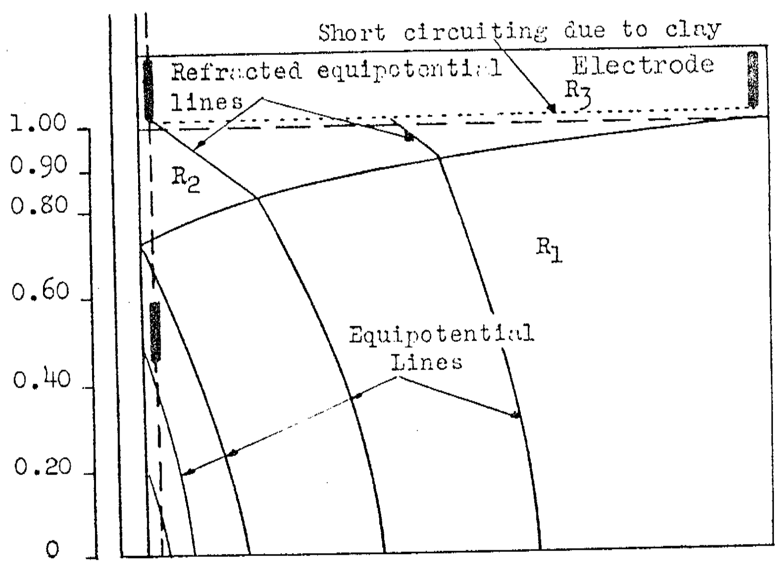

Fig. 17- Equipotential lines for streaming potential in an unconfined aquifer.

water, the water level is lowered and water saturation of the sand is decreased. The void spaces in the sand fill with air, increase in the resistivity of the sand. Thus a potential drop can be expected, as the equi-potential lines are refracted away from the normal, as shown in Figure 17. R_1 is less resistive than R_2 because the sand remains saturated.

A third cause results from the shorting effect of the clay layer. The clay layer of resistivity R_3 is less resistive than R_2. It acts as an external electrical load to the flow circuit and reduces the value of the streaming potential.

In consideration of these reasons, a decrease must occur if the electrodes are placed at the surface. It is thus clear that the V/P ratio obtained in case of the double layer suggests only qualitatively the decrease of potential to be expected in the field. But the model measurements in the sand may be used for some guidance.

Suppose there is an homogeneous unconfined aquifer of pure silica sand of mesh 20 - 30, overlying a very resistive bed, underlying a thin clay layer, and saturated by a water of resistivity of 24 ohm-meter, with the initial water level very close to the surface. Assume it to be pumped by a fully penetrating well having a head difference of 20 feet. It two electrodes are placed at the water level, and if the head difference between the electrode position is 20 feet, a streaming potential of 24 mv may be expected. This potential is estimated on the basis of the constancy of the V/P ratio and on independence of the streaming potential of the thickness of an aquifer. For an aquifer of 80 to 120 mesh, 32 mv streaming potential would be generated by the same head difference. This analysis assumes ideal sand conditions -- an aquifer of pure silica sand and a water table very close to the surface within a few feet. However, under actual field conditions, an aquifer is not usually homogeneous, and does not consist of pure silica sand; nor is there always a very high resistive bed beneath it. Often a clay layer is likely to be found. It would be logical, therefore, to say that the streaming potential values should be lower than the values obtained in the above analysis. In addition, on the basis of the data from the clay-sand section, if the potential is measured at the surface, a further decrease by a factor of at least three in these values may be expected. It is not possible to predict how much actual decrease may be expected. Only an upper limit may be predicted on the basis of the ratio of V/P from the flow tube experiment for the three natural sand grains.

SUMMARY AND CONCLUSIONS

The investigation of the streaming potential under the different conditions presented with models, gives rise to the following conclusions:

1. Measuring technique is of vital importance in the streaming potential determination.
2. In every case, the streaming potential, V, is found to be directly proportional to the pressure difference, P.
3. The geometry of the model which controls the flow pattern does not materially affect the generation of the streaming potential.
4. The greater the size of the grain, the smaller the streaming potential, an observation which contradicts the results reported earlier, but does agree with a theoretical solution based on grain size proposed by Zucker. A possible explanation based on double layer model is suggested.
5. A very slight increase in the streaming potential value is indicated with a decrease of permeability.
6. The greater the concentration of the electrolyte, the smaller the streaming potential. Saline water in an unconfined aquifer will generate almost no streaming potential.
7. Finally, a thorough analysis of all the data and utilization of the above conclusions show that extremely low field values of

the streaming potential may be expected in the vicinity of a pumping well. Qualitative information obtained from such measurements should be treated with great caution.

