A PROBLEM TO CHECK AND DEVELOP THEORETICAL RESISTIVITY CURVES IN THE LABORATORY WITH A MODEL TANK

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for

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INTRODUCTION

Theoretical resistivity curves for two layers have been worked out by Roman[1] and for three-layers by Wetzel and McMurry[2] but the extension of an analytical solution of these curves to four or more layers involves mathematics so tedious that no attempt has been made to develop a set of theoretical curves for the purpose of interpretation of field curves. This problem was an attempt to check two and three-layer resistivity curves in the laboratory using a model tank and to extend the solution to four or more layers.

Probably the greatest problem encountered with experiments of this kind is to find suitable materials for the various layers so that the thickness and resistivity of each layer can be varied at will. It is not only essential that there be a sharp, clear cut boundary between layers, that each layer in itself be homogeneous and electrically isotropic but that there be a wide variation in resistivity between layers.

In this problem sand was used entirely as a base material. At first lamp black was mixed with the sand to lower the resistivity. Considerable time was spent

1. Roman, Irwin, Some Interpretations of Earth Resistivity Data: A.I.M.M.E., Vol. 110, pp 183-200, (1934)

2. Wetzel, W. W. and McMurry, H. V., A Set of Curves to Assist in the Interpretation of the Three Layer Resistivity Problem: Geophysics, Vol. 4, Oct. (1937)

working out the variation of resistivity with density and the effects of compaction. However when the lamp black and sand mixture was placed in the model box the results were not as anticipated.

The results indicate that the lamp black absorbed enough moisture from the air to form a highly conductive layer at the surface. Each of the curves has the appearance of a two-layer curve, the top layer being a thin highly conductive layer overlaying a highly resistive layer. At this point the lamp black and sand was abandoned and the problem was finished using sand and water. The resistivity of the sand and water was varied by varying the amount of water in the sand and by adding salt to the water.

Four curves were plotted using sand and water. The two-layer curves agree fairly well with the theoretical curves but the three-layer cases show considerable error.

The following conclusions were reached:

1. A lamp black and sand mixture is entirely unsatisfactory material to use for different layers.
2. The model box needs to be larger and especially deeper.
3. Sand and water, although giving fair results in some cases, are not entirely satisfactory materials to use for the different layers.

4, Very sensitive instruments are needed in experiments of this kind.

5. Although variations in ambient temperatures probably cause the largest variations in resistivity, the results indicate that very small currents should be used to avoid changes in resistivity due to temperature.

6. It is possible that the box used in this problem could be used to duplicate two-layer curves but the accuracy of a three-layer curve would be doubtful.

7. The results indicate that if the dimensions of the box were to be increased, so that the electrode spacing and depth of the upper layers could be increased, that better results could be obtained.

RESUME OF THEORY

The electric potential at a point on the surface of a homogeneous isotropic medium due to a point source placed at the surface as given by Ehrenburg and Watson[1] is:

$$V = \frac{\rho I}{2\pi R}$$

where R is the distance from the point to the source of current I,

3, Ehrenburg, D. O. and Watson, R. J., Mathematical Theory of Electrical Flow in Stratified Media with Horizontal, Homogeneous and Isotropic Layers: A.I.M.E., Vol. 97, pp 423-442, (1932) and ρ is the resistivity of the medium.

For the Wenner[4] configuration (Figure I), where C_1 and C_2 are the current electrodes and P_1 and P_2 are the potential electrodes, the four electrodes are placed in a straight line at equal distances from one another. Current is passed through the earth from C_1 to C_2 and the potential is measured across P_1 and P_2.

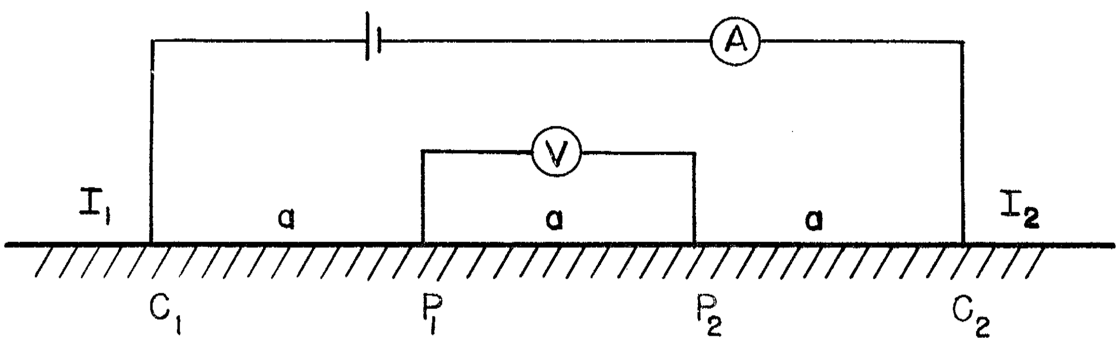


FIGURE I. WENNER CONFIGURATION

From the above equation the potential at P_1 due to current I_1 and I_2 is:

$$V_1 = \frac{\rho I}{2\pi}\left[\frac{1}{a} - \frac{1}{2a}\right] = \frac{\rho I}{4\pi a}$$

The potential at P_2 due to I_1 and I_2 is:

$$V_2 = \frac{\rho I}{2\pi}\left[\frac{1}{2a} - \frac{1}{a}\right] = -\frac{\rho I}{4\pi a}$$

4. Wenner, F., A Method of Measuring Earth Resistivity. U.S. Bur. Stds., Sci. Paper 258, pp469-478, (1915)

The difference in potential E which can be measured with a voltmeter or potentiometer is $V_1 - V_2$. Hence

$$E = V_1 - V_2 = \frac{\rho I}{2\pi a}$$

and transposing

$$\rho = \frac{2\pi a E}{I}$$

Since the potential E and the current I can be measured, the value of ρ can be calculated. However the above equation assumes a homogeneous isotropic medium which is seldom if ever encountered in the field. Thus the computed value of ρ is an apparent resistivity ρ_a rather than a true resistivity, that is, a weighted average of the various true resistivities of the soil strata.

In the paper by Wetzel and McMurry[5] theoretical curves for three-layer cases have been evaluated so that field curves may be interpreted by matching them with the theoretical curves. The values for the theoretical curves were evaluated with a Differential Analyzer using Slichter's[6] expression for the potential ϕ at a distance r from a disk electrode of radius b on the surface of the earth for any number of layers. (Figure II)

5. Wetzel and McMurry: op. cit.

6. Slichter, L. B., The Interpretation of the Resistivity Prospecting Method for Horizontal Structures: Physics: Vol. 4, (1933)

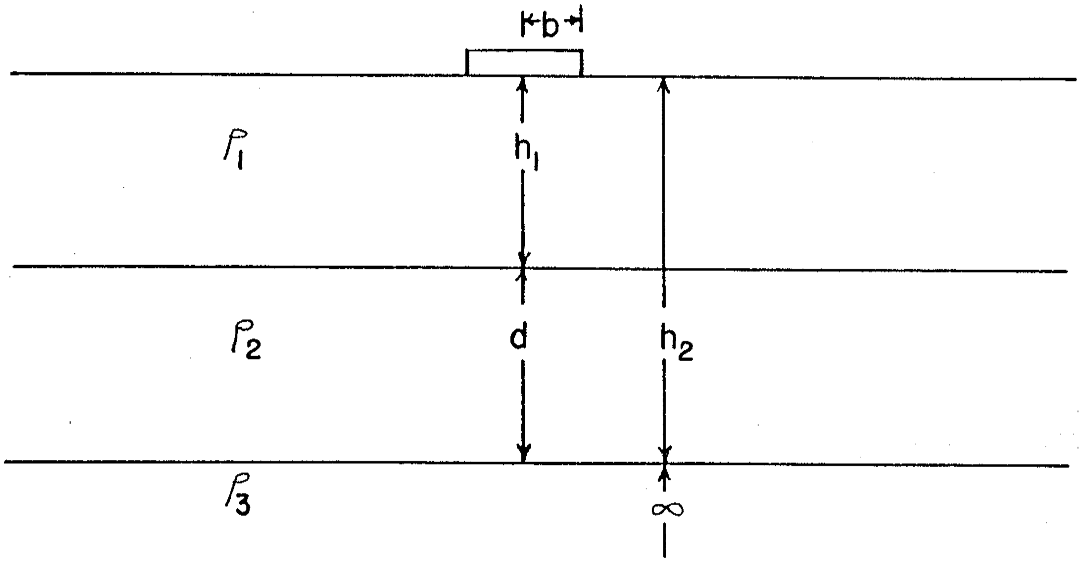


FIGURE II. LAYER THICKNESS FOR THREE-LAYER CASE

The equation for ϕ is:

$$\phi(r) = \frac{C\rho_1}{2\pi}\left[\frac{1}{r} + \int_0^\infty \frac{\sin\lambda b}{\lambda b} J_0(\lambda r)\; 2B_1(\lambda)\,d\lambda\right]$$

where

ρ_1 is the specific resistivity of the upper layer

C is the current

$J_0(\lambda r)$ is the Bessel function of zero order

B_1 is a function of the constants of the multiple layer earth under the electrode.

For the three-layer case shown in Figure II, assuming homogeneous uniform layers B_1 is given by

$$B_1(\lambda) = \frac{k_1 e^{-2\lambda h_1} + k_2 e^{-2\lambda h_2}}{1 - k_1 e^{-2\lambda h_1} - k_2 e^{-2\lambda h_2} - k_1 k_2 e^{-2\lambda d}}$$

where

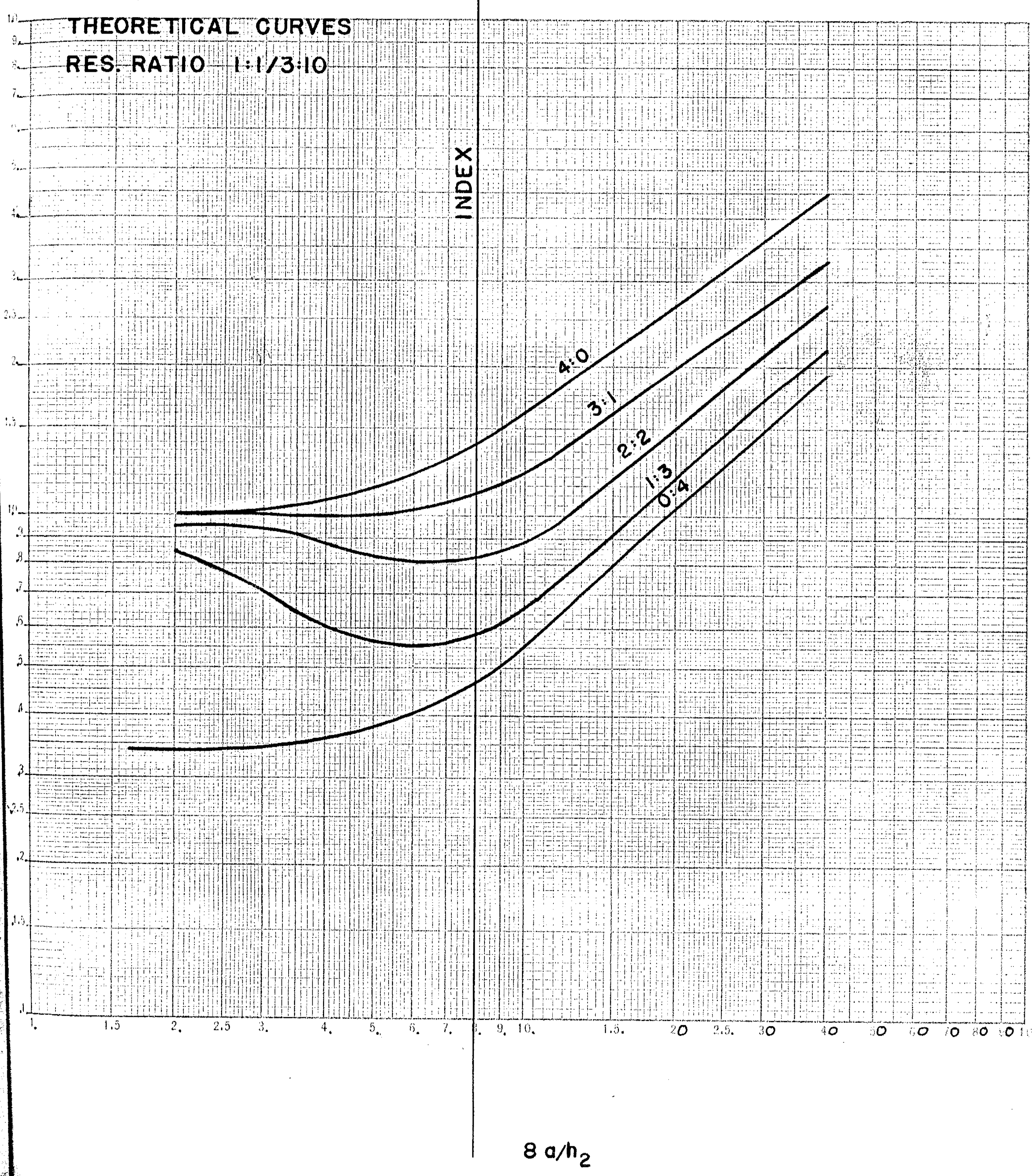
$$k_1 = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} \qquad k_2 = \frac{\rho_3 - \rho_2}{\rho_3 + \rho_2}$$

The relationship between the apparent resistivity and the potential ϕ is given by

$$\rho_a = \frac{2\pi a}{C} \left[2\phi(a) - 2\phi(2a) \right]$$

An example of these curves is shown by Plate 1. The apparent resistivity ρ_a is plotted as the ordinate and $8a/h_2$ as the abscissa. The labels shown on the curves are the ratios of h_1 to h_2 and those at the top of the page are the resistivity ratios as shown in Figure II. The curves are plotted on log log graph paper so that different field units may be used. The effect of changing units only moves the curve parallel to itself and has no effect on the shape of the curve. The abscissa is plotted as $8a/h_2$ as multiplying the ratio a/h_2 by a constant has no effect on the curve except to shift it one way or the other. Therefore the index of these curves is placed at 8, where at this point on the curve a/h_2 is equal to 1, that is, the electrode spacing is equal to the depth to the boundary between the second and third layer. When a field curve is matched with a theoretical curve, the point on the field curve which coincides with the index of the theoretical curve cor-

PLATE I



responds to h_2, the depth to the boundary of the second and third layer. If the field curve is displaced parallel to the abscissa, the resistivity ρ_1 is read from the point on the field curve that coincides with the value of 1 on the ordinate of the theoretical curve. As the ratio of h_1 to d is known from the matched type curve, the values of h_1, h_2, ρ_1, ρ_2, and ρ_3 can be calculated.

DETAILS OF CONSTRUCTION

The model box for this problem was constructed of 5/8-inch marine plywood with inside dimensions as shown in Figure III. In order to make the box watertight the corners and joints were filled with a commercial caulking compound and the whole box was covered with clear marine varnish. It was found that 5/8-inch plywood was not heavy enough to withstand the pressure of so much water without a great deal of bracing.

A removable shelf was fitted to the top of the box and the electrode shelf suspended below it. The electrode shelf was constructed of lucite and the suspension brackets of aluminum. The brackets were slotted so that the electrode shelf could be raised or lowered to make contact with the material in the box.

In the electrode shelf 3/16-inch holes were drilled through the lucite at carefully measured intervals to hold the electrodes. The holes were spaced so as to hold the electrodes at electrode intervals of 10, 20, 30, 40, 60, 80, 100, 120, 160, 200, 240, 320, 400 and 480 millimeters. This made it possible to change the electrode interval by simply moving the electrodes from one hole to another.

The electrodes were constructed from 1/4-inch round copper stock six inches long. Three inches of the electrode was turned down to 3/16-inch on a lathe so as to fit snugly in the holes in the electrode shelf. Holes were then drilled into the small end of the electrode to serve as a chuck to hold a sewing machine needle. The needle was held in place by a 4-40 Allen screw as shown by Figure IV-a. However it was found that the contact resistance between the needles and water was

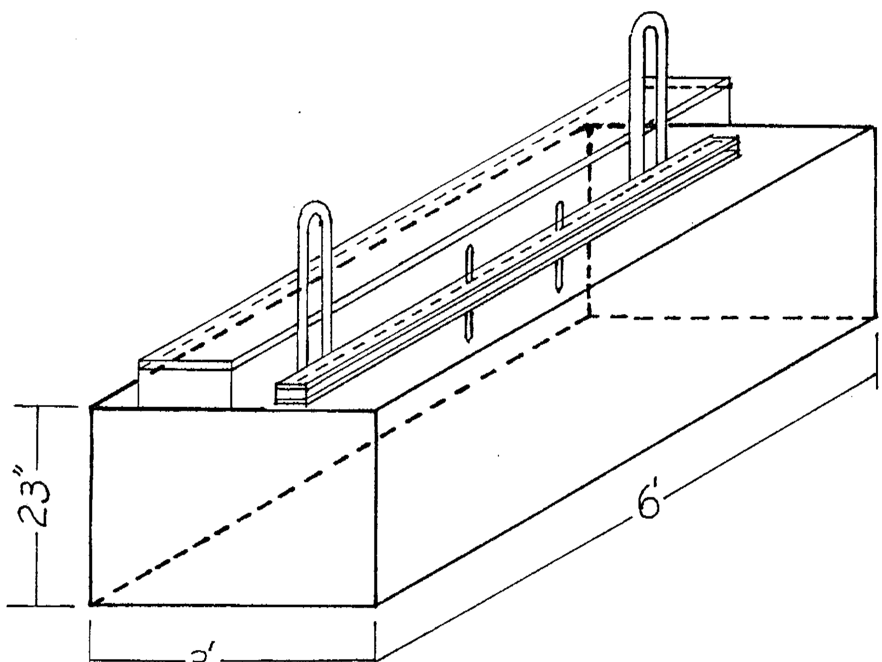


FIGURE III. MODEL BOX

too high. The contact resistance was reduced by constructing new electrodes similar to the first ones except that the small end was ground to a point and the tip coated with solder. (Figure IV-b) However with the lamp black and sand mixture the contact resistance was again too high so pencil lead was placed in the first electrodes with no success. Finally 48-mesh copper screen was rolled into a cylinder about 1/8-inch in diameter and used with the first electrodes. This reduced the contact resistance with the lamp black and sand so that the sensitivity of the instruments was not impaired.

After the lamp black and sand was abandoned and the problem was finished with sand and water the electrodes shown in Figure IV-b were used.

A considerable error in the results can be attributed to the electrodes. The theory of potential methods assumes a point source. At the close electrode spacings the electrodes used no longer approximate a

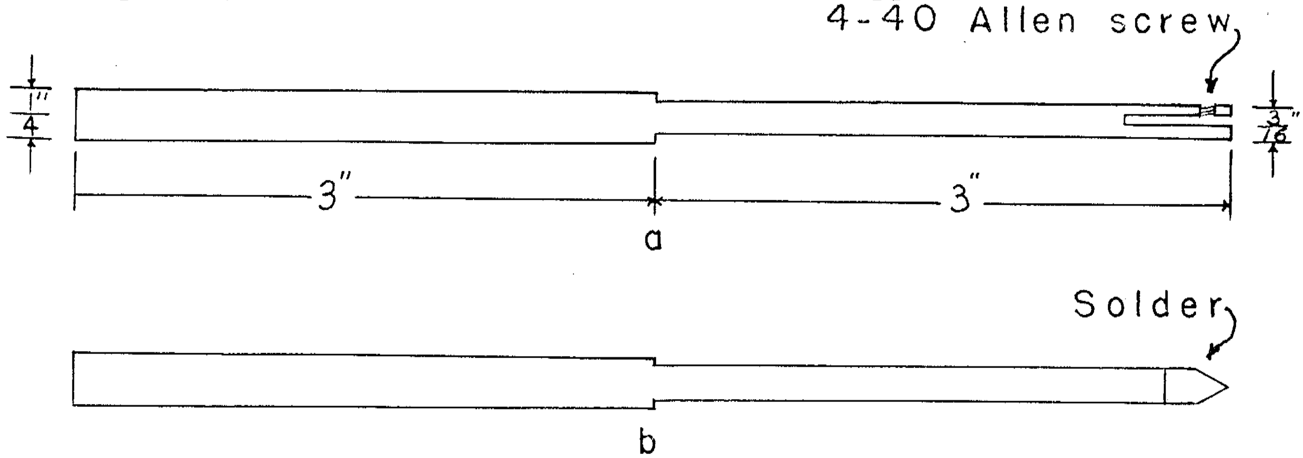


Figure IV
ELECTRODES

point source. Also at the closer electrode spacings a small error in the distance between the electrodes can cause considerable error in the calculated resistivity. At the 10 millimeter spacing an error of one millimeter in the electrode spacing could cause a ten percent error in the calculated resistivity.

INSTRUMENTATION

In the initial compensation of the model box it was found that, because of the high contact resistance encountered, the field instrument after Gish and Rooney[7] from the Research and Development Division was much too insensitive.

For compensating the model box with water, a 60 cycle transformer controlled with a variac was used for the current source. Rough measurements of the potential were made with a Volt-ohmyst voltmeter but were unreliable because of the current drain. Even though the input impedance of the voltmeter was 20,000 ohms per volt it still drew enough current to cause a considerable voltage drop across the contact resistance.

An oscilloscope was calibrated to read 10 volts full scale and a check made to determine the current drain. It was found to be of the order of one to two microamperes. With the contact resistance encountered

7. Gish, O.H. and Rooney, W.J., Measurement of Resistivity of Large Masses of Undisturbed Earth: Terr. Mag. and Atmos. Elec., Vol. 30, No. 4, pp 161-188, Dec. (1925)

between the electrodes and water the voltage drop across the contact resistance could be neglected.

When lamp black and sand were used in the box it was necessary to calibrate the oscilloscope to read 100 volts full scale. This was due to the higher resistivities encountered and the fact that there was no meter available for reading alternating currents of the order of a few microamperes. With the oscilloscope calibrated to read 100 volts full scale it was found that the current drain increased considerably, thereby increasing the voltage drop across the contact resistance and increasing the error of the results.

If the sand were perfectly dry and lamp black were to act as a metallic rather than an electrolytic conductor, direct current could be used. The instruments used were a Leeds and Northrup student potentiometer with ranges of 0 to 16 and 0 to 1.6 millivolts, a Leeds and Northrup galvanometer with a sensitivity of 0.036 microamperes per millimeter scale division, and a voltage-regulated power supply.

It was later found that evidently some polarization was taking place due to small amounts of moisture in the sand. Curve M, Plate 10 and Curve N, Plate 11 were both obtained using direct current but for Curve N the Gish and Rooney[8] double commutator was placed in the circuit. Curve N has higher values along the curve showing that some polarization must have been taking place. After abandoning the lamp black and sand, the same instruments were used, including the double commutator, with sand and water although with considerably more sensitivity than was actually needed.

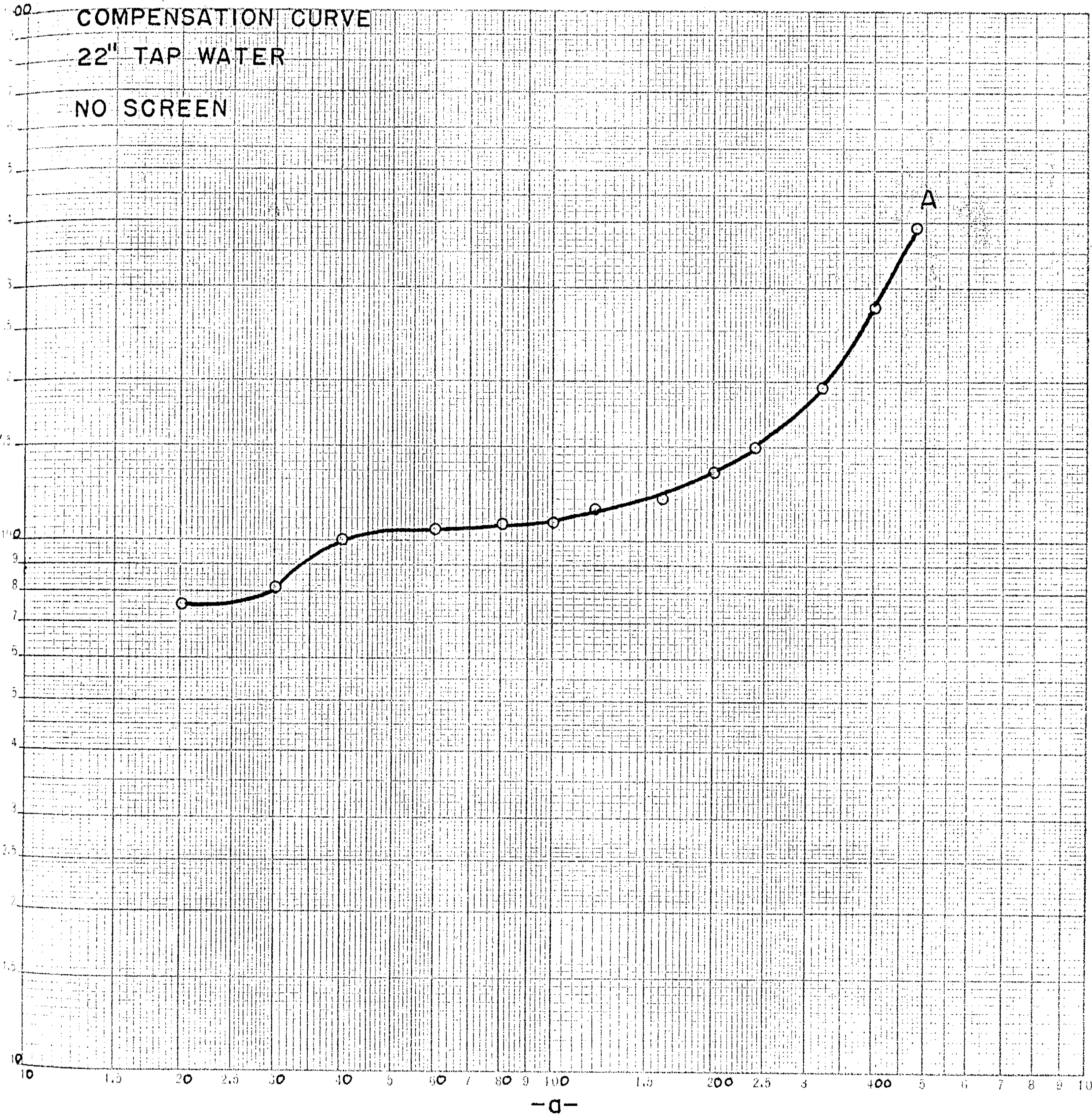
COMPENSATION OF THE MODEL BOX

The theory of the resistivity method assumes horizontal layers of isotropic, homogeneous materials of infinite lateral extent. Since the model box is not infinitely large, it was necessary to compensate according to the method of T. A. Manhart[9]. This was done by filling the box with water and making a series of curves, using varying sizes and shapes of screen on the bottom and sides of the box. With no screen in the box the bottom acts as a second layer of infinite resistance as shown by Curve A, Plate 2. It was found that when the bottom of the box was covered completely and the ends were covered from the bottom up to a height of 15 inches, the curve approached a straight line when plotted on log log graph paper, varying about four ohm-

8. Ibid

9. Manhart, T.A., Model Tank Experiments and Methods for Interpretation of Resistivity Curves: Colo. Sch. of Min. Quart., Vol. 32, No. 1, pp 139-158, Jan. (1937)