TABLE V DATA FROM EXPERIMENT

USING RECTANGULAR BOX

Data from Rectangular Box

Length = 30.5 cms
Breadth = 7.5 cms
Depth = 8.8 cms

Length of the sand column = 25 cms
Electrode resistance = 3 megohms

Sand 20 to 30 mesh

Head difference in cms of water	Q. Volume of water in cc per min.	Streaming potential in mv	Resistivity of water in ohm-meters
1.6		4.5	
1.7		3.8	2700
3.0	186	7.0	
4.7		10.5	
4.9		12.0	
5.3		12.0	
6.0		14.5	
5.5		11.0	1250
5.5		8.0	680
5.0		0.2	24

TABLE VI DATA FROM EXPERIMENT

USING THE FLOW TUBE

Flow tube data

Length of the tube = 19 cms   Electrode resistance = 300 K-ohms
Electrode distance = 11.5 cms   Electrode drift in 3 hours = ±0.04 mv
Radius of the tube = 0.8 cms   Piezometric distance = 11.5 cms
Resistivity of water = 24 ohm-meter

A. Sand 20 to 30 mesh

Head difference in inches of water at the electrodes 11.5 cms apart	Q. Volume of water in c.c. per min.	Streaming potential in mv
3.1	28	0.46
3.2	27	0.44
4.1	37	0.56
5.1	45	0.66
7.4	63	0.92
9.2	76	1.16
9.9	83	1.26
11.1	89.5	1.38
12.3	101	1.56

B. Sand 35 to 42 Mesh

Head difference 11.5 cms apart	Q. Volume	Streaming potential
2.8	9.3	0.40
3.7	12.3	0.52
6.8	20.3	1.00
6.9	20.0	1.00
8.0	24.0	1.20
9.5	28.0	1.40
9.8	29.3	1.40
10.4	32.0	1.46

TABLE VI (cont'd)

C. Sand 80 to 120 mesh

11.5 cms apart		
3.7	2.3	0.58
5.8	3.3	0.92
7.8	4.6	1.00
8.1	5.3	1.20
8.8	6.0	1.36
10.3	7.5	1.60
12.0		1.92

D. Glass beads 3 mm diameter

11.5 cms apart		
0.3	425	0.02
3.2	432	0.30
3.2	475	0.32
3.5		0.36
4.0		0.36

E. Glass beads 3 mm diameter
Distilled water resistivity 5600 ohm-meter

11.5 cms apart		
0.4	108	36
1.3	216	96
1.4	219	96
3.7	488	216
3.9	492	216
4.2		234
4.3		236

F. Glass beads 6 mm diameter
Distilled water resistivity 5600 ohm-meters

11.5 cms apart		
0.4	296	20
0.4	296	20
2.1	950	40
2.2	950	40
2.2	960	48

TABLE VII MODEL

EXPERIMENT DATA

Model tank of 60° sector
Sand length = 18 inches
Sand thickness = 17 inches

Well diameter = 2 inches
Piezometers at 2", 4", 8", 12", 16",
Electrode resistance = 300 K-ohms

Sand 20 to 30 mesh

Diameter of siphon tube in inches	Pumping rate	Depth of water at the radius of influence in inches	Depth of water at 2" well in inches	Draw-down in inches	Streaming potential in mv
0.25	0.32	16.75	15.75	1.00	0.08
0.25	0.33	17.00	16.125	0.875	0.08
0.375	0.92	16.75	13.275	3.375	0.34
0.50	1.33	16.75	12.0	4.75	0.44
0.50	1.32	16.75	10.0	4.75	0.48
0.625	1.40	16.0	10.0	6.00	0.60
0.625	2.09	17.0	9.75	7.25	0.72
0.625	2.30	17.25	9.00	8.25	0.82

Clay 2.75 inches thick over 17 inches sand

Diameter of siphon tube in inches	Pumping rate	Depth of water at the radius of influence in inches	Depth of water at 2" well in inches	Draw-down in inches	Streaming potential in mv
0.375	0.89	17.125	14.125	3.00	0.10
0.375	0.93	17.25	13.75	3.50	0.12
0.375	0.84	17.25	14.0	3.25	0.10
0.50	1.31	16.50	10.75	5.75	0.20
0.50	1.35	16.50	10.25	6.25	0.20
0.50	1.33	16.50	10.5	6.00	0.20
0.625	1.80	17.75	9.5	8.25	0.28
0.625	1.89	17.50	9.0	8.50	0.28
0.625	2.17	17.25	7.5	9.75	0.32

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This thesis is accepted on behalf of the faculty of the Institute by the following committee:

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