PLATE 2



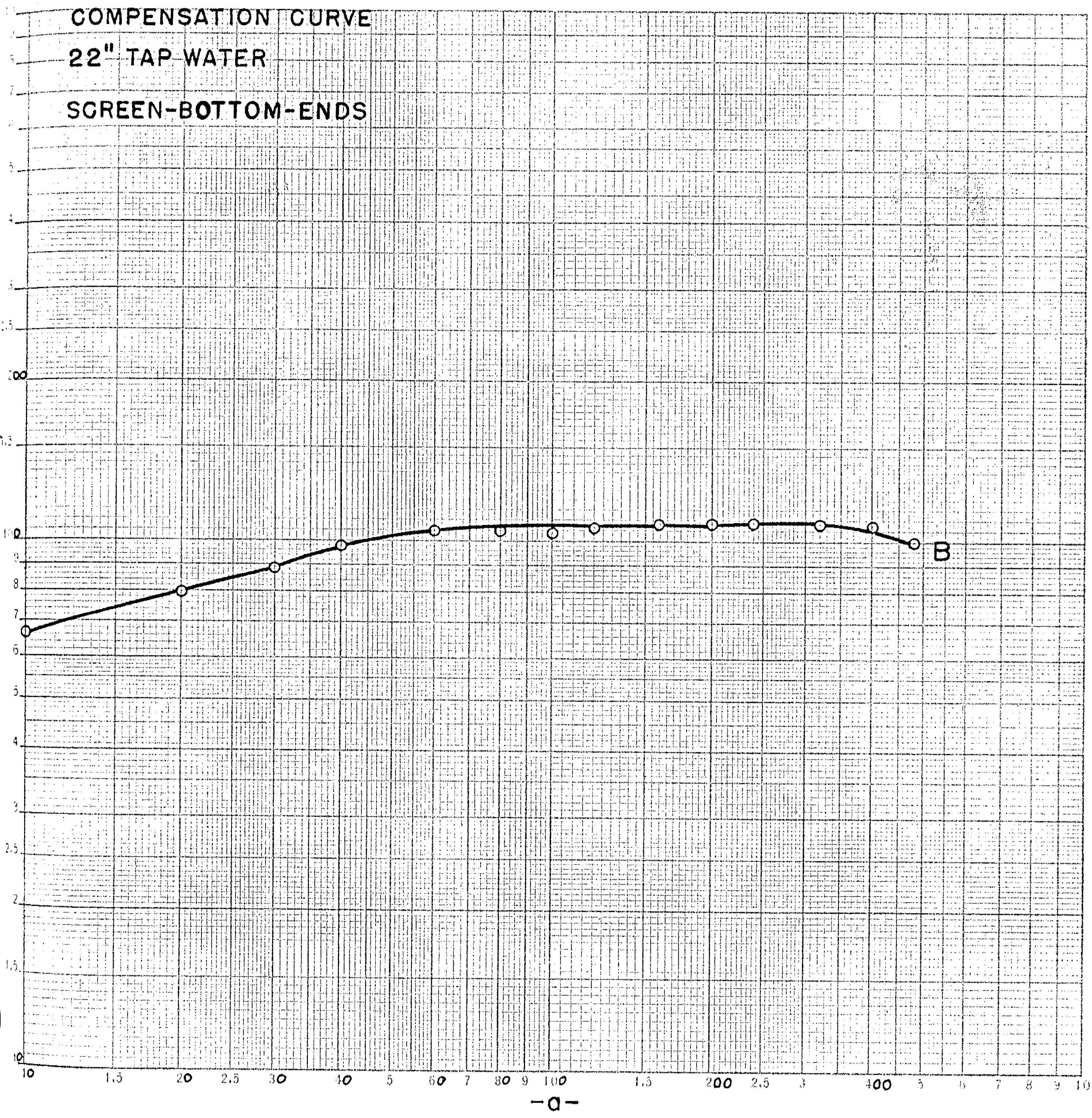
ELECTRODE INTERVAL in MILLIMETERS

feet from the 40 to the 480 millimeter electrode spacing. (Curve B, Plate 3)

After the lamp black and sand was abandoned, a re-check of the compensation was tried using sand saturated to the point where a very thin layer of water remained on top of the sand. Screen was placed on the bottom and on all four sides from the bottom up to a height of 15 inches. Curve C, Plate 4 shows the result when the box was filled to the top of the side screens or 15 inches deep. Curve D, Plate 4 was obtained when the box was filled to within an inch of the top of the box or 22 inches deep. These two curves show that the box was over-compensated so the two screens on the sides were removed leaving the bottom and two ends as they were when the box was compensated with water. Curve E, Plate 5 shows the result. The curve has a downward tendency with a sharp drop between the 400 and 480 millimeter spacing. In order to level out the curve the two end screens were lowered two inches. Curve F, Plate 5 shows that this helped to smooth the curve between the 400 and 480 millimeter spacing but did not help the downward tendency of the whole curve.

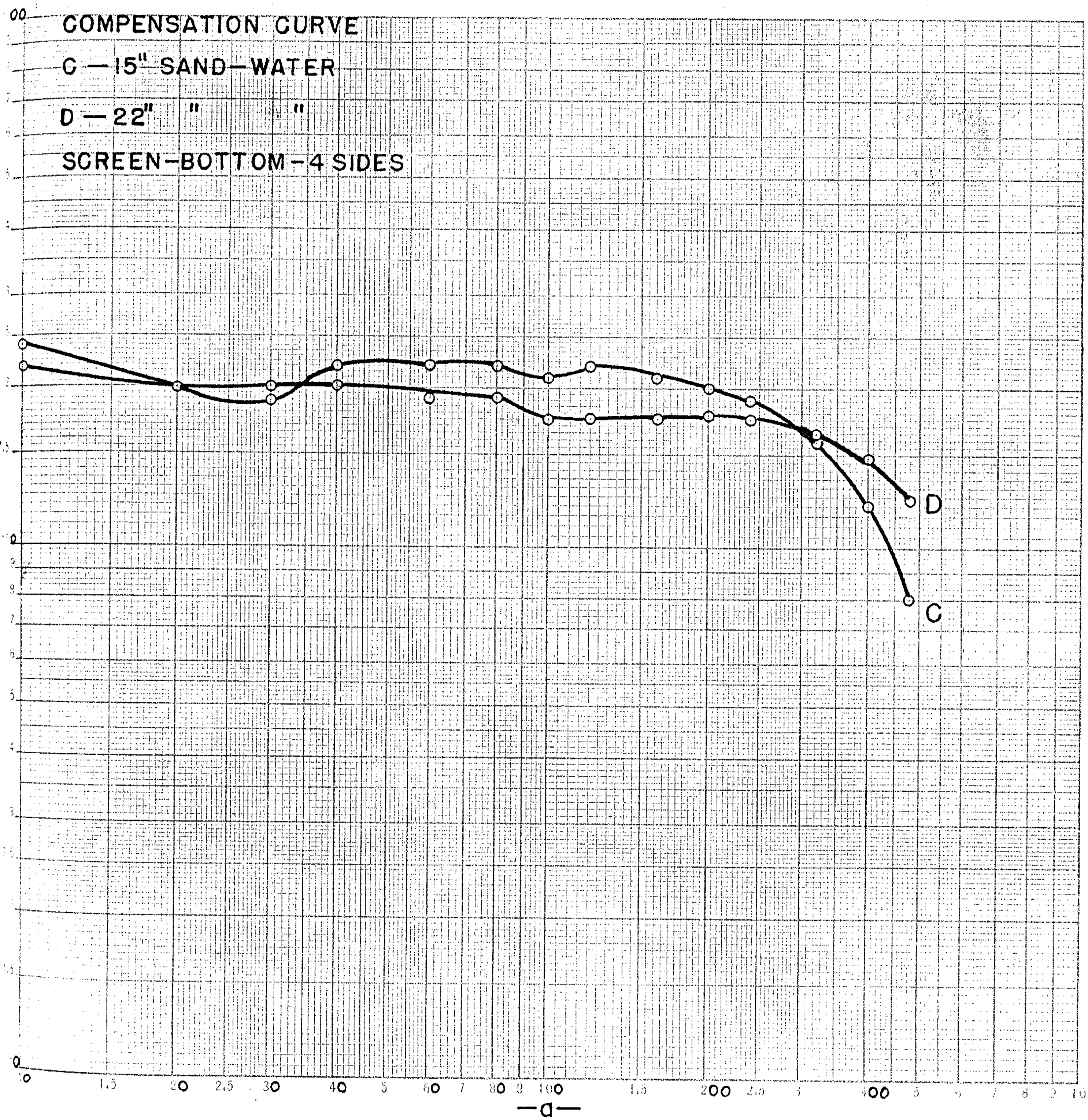
Evidently the drop in resistivity was due either to compaction of the sand or from a rise in temperature. Although the currents used were small, the maximum being

PLATE 3



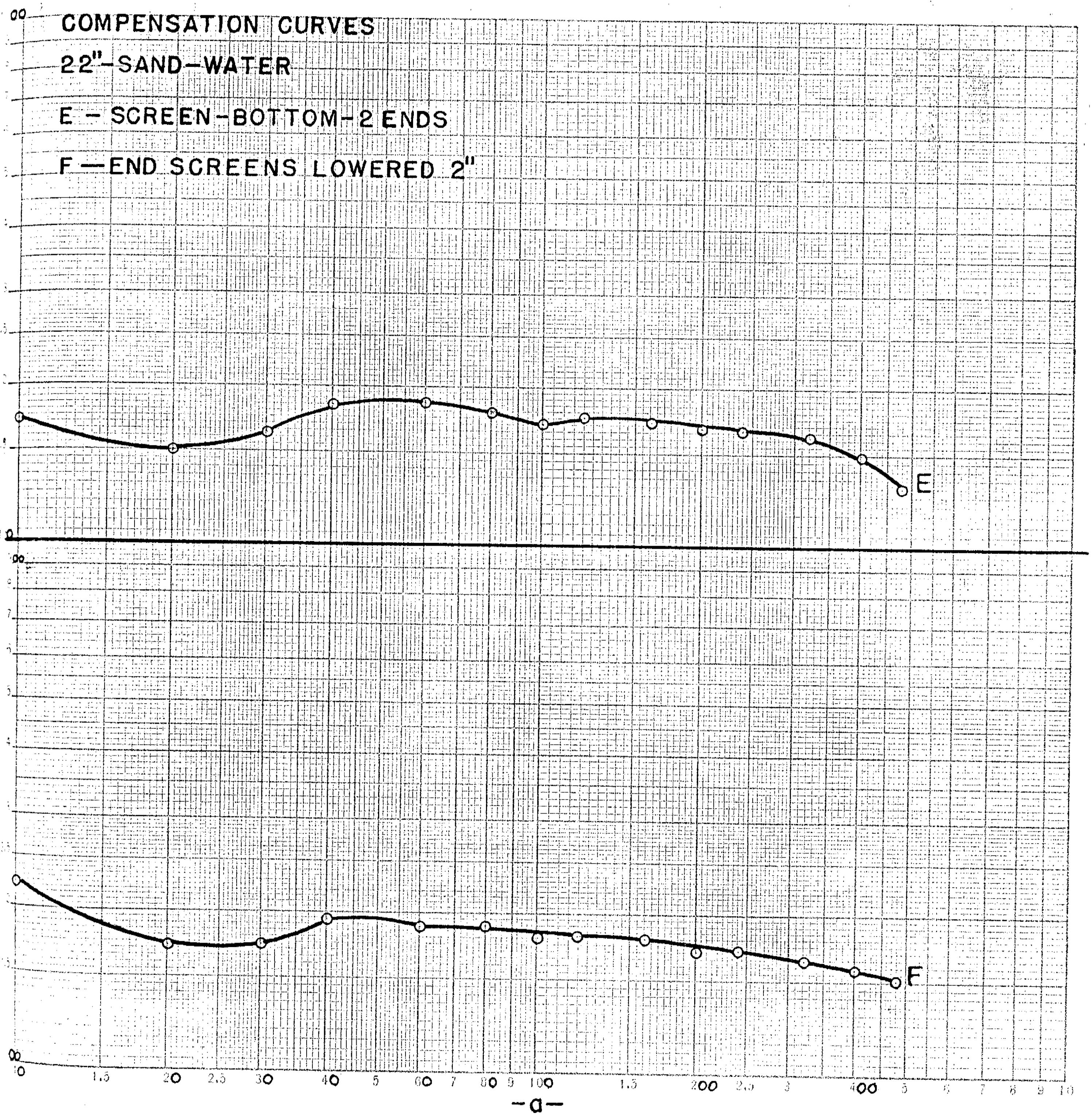
ELECTRODE INTERVAL in MILLIMETERS

PLATE 4



ELECTRODE INTERVAL in MILLIMETERS

PLATE 5



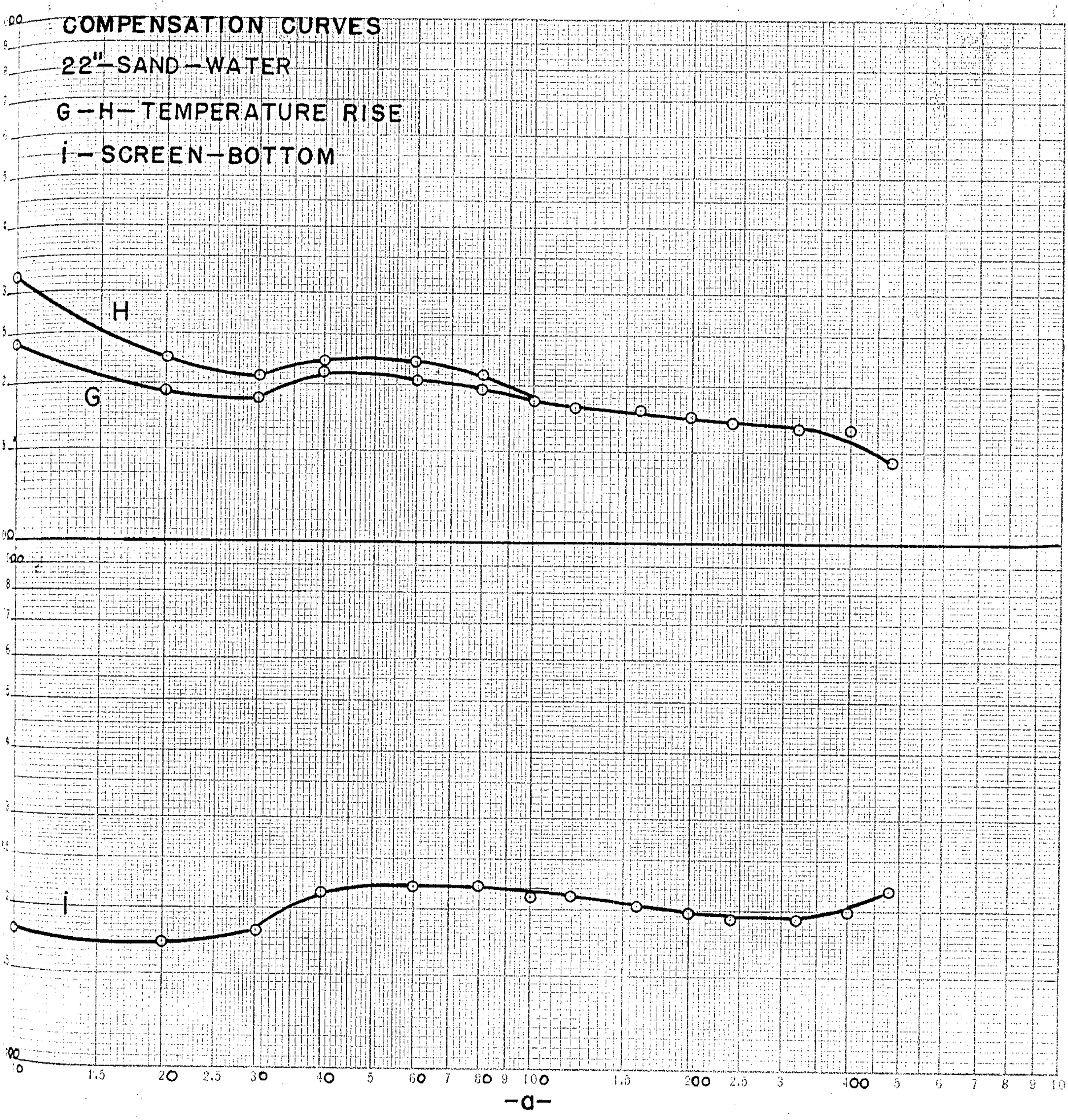
ELECTRODE INTERVAL in MILLIMETERS

five milliamperes in most cases, a small temperature rise could occur if the current were left on for a long enough period of time.

To check for a change in resistivity due to temperature two curves were plotted. Variations in ambient temperature were not taken into consideration but the materials were placed in the box two days before so the temperature should have been in equilibrium. The first curve was made starting with the 10 millimeter spacing and increasing the electrode spacing out to 480 millimeters. The second was made starting with the 480 millimeter spacing and decreasing the electrode spacing down to 10 millimeters. The second curve was made immediately after the first. Curves G and H, Plate 6 show the result of the two curves. Evidently there is a small temperature rise at the closer spacings but becomes negligible beyond the 100 millimeter spacing.

As the curve still had a downward tendency the two end screens were removed. Curve I, Plate 6 shows that the curve bends upward beyond the 320 millimeter spacing but the downward trend remains over the rest of the curve. As the decrease in resistivity with increased electrode spacing is practically linear, it is apparently caused from compaction of the sand with depth. As it was impossible to overcome compaction it was decided to use

PLATE 6



ELECTRODE INTERVAL in MILLIMETERS

the box with screen on the bottom and none on the sides. When analyzing curves it is necessary to take this compensation error into consideration.

MATERIALS USED

For this problem sand was used entirely as a base material. The sand came from the sand dunes about fifteen miles north of Socorro. The grains are well rounded from wind erosion and small enough that most of it can be sifted through 48-mesh screen.

It was first decided to mix lamp black with the sand to reduce the resistivity. If it were possible to use the lamp black it would have three advantages. First by varying the percentage of lamp black, the resistivity could be varied from infinity down to about four ohm-feet, that being the resistivity of pure lamp black when it is compacted as much as possible. Second lamp black should be a metallic rather than an electrolytic conductor. This would make it possible to use direct rather than alternating current or commutated direct current. Third it would be possible to make definite boundaries between the layers with no mixing of materials of the different layers.

In order to find a ratio of lamp black and sand for a given resistivity, a box with inside dimensions of

3x3x3 inches was constructed. The bottom and two sides were constructed of lucite and the other two ends of aluminum to be used as electrodes. Resistivity of any material placed in the box could then be calculated according to the formula ρ=RA/L where R is calculated from the voltage and current, A is the area of the electrodes and L is the distance between them. To measure the resistivity of different ratios of lamp black and sand, the electrodes were connected in series with an alternating current milliammeter and the voltage was measured across the electrodes with a Volt-ohmyst voltmeter. Except for very high resistivities it was found that the current drawn by the voltmeter, and for very low resistivities the voltage drop across the milliammeter, could be neglected.

Eight samples of lamp black and sand mixtures were mixed according to Table I and the densities were calculated based on 3.7 grams per cubic inch for lamp black and 25.15 grams per cubic inch for sand.

TABLE I

Sample	Grams sand	Grams lamp black	Percent lamp black	Density g/in^3
I	690	10	1.43	23.2
II	680	20	2.86	21.8
III	670	30	4.30	20.0
IV	660	40	5.72	18.9
V	650	50	7.15	17.8
VI	640	60	8.09	16.8
VII	630	70	10.00	15.9
VIII	620	80	11.40	15.1

It was found that the resistivities were not constant for the same sample due to the impossibility of packing the same amount into the box for each trial. In order to find a constant resistivity for a given density the resistivity was measured four times for each sample. For the first measurement the mixture was poured into the box and leveled with a straight edge. For the second measurement the mixture was packed a small amount by shaking. For the third measurement the mixture was packed more thoroughly by pressing down from the top with a knife blade. For the fourth measurement the mixture was packed as much as possible by tamping with the knife blade. (Table II)

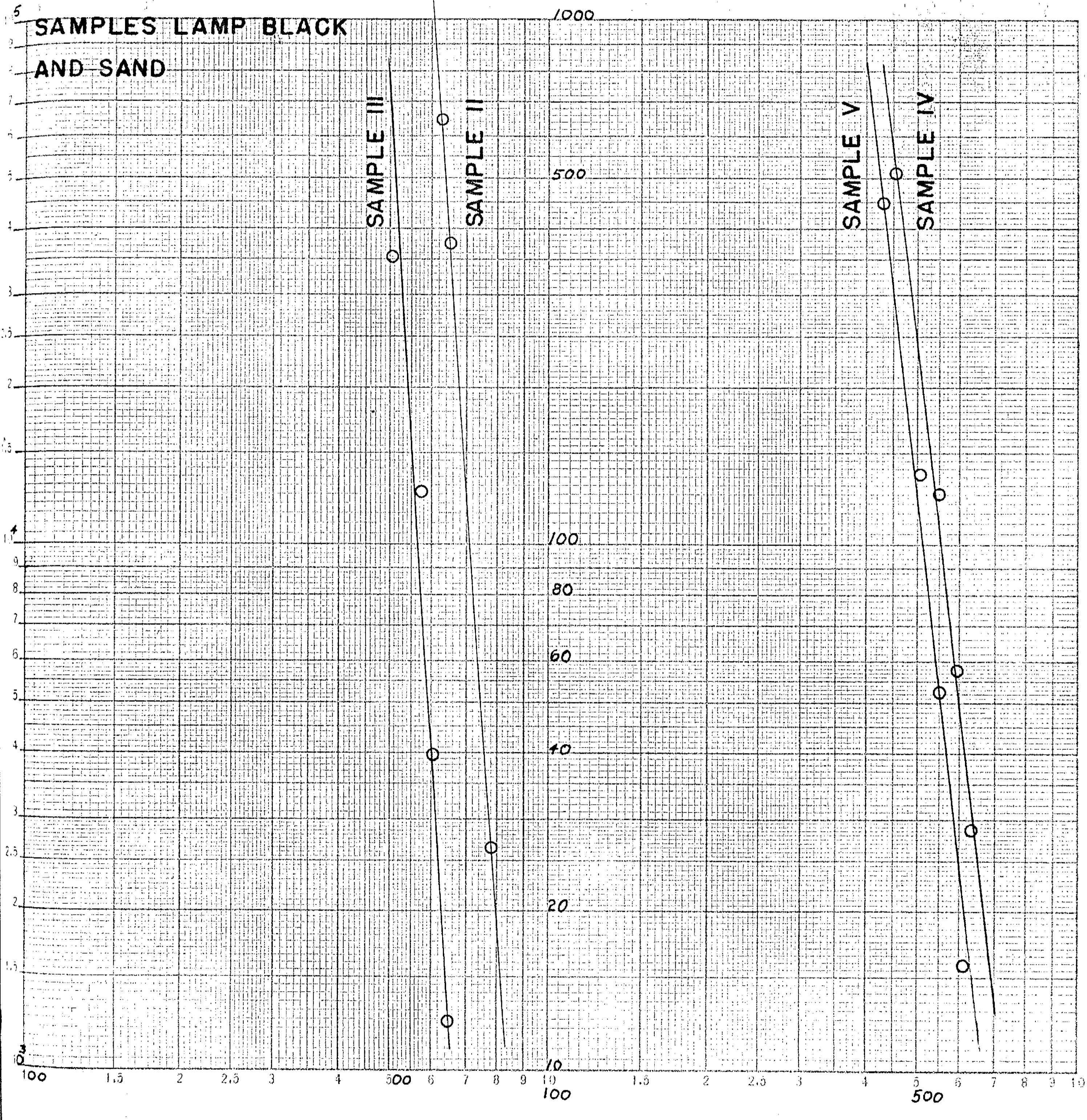
The values of resistivity for each of the four measurements were then plotted against grams per 27 cubic inches of mixture on log log graph paper. In each case a straight line was obtained. (Plates 7 and 8) The resistivity for the calculated density of each given in Table I was then taken from these curves and plotted on log log graph paper against percentage of lamp black, and again a straight line was obtained. (Curve J, Plate 9) From this curve it is possible to read the percentage of lamp black for any resistivity between 100 and 10,000 ohm-feet.

These curves would now give the resistivity for a given density but the problem remained to find a way to

TABLE II

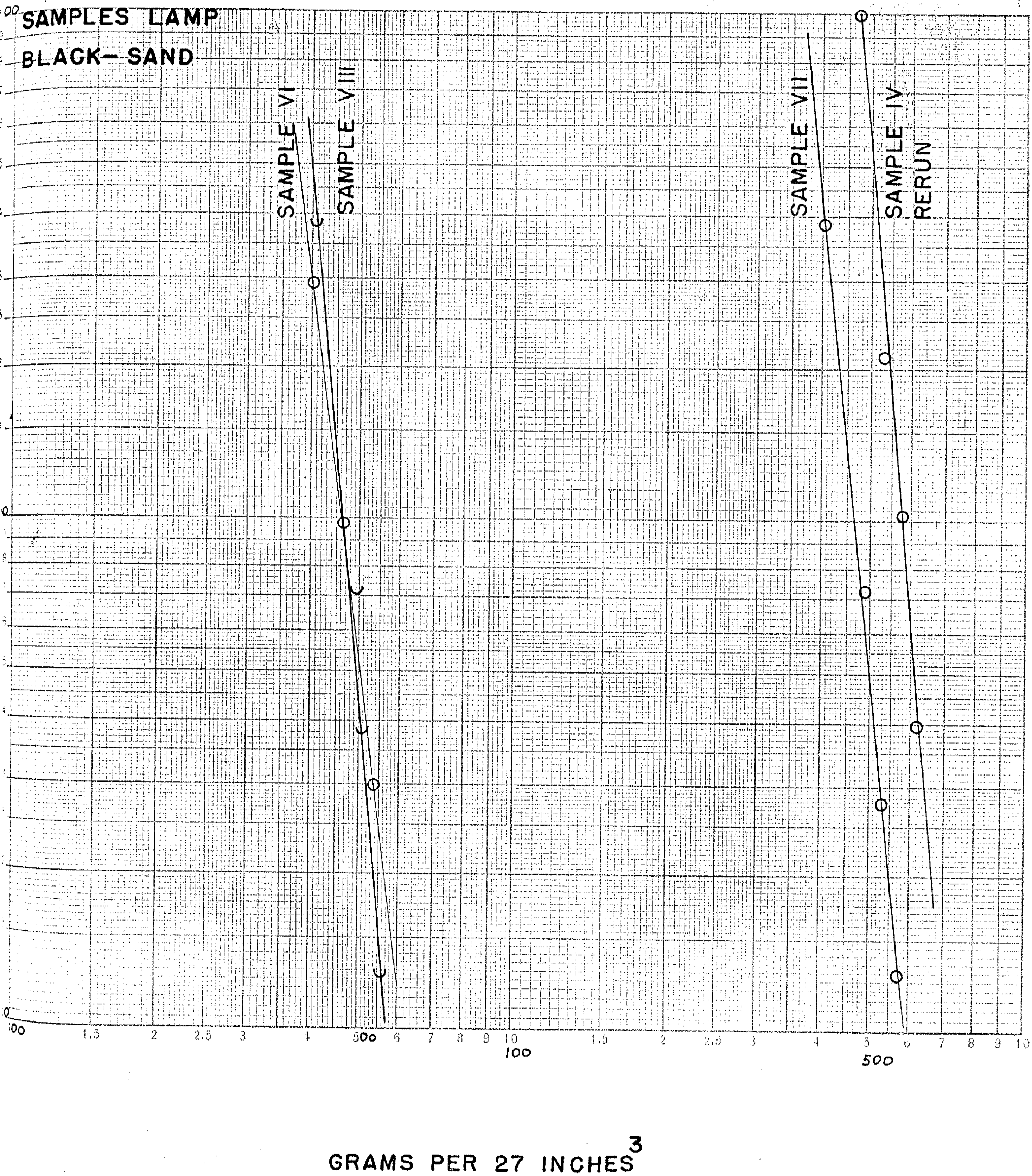
Grams in	E in volts	I in milliamp	ρ in ohm-feet	Calculated ρ in ohm-feet
Sample I				
604	1000	0		
673	1000	0		
686	1000	0		
655	1000	0		
Sample II				
572	1000	1.0	250000	
647	1000	6.6	37000	17000
655	1000	4.0	62500	
673	1000	96.0	2600	
Sample III				
506	1000	70	3570	
573	500	98	1275	1300
603	320	200	400	
635	100	200	125	
Sample IV				
472	800	200	1000	
535	170	200	213	400
579	82	200	103	
618	32	200	40	
Sample V				
428	360	200	450	
508	110	200	138	170
551	42	200	53	
607	13	200	16	
Sample VI				
405	230	200	288	
467	78	200	98	120
539	24	200	30	
560	8.3	200	10	
Sample VII				
403	310	200	388	
493	58	200	73	220
534	22	200	28	
575	10	200	13	
Sample VIII				
393	310	200	388	
446	120	200	150	250
510	31	200	39	
552	10	200	13	

PLATE 7



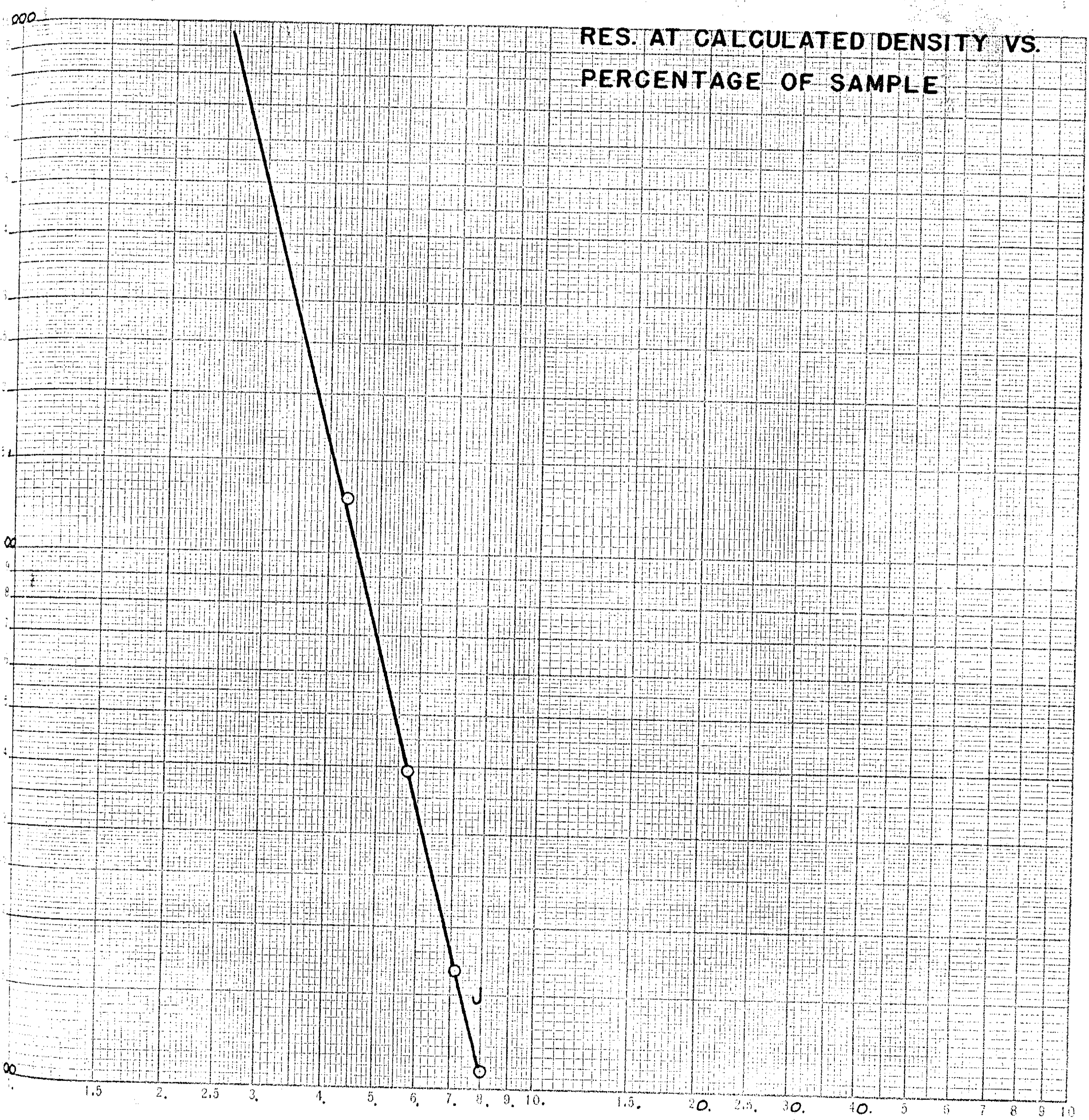
GRAMS PER 27 INCHES^3

PLATE 8



SAMPLES LAMP
BLACK– SAND
SAMPLE VI
SAMPLE VIII
SAMPLE VII
SAMPLE IV
RERUN
100
500
GRAMS PER 27 INCHES 3

PLATE 9



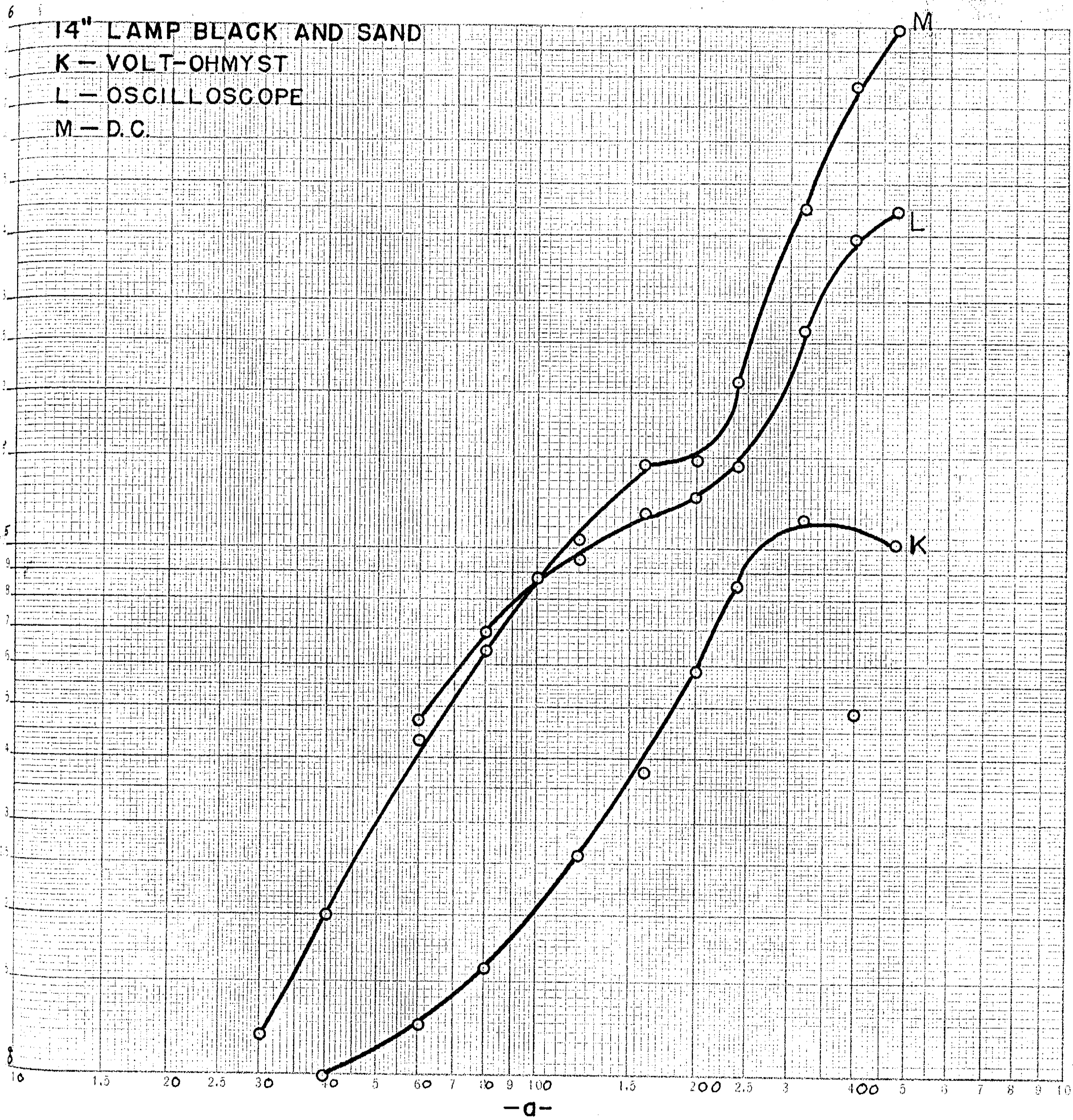
PERCENTAGE OF LAMP BLACK

calculate the change in density due to compaction. It was thought that by using a constant percentage of lamp black and sand and measuring the resistivity as it was placed into the model box, say every three inches, that a curve could be plotted to take compaction into effect. This would make it possible to read from Curve J, Plate 9 the percentage of lamp black to use for a given resistivity and then from the other curve the amount the resistivity would be lowered from compaction depending on the number of inches of material to be used for that given layer.

Fourteen inches of five percent lamp black and sand mixture were poured into the model box. However it was found that instead of decreasing resistivity with depth as would be expected, that the resistance remained practically constant, thus making the resistivity increase proportionally to the increase in electrode spacing.

The first measurements were made with 60 cycle alternating current and the Volt-ohmyst voltmeter. The results are shown by Curve K, Plate 10. The second measurements were made using the 60 cycle alternating current source but the potential was measured with the calibrated oscilloscope. The results are shown by Curve L, Plate 10. The next measurements were made using the direct current source and measuring the potential with the potentiometer

PLATE 10



ELECTRODE INTERVAL in MILLIMETERS

and galvanometer. The results are shown by Curve M, Plate 10.

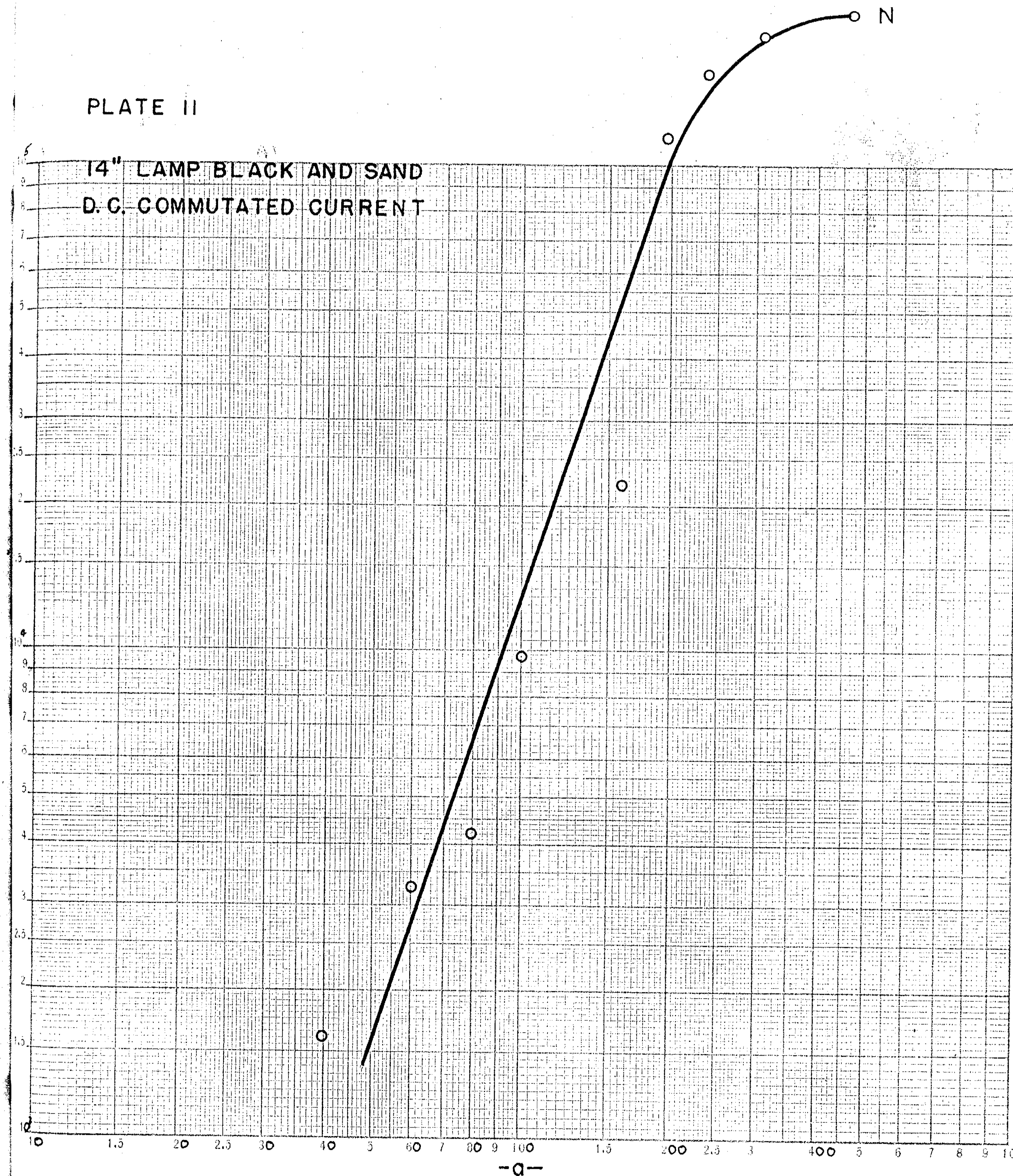
At this time it appeared that the trouble might be in instrumentation so the double commutator was connected with the direct current instruments. Curve N, Plate 11 shows the result. It is practically the same as the one made with direct current (M) except that it has higher values of resistivity all along the curve showing that evidently some polarization was taking place.

Each of the curves made with lamp black and sand show increasing resistivity with depth. Evidently the lamp black was absorbing moisture from the air. The surface would naturally contain the most moisture and thereby have the lower resistance. As the mixture was in the box for several days before any readings were taken, the moisture content would probably vary approximately linearly with depth. Whatever the reason for the behavior of the mixture it was unsatisfactory as a layer material.

As a further check a metal pan 14x18x4 inches with three inches of water in it was placed in the box under the electrodes. (Curve O, Plate 12) As was expected the curve breaks sharply downward so the instruments were considered to be working properly.

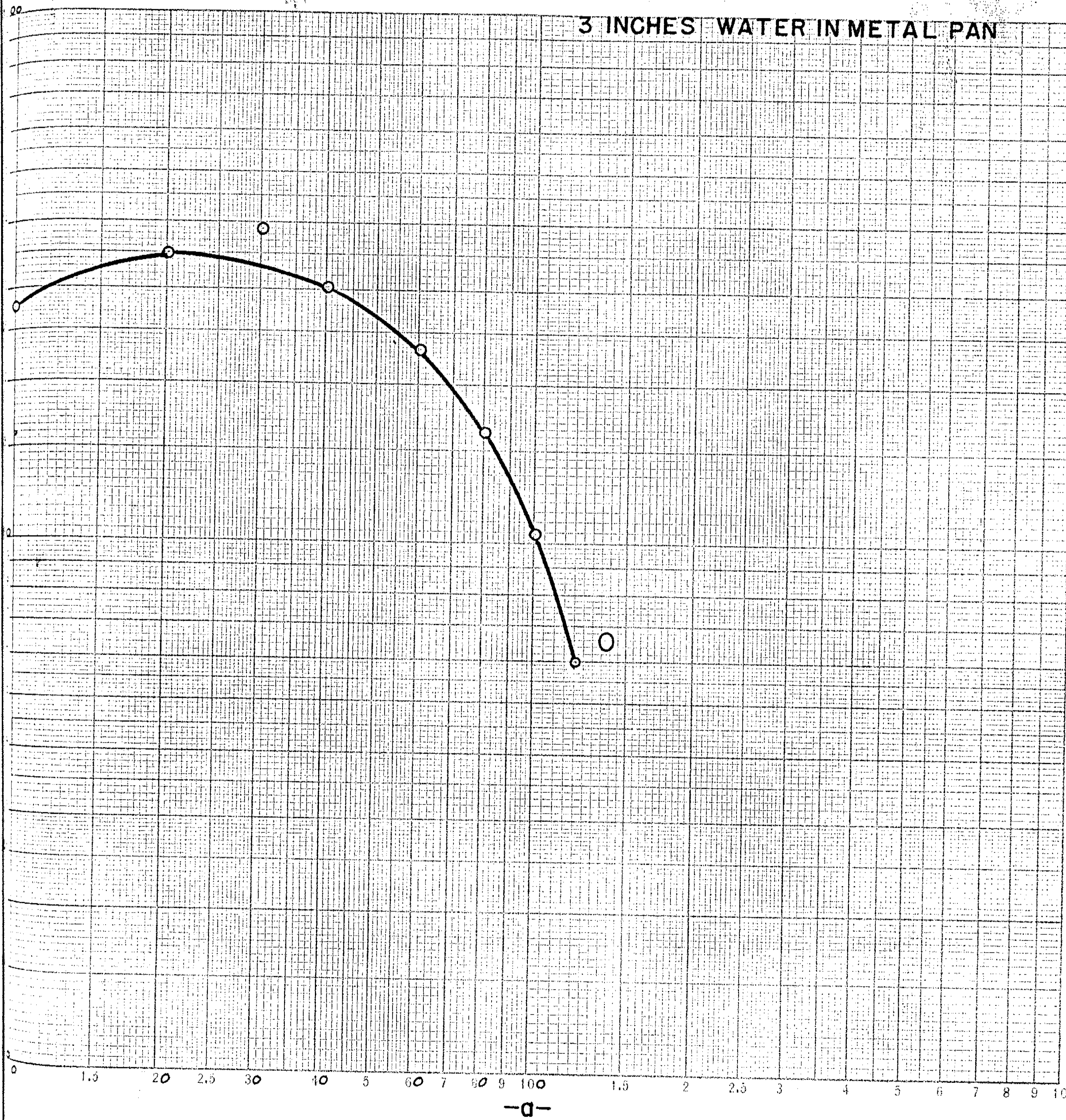
With sand and water it was found that there were several errors that must be taken into consideration.

PLATE II



ELECTRODE INTERVAL in MILLIMETERS

PLATE 12



ELECTRODE INTERVAL in MILLIMETERS

Probably the greatest error occurred because of the impossibility of making each layer evenly compacted. When all the material for one layer is placed into the box at one time and then the top of the layer leveled and smoothed out, the top of the layer is compacted more than it is at the bottom.

The layers were separated by placing heavy papyrus wrapping paper between the layers. In order to be sure that the paper had no effect on the curves, measurements were made on a layer of water over sand. Then the paper was inserted at the water-sand boundary and the same measurements made. The results showed that the paper would have no effect on the resistivity curves.

RESULTS

The results of each of the four curves will be treated separately. In each case a preliminary check of the resistivity of each of the layers was made. This was done by using the small box memtioned before. The results obtained from measuring the resistivity by this method are very unreliable because of the impossibility of packing the sand the same amount in each case. Therefore the resistivity ratio, the upper layer to the lower layer, given in each case is an approximation. Also it is impossible to make each layer a certain depth with any great accuracy. It depends on the amount of compaction that takes place when the upper layer or layers are placed in the box. As the bottom layer compacts it will decrease in depth while the upper layer will increase in depth. If the depth of each layer is measured as the sand is taken from the box the depth of each layer can be determined with a probable accuracy of one-eighth inch. It can be seen from the following curves that, as mentioned before, the values for the resistivity from the 10 to 40 millimeter electrode spacings are questionable.

Curve P, Plate 14

is a two-layer case where the top layer was about six inches thick and the lower layer was considered infinite. The resistivity ratio was approximately 10:1 where the upper layer was damp sand and the lower layer was sand saturated with salt solution.

The best fit was found to be the 0:4 curve from the theoretical curves with a resistivity ratio of 1:100:10. (Plate 13) The curve fits very well out to the 300 millimeter spacing where it starts deviating upward from the theoretical curve. This deviation was probably caused from the box as shown by the compensation curve. (Curve B, Plate 3)

The index of the theoretical curve lies at the 150

PLATE 13

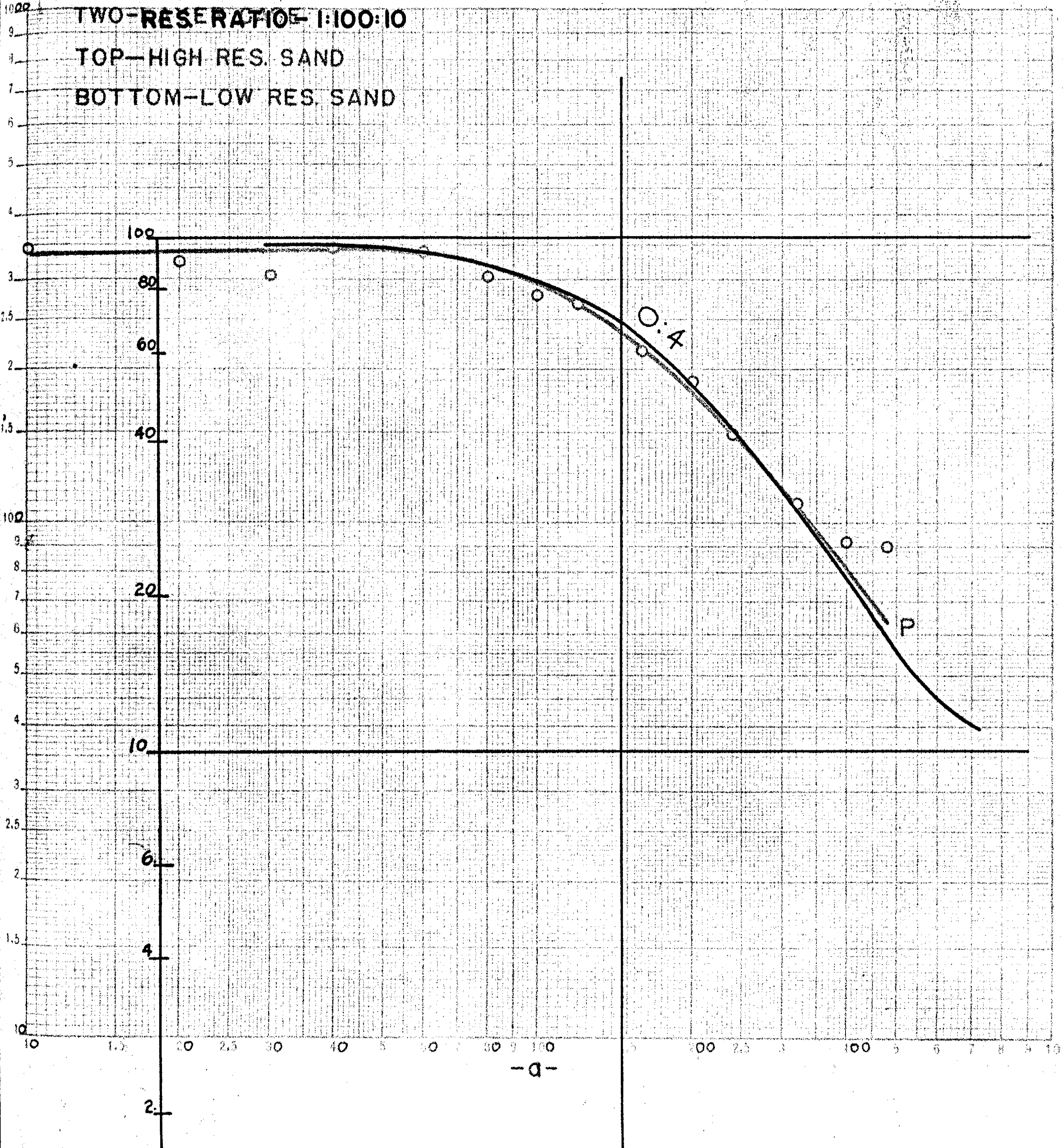
PLATE 14

THEORETICAL CURVE

TWO-**RES. RATIO — 1:100:10**

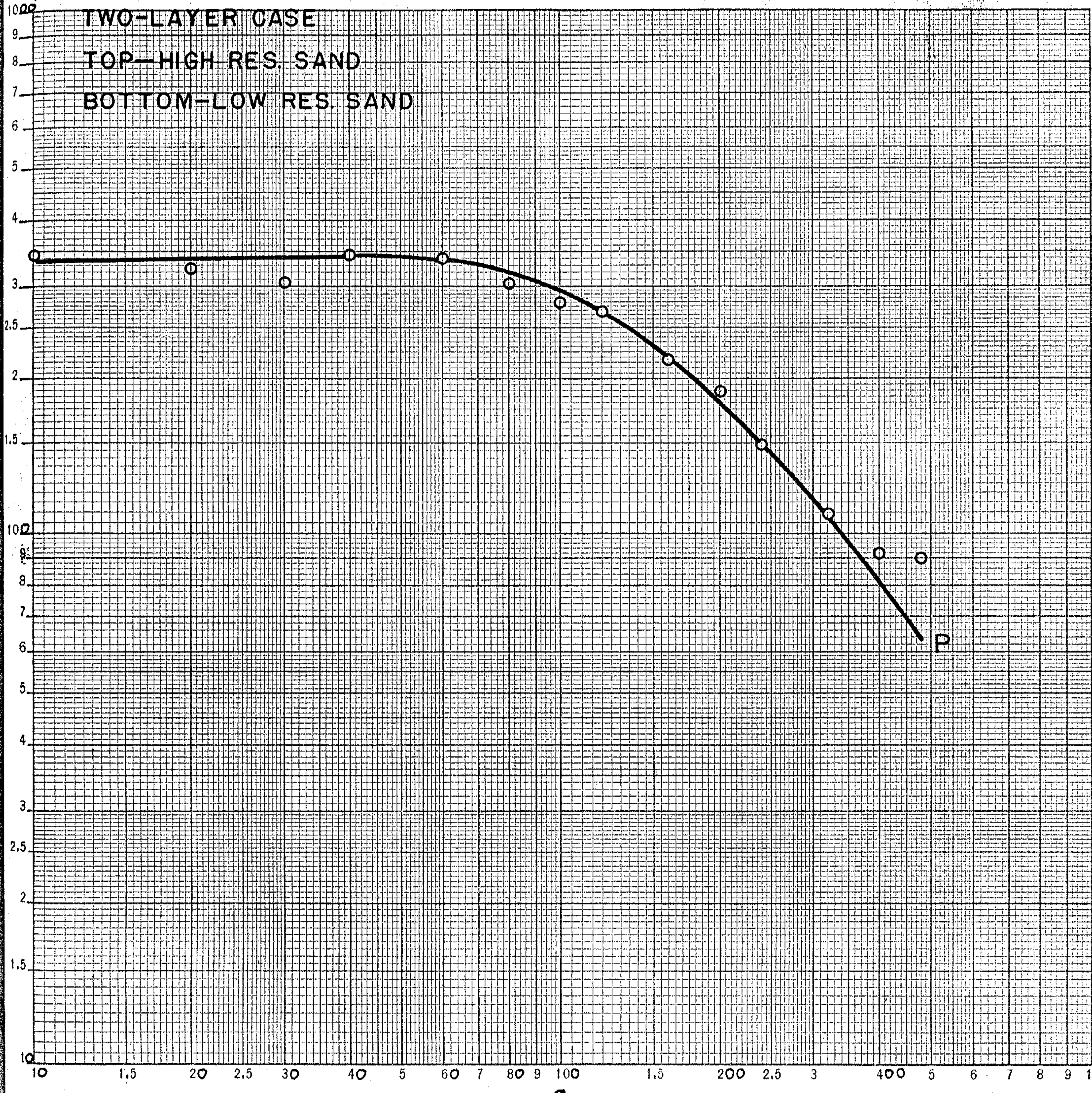
TOP—HIGH RES. SAND

BOTTOM—LOW RES. SAND



ELECTRODE INTERVAL in MILLIMETERS

PLATE 14



ELECTRODE INTERVAL in MILLIMETERS

millimeter or 5.9 inch electrode spacing which corresponds to the depth of the top layer. This is in good agreement with the actual depth of six inches of the top layer.

Curve Q, Plate 16

is a two-layer case where the top layer was about six inches thick and the lower layer was considered infinite. The resistivity ratio was approximately 1:100 with the upper layer of sand that had been saturated in salt solution and the excess solution drained off. The lower layer was damp sand.

The best fit for the curve was found to be the 0:4 curve from the theoretical curves with a resistivity ratio of 1:1/100:1. (Plate 15) The curve fits fairly well out to the 200 millimeter electrode spacing where it starts deviating upward from the theoretical curve. The discrepancy beyond the 200 millimeter electrode spacing may have been caused by compaction. When the sand was placed in the box it was just damp enough to barely stick together. All the sand for the lower layer was placed into the box at one time and then leveled. Thus it is possible that while leveling the bottom layer that it was compacted more at the top than near the bottom. This would lower the resistivity near the top of the bottom layer and in effect make the resistivity

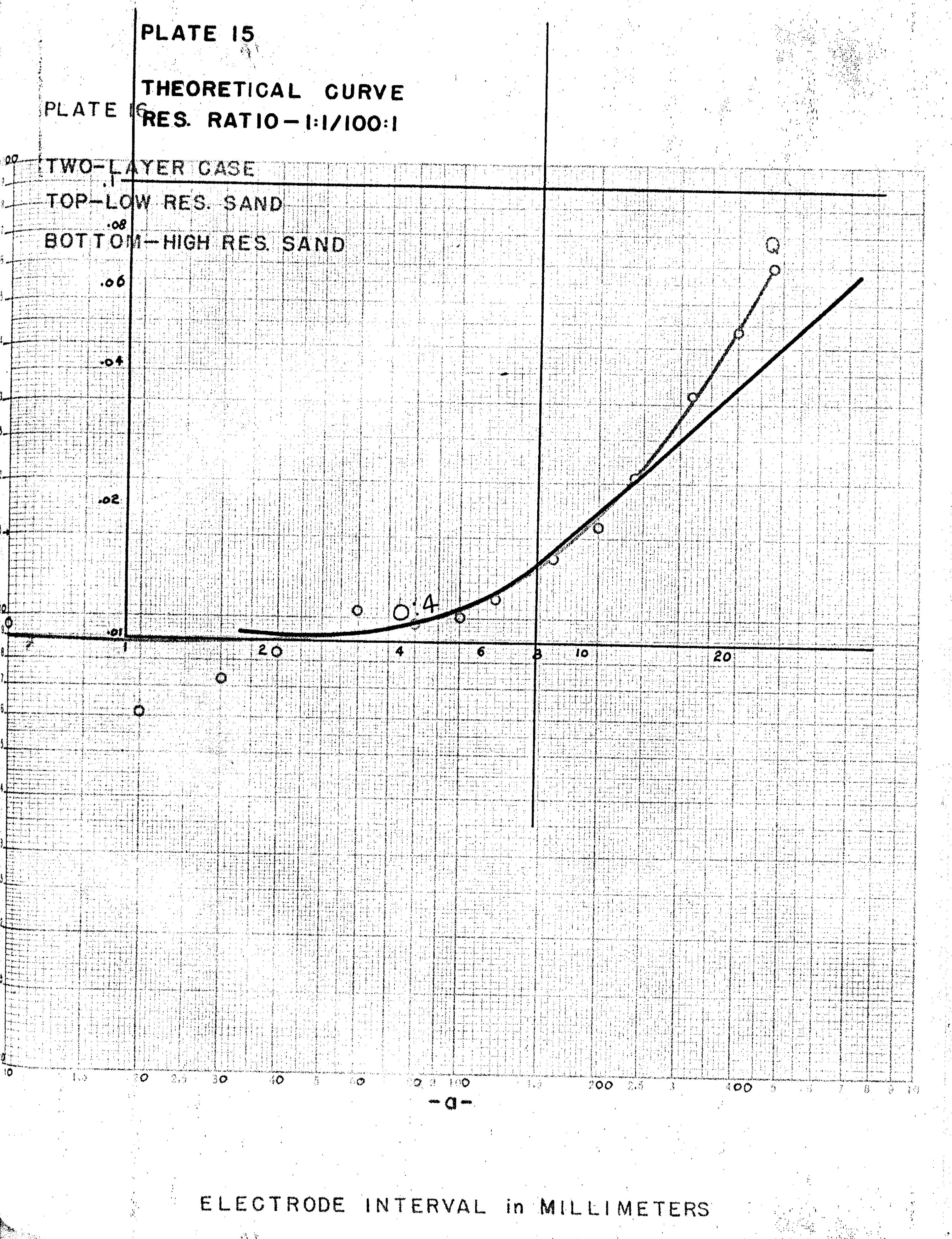
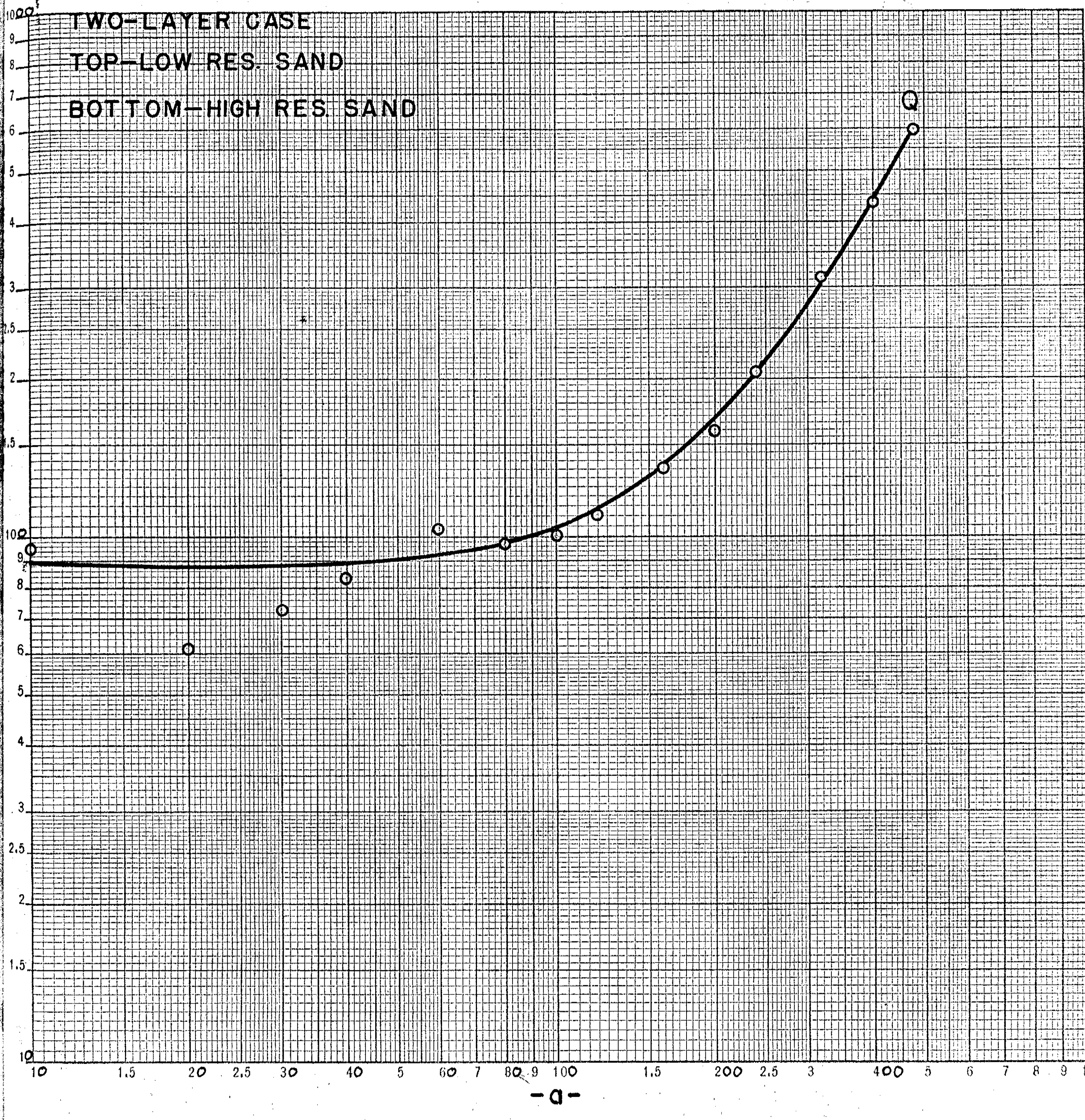


PLATE 15
THEORETICAL CURVE
RES. RATIO — 1:1/100:1
PLATE 16
TWO-LAYER CASE
TOP-LOW RES. SAND
BOTTOM-HIGH RES. SAND
Q
.1
.08
.06
.04
.02
.01
1
2
4
6
8
10
20
20
30
40
60
100
200
400
-a-
ELECTRODE INTERVAL in MILLIMETERS

PLATE 16



ELECTRODE INTERVAL in MILLIMETERS

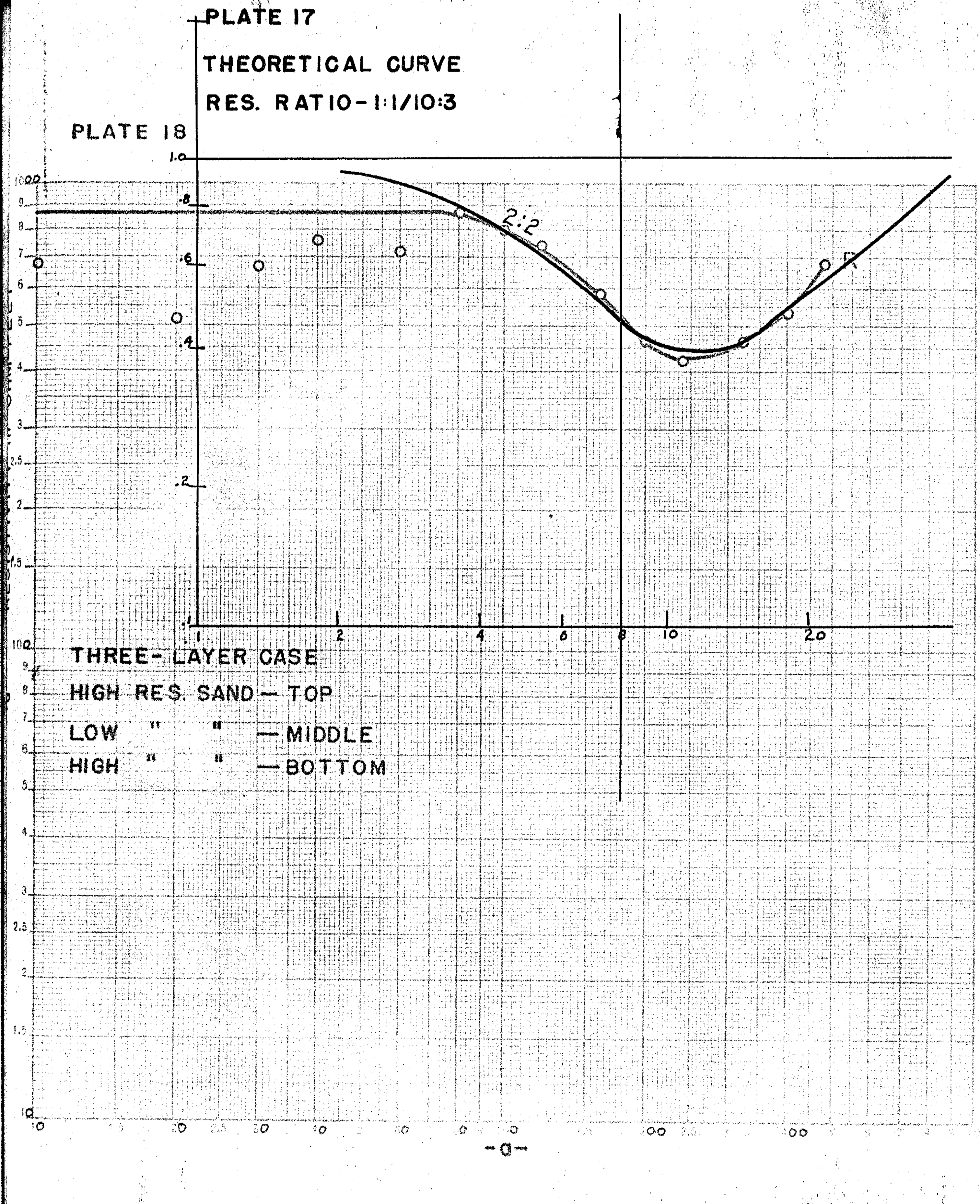
of the bottom layer increase with depth. Also as in the case of Curve P, the compensation of the box could account for part of the discrepancy in the two curves.

The index of the theoretical curve lies at the 150 millimeter or 5.9 inch electrode spacing which corresponds to the depth of the top layer. This is in good agreement with the actual depth of six inches.

Curve R, Plate 18

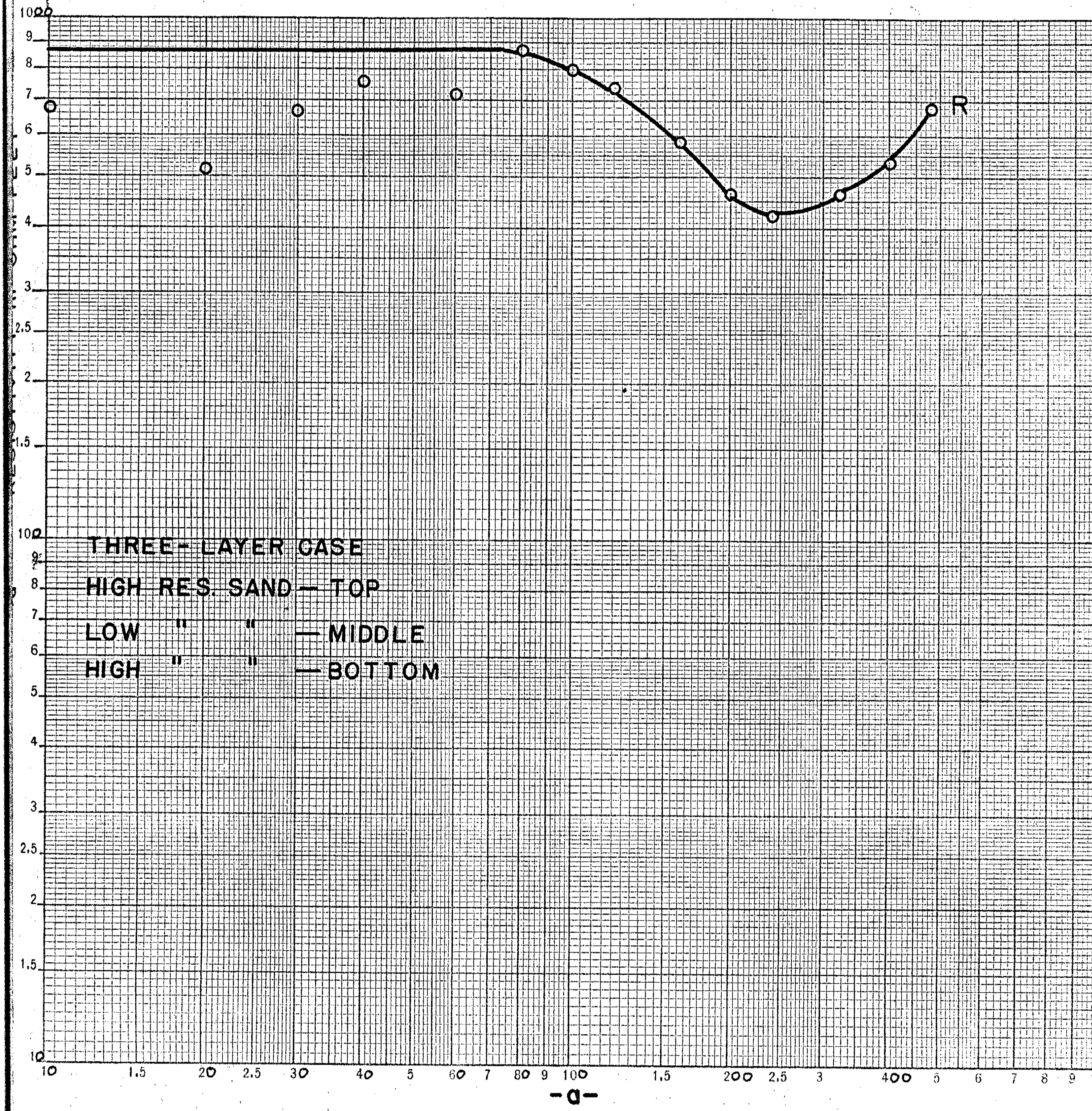
is a three-layer case where the top and middle layers were each approximately four inches thick and the lower layer was considered to be infinite. The resistivity ratio was found to be approximately 1:1/10:3 where the upper and lower layer were composed of damp sand and the middle layer of sand that had been saturated with salt solution and the excess solution drained off. The resistivity of the upper layer was less than that of the lower layer as some water was added to lower the contact resistance.

The best fit was found to be the 2:2 curve from the theoretical curves with a resistivity ratio of 1:1/10:3. (Plate 17) The curve fits fairly well beyond the 100 millimeter spacing but fits very poorly from the 10 to 100 millimeter spacings. Apparently when more water was added to the top layer the resistivity was lowered more than was expected and the 1:1/10 ratio does not hold.



ELECTRODE INTERVAL in MILLIMETERS

PLATE 18



ELECTRODE INTERVAL in MILLIMETERS

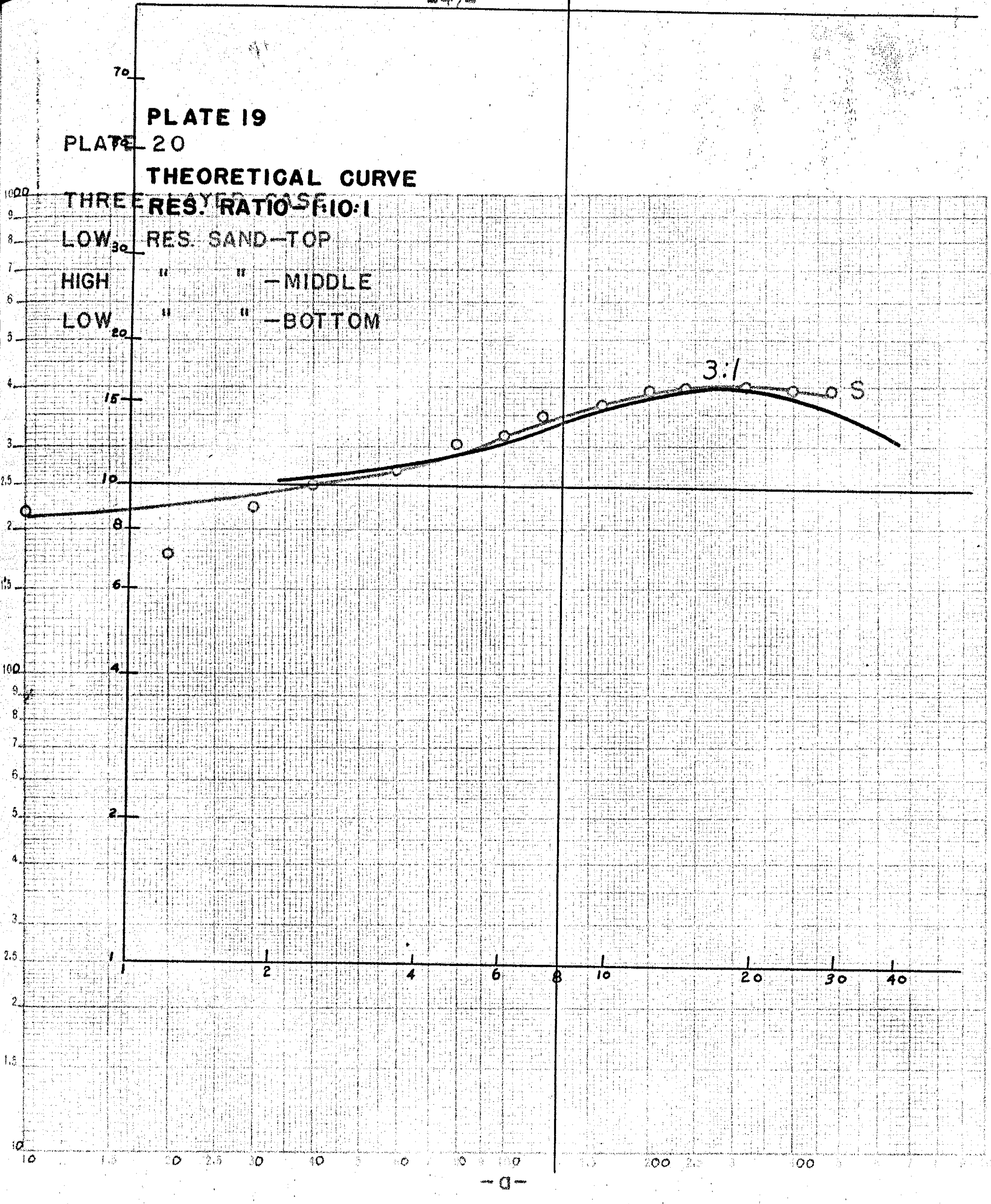
Probably the ratio for the two upper layers should be of the order of 1/2:1/10.

The index of the theoretical curve lies at the 180 millimeter of 7.1 inch electrode spacing which should correspond to the depth of the two top layers. This agrees very poorly with the actual depth of eight inches so the error is approximately ten percent.

Curve S, Plate 20

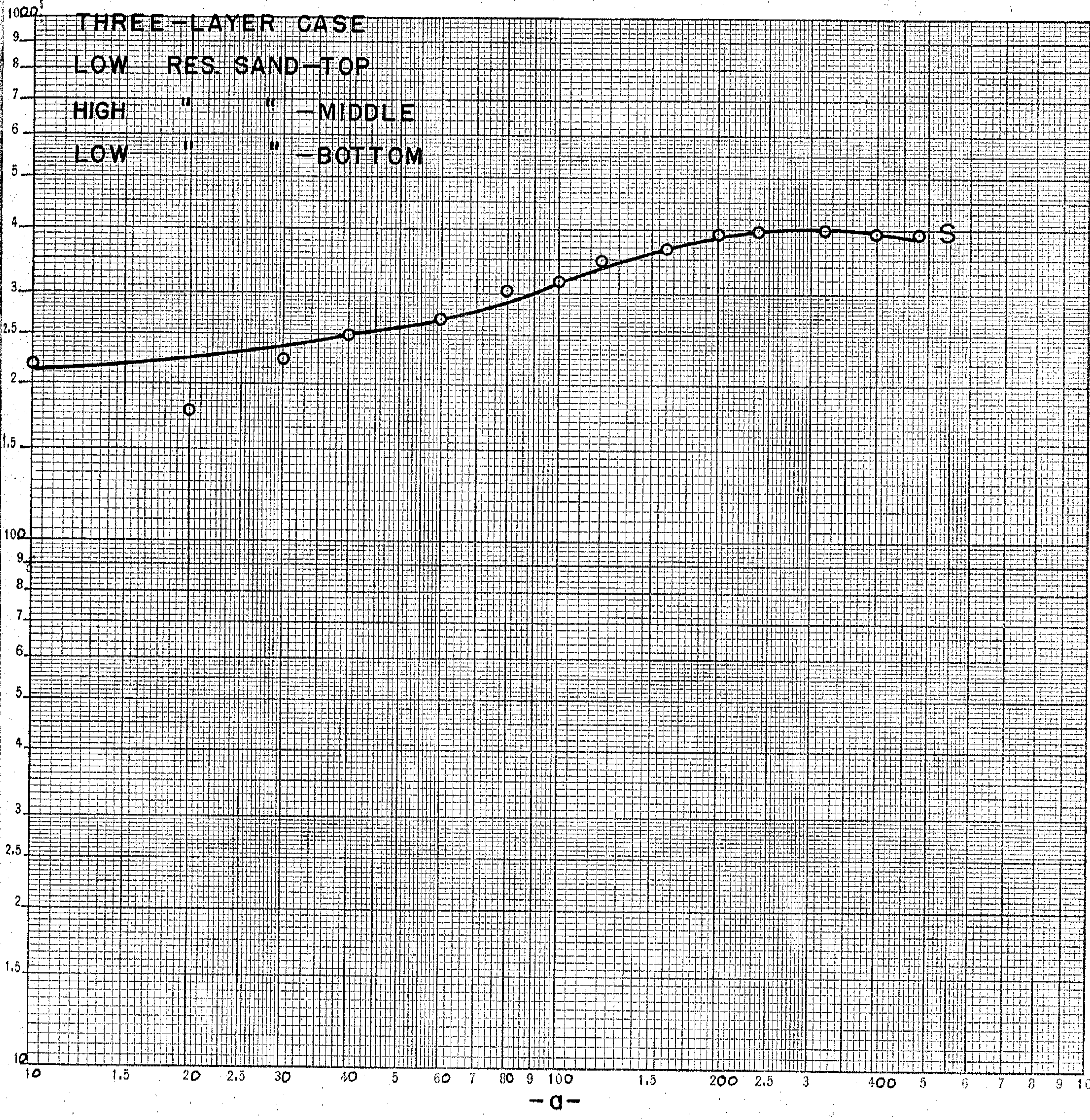
is a three-layer case where the top layer was approximately four inches thick, the middle layer approximately one and one-half inches thick and the lower layer was considered to be infinite. The resistivity ratio was found to be approximately 1:10:1 where the upper and lower layers were sand which had been saturated with salt solution and the excess drained off and the middle layer was damp sand.

The best fit was found to be the 3:1 curve from the theoretical curves with a resistivity ratio of 1:10:1. (Plate 19) The curve as a whole does not fit very well especially beyond the 320 millimeter spacing. Instead of breaking downward at this point as the theoretical curve does, it remains practically constant. Part of the deviation was probably caused from the box as shown by the compensation curve. Also as with Curve Q, part of the error could have been caused from compaction at



ELECTRODE INTERVAL in MILLIMETERS

PLATE 20



ELECTRODE INTERVAL in MILLIMETERS

the top of the bottom layer.

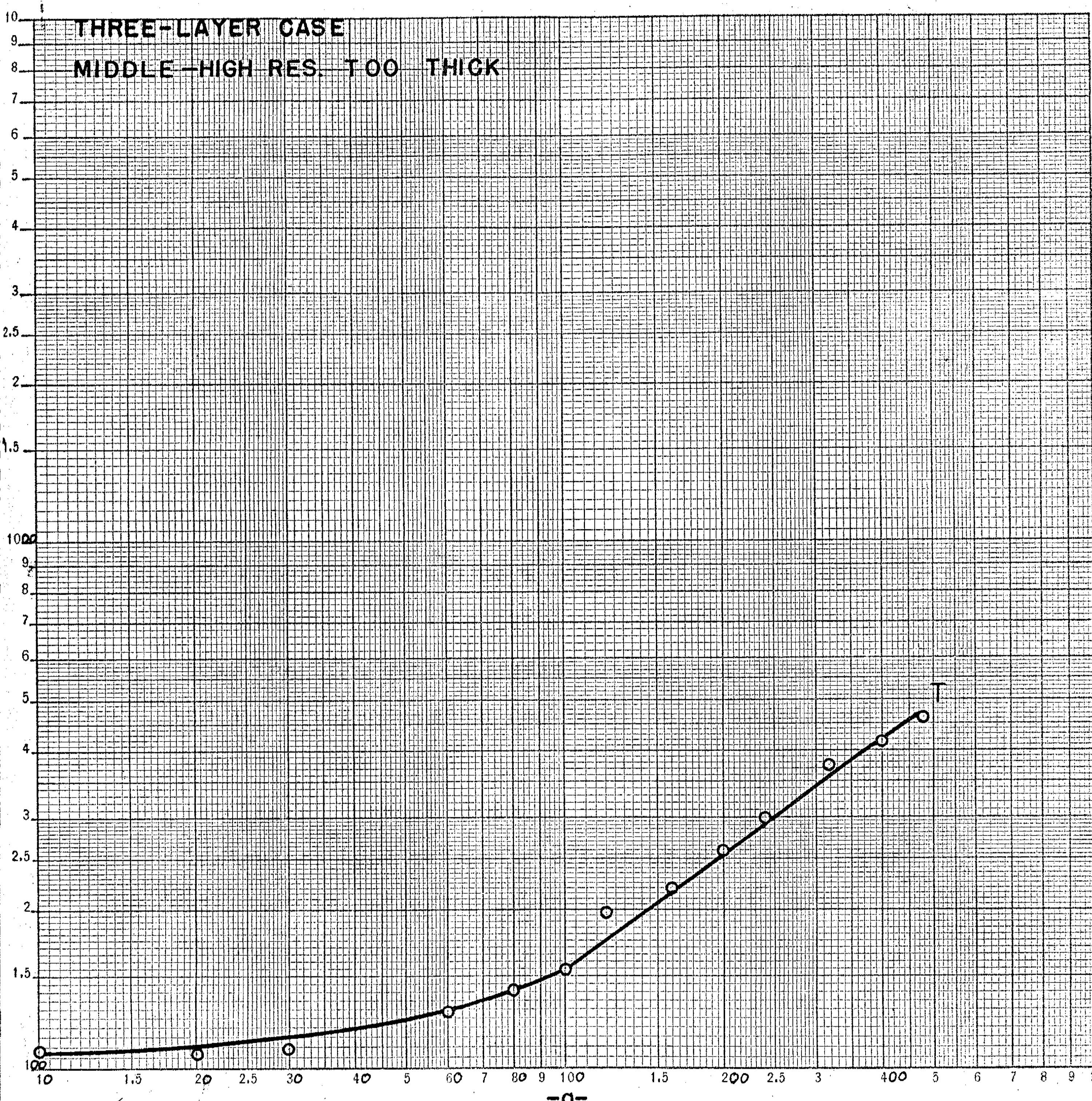
However it is possible that the boundary was too deep in this case where the high resistivity layer is in the middle. Curve T, Plate 21 shows a case where the middle layer was about six inches thick and the upper layer was about four inches thick. The curve has the appearance of a two-layer curve showing that the effect of the third layer was not reflected in the experimental curve for the spacings used. This means that for a three layer curve with the high resistivity layer in the middle, the upper layers must be very thin to show the boundary between the second and third layer.

CONCLUSIONS

The following conclusions were drawn from this problem:

1. Very sensitive instruments are needed in experimental work of this kind. Because of the high contact resistance encountered, the potential must be measured without drawing any current.
2. The results indicate that very small currents should be used to avoid changes in resistivity due to temperature. As the material had been placed in the box two days before, ambient temperatures were neglected since the temperature of the material was approximately in equilibrium and the temperature changes were small.

PLATE 21



ELECTRODE INTERVAL in MILLIMETERS

3. Lamp black and sand are unsatisfactory as a material for layers of different resistivities. Evidently the lamp black absorbs moisture from the air changing the resistivity a considerable amount. It is possible that lamp black and sand could be used under the right conditions, that is, where it could be kept perfectly dry. However this would entail very careful control of the humidity and all of the sand would have to be dried very carefully to reduce the moisture content to a fraction of one percent. Another very undesirable characteristic of lamp black is that it blackens everything in the area where it is being used.
4. The model box should be a great deal larger than the one used for this problem. In order to overcome the high contact resistance it was necessary to use electrodes much too large to be considered a point source, especially at the closer electrode spacings. Curve T shows that, unless the upper layers are kept thin, the effect of the lower layer will not be reflected in the experimental curve. In order to overcome this difficulty the box should be longer to allow a longer electrode spacing and deeper so that the upper layers could be made thicker.

5. Although sand and water will give fair results it is not a perfect layer material. Great care must be used when placing the sand in the box to keep it from compacting too much at one depth and not enough at another. Care must also be exercised when the sand is saturated with a salt solution to make sure that saturation is complete throughout the sand.

6. It is doubtful that the box could be used for a three-layer case with any accuracy with the exception of a very thin low resistive middle layer. The results indicate that fair results could be obtained for a two-layer case providing that great care is used in mixing the materials and in placing them into the box.

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